Flavour physics

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Karlsruhe Institute of Technology

Corfu Summer Institute, September 2010

in memory of Nicola Cabibbo
(10 Apr 1935 – 16 Aug 2010)

May 14, 2010

Fermilab Wine&Cheese seminar, talk by Guennadi Borrisov:

Evidence for an anomalous like-sign dimuon charge asymmetry

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>
> *Joe Lykken, a theorist at Fermilab, said, "So I would not say that this announcement is the equivalent of seeing the face of God, but it might turn out to be the toe of God."*

Contents

Basics

Flavour physics

studies transitions between fermions of different generations.

flavour = fermion species

$$\begin{pmatrix} u_L, u_L, u_L \\ d_L, d_L, d_L \end{pmatrix} \quad \begin{pmatrix} c_L, c_L, c_L \\ s_L, s_L, s_L \end{pmatrix} \quad \begin{pmatrix} t_L, t_L, t_L \\ b_L, b_L, b_L \end{pmatrix}$$

$$\begin{matrix} u_R, u_R, u_R \\ d_R, d_R, d_R \end{matrix} \quad \begin{matrix} c_R, c_R, c_R \\ s_R, s_R, s_R \end{matrix} \quad \begin{matrix} t_R, t_R, t_R \\ b_R, b_R, b_R \end{matrix}$$

$$\begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu,L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau,L} \\ \tau_L \end{pmatrix}$$

$$e_R \quad \mu_R \quad \tau_R$$

Flavour quantum numbers:

quantum number	d	u	s	c	b	t	e,ν_e	μ,ν_μ	τ,ν_τ
D	-1	0	0	0	0	0	0	0	0
U	0	1	0	0	0	0	0	0	0
strangeness S	0	0	-1	0	0	0	0	0	0
charm C	0	0	0	1	0	0	0	0	0
beauty B	0	0	0	0	-1	0	0	0	0
T	0	0	0	0	0	1	0	0	0
electron number L_e	0	0	0	0	0	0	1	0	0
muon number L_μ	0	0	0	0	0	0	0	1	0
tau number L_τ	0	0	0	0	0	0	0	0	1

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lepton number $L = L_e + L_\mu + L_\tau$

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antifermions have opposite quantum numbers

Flavour quantum numbers are respected by the strong interaction, so we can use them to categorise hadrons. E.g. a B^+ meson has $B = U = 1$, shorthand notation:

$$B^+ \sim \overline{b}u$$

For a $B_d \equiv B^0$ (with $B = -D = 1$) we write

$$B_d \sim \overline{b}d$$

Some flavoured mesons

charged:

$$K^+ \sim \bar{s}u, \quad D^+ \sim c\bar{d}, \quad D_s^+ \sim c\bar{s}, \quad B^+ \sim \bar{b}u, \quad B_c^+ \sim \bar{b}c,$$
$$K^- \sim s\bar{u}, \quad D^- \sim \bar{c}d, \quad D_s^- \sim \bar{c}s, \quad B^- \sim b\bar{u}, \quad B_c^- \sim b\bar{c},$$

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neutral:

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In flavour physics only the ground-state hadrons which decay weakly rather than strongly are interesting.
Weakly decaying baryons are less interesting, because they are produced in smaller rates and are theoretically harder to cope with.

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The neutral K, D, B_d and B_s mesons mix with their antiparticles, $\bar{K}$, $\bar{D}$, $\bar{B}_d$ and $\bar{B}_s$ thanks to the weak interaction (quantum-mechanical two-state systems).

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$\Rightarrow$ gold mine for fundamental parameters

Strong isospin: Instead of U and D use (I, I_3):

Fundamental doublets ($I = \frac{1}{2}$): $\begin{pmatrix} u \\ d \end{pmatrix}$ and $\begin{pmatrix} \overline{d} \\ -\overline{u} \end{pmatrix}$.

For $m_u = m_d$ the QCD lagrangian is invariant under SU(2) rotations of $\begin{pmatrix} u \\ d \end{pmatrix}$ and $\begin{pmatrix} \overline{d} \\ -\overline{u} \end{pmatrix}$.

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Owing to $m_d - m_u \ll \Lambda_{\rm had} \sim 500\,\text{MeV}$, strong isospin holds to $\sim 2\%$ accuracy. E.g. $M_{B_d} - M_{B^+} = (0.37 \pm 0.24)\,\text{MeV}$.

Isospin triplet:

$$\pi^+ = u\overline{d}, \qquad \pi^0 = \frac{u\overline{u} - d\overline{d}}{\sqrt{2}}, \qquad \pi^- = d\overline{u}.$$

Compare with spin triplet

$$\uparrow\uparrow, \qquad \frac{\uparrow\uparrow + \downarrow\downarrow}{\sqrt{2}}, \qquad \downarrow\downarrow$$

Flavour–$SU(3)$:

Since $m_s - m_{u,d} < \Lambda_{\rm had}$ we can try to enlarge isospin–SU(2) to SU(3)$_{\rm F}$ with fundamental triplet $\begin{pmatrix} u \\ d \\ s \end{pmatrix}$

U-spin subgroup: SU(2) rotations of $\begin{pmatrix} d \\ s \end{pmatrix}$

Pedestrian's use of U-spin:
(i) Draw all diagrams contributing to some process.
(ii) Replace $s \leftrightarrow d$ to connect the hadronic interaction in different processes.

Example: One can relate the strong interaction effects in $B_s \to K^+K^-$ and $B_d \to \pi^+\pi^-$. Dunietz; Fleischer

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Accuracy of $\mathrm{SU(3)_F}$: 30% per $s \leftrightarrow d$ exchange.

Elektroweak interaction

Gauge group:

$SU(2) \times U(1)_Y$

doublets: $Q_L^j = \begin{pmatrix} u_L^j \\ d_L^j \end{pmatrix}$ und $L^j = \begin{pmatrix} \nu_L^j \\ \ell_L^j \end{pmatrix}$

$j = 1, 2, 3$ labels the generation.

Examples: $Q_L^3 = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$, $L^1 = \begin{pmatrix} \nu^{eL} \\ e_L \end{pmatrix}$

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Important: Only left-handed fields couple to the W boson.

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- three gauge interactions
- Yukawa interaction of Higgs with quarks and leptons
- Higgs self-interaction

Yukawa interaction

Higgs doublet $H = \begin{pmatrix} G^+ \\ v + \frac{h^0 + iG^0}{\sqrt{2}} \end{pmatrix}$ with $v = 174\,\mathrm{GeV}$.

Charge-conjugate doublet: $\widetilde{H} = \begin{pmatrix} v + \frac{h^0 - iG^0}{\sqrt{2}} \\ -G^- \end{pmatrix}$

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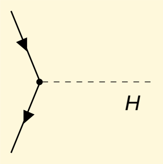

Yukawa lagrangian:

$$-L_Y = Y^d_{jk}\, \overline{Q}^j_L\, H\, d^k_R \; + \; Y^u_{jk}\, \overline{Q}^j_L\, \widetilde{H}\, u^k_R \; + \; Y^l_{jk}\, \overline{L}^j_L\, H\, e^k_R \; + \; \text{h.c.}$$

Here neutrinos are (still) massless.
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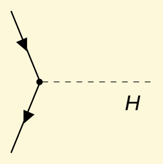

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The Yukawa matrices Y^f are arbitrary complex 3×3 matrices.
The mass matrices $M^f = Y^f v$ are not diagonal!

$\Rightarrow$ $u^j_{L,R}, d^j_{L,R}$ do not describe physical quarks!
We must find a basis in which Y^f is diagonal!

Any matrix can be diagonalised by a bi-unitary transformation. Start with

$$\widehat{Y}^u = S_Q^\dagger Y^u S_u \quad \text{with } \widehat{Y}^u = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} \quad \text{and } y_{u,c,t} \geq 0$$

This can be achieved via

$$Q_L^j = S_{jk}^Q Q_L^{k\prime}, \qquad u_R^j = S_{jk}^u u_R^{k\prime}$$

with unitary 3×3 matrices S^Q, S^u.
This transformation leaves $L_{\rm gauge}$ invariant (“flavour-blindness of the gauge interactions”)!

