HIERARCHICAL NEUTRINO MASSES AND MIXING IN GUTS

Kyriakos Tamvakis

Physics Department, University of Ioannina
tamvakis@uoi.gr

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The Seesaw Mechanism

The wealth of Experimental Data on **Neutrino Masses** and **Mixing** have motivated the study of possible relevant mechanisms. These attempts are more appealing if they are developed within the existing frameworks of **Unification** and **Supersymmetry**.

Among existing proposals the ***most appealing*** is the so-called

Seesaw Mechanism

giving an elegant answer to the smallness of the neutrino mass.

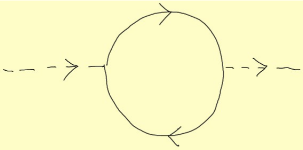
Simplest version of Seesaw (Type-I) is when N^c is a SM-singlet.

$$\mathbf{m}\,\nu\,N^c + \mathbf{M}\,N^c\,N^c \;=\; [\,\nu,\;N^c\,]\begin{bmatrix} 0 & \mathbf{m} \\ \mathbf{m} & \mathbf{M} \end{bmatrix}\begin{bmatrix} \nu \\ N^c \end{bmatrix} \Longrightarrow_{(\mathbf{M}>>\mathbf{m})} \begin{cases} \dfrac{\mathbf{m}^2}{\mathbf{M}} \\ \mathbf{M} \end{cases}$$

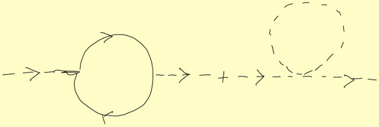
Supersymmetry

Supersymmetry is important:

The interaction of the Higgs doublet with heavy degrees of freedom (namely, $\mathbf{N^c}$) gives corrections $\delta m_H^2 = \frac{Y^2}{16\pi^2} M_R^2$ from



In the SUSY version of the seesaw, we have



with the acceptable $\delta m_H^2 = \frac{Y^2}{16\pi^2} m_s^2 \log \frac{M_R^2}{Q^2}$.

Varieties of Seesaw

Type-I Seesaw $\Longrightarrow$ If N^c is a SM-singlet:

$$\ell\, N^c\, H^c \;+\; M_R\, N^c\, N^c \;=\; \nu\, N^c\, m_D \;+\; M_R\, N^c\, N^c \;+\; \ldots$$

Type-II Seesaw $\Longrightarrow$ If a Charged Isotriplet is present :

$$T \;=\; \begin{pmatrix} T_{++} & T_{+} \\ T_{+}^{\dagger} & T_0 \end{pmatrix}.$$

$$\ell\, T\, \ell \;+\; H\, T\, H \;+\; M\, T^2 \;\Longrightarrow\; \langle T_0 \rangle \;\sim\; \langle H_0 \rangle^2 / M$$

$$\Longrightarrow\; \nu\, \nu\, \frac{\langle H_0 \rangle^2}{M}\,.$$

Note that there is no right-handed neutrino in type-II seesaw.

Varieties of Seesaw Continued

Type-III Seesaw $\Longrightarrow$ If a Neutral (Matter) Isotriplet is present :

$$\Psi = \begin{pmatrix} \Psi_0 & \Psi_+ \\ \Psi_- & -\Psi_0 \end{pmatrix} .$$

$$\ell \, \Psi \, H^c + M \, Tr(\Psi\Psi) = m_D \, \nu \, \Psi_0 + M \, \Psi_0^2 + \ldots .$$

Mixed Seesaw is possible :

For example **I** + **II** in $SO(10)$

$$(16)_F (16)_F (10)_H + (16)_F (16)_F (\overline{126})_H = \nu \, N^c \, \langle 10_{(2,1/2)} \rangle +$$

$$N^c \, N^c \, \langle \overline{126}_{(1,0)} \rangle + \nu \, \nu \, \langle \overline{126}_{(3,1)} \rangle + \ldots .$$

A hierarchy of the vevs is required in order to achieve a small neutrino mass.

