Structure of the broken phase of the sine-Gordon model

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   - General features
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Main features at LPA [1]

Lagrangian

In Euclidean space time $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial_\mu\phi + u_\Lambda cos(\beta\phi)$

Perturbative considerations

- The Coleman perturbative UV fixed point : $\beta_c^2 = 8\pi$
- Asymptotic freedom of the broken phase $\beta < \beta_c$
  $\Rightarrow$ the IR is non-perturbative.
- Perturbative non-renormabizability of the symmetric phase $\beta > \beta_c$

1. The following is based on V. P. *hep-th/1008.0281* and V. P. & al. (to appear) PLB 2010 *hep-th/0907.0496*

Wegner-Hougthon RG

Properties

- $k\partial_k \tilde{V} + 2\tilde{V} = -\frac{1}{4\pi} Log\left(1 + \tilde{V}''\right)$
- For a periodic initial condition, the potential remains periodic.
- The flow equation is valid as long as $\forall(k,\phi)$, $k^2 + V_k''(\phi) > 0$
- The gradient expansion is ill-defined for higher order : only LPA

Perturbative features in $d = 2$

- in the UV for weak coupling

$$k\partial_k \tilde{u}(k) = 2\left(\frac{\beta^2}{8\pi} - 1\right)\tilde{u}(k) + O(\tilde{u}(k)^2)$$

- The dimensionless quantities are the important ones.
- The Coleman frequency is easily reproduced.

IR of the broken phase

Convexity

- We expect $V_k \simeq -\frac{k^2}{2}\phi^2$ in the concave regions and periodically repeated [a].
- $V_k(\phi)$ is "maximally" concave at $\phi_n = \frac{2n\pi}{\beta}$
- The curvature $1 + \tilde{V}''(\phi_n) > 0$ tests the validity of loop-expansion. Instability ?

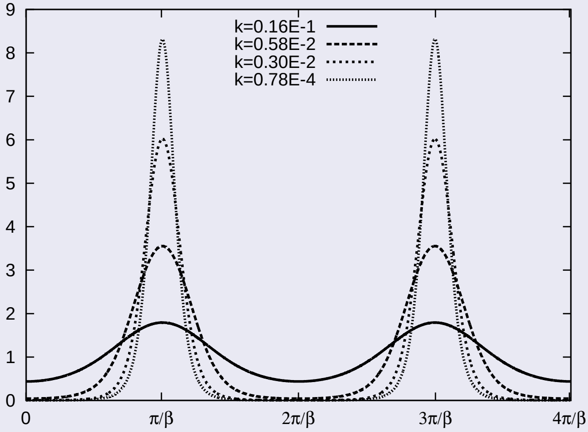
a. Alexandre-Branchina-Polonyi, *PLB* (1999)

Order parameter

- The curvature $1 + \tilde{V}''(\phi_n) \to 1$ for $\beta_r = \frac{\beta}{\sqrt{8\pi}} > 1$
- The curvature $1 + \tilde{V}''(\phi_n) \to 0^+$ for $\beta_r < 1$?

Fixed points of the broken phase

Curvature for $\beta_r = 0.70$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$ as a function of ϕ



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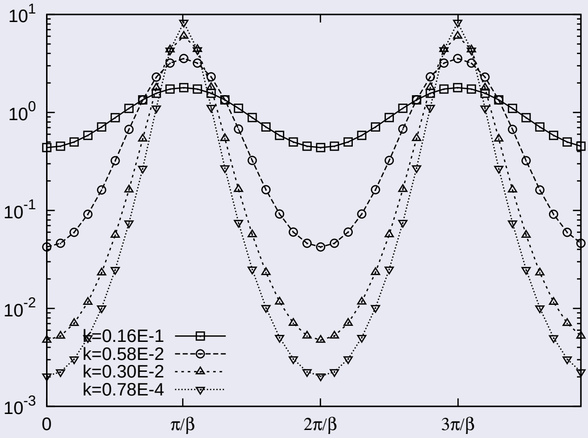
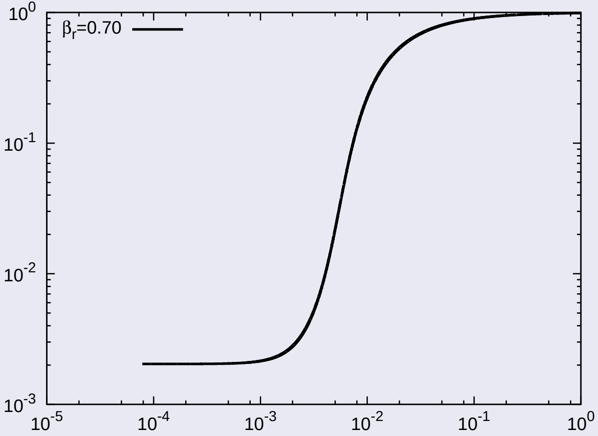


