Effects of non-decoupled mass hierarchies on cosmology

based on arXiv:1005.3848 and work in preparation

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Outline

Work in progress...

Goal: Classify supergravity models for cosmology by their observational implications

In this talk: Effects of non-decoupled mass hierarchies on cosmology

- Context: inflation
- Motivation: supergravity and its multiple fields
- Results: signatures of curved trajectories in field space
  - Speed of sound
  - Numerical results for trajectories and power spectra
- Conclusions

History of our universe?

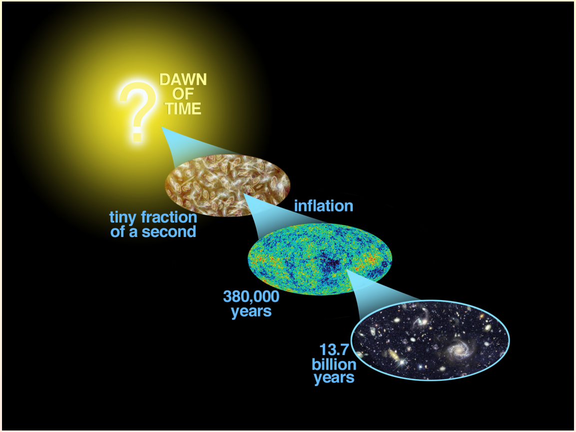

Figure: History of our universe (image courtesy of LAMBDA)

$E \sim 10^{19}$ GeV

$E \sim 10^{16}$ GeV
→ inflation

$E \sim 10^{3}$ GeV → LHC

$E \sim 1$ eV → CMB

$E \sim 10^{-5}$ eV → now

History of our universe?

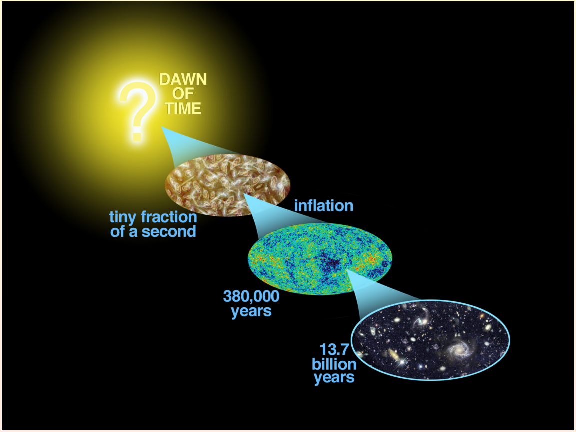

Figure: History of our universe (image courtesy of LAMBDA)

Figure: Launch of the Planck satellite, May 14th, 2009

What is inflation?

Cosmic inflation

- is a period of exponential expansion,
- is generated by a non-zero potential $V(\phi)$,
- leads to a homogeneous, isotropic and flat universe,
- generates density perturbations from quantum field perturbations.

Definition

Big bang cosmology starts when inflation ends.

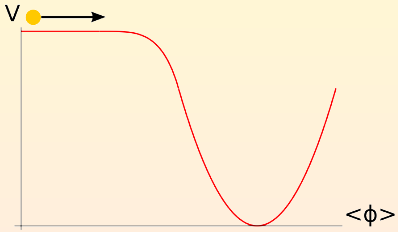

Figure: Typical inflationary potential

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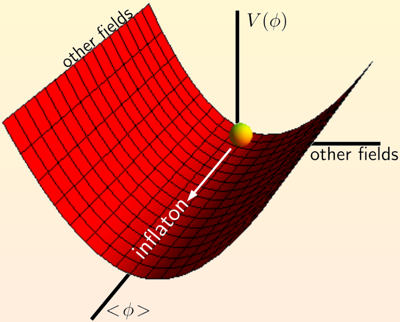


Figure: Typical inflationary potential with stabilised fields

Inflation?

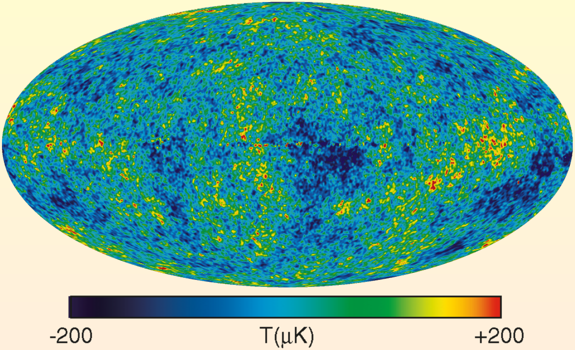


Figure: CMB image from 5-year WMAP data, $\langle T \rangle \sim 2.7\,\mathrm{K}$ (image courtesy of LAMBDA.)

