The Phase Structure of the Polyakov–Quark-Meson Model beyond Mean Field

Tina Katharina Herbst

In Collaboration with B.-J. Schaefer and J.M. Pawlowski

arXiv: 1008.0081 [hep-ph]

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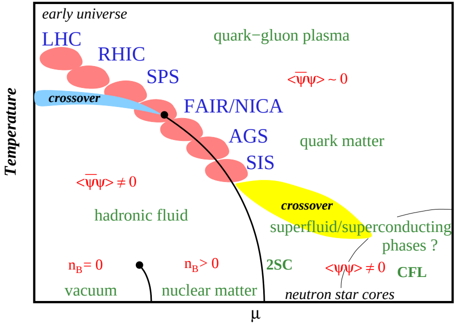
at the Institute of Physics.

QCD Phase Structure



Chiral Symmetry

- $m_q \to 0$
- **chiral condensate** $\langle \bar{q} q \rangle$

Z_{N_c} Center Symmetry

- $m_q \to \infty$
- **Polyakov loop** $\Phi = \langle \ell(\vec{x}) \rangle_\beta$

$$\ell(\vec{x}) = \frac{1}{N_c} \mathrm{Tr}_c \mathcal{P} \exp\{i \int_0^\beta d\tau A_4(\vec{x}, \tau)\}$$

- **relation to confinement:**

$$\Phi \sim e^{-\beta F_q}$$

(approx.) order parameters

$$\langle \bar{q} q \rangle \begin{cases} = 0 & \text{symmetric} \\ \neq 0 & \text{broken} \end{cases} \qquad \Phi \begin{cases} = 0 & \text{confined} \\ \neq 0 & \text{deconfined} \end{cases}$$

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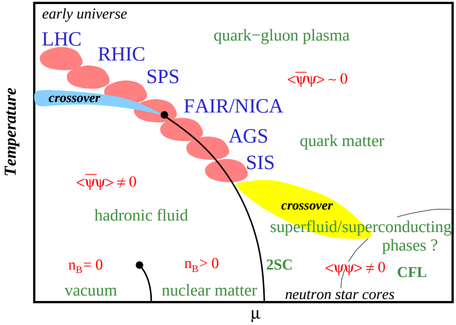

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Other Phases ?

Polyakov-Quark-Meson Model

Lagrangian

$$\mathcal{L}_{PQM} = \bar{q}\left[i\not{D} - h(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\right]q + \frac{1}{2}(\partial_\mu\phi)^2 - U(\sigma,\vec{\pi}) - \mathcal{U}(\Phi,\bar{\Phi})$$

- $\phi = (\sigma, \vec{\pi})\ldots$ $O(4)$-representation of the meson field ($N_f = 2$)
- $\not{D}(\Phi) = \gamma_\mu\partial_\mu - i\,g\gamma_0 A_0(\Phi)$
- $g\ldots$ gauge coupling
- $h\ldots$ Yukawa coupling

Meson Potential

$$U(\sigma,\vec{\pi}) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

Polyakov Loop Potential

[C. Ratti, M.A. Thaler, W. Weise, Phys.Rev. **D73**, 014019 (2006)]

Polynomial Ansatz

$$\frac{\mathcal{U}(\Phi,\bar{\Phi})}{T^4} = -\frac{b_2(T)}{2}\Phi\bar{\Phi} - \frac{b_3}{6}(\Phi^3+\bar{\Phi}^3) + \frac{b_4}{4}(\Phi\bar{\Phi})^2$$

coefficients fitted to lattice data (pure glue):

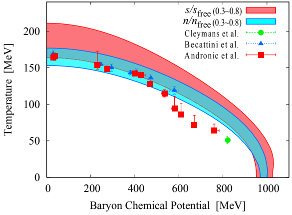
$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$$

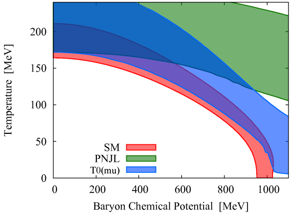
a_0	a_1	a_2	a_3	b_3	b_4
6.75	−1.95	2.625	−7.44	0.75	7.5

$$T_0 = \begin{cases} 270\,\text{MeV (pure glue) ?} \\ 208\,\text{MeV ?} \\ \text{something else ?} \end{cases}$$