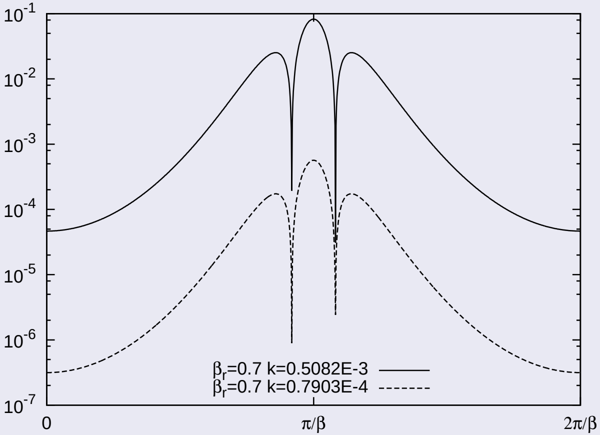
Fixed points of the broken phase

Flow of the curvature for $\beta_r = 0.70$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$



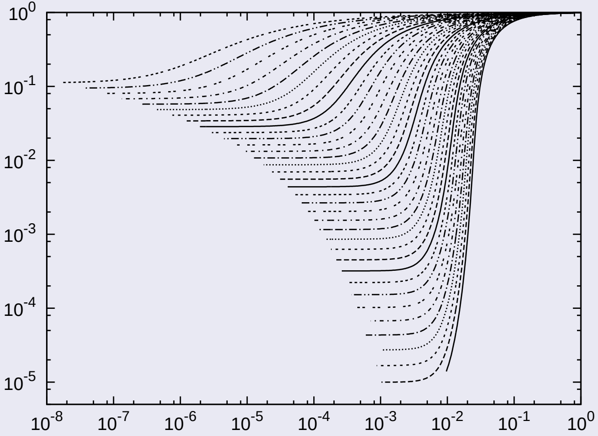
Fixed points of the broken phase

β-function of the curvature for $\beta_r = 0.70$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$ as a function of ϕ



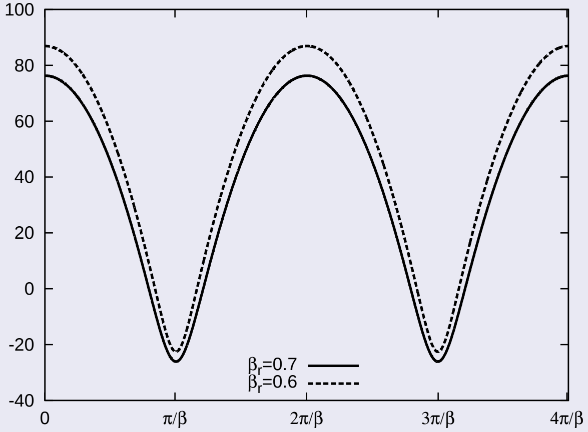
Fixed points of the broken phase

Flow of the curvature in ϕ_n for $\beta_r \in [0.55 : 0.90]$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$



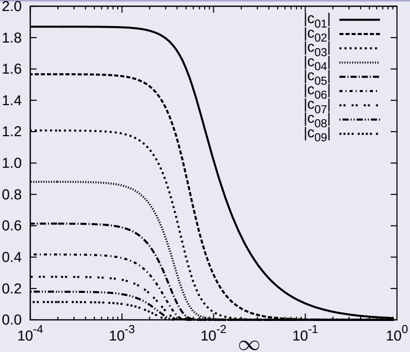
Fixed points of the broken phase

Effective potential for $\beta_r = 0.70$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$ as a function of ϕ