Unification

In a unified framework, **in general, N^c participates in a GUT representation and the matrices M and $m_D = Y^{(\nu)} v$ are constrained.** Nevertheless, the seesaw-GUT scenario does not lead by default to an understanding of the neutrino mass, and extra ingredients are required provided by the specific model.

The simplest case of **Type-I in SU(5)**, is not appealing since the right-handed neutrino is a gauge singlet

$$Y^{(\nu)} \mathcal{F}_{\overline{5}} N^c \mathcal{H}_5 + M N^c N^c = Y^{(\nu)} \langle H^c \rangle \nu N^c + M N^c N^c + \ldots .$$

and the matrix M is unconstrained. Nevertheless, **Type I+III in $SU(5)$ in terms of a $\underline{24} + \underline{1}$ is interesting:**

$$\mathcal{F}_{\overline{5}} \left(Y^{(\nu)} \Psi_{24} + Y^{(\nu)'} S \right) \mathcal{H}_5 + M\, Tr(\Psi_{24}^2) + M' S^2$$

$$= \nu \left(Y^{(\nu)} \Psi_{(1,3,0)} + Y^{(\nu)} \Psi_{(1,1,0)} + Y^{(\nu)'} S \right) v + M \left(\Psi^2_{(1,3,0)} + \Psi^2_{(1,1,0)} \right) + M' S^2 + \ldots .$$

SO(10)

In **SO(10)**, the right-handed neutrino is unified in the **16** representation with the rest of matter fermions. The Dirac neutrino mass, arising from

$$Y\,\Psi_{16}\,\Psi_{16}\,\mathcal{H}_{10} \;=\; Y\,\nu\,N^c\,\langle H\rangle \;+\; \ldots$$

is constrained with the rest of matter fermions. The right-handed neutrino Majorana mass can arise either through the introduction of $\overline{\mathbf{126}}$

$$Y'\,\Psi_{16}\,\Psi_{16}\,\mathcal{H}_{\overline{126}}$$

or through non-renormalizable interactions of pairs $\mathbf{16}+\overline{\mathbf{16}}$

$$\Psi_{16}\,\Psi_{16}\,\mathcal{H}_{\overline{16}}\,\mathcal{H}_{\overline{16}}\,.$$

The latter alternative is equivalent to introducing heavy $SO(10)$-singlets **S** as

$$f\,\Psi_{16}\,S\,\mathcal{H}_{\overline{16}} \;+\; M\,S^2$$

and integrating them out.

More SO(10)

An acceptable neutrino Dirac mass M_D requires an extended Higgs sector $\underline{\mathbf{10}} + \underline{\mathbf{16}}' + \underline{\overline{\mathbf{16}}}' + \underline{\mathbf{45}}$ with interactions (K.S. Babu, J.C. Pati, F. Wilczek)

$$\Psi_3 \Psi_3 \mathcal{H}_{10} + \Psi_2 \Psi_3 \mathcal{H}_{10} \mathcal{H}_{45} + \ldots . \Longrightarrow M_D \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix} .$$

The overall neutrino mass matrix is

$$\frac{1}{2} (\nu, N^c, S) \begin{pmatrix} 0 & M_D & fu \\ M_D^{\perp} & 0 & fV \\ f^{\perp} u & f^{\perp} V & M \end{pmatrix} \begin{pmatrix} \nu \\ N^c \\ S \end{pmatrix}$$

The singlet interaction terms arise from

$$f \Psi S \mathcal{H}_{\overline{16}} = (\nu + N^c + \ldots .) S (\langle H \rangle + \langle \overline{N}_H^c \rangle + \ldots .)$$

$$= f u \nu S + f V N^c S + \ldots .$$

Type-I + III Contributions

The resulting light neutrino mass is (S. Barr)

$$\mathcal{M}_\nu = -\mathbf{M_D}\,\mathbf{M_R^{-1}}\,\mathbf{M_D^\perp} - \frac{\mathbf{u}}{\mathbf{V}}\left(\mathbf{M_D} + \mathbf{M_D^\perp}\right) ,$$

where

$$\mathbf{M_R} \equiv = -\mathbf{V^2}\,\mathbf{f}\,\mathbf{M^{-1}}\,\mathbf{f^\perp} .$$