Next diagonalise Y^d:

$$\widehat{Y}^d = V^\dagger S_Q^\dagger Y^d S_d \quad \text{with } \widehat{Y}^d = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} \quad \text{and } y_{d,s,b} \geq 0$$

with unitary 3×3 matrices V, S^d.
Via $d_R^j = S_{jk}^d d_R^{k\prime}$ we leave L_{gauge} unchanged, while

$$-L_Y^{\text{quark}} = \overline{Q}_L V \widehat{Y}^d H\, d_R \;+\; \overline{Q}_L \widehat{Y}^u \widetilde{H}\, u_R \;+\; \text{h.c.}$$

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To diagonalise $M^d = V \widehat{Y}^d v$ transform

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$\Rightarrow$ Also the neutral Higgs fields h^0 and G^0 have only flavour-diagonal couplings!

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The transformation $d_L^j = V_{jk} d_L^{k\prime}$ changes the W-boson couplings in L_{gauge}:

$$L_W = \frac{g_2}{\sqrt{2}} \left[\overline{u}_L V \gamma^\mu\, d_L\, W_\mu^+ \; + \; \overline{d}_L V^\dagger \gamma^\mu\, u_L\, W_\mu^- \right]$$

The Z-boson couplings stay flavour-diagonal because of $V^\dagger V = 1$.

V is the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

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$\Rightarrow$ Add a ν_R to the SM to mimick the quark sector or add a Majorana mass term $Y^M \dfrac{\overline{L} H H^T L^c}{M}$.

The lepton mixing matrix is the
Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix.

Discrete symmetries

Parity transformation P: $\vec{x} \to -\vec{x}$

Charge conjugation C: Exchange particles and antiparticles, e.g. $e^- \leftrightarrow e^+$

Time reversal T: $t \to -t$

C and P

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1964:	CP is not a symmetry of the microscopic laws of nature!
$\Rightarrow$	Also the T symmetry must be violated, there is a microscopic arrow of time!

K and M

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Also QED respects C,P, and T.

Parity violation

1956: $\theta - \tau$ *puzzle*:
A seemingly degenerate pair (θ, τ) of two mesons with P$= +1$ and P$= -1$, weakly decaying as

$$\begin{array}{lcl} \text{“}\theta\text{”} & \to & \pi\pi \qquad\qquad \mathrm{P} = +1 \\ \text{“}\tau\text{”} & \to & \pi\pi\pi \qquad\quad \mathrm{P} = -1 \end{array}$$

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$$K^+ = \text{“}\theta\text{”} = \text{“}\tau\text{”}.$$

Maximal P violation

In the SM only left-handed fields feel the charged weak interaction, no couplings of the W-boson to u_R^j, d_R^j, and e_R^j.

Early monograph on parity violation:

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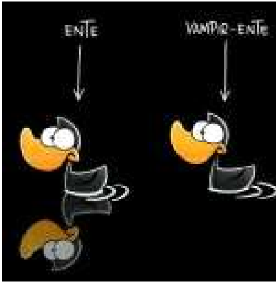
Lewis Carroll:
Alice through the looking glass

Maximal parity violation



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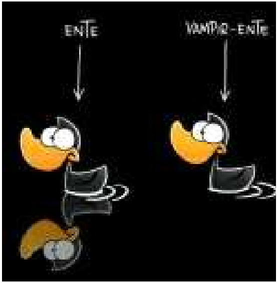

Charge conjugation C maps left-handed (particle) fields on right-handed (antiparticle) fields and vice versa:

$\psi_L \xleftrightarrow{C} \psi_L^C$, where $\psi_L^C \equiv (\psi^C)_R$ is right-handed.

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But: Nothing prevents CP and T from being good symmetries. . .

. . . except experiment!

CP violation

Neutral K mesons:

K_{long} and K_{short} (linear combinations of K and $\overline{K}$).

Dominant decay channels:

$$K_{\text{long}} \to \pi\pi\pi \qquad \text{CP} = -1$$
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1964: Christenson, Cronin, Fitch and Turlay observe

$$K_{\text{long}} \to \pi\pi$$

and therefore discover CP violation.

$$\epsilon_K \equiv \frac{\langle (\pi\pi)_{I=0} | H_{weak} | K_{\text{long}} \rangle}{\langle (\pi\pi)_{I=0} | H_{weak} | K_{\text{short}} \rangle} = (2.229 \pm 0.010) \cdot 10^{-3} e^{i0.97\pi/4}.$$


M. C. E S C H E R.

CP violation in the SM

Example: W coupling to b and u:

$$L_W = \frac{g_2}{\sqrt{2}} \left[V_{ub} \overline{u}_L \gamma^\mu b_L W_\mu^+ + V_{ub}^* \overline{b}_L \gamma^\mu u_L W_\mu^- \right]$$

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Is CP violated?

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Is CP violated? Not yet. . .
Rephasing $b_L \to e^{i\phi}b_L$, $u_L \to e^{i\phi'}u_L$ amounts to

$$L_W \xrightarrow{CP+\text{reph.}} \frac{g_2}{\sqrt{2}}\left[V_{ub}e^{i(\phi'-\phi)}\overline{b}_L\gamma^\mu u_L W_\mu^- + V_{ub}^*e^{i(\phi-\phi')}\overline{u}_L\gamma^\mu b_L W_\mu^+\right],$$

so that we can achieve $V_{ub}e^{i(\phi'-\phi)} = V_{ub}^*$.

Alternatively we could have used the rephasing to render V_{ub} real from the beginning.

Observation by Kobayashi and Maskawa:

A unitary $n \times n$ matrix has $\frac{n(n+1)}{2}$ phases. In an n-generation SM one can eliminate $2n - 1$ phases from V by rephasing the quark fields. The remaining $\frac{(n-1)(n-2)}{2}$ phases are physical, CP-violating parameters of the theory!

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n	$\frac{(n-1)(n-2)}{2}$
1	0
2	0
3	1
4	3

Kobayashi-Maskawa phase δ_{KM}

CKM metrology

The Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

involves 4 parameters: 3 angles and the KM phase $\delta_{\rm KM}$.
Best way to parametrise V: Wolfenstein expansion

Expand the CKM matrix V in $V_{us} \simeq \lambda = 0.2246$:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3\left(1+\frac{\lambda^2}{2}\right)(\overline{\rho}-i\overline{\eta}) \\ -\lambda-iA^2\lambda^5\overline{\eta} & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\overline{\rho}-i\overline{\eta}) & -A\lambda^2-iA\lambda^4\overline{\eta} & 1 \end{pmatrix}$$

with the Wolfenstein parameters λ, A, $\overline{\rho}$, $\overline{\eta}$

CP violation $\Leftrightarrow \overline{\eta} \neq 0$

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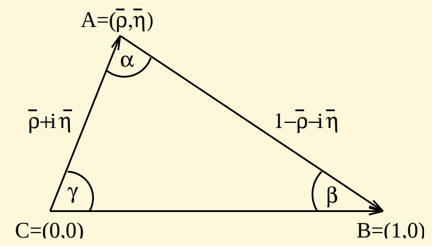
Unitarity triangle:

Exact definition:

$$\overline{\rho}+i\overline{\eta} = -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} = \left|\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right| e^{i\gamma}$$



In the SM the flavour violation only occurs in the couplings of $W_\mu^\pm$ and $G^\pm$ to fermions.