Fixed-points as a finely tune competition

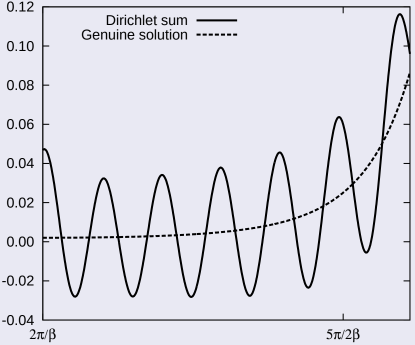
Flow of the Fourier coefficients of $\tilde{V}''$ for $\beta_r = 0.70$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$



$$\tilde{V}''(\phi) = \sum_{n=1}^{\infty} c_n cos(n\beta\phi)$$

Gibbs phenomenom

Curvature for $\beta_r = 0.70$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$ as a function of ϕ



The overshoot does not die out and violates loop-expansion validity.

Strong coupling

Properties

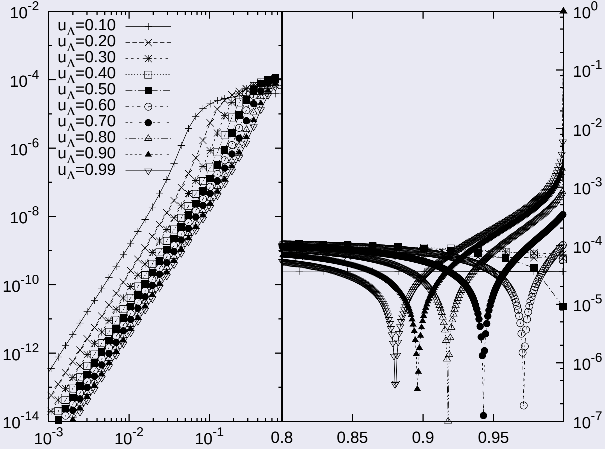
- Defined by :

$$\tilde{u}_\Lambda \lesssim \frac{1}{\beta^2} \Leftrightarrow k\partial_k \tilde{V} + 2\tilde{V} = \frac{2}{\beta_c^2} Log \left(1 - \beta^2 \tilde{u}_\Lambda cos(\beta\phi)\right)$$

  $\Rightarrow$ Soft modes in ϕ_n arising in the UV.
- Linearization of the flow equation fails.
- Location of the phase boundary ?
- Change of behavior wrt weakly coupled ?

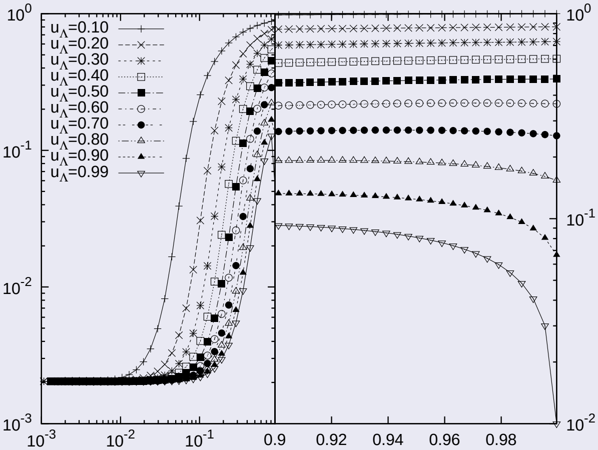
Universality

β-function of the curvature in ϕ_n for $\beta_r = 0.70$ for different $\tilde{u}_\Lambda$



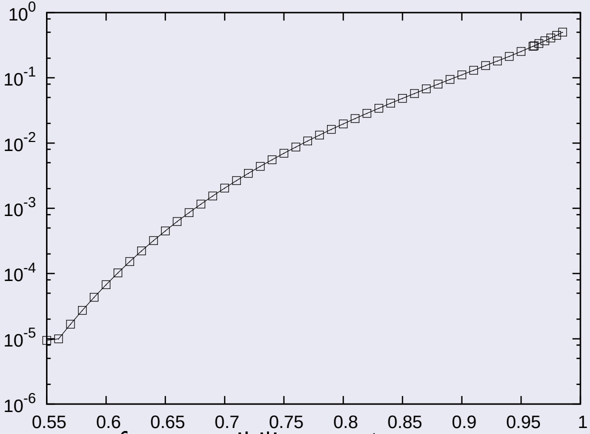
Generalized universality

Flow of the curvature in ϕ_n for $\beta_r = 0.70$ for different $\tilde{u}_\Lambda$



Fixed-points line : summary

Fixed-point curvature in ϕ_n for different β_r



Critical exponent for suceptibility $\gamma \simeq 1$

Conclusion

What Average action at LPA should reproduce :

- Coleman fixed point in the perturbative UV.
- The line of fixed-points in the IR of the broken phase.
- The strict independence on $\tilde{u}_\Lambda$ of the β-functions and the renormalized quantities.
- The phase boundary for strongly coupled bare theories.