$T_0(\mu)$ - One Motivation: Experiment

- experimental information on the QCD phase diagram: **chemical freezeout points**
- *not* raw data, but interpretation using **Statistical Model**
- increase of entropy (red band) and density suggests position of phase transition
- PNJL computation with $T_0 = 200\,\text{MeV}$ inconsistent (green band)
- polynomial ansatz for $T_0(\mu)$ $\Rightarrow$ greater overlap (blue band)





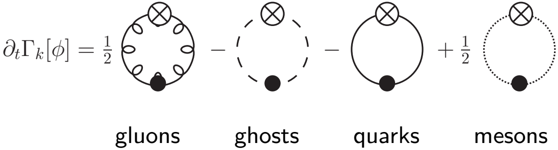
[in these plots: $N_c = N_f = 3$]

[K. Fukushima, arXiv:1006.2596]

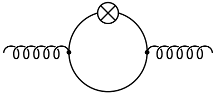
$T_0(\mu)$ - Another Motivation: Theory

[B.-J. Schaefer, J.M. Pawlowski, J. Wambach, Phys.Rev. **D76**, 074023 (2007)]

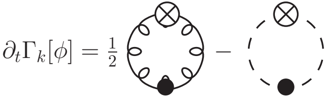
FRG flow for QCD: → Talks by Jan Pawlowski and Lisa Haas



dynamical quarks modify
the gluon contribution:

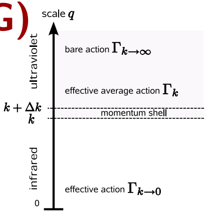


Polyakov Loop potential:
from pure YM contribution



$$T_0 \to T_0(N_f, \mu) = T_\tau e^{-1/(\alpha_0 b(N_f, \mu))}$$

Functional Renormalization Group (FRG)



Flow Equation

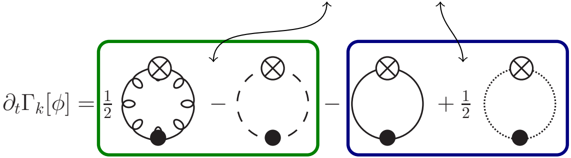
[C. Wetterich, 1993]

$$\partial_t \Gamma_k[\varphi] \quad = \quad \frac{1}{2}\,\mathrm{Tr}\left\{\partial_t R_{k,B}\left(\Gamma_k^{(2,0)}[\varphi] + R_{k,B}\right)^{-1}\right\}$$

Polyakov–Quark-Meson Truncation

$$\Gamma_k = \int d^4x \left\{\bar{q}\left(\not{D} + \mu\gamma_0 + ih(\sigma + i\gamma_5\vec{\tau}\vec{\pi})\right)q + \frac{1}{2}(\partial_\mu\phi)^2 + \Omega_k[\sigma,\vec{\pi},\Phi,\bar{\Phi}]\right\}$$

at initial scale Λ: $\Omega_\Lambda[\sigma,\vec{\pi},\Phi,\bar{\Phi}] = \mathcal{U}(\Phi,\bar{\Phi}) + U(\sigma,\vec{\pi}) + \Omega_\Lambda^\infty[\sigma,\vec{\pi},\Phi,\bar{\Phi}]$



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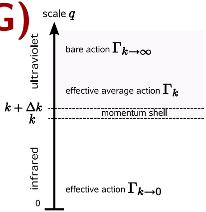

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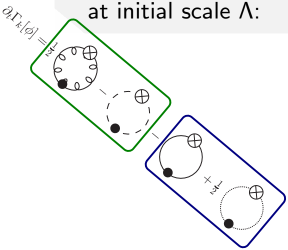


$$\partial_k \Omega_\Lambda^k = \frac{-N_c N_f k^4}{3\pi^2 E_q}\left[1 - N_q(\Phi, \bar{\Phi}) - N_{\bar{q}}(\Phi, \bar{\Phi})\right]$$

[J. Braun, K. Schwenzer, H.J. Pirner, Phys.Rev. **D70**, 085016 (2004)]
[V. Skokov, B. Stokic, B. Friman, Phys.Rev. **C82**, 015206 (2010)]