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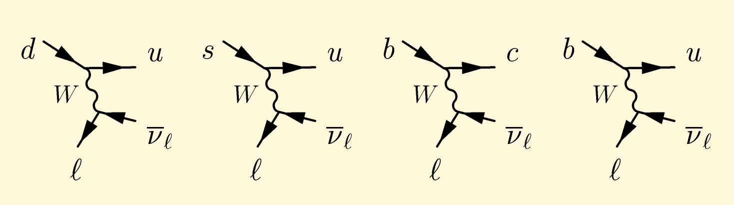
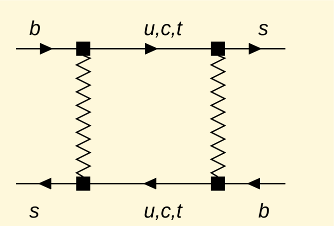
Semileptonic decays:



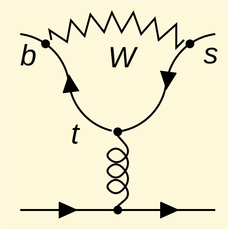
determining $|V_{ud}|$ $|V_{us}|$ $|V_{cb}|$ $|V_{ub}|$.

Flavour-changing neutral current (FCNC) processes

Examples:



$B_s - \overline{B}_s$ mixing



penguin diagram

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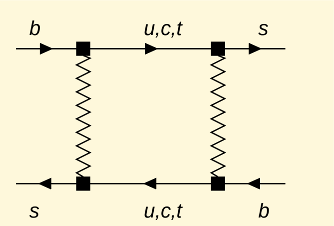

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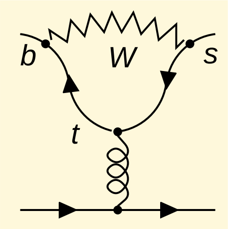

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FCNC processes are the only possibility to gain information on V_{td} and V_{ts}. However: FCNC processes are highly sensitive to physics beyond the SM.

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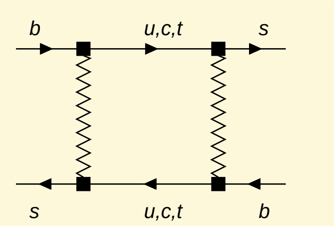

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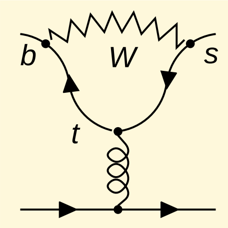


penguin diagram

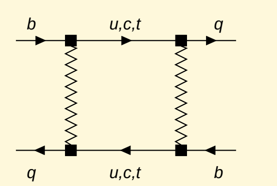
FCNC processes are the only possibility to gain information on V_{td} and V_{ts}. However: FCNC processes are highly sensitive to physics beyond the SM.

In principle can determine all parameters λ, A, $\overline{\rho}$, $\overline{\eta}$ from tree-level processes.

$\Rightarrow$ View FCNC processes as new physics analysers rather than ways to measure V_{td} and V_{ts}.

$\mathrm{B}-\overline{\mathrm{B}}$ mixing basics

Consider $\mathrm{B_q}-\overline{\mathrm{B}}_\mathrm{q}$ mixing with $q = d$ or $q = s$:
A meson identified ("tagged") as a B_q at time $t = 0$ is described by $|B_q(t)\rangle$.



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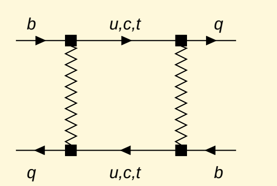

For $t > 0$:

$$|B_q(t)\rangle = \langle B_q|B_q(t)\rangle |B_q\rangle + \langle \overline{B}_q|B_q(t)\rangle|\overline{B}_q\rangle + \ldots ,$$

with "..." denoting the states into which $B_q(t)$ can decay.

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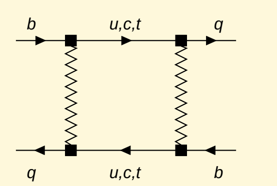

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Analogously: $|\overline{B}_q(t)\rangle$ is the ket of a meson tagged as a $\overline{B}_q$ at time $t = 0$.

Schrödinger equation:

$$i\,\frac{d}{dt}\begin{pmatrix}\langle B_q|B_q(t)\rangle\\ \langle\overline{B}_q|B_q(t)\rangle\end{pmatrix} \;=\; \left(M^q - i\,\frac{\Gamma^q}{2}\right)\begin{pmatrix}\langle B_q|B_q(t)\rangle\\ \langle\overline{B}_q|B_q(t)\rangle\end{pmatrix}$$

with the 2×2 mass and decay matrices $M^q = M^{q\dagger}$ and $\Gamma^q = \Gamma^{q\dagger}$.

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3 physical quantities in $\mathrm{B_q}-\overline{\mathrm{B}}_\mathrm{q}$ mixing:

$$\left|M_{12}^q\right|, \quad \left|\Gamma_{12}^q\right|, \quad \phi_q \equiv \arg\left(-\frac{M_{12}^q}{\Gamma_{12}^q}\right)$$

Diagonalise $M^q - i\,\frac{\Gamma^q}{2}$ to find the two mass eigenstates:

$$\begin{aligned} \text{Lighter eigenstate:} \quad |B_L\rangle &= p|B_q\rangle + q|\overline{B}_q\rangle. \\ \text{Heavier eigenstate:} \quad |B_H\rangle &= p|B_q\rangle - q|\overline{B}_q\rangle \end{aligned}$$

with masses $M^q_{L,H}$ and widths $\Gamma^q_{L,H}$.

Further $|p|^2 + |q|^2 = 1$.

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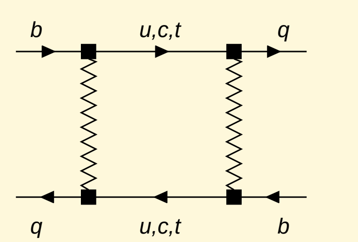
Further $|p|^2 + |q|^2 = 1$.

Relation of Δm_q and $\Delta\Gamma_q$ to $|M^q_{12}|$, $|\Gamma^q_{12}|$ and ϕ_q:

$$\begin{aligned}\Delta m_q &= M_H - M_L \simeq 2|M^q_{12}|, \\ \Delta\Gamma_q &= \Gamma_L - \Gamma_H \simeq 2|\Gamma^q_{12}|\cos\phi_q\end{aligned}$$

M_{12}^q stems from the dispersive (real) part of the box diagram, internal t.
Γ_{12}^q stems from the absorpive (imaginary) part of the box diagram, internal c, u.



Solve the Schrödinger equation to find the desired $\mathrm{B_q}-\overline{\mathrm{B}}_\mathrm{q}$ oscillations:

$$
\begin{aligned}
|\langle B_q|B_q(t)\rangle|^2 = |\langle \bar{B}_q|\bar{B}_q(t)\rangle|^2 &= \frac{e^{-\Gamma_q t}}{2}\left[\cosh\frac{\Delta\Gamma_q\, t}{2} + \cos(\Delta m_q\, t)\right] \\
|\langle \bar{B}_q|B_q(t)\rangle|^2 \simeq |\langle B_q|\bar{B}_q(t)\rangle|^2 &\simeq \frac{e^{-\Gamma_q t}}{2}\left[\cosh\frac{\Delta\Gamma_q\, t}{2} - \cos(\Delta m_q\, t)\right]
\end{aligned}
$$

with $\Gamma_q \equiv \dfrac{\Gamma_L^q + \Gamma_H^q}{2}$

Time-dependent decay rate:

$$\Gamma(B_q(t) \to f) = \frac{1}{N_B} \frac{d\,N(B_q(t) \to f)}{d\,t}\,,$$

where $d\,N(B_q(t) \to f)$ is the number of $B_q(t) \to f$ decays within the time interval $[t, t + d\,t]$.
N_B is the number of B_q's present at time $t = 0$.