What Average action is not expected to reproduce accurately :

- the fixed-point curvature values
- the power-law of the decay towards fixed points.
- the critical exponent for suceptibility γ.

Average action

Periodicity

- The bare action $S_\Lambda[\phi]$ is periodic
- Z_k involves a non-periodic contribution : $\frac{1}{2}\int_p \phi(p)R_k(p)\phi(-p)$
- The one-computation of the average action gives :

$$\Gamma_k[\phi] = S_\Lambda[\phi] + \frac{1}{2} TrLog\left(S'' + R_k\right)$$

- The initial condition is actually periodic !
- Periodicity is broken only by the "mass" of the fluctuations.
- The genuine flow equation still preserves periodicity :

$$k\partial_k\Gamma_k = \frac{1}{2} Tr\left(\frac{k\partial_k R_k}{\Gamma^{(2)} + R_k}\right)$$

Average action

Convexity

- The only assumption needed to obtain the flow equation :

$$\Gamma^{(2)} + R_k > 0$$

- At LPA, it reads :

$$\overline{k}^2 + V_k'' \quad > 0 \quad , \quad \overline{k}^2 = min_p \Big\{ p^2 + R_k(p) \Big\}$$

$$\tilde{k}^2 + \tilde{V}''(\phi_n) \quad > 0 \quad , \quad \text{e. g. } \tilde{k}^2 = 1 \text{ for Litim regulator}$$

- The limit $V_k = -\frac{\overline{k}^2}{2}\phi^2$ is IR attractive in concave regions [a]
- $\tilde{k}^2 + \tilde{V}''(\phi_n)$ will also be used as order parameter.

a. Tetradis-Wetterich, *NPB* (1992)

Phase boundary

Coleman fixed point

- Linearizing the flow equation

$$k\partial_k \tilde{V} + 2\tilde{V} = \left(\frac{1}{4\pi} \int_0^\infty dy \frac{r'(y)}{(r(y)+1)^2} \right) \tilde{V}''$$

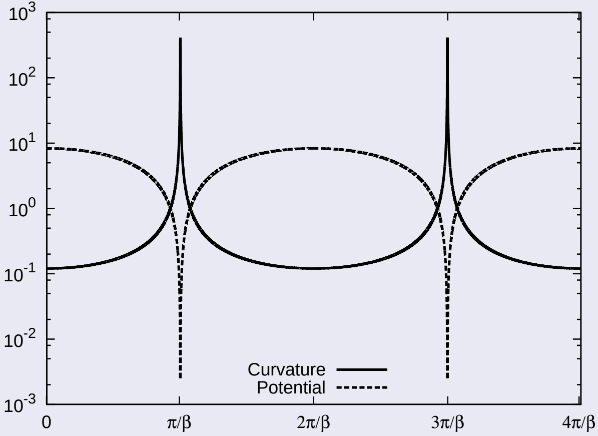
- Periodic UV fixed point characterized by :

$$\beta_c^2 = 8\pi \left(- \left[\frac{-1}{r(y)+1} \right]_0^\infty \right)^{-1} = 8\pi$$

- All the regulators reproduce the Coleman fixed point !

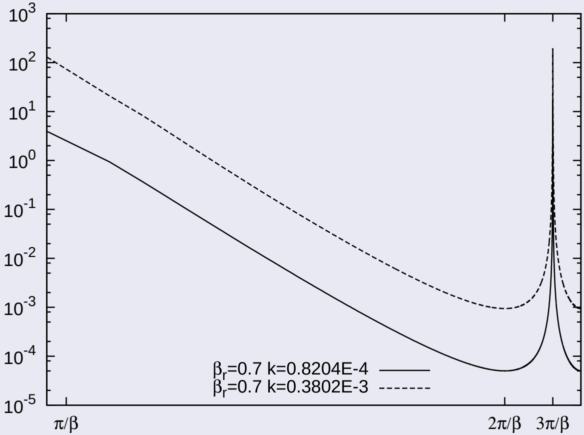
Fixed points of the broken phase

Curvature for $\beta_r = 0.70$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$ as a function of ϕ