For $M_D \sim O(m_f)$, the ratio of the two terms is

$$\frac{m_f}{u} : f^2 \frac{V}{M} ,$$

which indicates that for a singlet mass $M >> M_X$, the quadratic term dominates. In the case of singlet masses comparable to M_X, the second term could be dominant. **Note that the linear term is highly constrained, since it does not involve the arbitrary right-handed neutrino mass.**

Flipped SU(5)

In the model based on the $\mathbf{SU(5)} \times \mathbf{U(1)}$ group, the so-called ***flipped SU(5) GUT*** the right-handed neutrino is part of the $(\underline{\mathbf{10}},\ \mathbf{1})$ representation. Matter and Higgs content is

$$\Psi(\underline{10},1) = \left[\begin{pmatrix} u \\ d \end{pmatrix}, d^c, \mathbf{N^c}\right], \Psi^c(\underline{\bar{5}},-3) = \left[\begin{pmatrix} e \\ \nu \end{pmatrix}, u^c\right], L^c(\underline{1},5) \ = \ e^c$$

$$\mathcal{H}(\underline{10},\ 1) + \overline{\mathcal{H}}(\underline{\overline{10}},\ -1) \ = \ \left[\begin{pmatrix} u_H \\ d_H \end{pmatrix}, d^c_H, N^c_H\right] + \left[\begin{pmatrix} \bar{u}_H \\ \bar{d}_H \end{pmatrix}, \bar{d}^c_H, \overline{N}^c_H\right],$$

$$h(\underline{5},\ -2) \ + \ h^c(\underline{\bar{5}},\ 2) \ = \ (\mathcal{H}_1,\ D_H) \ + \ (\mathcal{H}_2, \bar{D}_H) \ .$$

$$\langle \mathbf{N^c_H} \rangle \ = \ \langle \overline{\mathbf{N}}^c_\mathbf{H} \rangle \ \equiv \ \mathbf{V} \ \Longrightarrow \ \mathbf{SU(5)} \times \mathbf{U(1)} \ \to \ \mathbf{SU(3)} \times \mathbf{SU(2)} \times \mathbf{U(1)}$$

$(d_H,\ u_H)$, $(\bar{d}_H,\ \bar{u}_H)$ and a combination of N^c_H, $\overline{N}^c_H$ are higgsed away while the triplets d^c_H, $\bar{d}^c_H$, D_H, $\bar{D}_H$ obtain large masses λV, $\lambda' V$ through the couplings $\lambda \mathcal{H}\mathcal{H}\, h$ and $\lambda' \overline{\mathcal{H}\mathcal{H}}\, h^c$. Thus, the triplets are split from the (massless) doublets.

The Superpotential

$$\mathcal{W} \;=\; Y^{(d)}\,\Psi\Psi\,h \,+\, Y^{(u)}\,\Psi\,\Psi^c\,h^c \,+\, Y^{(\ell)}\,\Psi^c\,L^c\,h^c$$
$$+\,\lambda\mathcal{H}\mathcal{H}\,h + \lambda'\overline{\mathcal{H}\mathcal{H}}\,h^c + \mu\,h\,h^c \;\Longrightarrow\; Y^{(u)}\,\Psi\,\Psi^c\,h^c \;=\; Y^{(u)}\,N^c\,\ell\,H_2 + \ldots .$$

A right-handed neutrino mass can arise through the non-renormalizable term

$$\Psi\cdot\frac{\overline{\mathcal{H}\mathcal{H}}}{M}\cdot\Psi \;=\; N^c\,\frac{\left(\overline{N}^c_H\right)^2}{M}N^c \;+\; \ldots .$$