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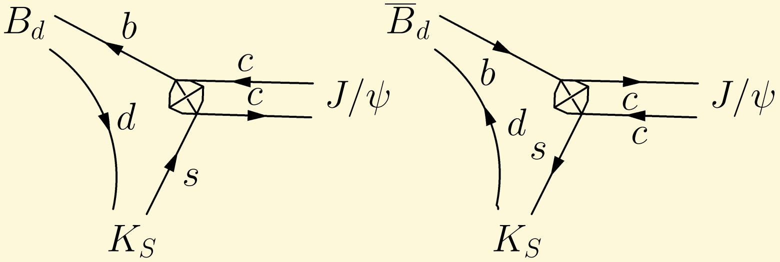
With $|\overline{f}\rangle \equiv CP|f\rangle$ define the time-dependent CP asymmetry:

$$a_f(t) = \frac{\Gamma(\overline{B}_q(t) \to f) - \Gamma(B_q(t) \to \overline{f})}{\Gamma(\overline{B}_q(t) \to f) + \Gamma(B_q(t) \to \overline{f})}$$

Example 1:

$B_d \to J/\psi K_S \quad \Rightarrow \quad |\bar{f}\rangle = -|f\rangle$ (CP-odd eigenstate)



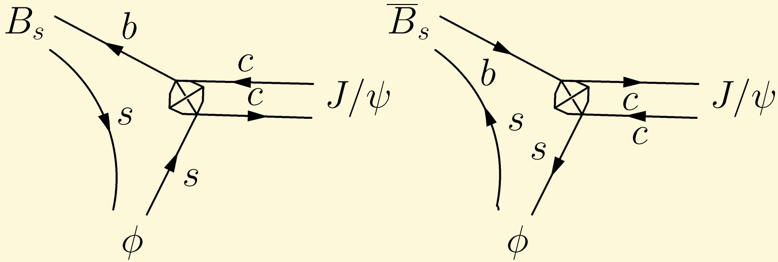
$$a_{J/\psi K_S}(t) \simeq -\sin(2\beta)\sin(\Delta m_d t),$$

where

$$\beta = \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

Example 2:

$B_s \to (J/\psi\phi)_{L=0}$ $\Rightarrow$ $|\bar{f}\rangle = |f\rangle$ (CP-even eigenstate)



$$a_{(J/\psi\phi)_{L=0}}(t) = -\frac{\sin(2\beta_s)\sin(\Delta m_s t)}{\cosh(\Delta\Gamma_s t/2) - \cos(2\beta_s)\sinh(\Delta\Gamma_s t/2)},$$

where

$$\beta_s = \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right] \simeq \lambda^2\overline{\eta}$$

The Wolfenstein parameters λ and A are well determined from the semileptonic decays $K \to \pi\ell^+\nu_\ell$ and $B \to X_c\ell^+\nu_\ell$, $\ell = e, \mu$.

Metrology of the unitarity triangle:
The apex $(\bar{\rho},\bar{\eta})$ is currently constrained from the following experimental input:

- $|V_{ub}| \propto \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$ measured in $B \to \pi\ell\nu_\ell$, $B \to X_u\ell\nu_\ell$ and $B^+ \to \tau^+\nu_\tau$.

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- $\Delta m_d / \Delta m_s \propto \sqrt{(1-\overline{\rho})^2 + \overline{\eta}^2}$
- $\sin(2\beta)$ from $a_{J/\psi K_S}(t)$ and other $b \to c\overline{c}s$ decays

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- $|V_{ub}| \propto \sqrt{\overline{\rho}^2 + \overline{\eta}^2}$ measured in $B \to \pi \ell \nu_\ell$, $B \to X_u \ell \nu_\ell$ and $B^+ \to \tau^+ \nu_\tau$.
- γ extracted from $B^\pm \to \overset{(-)}{D} K^\pm$
- $\Delta m_d \propto \sqrt{(1-\overline{\rho})^2 + \overline{\eta}^2}$
- $\Delta m_d / \Delta m_s \propto \sqrt{(1-\overline{\rho})^2 + \overline{\eta}^2}$
- $\sin(2\beta)$ from $a_{J/\psi K_S}(t)$ and other $b \to c\overline{c}s$ decays
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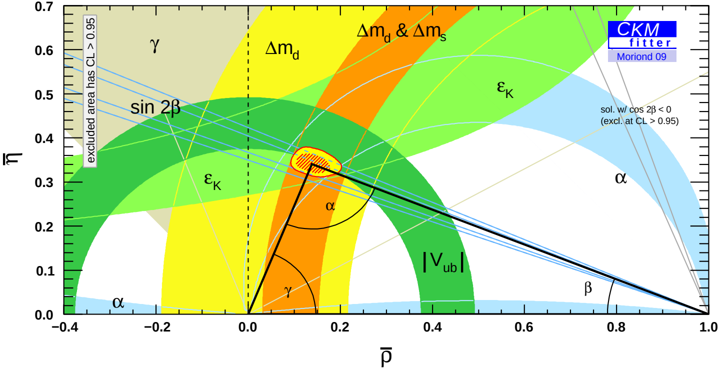
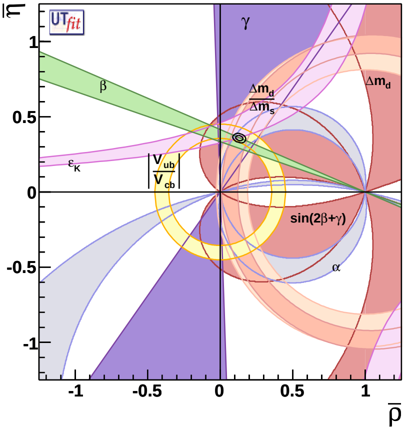
- ϵ_K (the measure of CP violation in $\mathrm{K}-\overline{\mathrm{K}}$ mixing), which defines a hyperbola in the $(\overline{\rho},\overline{\eta})$ plane.

Global fit in the SM from CKMfitter:



Statistical method: Rfit, a Frequentist approach.

Global fit in the SM from UTfit:



Statistical method: Bayesian.

Flavour experiments

B,D,τ: BELLE (upgrade: BELLE-II)
CDF, DØ
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$\mu \to e\gamma$ search: MEG (at PSI)

...plus neutrino experiments like MINOS

Future: Project X at Fermilab for rare K and μ decays.

New physics

In the LHC era CKM metrology is less important and constraints on physics beyond the SM is the main focus of flavour physics.

In the flavour-changing neutral current (FCNC) processes of the Standard Model several suppression factors pile up:

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Spectacular: In FCNC transitions of charged leptons the GIM suppression factor is even m_ν^2/M_W^2!

$\Rightarrow$ The SM predictions for charged-lepton FCNCs are essentially zero!

The suppression of FCNC processes in the Standard Model is not a consequence of the $SU(3) \times SU(2)_L \times U(1)_Y$ symmetry. It results from the particle content of the Standard Model and the accidental smallness of most Yukawa couplings. It is absent in generic extensions of the Standard Model.

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Examples:

extra Higgses	$\Rightarrow$	Higgs-mediated FCNC's at tree-level , helicity suppression possibly absent,
squarks/gluinos	$\Rightarrow$	FCNC quark-squark-gluino coupling, no CKM/GIM suppression,
vector-like quarks	$\Rightarrow$	FCNC couplings of an extra Z',
$\mathrm{SU}(2)_{\mathrm{R}}$ gauge bosons	$\Rightarrow$	helicity suppression absent

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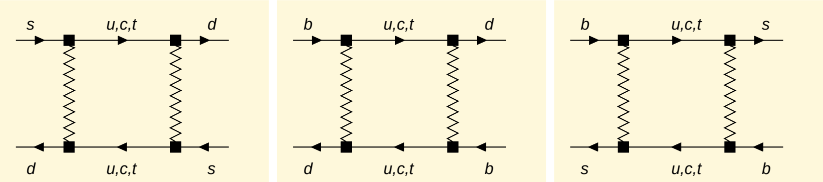
$\mathrm{B_d} - \overline{\mathrm{B}}_\mathrm{d}$ mixing and $\mathrm{B_s} - \overline{\mathrm{B}}_\mathrm{s}$ mixing are sensitive to scales up to $\Lambda \sim 100\,\mathrm{TeV}$.

New-physics analysers:

- Global fit to UT: overconstrain $(\bar{\rho}, \bar{\eta})$,
  probes FCNC processes $\mathrm{K}-\overline{\mathrm{K}}$, $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ and $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing.