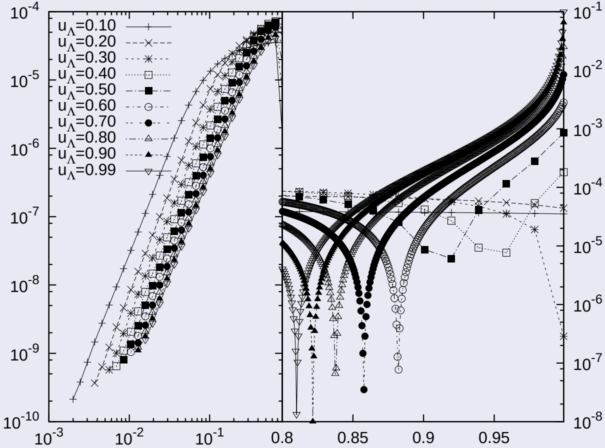
Fixed points of the broken phase

β-function of the curvature for $\beta_r = 0.70$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$ as a function of ϕ



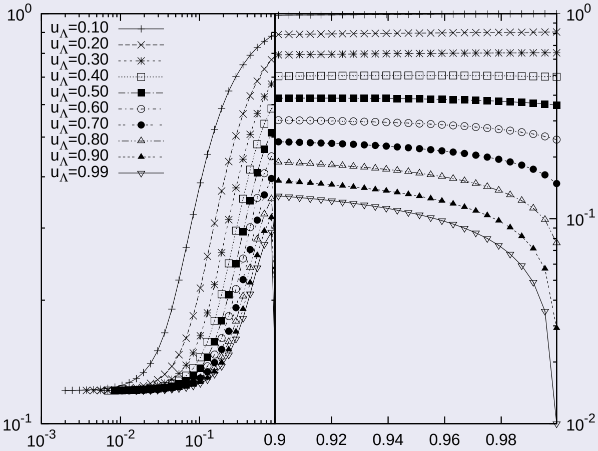
Universality

β-function of the curvature in ϕ_n for $\beta_r = 0.70$ for different $\tilde{u}_\Lambda$



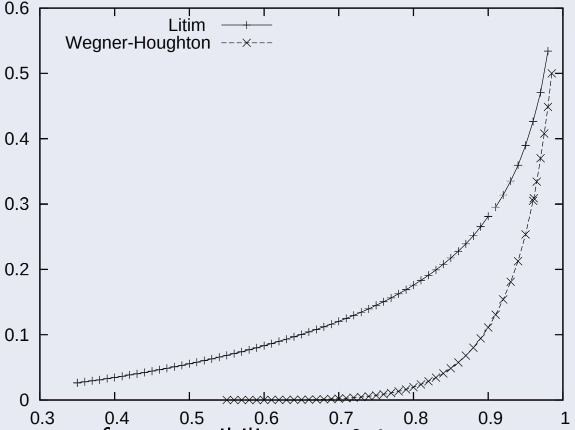
Generalized universality

Flow of the curvature in ϕ_n for $\beta_r = 0.70$ for different $\tilde{u}_\Lambda$



Fixed-points line : summary

Fixed-point curvature in ϕ_n for different β_r



Critical exponent for suceptibility $\gamma \simeq 0.4$

Smooth cut-off

Power-law results

- Testing if the results hold for smooth cut-off needed for inclusion of Z.
- The regulator is parametrized :

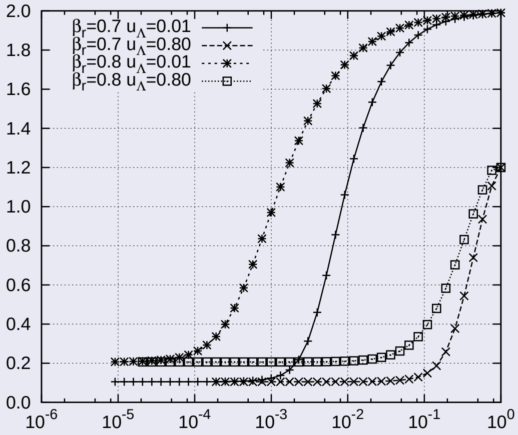
$$r(y) = a_p y^{-b_p},\ (a_p, b_p) \in \mathbb{R}^{+*} \times [1; +\infty]$$