In a renormalizable framework, such a term can arise if we introduce a set of superheavy singlets S with negative matter-parity. Thus, the most general renormalizable superpotential that can be added to $\mathcal{W}$ is

$$Y^{(s)}\,\mathcal{S}\,\Psi\,\overline{\mathcal{H}} \;+\; \frac{1}{2}M_S\mathcal{S}\mathcal{S}\,.$$

A singlet-left-handed neutrino interaction term can arise from a non-renormalizable interaction

$$\frac{\tilde{Y}^{(s)}}{M}\,\Psi^c\,\mathcal{H}\,h^c\,\mathcal{S} \;\Longleftarrow\; \Psi^c\,\mathcal{H}'\,h^c \;+\; M\,\mathcal{H}'\,\overline{\mathcal{H}}' \;+\; \mathcal{S}\overline{\mathcal{H}}'\,\mathcal{H} \;+\; \ldots .$$

The Overall Neutrino Mass

$$\frac{1}{2}\left(\nu,\, N^c,\, S\right)\begin{pmatrix} 0 & M_D & \tilde{Y}^{(s)}\, u \\ M_D^{\perp} & 0 & Y^{(s)}\, V \\ \left(\tilde{Y}^{(s)}\right)^{\perp} u & \left(Y^{(s)}\right)^{\perp} V & M_s \end{pmatrix}\begin{pmatrix} \nu \\ N^c \\ S \end{pmatrix}$$

with $\tilde{Y} \neq Y$ and $u = v\,\frac{V}{M_s} \sim 10^2 \left(\frac{M_X}{M_s}\right)$ GeV. The rotation $\nu = \nu' + \frac{u}{V} H\, N^{c\prime}$, $N^c = N^{c\prime} - \frac{u}{V} H \nu'$ in terms of the matrix $H \equiv \left(\tilde{Y}^{(s)} \left(Y^{(s)}\right)^{-1}\right)^{\perp}$, removes the $\nu - S$ coupling.

$$\Longrightarrow \begin{pmatrix} -\frac{u}{V}\left(M_D\, H + H^{\perp} M_D^{\perp}\right) & M_D & 0 \\ M_D^{\perp} & \frac{u}{V}\left(H^{\perp} M_D + M_D^{\perp} H\right) & Y^{(s)} V \\ 0 & \left(Y^{(s)}\right)^{\perp} V & M_s \end{pmatrix}$$

A Type I+III Seesaw

The light neutrino mass is

$$\mathcal{M}_\nu = -M_D M_R^{-1} M_D^\perp - \frac{u}{V}\left(M_D H + H^\perp M_D^\perp\right) ,$$

with

$$M_D = Y^{(u)} v, \qquad M_R = -V^2 Y^{(s)} M_s^{-1} \left(Y^{(s)}\right)^\perp .$$

Can the linear term be non-negligible? In the case $M_s \sim V$, we have

$$\frac{M^{(2)}}{M^{(1)}} \sim \frac{m_f}{v\, Y\, \tilde{Y}} ,$$

which can be of $O(1)$.

Depending on the scale of the singlet masses in relation to the GUT scale, either of the two terms could dominate. In the quadratic term, an extra source of hierarchical masses and mixing is provided by $Y^{(s)}$ beyond that of M_D. Similarly, in the linear term, since $H \neq 1$, now there is an extra source of mixing beyond the one present in M_D.