PQM Flow Equation

[V. Skokov, B. Stokic, B. Friman, Phys.Rev. C82, 015206 (2010)]

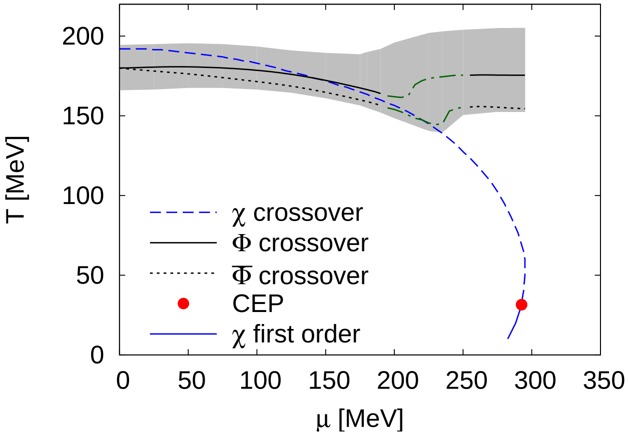
$$\partial_k \Omega_k(T,\mu) = \frac{k^4}{12\pi^2}\left[\frac{3}{E_\pi}\coth\left(\frac{E_\pi}{2T}\right) + \frac{1}{E_\sigma}\coth\left(\frac{E_\sigma}{2T}\right) - \frac{2\nu_q}{E_q}\left\{1 - N_q(T,\mu;\Phi,\bar\Phi) - N_{\bar q}(T,\mu;\Phi,\bar\Phi)\right\}\right]$$

$$N_q(T,\mu;\Phi,\bar\Phi) = \frac{1 + 2\bar\Phi e^{(E_q-\mu)/T} + \Phi e^{2(E_q-\mu)/T}}{1 + 3\bar\Phi e^{(E_q-\mu)/T} + 3\Phi e^{2(E_q-\mu)/T} + e^{3(E_q-\mu)/T}}$$

$$N_{\bar q}(T,\mu;\Phi,\bar\Phi) \equiv N_q(T,-\mu;\bar\Phi,\Phi)$$

$$E_\pi = \sqrt{k^2 + 2\Omega_k'}, \quad E_\sigma = \sqrt{k^2 + 2\Omega_k' + 4\sigma^2\Omega_k''}, \quad \nu_q = 2N_cN_f$$

Phase Structure $T_0 = 208\,\text{MeV}$ const.



[TKH, J.M. Pawlowski, B.-J. Schaefer, arXiv:1008.0081]

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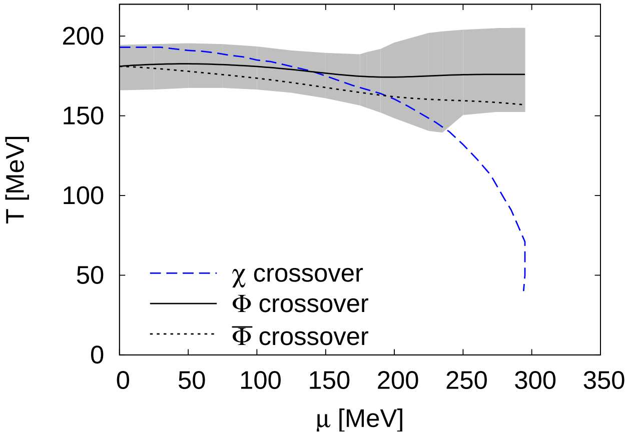
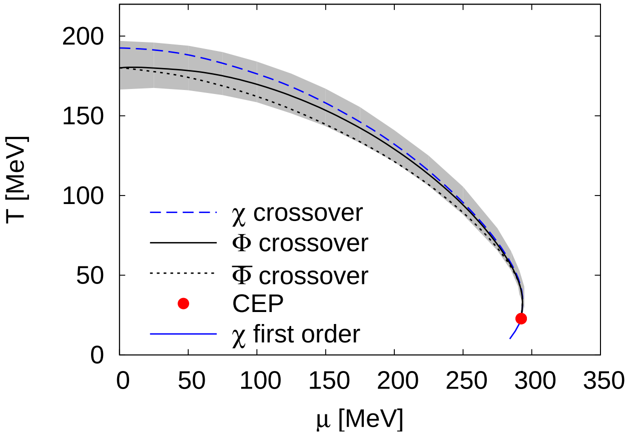