- Special case $b_p = 1$ defines Callan Symanzik RG and matches in $d = 2$ the Wegner-Houghton equation.
- Loop-integral analytical for $b_p = 2$.
- The important quantity for the curvature is :

$$\tilde{k}^2 = (a_p(b_p - 1))^{\frac{1}{b_p}} \left[1 + \frac{1}{b_p - 1}\right] \overset{b_p \to \infty}{\to} 1$$

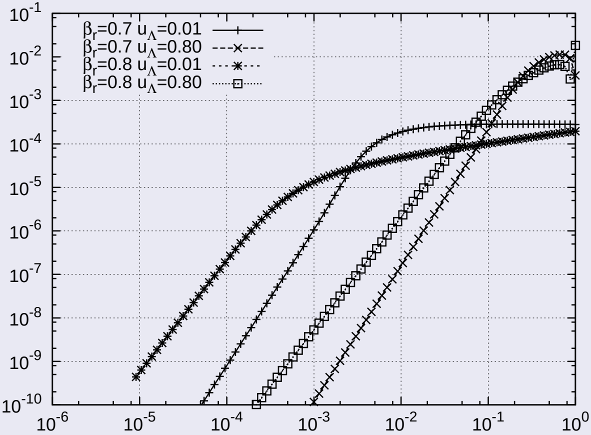
Fixed points of the broken phase

Flow of the curvature in ϕ_n



Fixed points of the broken phase

β-function of curvature in ϕ_n



Fixed points of the broken phase

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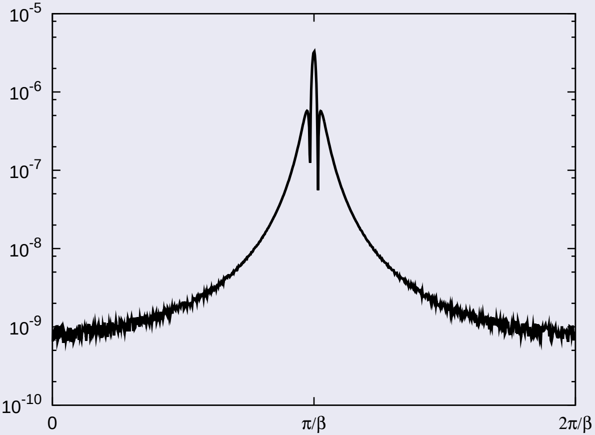
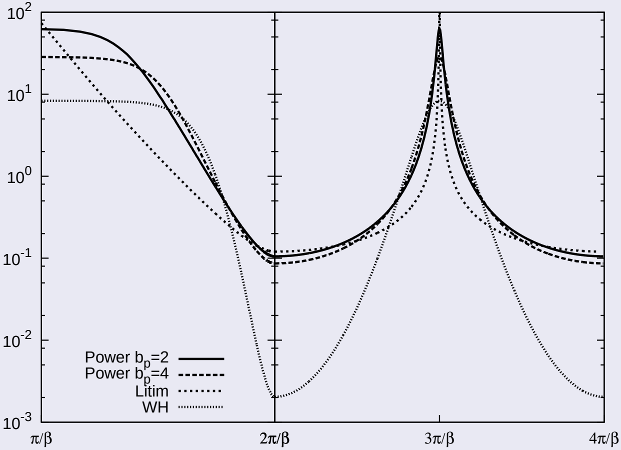


Comparison of RG schemes

Curvature for $\beta_r = 0.70$, $\tilde{u}_\Lambda = \frac{0.01}{\beta^2}$ as a function of ϕ



Conclusions

Structure of the broken phase

- The IR of the broken phase is highly non-trivial
- It exhibits a line of fixed points for $\beta_r > 0.55$
- No instability in the IR flow
- Generalized universality
- Bad convergence of the Fourier series

Using Average action

- The Coleman frequency is always reproduced
- All the features of the broken phase are preserved when adding a regulator.
- The qualitative behavior is regulator-independent.

Conclusion

Outlook

- No conceptual problem to study Z
- Possible dependence on the regulator ?
- What happens in other (higher) dimensions ?
- Test of propertime flows
- Study of SU(2) deconfinement transition.
- ...

Thank you for your attention !