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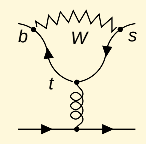
- Penguin decays: $B \to X_s\gamma$, $B \to X_s\ell^+\ell^-$, $B \to K\pi$, $B_d \to \phi K_{\mathrm{short}}$, $B_s \to \mu^+\mu^-$, $K \to \pi\nu\overline{\nu}$.



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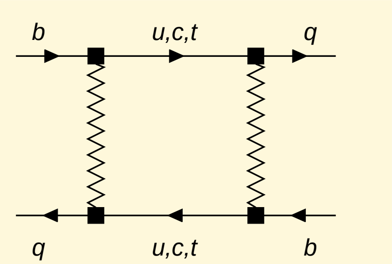
- CKM-suppressed or helicity-suppressed tree-level decays: $B^+ \to \tau^+\nu$, $B \to \pi\ell\nu$, $B \to D\tau\nu$, probe charged Higgses and right-handed W-couplings.

$\mathrm{B}-\overline{\mathrm{B}}$ mixing and new physics

New physics can barely affect Γ_{12}^q, which stems from tree-level decays.
M_{12}^q is very sensitive to virtual effects of new heavy particles.



Generic new physics

The phase $\phi_s = \arg(-M_{12}^s/\Gamma_{12}^s)$ is negligibly small in the Standard Model:

$$\phi_s^{\mathrm{SM}} = 0.2^\circ .$$

Define the complex parameter Δ_s through

$$M_{12}^s \equiv M_{12}^{\mathrm{SM},s} \cdot \Delta_s , \qquad \Delta_s \equiv |\Delta_s| e^{i\phi_s^\Delta} .$$

In the Standard Model $\Delta_s = 1$. Use $\phi_s = \phi_s^{\mathrm{SM}} + \phi_s^\Delta \simeq \phi_s^\Delta$.

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In the Standard Model $\Delta_s = 1$. Use $\phi_s = \phi_s^{\rm SM} + \phi_s^\Delta \simeq \phi_s^\Delta$.
The CDF measurement

$$\Delta m_s = (17.77 \pm 0.10 \pm 0.07)\ \text{ps}^{-1}$$

implies

$$|\Delta_s| = 0.92 \pm 0.14_{\rm (th)} \pm 0.01_{\rm (exp)}$$

Flavour-specific decay: $B_s \to f$ is allowed, while
$\bar{B}_s \to f$ is forbidden

CP asymmetry in flavour-specific decays (semileptonic CP asymmetry):

$$a_{\text{fs}}^{s} = \frac{\Gamma(\bar{B}_s(t) \to f) - \Gamma(B_s(t) \to \bar{f})}{\Gamma(\bar{B}_s(t) \to f) + \Gamma(B_s(t) \to \bar{f})}$$

with e.g. $f = X\ell^+\nu_\ell$ and $\bar{f} = \overline{X}\ell^-\overline{\nu}_\ell$. Untagged rate:

$$a_{\text{fs,unt}}^{s} \equiv \frac{\int_0^\infty dt \left[\Gamma(\overset{(-)}{B}_s \to \mu^+ X) - \Gamma(\overset{(-)}{B}_s \to \mu^- X)\right]}{\int_0^\infty dt \left[\Gamma(\overset{(-)}{B}_s \to \mu^+ X) + \Gamma(\overset{(-)}{B}_s \to \mu^- X)\right]} \simeq \frac{a_{\text{fs}}^{s}}{2}$$

Relation to M_{12}^s:

$$a_{\rm fs}^s = \frac{|\Gamma_{12}^s|}{|M_{12}^s|}\sin\phi_s = \frac{|\Gamma_{12}^s|}{|M_{12}^{\rm SM,s}|}\cdot\frac{\sin\phi_s}{|\Delta_s|} = (4.97\pm 0.94)\cdot 10^{-3}\cdot\frac{\sin\phi_s}{|\Delta_s|}$$

A. Lenz, UN, 2006

Dilepton events:

Compare the number N_{++} of decays $(B_s(t), \bar{B}_s(t)) \to (f, f)$ with the number N_{--} of decays to $(\bar{f}, \bar{f})$.

Then $a_{\mathrm{fs}}^{s} = \dfrac{N_{++} - N_{--}}{N_{++} + N_{--}}$.

At the Tevatron all b-flavoured hadrons are produced. Still only those events contribute to $(N_{++} - N_{--})/(N_{++} + N_{--})$, in which one of the b hadronises as a B_d or B_s and undergoes mixing.

May 15, 2010: DØ presents

$$a_{\rm fs} \;=\; (-9.57 \pm 2.51 \pm 1.46) \cdot 10^{-3}$$

for a mixture of B_d and B_s mesons with

$$a_{\rm fs} \;=\; (0.506 \pm 0.043) a_{\rm fs}^d + (0.494 \pm 0.043) a_{\rm fs}^s$$

The result is 3.2σ away from $a_{\rm fs}^{\rm SM} = \left(-0.23^{+0.05}_{-0.06}\right) \cdot 10^{-3}$.

A. Lenz, UN, 2006

Averaging with an older CDF measurement yields

$$a_{\rm fs} = (-8.5 \pm 2.8) \cdot 10^{-3},$$

which is 3.0σ away from $a_{\rm fs}^{\rm SM}$.

$$a_{\mathrm{fs}}^{s} = (4.97 \pm 0.94) \cdot 10^{-3} \cdot \frac{\sin \phi_s}{|\Delta_s|}$$

If there is no new physics in a_{fs}^{d}, the Tevatron measurement of $a_{\mathrm{fs}} = (-8.5 \pm 2.8) \cdot 10^{-3}$ roughly implies $a_{\mathrm{fs}}^{s} = (-17 \pm 6) \cdot 10^{-3}$. With $|\Delta_s| \geq 0.78$ find

$$\sin \phi_s \leq -2.2 \pm 0.7.$$

Closer look: Allow for new physics in $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ mixing as well:

$$\frac{M_{12}^d}{M_{12}^{\mathrm{SM,d}}} \equiv \Delta_d = |\Delta_d| e^{i\phi_d^\Delta}$$

Measurement by B factories: $a_{\mathrm{fs}}^d = (-4.7 \pm 4.6) \cdot 10^{-3}$

However: a_{fs}^d can be better determined indirectly through

$$a_{\mathrm{fs}}^d = \frac{|\Gamma_{12}^d|}{M_{12}^{\mathrm{SM,d}}} \frac{\sin(\phi_d^{\mathrm{SM}} + \phi_d^\Delta)}{|\Delta_d|} \qquad \text{with} \quad \phi_d^{\mathrm{SM}} = (-5 \pm 2)^\circ$$

using the measurements of $\Delta m_d = 2|M_{12}^d|$ and of $2\beta + \phi_d^\Delta = (21 \pm 1)^\circ$ from $A_{CP}^{\mathrm{mix}}(B_d \to J/\psi K_{\mathrm{short}})$.

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$\Rightarrow$ requires fit to unitarity triangle to find β

Other connection between B_d and B_s mixing:

The global fit to the unitarity triangle involves $\frac{\Delta m_d}{\Delta m_s}$ from which hadronic uncertainties cancel to a large extent.

Global analysis of $B_s - \overline{B}_s$ mixing and $B_d - \overline{B}_d$ mixing

Based on work with A. Lenz and the CKMfitter Group
(J. Charles, S. Descotes-Genon, A. Jantsch, C. Kaufhold,
H. Lacker, S. Monteil, V. Niess) arXiv:1008.1593

Rfit method: No statistical meaning is assigned to systematic errors and theoretical uncertainties.

We have performed a simultaneous fit to the Wolfenstein parameters and to the new physics parameters Δ_s and Δ_d in three scenarios.