[TKH, J.M. Pawlowski, B.-J. Schaefer and M. Wagner, Work in Progress]

Phase Structure $T_0(\mu)$, $T_0(0) = 208\,\mathbf{MeV}$



[TKH, J.M. Pawlowski, B.-J. Schaefer, arXiv:1008.0081]

Normalized Pressure

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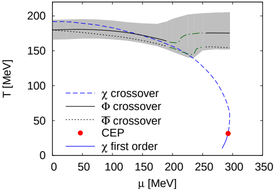

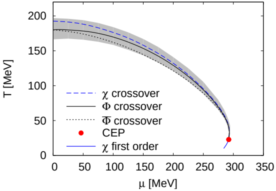

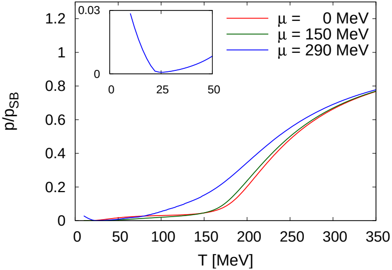

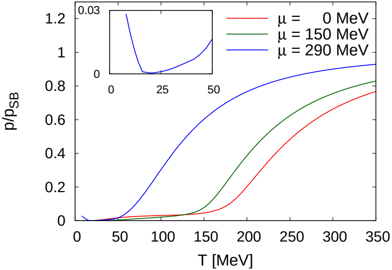

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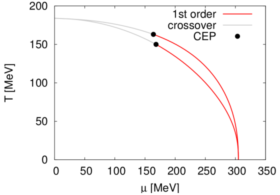
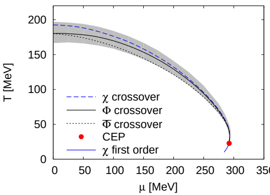
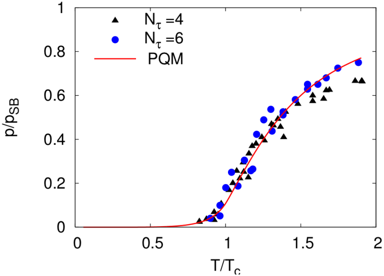
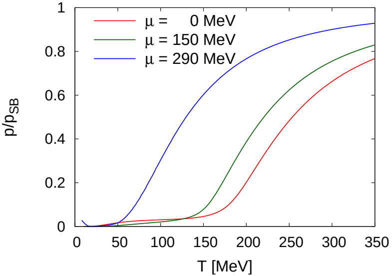


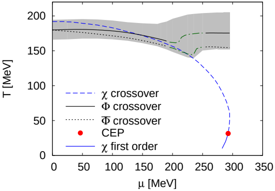


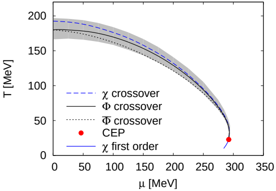


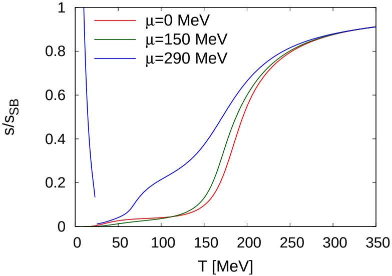


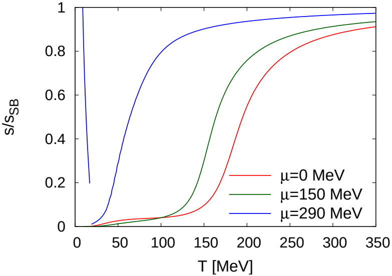
Normalized Entropy Density

[TKH, J.M. Pawlowski, B.-J. Schaefer, arXiv:1008.0081]



