Linear realization of the BRST symmetry

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Introduction

- Nonlinear realization of a symmetry in the simple case of the nonlinear σ model. Relation with NPRG.
- Comparison with the case of QCD (Yang-Mills theory with gauge fixing)
- Description of the linear realization.
- Some consequences and further developments.

Nonlinear σ model

- Consider the action for the N-component field $\vec{\pi}$ in $d = 2$:

$$S[\vec{\pi}] = \int d^2x \, \frac{1}{2} \left((\partial_i \vec{\pi})^2 + \frac{(\vec{\pi}.\partial_i \vec{\pi})^2}{1 - \vec{\pi}^2} \right)$$

- On top of the O(N) symmetry group, there are N symmetries realized non-linearly:

$$\pi_i \to \pi_i + \delta\pi_i = \pi_i + \epsilon_i \sqrt{1 - \vec{\pi}^2}$$

  Symmetry group: $\mathrm{O}(N+1)$.

- In relation with NPRG: any quadratic regulator break the nonlinear symmetries!

Linear realization

- Model equivalent in the low energy limit: add an extra massive field σ:

$$S[\vec{\pi},\sigma] = \int d^2x \, \left\{ \frac{1}{2}(\partial_i \vec{\pi})^2 + \frac{1}{2}(\partial_i \sigma)^2 + \lambda(\sigma^2 + \vec{\pi}^2 - 1)^2 \right\}$$

- The σ mode is massive. At low energy the σ is frozen. Replace σ by $\sqrt{1-\vec{\pi}^2}$ and we get the previous action!
- Nonlinear symmetries are now realized linearly:

$$\begin{cases} \delta\vec{\pi} = \vec{\epsilon}\,\sigma \\ \delta\sigma = -\vec{\epsilon}\cdot\vec{\pi} \end{cases}$$

- We can write a symmetric quartic regulating term.
- Linear realization enables to understand the spectrum of the theory in $d = 2$ (restoration of the symmetry).

- Model with a nonabelian gauge symmetry.

$$\mathcal{L}_{\rm YM} = \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 = \frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu)^2$$

- In most calculations in Yang-Mills theory: Faddeev-Popov gauge fixing.
- Introduce new fields: 2 fermionic Faddeev-Popov ghost fields c and $\bar{c}$ and 1 bosonic Nakanishi-Laudrup field h.
- Adding the linear gauge fixing lagrangian:

$$\mathcal{L}^{\rm linear}_{\rm GF} = \partial_\mu \bar{c}^a (D_\mu c)^a + \frac{\xi_0}{2} h^a h^a + ih^a \partial_\mu A^a_\mu. \qquad (1)$$

or the Curci-Ferrari gauge fixing

$$\mathcal{L}^{\rm CF}_{\rm GF} = \frac{1}{2}\partial_\mu \bar{c}^a (D_\mu c)^a + \frac{1}{2}(D_\mu \bar{c})^a \partial_\mu c^a + \frac{\xi_0}{2} h^a h^a + ih^a \partial_\mu A^a_\mu - \xi_0 \frac{g_0^2}{8}(f^{abc}\bar{c}^b c^c)^2. \qquad (2)$$

lead to the same gauge invariant predictions as Yang-Mills.

Symmetries

- The previous action has nontrivial nonlinear supersymmetries, BRST and anti BRST, which is the quantum equivalent of the gauge invariance.

$$sA_\mu^a = (D_\mu c)^a, \qquad \bar{s}A_\mu^a = (D_\mu \bar{c})^a$$

$$sc^a = -\frac{g_0}{2} f^{abc} c^b c^c, \qquad \bar{s}c^a = -ih^a - \frac{g_0}{2} f^{abc} \bar{c}^b c^c$$

$$s\bar{c}^a = ih^a - \frac{g_0}{2} f^{abc} \bar{c}^b c^c, \qquad \bar{s}\bar{c}^a = -\frac{g_0}{2} f^{abc} \bar{c}^b \bar{c}^c$$

$$s\, ih^a = \frac{g_0}{2} f^{abc} \left(ih^b c^c + \frac{g_0}{4} f^{cde} \bar{c}^b c^d c^e \right),$$

$$\bar{s}\, ih^a = \frac{g_0}{2} f^{abc} \left(ih^b \bar{c}^c - \frac{g_0}{4} f^{cde} c^b \bar{c}^d \bar{c}^e \right).$$

- symplectic group SP(2) with 3 generators:

$$t\bar{c} = c \qquad tc = th = tA_\mu = 0$$

$$\bar{t}c = \bar{c} \qquad \bar{t}\bar{c} = \bar{t}h = \bar{t}A_\mu = 0$$

$$Nc = c \qquad N\bar{c} = -\bar{c} \qquad Nh = NA_\mu = 0$$

Comparison NLσ/YM

- Both have a nonlinearly realized symmetry,
- No symmetric quadratic cutoff,
- Both present asymptotic freedom
- Both present a scale, generically small compared with the UV cut-off, where the relevant excitations change.
- However, no known linear realization of BRST!
- With such a realization, we hope to:
  - implement the NPRG is a standard manner
  - understand better the IR physics of QCD.