Scenario I: arbitrary complex parameters Δ_s and Δ_d

Scenario II: new physics is minimally flavour violating (MFV) (meaning that all flavour violation stems from the Yukawa sector) and y_b is small: one real parameter $\Delta = \Delta_s = \Delta_d$

Scenario III: MFV with a large y_b: one complex parameter $\Delta = \Delta_s = \Delta_d$

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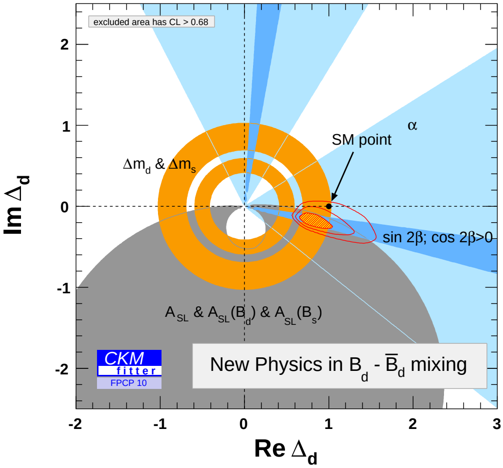
Examples: Scenario I covers the MSSM with generic flavour structure of the soft terms and small $\tan\beta$.
Scenario II covers the MSSM with MFV and small $\tan\beta$.
Scenario III covers certain two-Higgs models (but not the MFV-MSSM).

Results in scenario I:



SM point $\Delta_d = 1$ disfavoured by $\geq 2.5\sigma$.

$\phi_d^{\Delta} < 0$ helps to explain DØ dimuon asymmetry.

Reason for the tension with the SM: $B(B^+ \to \tau^+ \nu_\tau)$

SM prediction (CL$= 2\sigma$):

$$B(B^+ \to \tau^+ \nu_\tau) = \left(0.763^{+0.214}_{-0.097}\right) \cdot 10^{-4}$$

Average of several measurements by BaBar and Belle:

$$B^{\text{exp}}(B^+ \to \tau^+ \nu_\tau) = (1.68 \pm 0.31) \cdot 10^{-4}$$

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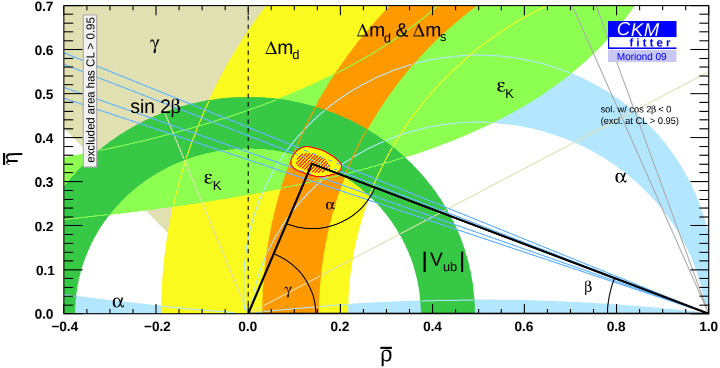
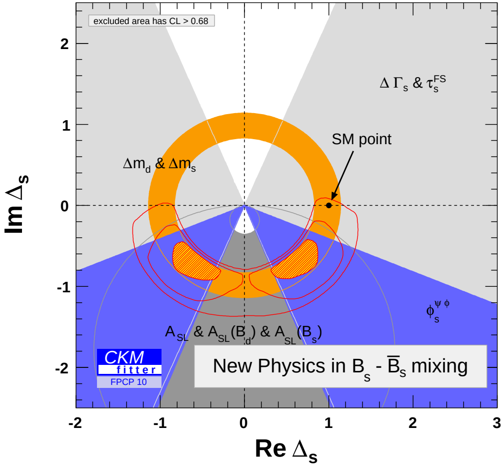
$$B^{\mathrm{SM}}(B^+ \to \tau^+ \nu_\tau) = \frac{G_F^2 m_{B^+} m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_{B^+}^2}\right)^2 |V_{ub}|^2 f_B^2 \tau_{B^+}.$$

But with e.g. $f_B = 210\,\mathrm{MeV}$ and $|V_{ub}| = 4.4 \cdot 10^{-3}$ find $B^{\mathrm{SM}}(B^+ \to \tau^+ \nu_\tau) = 1.51 \cdot 10^{-4}$. These parameters comply with the global fit to the UT only, if new physics changes the constraints from $A_{CP}^{\mathrm{mix}}(B_d \to J/\psi K_{\mathrm{short}})$, Δm_d or $\Delta m_d/\Delta m_s$.

Global fit in the SM:





SM point $\Delta_s = 1$ dis-favoured by $\geq 2.7\sigma$.

without 2010 CDF/DØ data on $B_s \to J/\psi\phi$

Global fit to UT hinting at $\phi_d^\Delta < 0$:
Other authors have seen a tension with the SM in the same direction stemming from ϵ_K.

Lunghi,Soni; Buras,Guadagnoli

In our fit the tension with ϵ_K is mild, because we use a more conservative error on the hadronic parameter $\widehat{B}_K = 0.724 \pm 0.004 \pm 0.067$ and because the Rfit method is more conservative.

p-values:

Calculate χ^2/N_{dof} with and without a hypothesis to find:

Hypothesis	p-value
$\Delta_d = 1$	$2.5\,\sigma$
$\Delta_s = 1$	$2.7\,\sigma$
$\Delta_d = \Delta_s = 1$	$3.4\,\sigma$
$\Delta_d = \Delta_s$	$2.1\,\sigma$

Fit result at $95\%\mathrm{CL}$:

$$\phi_s^{\Delta} = (-51^{+32}_{-25})^\circ \qquad \text{(and } \phi_s^{\Delta} = (-129^{+28}_{-27})^\circ\text{)}$$

Compare with the 2010 CDF/DØ result from $B_s \to J/\psi\phi$:

CDF: $\phi_s^{\Delta} = -29^{+44}_{-49}$ at $95\%\mathrm{CL}$

DØ: $\phi_s^{\Delta} = -44^{+59}_{-51}$ at $95\%\mathrm{CL}$

Naive average: $\phi_s^{\mathrm{avg}} = (-36 \pm 35)^\circ$ at $95\%\mathrm{CL}$

Is the result driven by the DØ dimuon asymmetry?
One can remove a_{fs} as an input and instead predict it from the global fit:

$$a_{\mathrm{fs}} = \left(-4.2^{+2.7}_{-2.6}\right) \cdot 10^{-3} \qquad \text{at } 2\sigma.$$

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$$a_{\rm fs} = \left(-4.2^{+2.7}_{-2.6}\right) \cdot 10^{-3} \qquad\qquad \text{at } 2\sigma.$$

This is just 1.5σ away from the DØ/CDF average

$$a_{\rm fs} = (-8.5 \pm 2.8) \cdot 10^{-3}.$$

The fit in scenario II (real $\Delta_s = \Delta_d$) is not better than the SM fit and gives $\Delta = 0.907^{+0.091}_{-0.067}$.

Scenario III (complex $\Delta_s = \Delta_d$) fits the data quite well irrespective of whether $B(B^+ \to \tau^+ \nu_\tau)$ is included or not.

Hypothesis	p-value
$\Delta = 1$	$3.1\,\sigma$

Supersymmetry

The MSSM has many new sources of flavour violation, all in the supersymmetry-breaking sector.

No problem to get big effects in $B_s - \overline{B}_s$ mixing, but rather to suppress the big effects elsewhere.