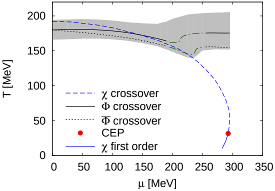


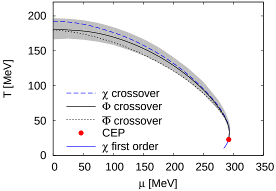


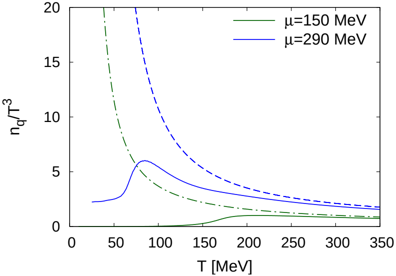


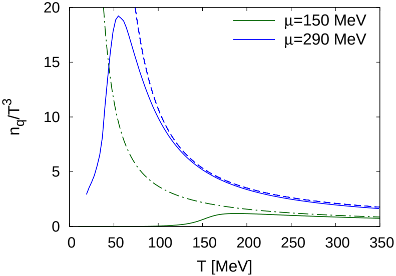
Quark Number Density

[TKH, J.M. Pawlowski, B.-J. Schaefer, arXiv:1008.0081]



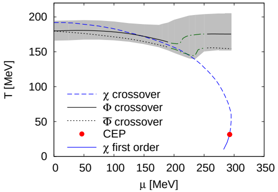


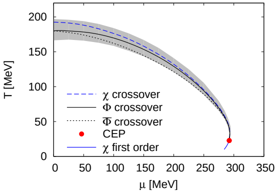


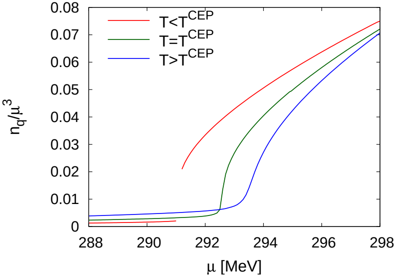


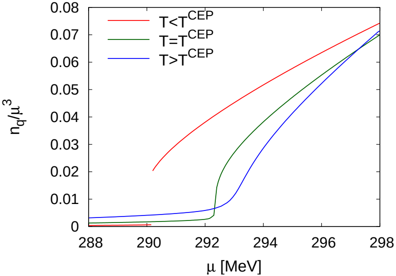
Vicinity of the CEP

[TKH, J.M. Pawlowski, B.-J. Schaefer, arXiv:1008.0081]









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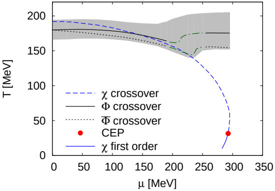
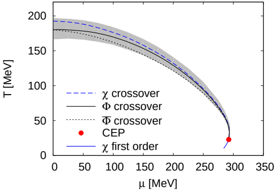
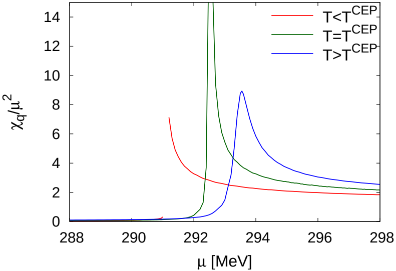
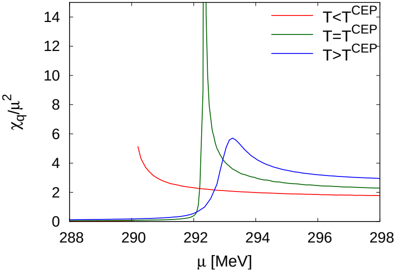






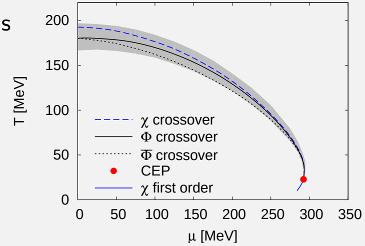


Summary

- PQM model beyond Mean Field
  - quark-meson fluctuations included within FRG approach
  - Important feature:
    back-reaction to gluonic sector

$$T_0 \longrightarrow T_0(N_f, \mu)$$

- Modifications of the Phase Structure
  - fluctuations push CEP downwards
  - $T_0(\mu)$: chiral and deconfinement transitions coincide
  - $\rightsquigarrow$ no quarkyonic phase



- Thermodynamics
  - agree well with lattice studies at $\mu = 0$