Hierarchical Masses from the Standard Seesaw

For $M_s \sim 10^{18}\, GeV$ and $V \sim 10^{16}\, GeV$, the quadratic term dominates and the scale of the light neutrino mass is

$$\mathcal{M}_\nu \sim \frac{(10^2)^2}{\frac{(10^{16})^2}{10^{18}}}\, GeV \sim O(0.1\, eV)\,.$$

Defining dimensionless quantities, we write

$$\hat{\mathcal{M}}_\nu = Y_u \left(Y_s^\perp\right)^{-1} \hat{M}_s\, Y_s^{-1}\, Y_u^\perp\,.$$

Next, we introduce an Ansatz for Y_u

$$Y_u = \begin{pmatrix} 0 & e_1\lambda^6 & 0 \\ e_1\lambda^6 & 0 & e_2\lambda^2 \\ 0 & e_2\lambda^2 & e_3 \end{pmatrix}$$

where $e_1,\ e_2,\ e_3$ parameters of $O(1)$ in λ and

$$\lambda \approx 0.22$$

is the Cabbibo mixing parameter.

Hierarchical Ansatz

At this point we must choose an Ansatz for the couplings Y_s. A simple choice is a diagonal matrix of the form $\delta_{ij}\, f_j\, \lambda^{n_j}$. We choose (J. Rizos, K.T.)

$$Y_s \;=\; \begin{pmatrix} f_1\,\lambda^5 & 0 & 0 \\ 0 & f_2\,\lambda^2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} .$$

For the singlet mass matrix M_s we may chose **a generic symmetric matrix with O(1) elements**. Thus, we obtain for $\hat{\mathcal{M}}_\nu$

$$\begin{pmatrix} O(\lambda^8) & O(\lambda^5) & O(\lambda^4) \\ O(\lambda^5) & A\,\lambda^2 & C\,\lambda \,+\, D\,\lambda^2 \\ O(\lambda^4) & C\,\lambda \,+\, D\,\lambda^2 & E \end{pmatrix}$$

Hierarchical Ansatz Continued

The neutrino eigenvalues are

$$\mathbf{M_3} \approx \mathbf{E} + \lambda^2 \frac{\mathbf{C^2}}{\mathbf{E}}, \quad \mathbf{M_2} \approx \lambda^2 \left(\mathbf{A} - \frac{\mathbf{C^2}}{\mathbf{E}} \right), \quad \mathbf{M_1} \approx \mathbf{O}(\lambda^8)\,.$$

in terms of

$$A = e_1^2 f_1^{-2} M_{11}, \quad C = e_1 e_2 f_1^{-1} f_2^{-1} M_{12} + e_1 e_3 f_1^{-1} f_3^{-1} M_{13}$$

$$D = e_2^2 f_2^{-1} f_3^{-1} M_{23} + e_2 e_3 f_3^{-2} M_{33}$$

$$E = e_2^2 f_2^{-2} M_{22} + 2 e_2 e_3 f_2^{-1} f_3^{-1} M_{23} + e_3^2 f_3^{-2} M_{33}$$

If $\mathbf{U}_\nu$ is the matrix that diagonalizes the neutrino mass

$$\mathbf{U}_\nu \mathcal{M}_\nu \mathbf{U}_\nu^\dagger = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix} \Longrightarrow \mathbf{U}_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix}$$

Neutrino Mixing

Substituting $\mathcal{M}_\nu$ and the eigenvalues, we obtain

$$\phi \approx \lambda \frac{C}{E} - \lambda^2 \frac{D}{E} \, .$$

Nevertheless, the neutrino charged current involves the ***PMNS-mixing matrix***

$$\mathcal{U}_{PMNS} \equiv \mathbf{U}_\ell \mathbf{U}_\nu^\dagger \, ,$$

which depends on the charged left-handed lepton diagonalizing matrix $\mathbf{U}_\ell$ aswell.
It is conceivable that the large atmospheric and solar neutrino mixing angles arise in the charged lepton sector. This can come about naturally in the so-called *lopsided models* (S. Barr, A. Khan) in which the lepton mass terms have the following structure

$$Y_{13}^{(\ell)} \Psi_1^c L_3^c h + Y_{23}^{(\ell)} \Psi_2^c L_3^c h + Y_{33}^{(\ell)} \Psi_3^c L_3^c h \, .$$