Squark mass matrix

Diagonalise the Yukawa matrices Y^u_{jk} and Y^d_{jk}

$\Rightarrow$ quark mass matrices are diagonal, super-CKM basis

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E.g. Down-squark mass matrix:

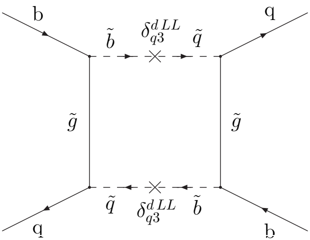
$$M^2_{\tilde d} = \begin{pmatrix}
\left(M^{\tilde d}_{1L}\right)^2 & \Delta^{\tilde d\,LL}_{12} & \Delta^{\tilde d\,LL}_{13} & \Delta^{\tilde d\,LR}_{11} & \Delta^{\tilde d\,LR}_{12} & \Delta^{\tilde d\,LR}_{13} \\
\Delta^{\tilde d\,LL\,*}_{12} & \left(M^{\tilde d}_{2L}\right)^2 & \Delta^{\tilde d\,LL}_{23} & \Delta^{\tilde d\,RL\,*}_{12} & \Delta^{\tilde d\,LR}_{22} & \Delta^{\tilde d\,LR}_{23} \\
\Delta^{\tilde d\,LL\,*}_{13} & \Delta^{\tilde d\,LL\,*}_{23} & \left(M^{\tilde d}_{3L}\right)^2 & \Delta^{\tilde d RL\,*}_{13} & \Delta^{\tilde d\,RL*}_{23} & \Delta^{\tilde d\,LR}_{33} \\
\Delta^{\tilde d\,LR\,*}_{11} & \Delta^{\tilde d RL}_{12} & \Delta^{\tilde d RL}_{13} & \left(M^{\tilde d}_{1R}\right)^2 & \Delta^{\tilde d\,RR}_{12} & \Delta^{\tilde d\,RR}_{13} \\
\Delta^{\tilde d\,LR\,*}_{12} & \Delta^{\tilde d\,LR*}_{22} & \Delta^{\tilde d RL}_{23} & \Delta^{\tilde d\,RR\,*}_{12} & \left(M^{\tilde d}_{2R}\right)^2 & \Delta^{\tilde d\,RR}_{23} \\
\Delta^{\tilde d\,LR\,*}_{13} & \Delta^{\tilde d\,LR\,*}_{23} & \Delta^{\tilde d\,LR\,*}_{33} & \Delta^{\tilde d\,RR\,*}_{13} & \Delta^{\tilde d\,RR\,*}_{23} & \left(M^{\tilde d}_{3R}\right)^2
\end{pmatrix}$$

Squark mass matrix

Diagonalise the Yukawa matrices Y^u_{jk} and Y^d_{jk}
$\Rightarrow$ quark mass matrices are diagonal, super-CKM basis
E.g. Down-squark mass matrix:

$$M^2_{\tilde{d}} = \begin{pmatrix} \left(M^{\tilde{d}}_{1L}\right)^2 & \Delta^{\tilde{d}\,LL}_{12} & \Delta^{\tilde{d}\,LL}_{13} & \Delta^{\tilde{d}\,LR}_{11} & \Delta^{\tilde{d}\,LR}_{12} & \Delta^{\tilde{d}\,LR}_{13} \\ \Delta^{\tilde{d}\,LL\,*}_{12} & \left(M^{\tilde{d}}_{2L}\right)^2 & \Delta^{\tilde{d}\,LL}_{23} & \Delta^{\tilde{d}\,RL\,*}_{12} & \Delta^{\tilde{d}\,LR}_{22} & \Delta^{\tilde{d}\,LR}_{23} \\ \Delta^{\tilde{d}\,LL\,*}_{13} & \Delta^{\tilde{d}\,LL\,*}_{23} & \left(M^{\tilde{d}}_{3L}\right)^2 & \Delta^{\tilde{d}RL\,*}_{13} & \Delta^{\tilde{d}\,RL*}_{23} & \Delta^{\tilde{d}\,LR}_{33} \\ \Delta^{\tilde{d}\,LR\,*}_{11} & \Delta^{\tilde{d}RL}_{12} & \Delta^{\tilde{d}RL}_{13} & \left(M^{\tilde{d}}_{1R}\right)^2 & \Delta^{\tilde{d}\,RR}_{12} & \Delta^{\tilde{d}\,RR}_{13} \\ \Delta^{\tilde{d}\,LR\,*}_{12} & \Delta^{\tilde{d}\,LR*}_{22} & \Delta^{\tilde{d}RL}_{23} & \Delta^{\tilde{d}\,RR\,*}_{12} & \left(M^{\tilde{d}}_{2R}\right)^2 & \Delta^{\tilde{d}\,RR}_{23} \\ \Delta^{\tilde{d}\,LR\,*}_{13} & \Delta^{\tilde{d}\,LR\,*}_{23} & \Delta^{\tilde{d}\,LR\,*}_{33} & \Delta^{\tilde{d}\,RR\,*}_{13} & \Delta^{\tilde{d}\,RR\,*}_{23} & \left(M^{\tilde{d}}_{3R}\right)^2 \end{pmatrix}$$

Not diagonal! $\Rightarrow$ new FCNC transitions.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q
$\tilde{b}$
δ^{dLL}_{q3}
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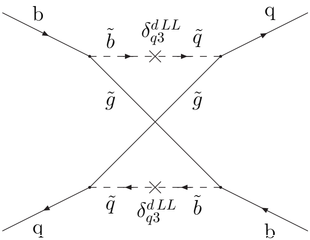
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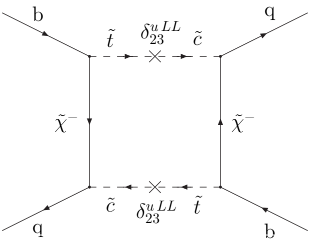
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q
b

Model-independent analyses constrain

$$\delta_{ij}^{q\,XY} = \frac{\Delta_{ij}^{\tilde{q}\,XY}}{\frac{1}{6}\sum_s \left[M_{\tilde{q}}^2\right]_{ss}} \qquad \text{with } XY = LL, LR, RR \text{ and } q = u, d$$

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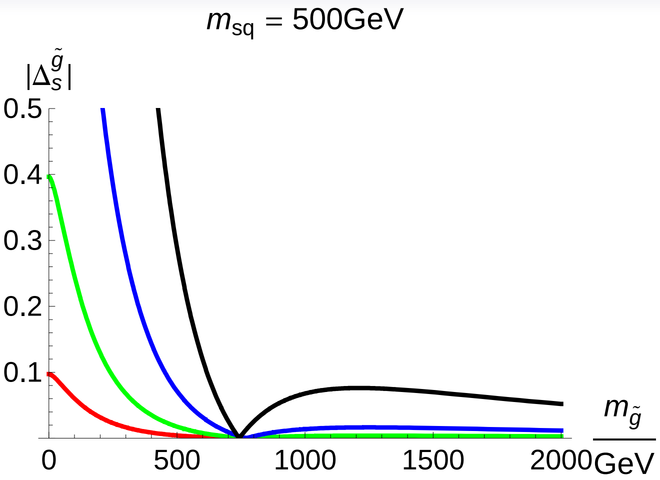
- For $M_{\tilde{g}} \gtrsim 1.5 M_{\tilde{q}}$ the gluino contribution is small for $AB = LL, RR$, so that chargino/neutralino contributions are important.
- To derive meaningful bounds on $\delta_{ij}^{q\,LR}$ chirally enhanced higher-order contributions must be taken into account.

A. Crivellin, UN, 2009



The gluino contribution vanishes for $M_{\tilde g} \approx 1.5 M_{\tilde q}$, independently of the size of $\Delta_{23}^{d\,LL}$ (curves correspond to 4 different values).

Are there natural ways to motivate sizable new flavour violation in $B_s - \overline{B}_s$ mixing and $B_d - \overline{B}_d$ mixing while simultaneous suppressing flavour violation elsewhere?

Flavour violation from trilinear terms

Origin of the SUSY flavour problem: Misalignment of squark mass matrices with Yukawa matrices.

Unorthodox solution: Set Y_{ij}^{u} and Y_{ij}^{d} to zero, except for $(i,j) = (3,3)$.

$\Rightarrow$ No flavour violation from $Y_{ij}^{u,d}$ and $V_{\mathrm{CKM}} = 1$.

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Radiative flavour violation: S. Weinberg 1972

flavour from soft SUSY terms:

W. Buchmüller, D. Wyler 1983,
T. Banks 1988,
F. Borzumati, G.R. Farrar,
N. Polonsky, S.D. Thomas 1998, 1999
J. Ferrandis, N. Haba 2004

Today: Strong constraints from FCNCs probed at B factories.