Linear realization of BRST I: the space

- Idea: add massive fields (equivalent of the σ field).
- Theory in a superspace *à la* Parisi & Sourlas: $(x^\mu, \theta, \bar{\theta})$:

$$\theta^2 = \bar{\theta}^2 = \bar{\theta}\theta + \theta\bar{\theta} = 0$$

- The space is flat in the euclidean directions and curved in the fermionic ones:

$$ds^2 = dx_\mu^2 + d\bar{\theta}d\theta(1 + M^2\bar{\theta}\theta)$$

- At large distances compared with M^{-1}, grassmann directions are wrapped around.
- At small distances, the superspace is effectively flat $\rightarrow$ dimensional reduction (Parisi Sourlas, etc) the theory behaves like a bosonic $d = 2$ theory in the UV.

Linear realization of BRST II: isometries of the superspace

- Isometries of the superspace associated with symmetries of the gauge-fixed action!
  - Poincaré group
  - "rotations" in the grassmann subspace (symplectic group SP(2))
  - "translations" in the grassmann directions (BRST and anti-BRST symmetries)

Linear realization of BRST III: field content and action

- Introduce a vector field
  $\mathcal{A} = \{\mathcal{A}_\mu(x^\nu,\theta,\bar\theta), \mathcal{A}_\theta(x^\nu,\theta,\bar\theta), \mathcal{A}_{\bar\theta}(x^\nu,\theta,\bar\theta)\}$
- The Taylor expansion of vector field is finite:

$$\begin{aligned}
\mathcal{A}^\alpha_\mu(x,\theta,\bar\theta) &= A^\alpha_\mu(x) + \bar\theta B^\alpha_\mu(x) - \theta\bar B^\alpha_\mu(x) + \bar\theta\theta E^\alpha_\mu(x),\\
\mathcal{A}^\alpha_\theta(x,\theta,\bar\theta) &= -\bar c^\alpha(x) + \theta\bar d^\alpha(x) - \bar\theta b^\alpha(x) - \bar\theta\theta\bar F^\alpha(x),\\
\mathcal{A}^\alpha_{\bar\theta}(x,\theta,\bar\theta) &= c^\alpha(x) + \bar\theta d^\alpha(x) + \theta\bar b^\alpha(x) + \bar\theta\theta F^\alpha(x).
\end{aligned}$$

- We choose the action

$$\mathcal{S}_{\rm YM} = \int_x -\frac{1}{4}(-1)^a \mathcal{F}_{AB}\, g^{BC}\mathcal{F}_{CD}\, g^{DA} + \left(\frac{m^2}{2} g_{AB} + \frac{\nu}{2} R_{AB}\right)\mathcal{A}^A\mathcal{A}^B$$

Decoupling of the massive modes

- In the large M limit, some modes decouple. The action has terms of the form:

$$\int d^d x \; M^2(\bar{B}_\mu^\alpha - D_\mu \bar{c}^\alpha)(B_\mu^\alpha - D_\mu c^\alpha)$$

- The B_μ and $\bar{B}_\mu$ fields are replaced by their classical solutions.
- Eventually get an action for A_μ, c, $\bar{c}$ and $h = b + \bar{b}$. Up to renormalization factors, these are the fields appearing in the Faddeev-Popov procedure.
- We finally obtain the gauge-fixed action with an extra mass term (Curci-Ferari model)

$$\mathcal{L} = \mathcal{L}_{\mathrm{YM}} + \mathcal{L}_{\mathrm{GF}}^{\mathrm{CF}} + m^2\left(\frac{1}{2}(A_\mu^\alpha)^2 + \xi \bar{c}^\alpha c^\alpha\right)$$

Further developments

- Add a symmetric regulating term, quadratic in the superfields and derive flow equations for a simple truncation.
- Understand better the influence of the mass term in the low energy limit.
- Study other linear realizations (other actions for the superfields). Does this give other gauge fixings in the IR?
- Renormalizability of the theory?
- Unitarity?
- Possible application to quantum gravity.