Left-handed Lepton Mixing

$$\Longrightarrow M^{(\ell)} = v \begin{pmatrix} 0 & 0 & Y_{13} \\ 0 & 0 & Y_{23} \\ 0 & 0 & Y_{33} \end{pmatrix}$$

The zeros correspond to elements much smaller that $Y_{i3}^{(\ell)} v$. This matrix is diagonalized by two successive finite rotations of left-handed leptons

$$M^{(\ell)} - - - - \mathbf{U}_{12}(\eta_s) \rightarrow - - - - - \mathbf{U}_{23}(\eta_a) \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Y \end{pmatrix}$$

with

$$\eta_s = \arctan(Y_{13}/Y_{23}), \quad \eta_a = \arctan(\sqrt{Y_{13}^2 + Y_{23}^2}/Y_{33})$$

and

$$Y = \sqrt{Y_{13}^2 + Y_{23}^2 + Y_{33}^2} \,.$$

The small 12 element is eliminated with a small additional rotation of the left-handed charged leptons, while all small elements below the main diagonal are eliminated with small rotations of the right-handed leptons.

Neutrino Mixing Hierarchy

The charged left-handed lepton matrix is

$$\mathbf{U}_\ell = \mathbf{U}_{12}(\eta_s)\,\mathbf{U}_{23}(\eta_a) = \begin{pmatrix} \cos\eta_s & \sin\eta_s & 0 \\ -\cos\eta_a\sin\eta_s & \cos\eta_a\cos\eta_s & \sin\eta_a \\ \sin\eta_a\sin\eta_s & -\sin\eta_a\cos\eta_s & \cos\eta_a \end{pmatrix}.$$

Comparing the total mixing matrix $\mathcal{U}_{PMNS} \approx \mathbf{U}_{12}(\eta_s)\,\mathbf{U}_{23}(\eta_a)\,\mathbf{U}_{23}(-\phi)$ with the general form $\mathcal{U}_{PMNS} = \mathbf{U}(\theta_a)\mathbf{U}(\theta_{13})\,\mathbf{U}(\theta_s)$, we conclude that

$$\sin\theta_s \approx \cos\phi\,\sin\eta_s \approx \sin\eta_s$$

$$\sin\theta_a \approx \sin\eta_a + \cos\eta_a\cos\theta_s\sin\phi\,\cos\phi \approx \sin\eta_a + \phi\,\cos\eta_a\,\cos\theta_s$$

$$\sin\theta_{13} \approx \sin\eta_s\sin\phi \approx \sin\theta_s\sin\phi \approx \phi\,\sin\theta_s,$$

$$\Longrightarrow \theta_a > \theta_s >> \theta_{13}$$

This hierarchy is controlled by the Cabbibo parameter, since

$$\phi \approx \lambda\frac{C}{E} + O(\lambda^2) = \lambda\left(\frac{\frac{e_1e_2}{f_1f_2}M_{12} + \frac{e_1e_3}{f_1f_3}M_{13}}{\frac{e_2^2}{f_2^2}M_{22} + 2\frac{e_2e_3}{f_2f_3}M_{23} + \frac{e_3^2}{f_3^2}M_{33}}\right) + O(\lambda^2).$$

Type-III Neutrino Mass Term

What if the linear neutrino mass term is dominant?

$$\hat{\mathcal{M}}_\nu \sim Y_u H + H^\perp Y_u^\perp .$$

Introducing the same Ansätze for Y_u and Y_s as before, we obtain

$$\hat{\mathcal{M}}_\nu = Y_u Y_s^{-1} \tilde{Y}_s^\perp + \tilde{Y}_s Y_s^{-1} Y_u$$

$$= \begin{pmatrix} 0 & e_1 f_2^{-1} \lambda^4 & 0 \\ e_1 f_1^{-1} \lambda & 0 & e_2 f_3^{-1} \lambda^2 \\ 0 & e_2 f_2^{-1} & e_3 f_3^{-1} \end{pmatrix} \tilde{Y}_s^\perp + h.c.$$