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But: Radiative flavour violation in the MSSM is still viable, albeit only with A^d_{ij} and A^u_{ij} entering

$$M^{\tilde{d}\,LR}_{ij} = A^d_{ij} v_d + \delta_{i3}\delta_{j3} y_b \mu v_u, \qquad M^{\tilde{u}\,LR}_{ij} = A^u_{ij} v_u + \delta_{i3}\delta_{j3} y_t \mu v_d.$$

Andreas Crivellin, UN, PRD 79 (2009) 035018

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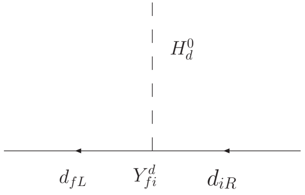

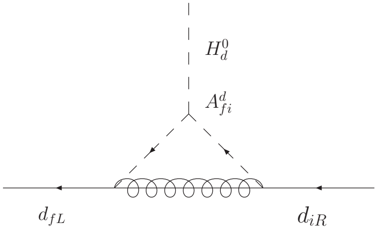

Electric dipole moments

Darkest corner of the MSSM: The phases of A_{ii}^{q} and μ generate too large EDMs. If light quark masses are generated radiatively through soft SUSY-breaking terms, this “supersymmetric CP problem” is substantially alleviated:

- The phases of A_{ii}^{q} and m_q are aligned, i.e. zero.
- The phase of μ (essentially) does not enter the EDMs at the one-loop level, because the Yukawa couplings of the first two generations are zero.

Borzumati, Farrar, Polonsky, Thomas 1998,1999

Flavour and SUSY GUTs

Linking quarks to neutrinos: Flavour mixing:
quarks: Cabibbo-Kobayashi-Maskawa (CKM) matrix
leptons: Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

Consider SU(5) multiplets:

$$\overline{\mathbf{5}}_1 = \begin{pmatrix} d_R^c \\ d_R^c \\ d_R^c \\ e_L \\ -\nu_e \end{pmatrix}, \qquad \overline{\mathbf{5}}_2 = \begin{pmatrix} s_R^c \\ s_R^c \\ s_R^c \\ \mu_L \\ -\nu_\mu \end{pmatrix}, \qquad \overline{\mathbf{5}}_3 = \begin{pmatrix} b_R^c \\ b_R^c \\ b_R^c \\ \tau_L \\ -\nu_\tau \end{pmatrix}.$$

If the observed large atmospheric neutrino mixing angle stems from a rotation of $\overline{\mathbf{5}}_2$ and $\overline{\mathbf{5}}_3$, it will induce a large $\tilde{b}_R - \tilde{s}_R$-mixing (Moroi; Chang,Masiero,Murayama).

$\Rightarrow$ new b_R–s_R transitions from gluino–squark loops possible.

Key ingredients: Some weak basis with

$$\mathsf{Y}_d = V_{\mathrm{CKM}}^* \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} U_{\mathrm{PMNS}}$$

and right-handed down squark mass matrix:

$$\mathsf{m}_{\tilde{d}}^2 (M_Z) = \mathrm{diag}\left(m_{\tilde{d}}^2, m_{\tilde{d}}^2, m_{\tilde{d}}^2 - \Delta_{\tilde{d}} \right).$$

with a calculable real parameter $\Delta_{\tilde{d}}$, typically generated by top-Yukawa RG effects.

Rotating Y_d to diagonal form puts the large atmospheric neutrino mixing angle into $\mathsf{m}^2_{\tilde{d}}$:

$$U^\dagger_{\mathrm{PMNS}}\, \mathsf{m}^2_{\tilde{d}}\, U_{\mathrm{PMNS}} = \begin{pmatrix} m^2_{\tilde{d}} & 0 & 0 \\ 0 & m^2_{\tilde{d}} - \frac{1}{2}\,\Delta_{\tilde{d}} & -\frac{1}{2}\,\Delta_{\tilde{d}}\, e^{i\xi} \\ 0 & -\frac{1}{2}\,\Delta_{\tilde{d}}\, e^{-i\xi} & m^2_{\tilde{d}} - \frac{1}{2}\,\Delta_{\tilde{d}} \end{pmatrix}$$

The CP phase ξ affects $\mathrm{B_s} - \overline{\mathrm{B}}_\mathrm{s}$ mixing!

Realistic GUTs involve further dimension-5 Yukawa terms to fix the Yukawa unification in the first two generations. One can use these terms to shuffle a part of the effect from $b_R \to s_R$ into $b_R \to d_R$ transitions. This "leakage" is strongly constrained by $\mathrm{K}-\overline{\mathrm{K}}$ mixing.

Trine, Wiesenfeldt, Westhoff 2009

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Similar constraints can be found from $\mu \to e\gamma$.
Borzumati,Yamashita 2009; Girrbach,Mertens,UN,Wiesenfeldt 2009

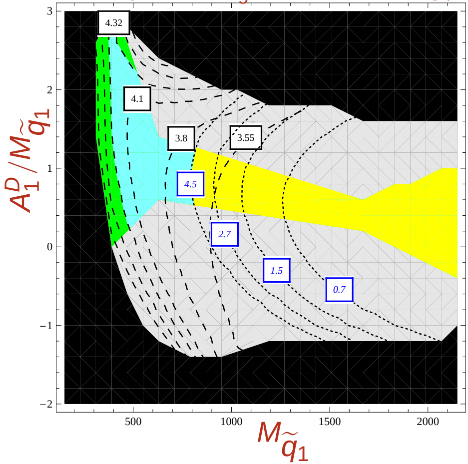
Chang-Masiero-Murayama model

We have considered $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing, $b \to s\gamma$, $\tau \to \mu\gamma$, vacuum stability bounds, lower bounds on sparticle masses and the mass of the lightest Higgs boson.
The analysis involves 7 parameters in addition to those of the Standard Model.

Generic results: Largest effect in $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing
tension with $M_h \geq 114\,\mathrm{GeV}$

J. Girrbach, S. Jäger, M. Knopf, W. Martens, UN, C. Scherrer, S. Wiesenfeldt

Contour plot for $M_{\tilde{g}} = 350\,\mathrm{GeV}$, $\arg \mu = 0$:



Black: negative soft masses[2]
Green: excluded by $\tau \to \mu\gamma$ and $b \to s\gamma$
Blue: excluded by $\tau \to \mu\gamma$
Gray: excluded by $B_s - \overline{B}_s$ mixing
Yellow: allowed

dashed lines: $10^4 \cdot Br(b \to s\gamma)$; dotted lines: $10^8 \cdot Br(\tau \to \mu\gamma)$.

Conclusions

- The DØ result for the dimuon asymmetry in B_s decays supports the hints for $\phi_s < 0$ seen in $B_s \to J/\psi\phi$ data. The central value is easier to accomodate if both $a_{\rm fs}^s$ and $a_{\rm fs}^d$ receive negative contributions from new physics.

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- A global fit to the UT indeed shows a slight preference for a new CP phase $\phi^\Delta_d < 0$, driven by $B(B^+ \to \tau^+\nu_\tau)$ (and possibly ϵ_K). In a simultaneously global fit to the UT and the $\mathrm{B_s\!-\!\overline{B}_s}$ mixing complex a plausible picture of new CP-violating physics emerges.

Conclusions

- Large CP-violating contributions to $B_s - \overline{B}_s$ mixing are possible in supersymmetry without violating constraints from other FCNC processes. If confirmed the DØ/CDF results imply physics beyond the MFV-MSSM.

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- Large CP-violating contributions to $\mathrm{B_s} - \overline{\mathrm{B}}_\mathrm{s}$ mixing are possible in supersymmetry without violating constraints from other FCNC processes. If confirmed the DØ/CDF results imply physics beyond the MFV-MSSM.
- An attractive variant is the MSSM with vanishing Yukawa couplings for the first two generations and radiative flavour violation.
- Models of GUT flavour physics with $\tilde{b}_R - \tilde{s}_R$ mixing driven by the atmospheric neutrino mixing angle can explain the Tevatron data on $\mathrm{B_s} - \overline{\mathrm{B}}_\mathrm{s}$ mixing without conflicting with $b \to s\gamma$ and $\tau \to \mu\gamma$.

A pinch of new physics in $B - \overline{B}$ mixing?