At this point we may adopt a *lopsided Ansatz* for $\tilde{Y}_s$

$$\tilde{Y}_s = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \tilde{Y}_{31} & \tilde{Y}_{32} & \tilde{Y}_{33} \end{pmatrix}$$

resulting from the couplings

$$\left(\tilde{Y}_{31} \Psi_3^c \mathcal{S}_1 + \tilde{Y}_{32} \Psi_3^c \mathcal{S}_2 + \tilde{Y}_{33} \Psi_3^c \mathcal{S}_3\right) \mathcal{H}\, h^c$$

Linear Mass Term Continued

All that gives a neutrino mass $\hat{\mathcal{M}}_\nu$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & e_1 f_1^{-1} \tilde{Y}_{31} \lambda + e_2 f_3^{-1} \tilde{Y}_{33} \lambda^2 \\ 0 & e_1 f_1^{-1} \tilde{Y}_{31} \lambda + e_2 f_3^{-1} \tilde{Y}_{33} \lambda^2 & 2 e_2 f_2^{-1} \tilde{Y}_{32} + 2 e_3 f_3^{-1} \tilde{Y}_{33} \end{pmatrix} + O(\lambda^4)$$

with eigenvalues

$$M_1 \approx 0, \quad M_2 \approx \lambda^2 \frac{e_1^2 f_1^{-2} \tilde{Y}_{31}^2}{\left(2 e_2 f_2^{-1} \tilde{Y}_{32} + 2 e_3 f_3^{-1} \tilde{Y}_{33}\right)}$$

$$M_3 \approx 2 e_2 f_2^{-1} \tilde{Y}_{32} + 2 e_3 f_3^{-1} \tilde{Y}_{33} + O(\lambda^2) \,.$$

Mixing Angle Hierarchy

The neutrino mass matrix is diagonalized by

$$\mathbf{U}_\nu \;=\; \mathbf{U}_{23}(-\phi)$$

with

$$\phi \;\approx\; \frac{e_1 f_1^{-1}\tilde{Y}_{31}\lambda + e_2 f_3^{-1}\tilde{Y}_{33}\lambda^2}{2e_2 f_2^{-1}\tilde{Y}_{32} + 2e_3 f_3^{-1}\tilde{Y}_{33}} \;\approx\; \sqrt{\frac{M_2}{M_3}} \;+\; O(\lambda^2)\,.$$

Note that, here, the Cabbibo mixing is directly related only to the neutrino mass ratios.

Assuming, that the lepton mixing proceeds as before, the total mixing will be again

$$\sin\theta_{\mathbf{s}} \;\approx\; \sin\eta_{\mathbf{s}}$$

$$\sin\theta_{\mathbf{a}} \;\approx\; \sin\eta_{\mathbf{a}} \;+\; \phi\,\cos\eta_{\mathbf{a}}\,\cos\theta_{\mathbf{s}}$$

$$\sin\theta_{\mathbf{13}} \;\approx\; \phi\,\sin\theta_{\mathbf{s}}\,,$$

$$\Longrightarrow\; \theta_{\mathbf{a}} \;>\; \theta_{\mathbf{s}} \;>>\; \theta_{\mathbf{13}}$$

SUMMARY

VARIETIES OF SEESAW MECHANISM IN GUTS

$\Downarrow$

TYPE I + III IN SU(5), SO(10) AND SU(5) $\times$ U(1)

$\Downarrow$

ANSATZE FOR THE COUPLINGS TO THE SINGLET SECTOR

$\Downarrow$

HIERARCHICAL NEUTRINO MASSES

$$1 \; : \; \lambda^{1,2} \; : \; \lambda^{n}$$

$\Downarrow$

HIERARCHICAL NEUTRINO MIXING

$$\theta_{23} \; > \; \theta_{12} \; >> \; \theta_{13} \; \sim \; O(\lambda)$$