- Inflation explains isotropy, homogeneity and flatness of the universe.
- Inflation explains the $O(10^{-5})$ perturbations.
- CMB opens a window to study the physics during inflation.

Inflation?

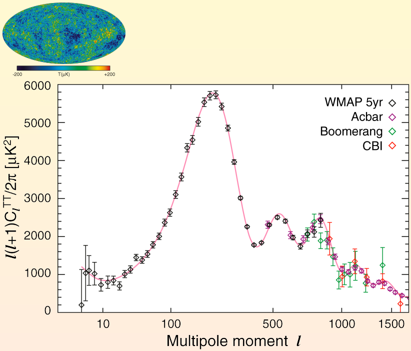
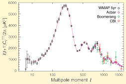


Figure: CMB power spectrum as a function of spherical harmonics P_l (image courtesy of LAMBDA)

Inflation should give

- $\gtrsim e^{55} \sim 10^{24}$ orders of expansion,
- scale invariant fluctuations, which can be studied from the CMB.

Inflation?



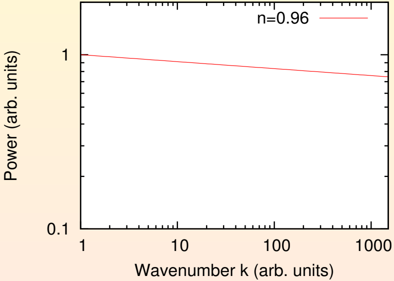


Figure: Scale invariant power spectrum k^{n_s-1}, with spectral index $n_s = 0.96$.

Inflation should give

- $\gtrsim e^{55} \sim 10^{24}$ orders of expansion,
- scale invariant fluctuations, which can be studied from the CMB.

Achieved by slow roll inflation, parametrised by

$$\epsilon = \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

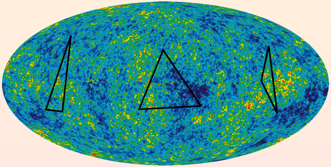
$$\eta = M_{Pl}^2 \frac{V''}{V} \ll 1$$

Non-Gaussianities and the speed of sound

- For Gaussian perturbations, the three-point function vanishes.
- Non-Gaussianity is the non-vanishing of the three-point function.

$$f(k_1 k_2 k_3) = \langle \varphi(k_1) \varphi(k_2) \varphi(k_3) \rangle$$

- Non-Gaussian perturbations $\Rightarrow$ interactions
- Different relations between k_1, k_2, k_3 represent different types of interactions[1]



[1] Maldacena, 2002; Criminelli, 2003 and many thereafter

Non-Gaussianities and the speed of sound

Generation of $c_s < 1$

A speed of sound for the inflaton perturbations $c_s < 1$ results from higher order derivatives.

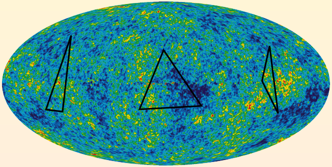
Examples

- K-inflation[2]

$$L_{\text{kin}} \sim K(\varphi)(\nabla\varphi) + L(\phi)(\nabla\varphi)^2 + \ldots$$

- DBI-inflation[3]

$$L_{\text{eff}} \sim f(\varphi)^{-1}\sqrt{1 + f(\varphi) g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi}$$



Non-Gaussianities from $c_s < 1$

Non-local interactions give largest contributions when $k_1 \approx k_2 \approx k_3$, creating equilateral non-Gaussianities[1]

[1] Criminelli, 2003
[2] Garriga, Mukhanov, 1999 and many thereafter
[3] Alishahiha, Silverstein, Tong, 2008 and many thereafter

Supergravity and cosmology

- Inflation is a good probe for the physics at the highest energy scales of $O(10^{13})\times$ LHC.
- For theoretical reasons supergravity/superstring theory is expected to play a role.

Global challenge

There is (worldwide) a large body of work trying to

- make inflation work within the supergravity/superstring framework,
- make testable predictions for cosmological observables.

Our approach

Try to classify/identify general features of supergravity models of inflation, such as

- multifield inflation,
- curved inflaton trajectories,
- decoupling - why did inflation only inflate $3+1d$ spacetime?[4]

[4] eg. work by Choi, Falkowski, Nilles, Olechowski, Pokorski, 2004; De Alwis, 2005; (Covi,) Gomez-Reino, Scrucca, (Palma) 2006-2008; Achúcarro, SH, Sousa 2007-2008

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$\mathcal{N} = 1$ supergravity

The $\mathcal{N} = 1$ supergravity action for uncharged scalar fields

$$S = M_{Pl}^2 \int \mathrm{d}^4 x \sqrt{g} \left(\frac{1}{2} R + K_{i\bar{j}}(\phi, \bar{\phi}) \nabla_\mu \phi^i \nabla^\mu \bar{\phi}^{\bar{j}} - V(\phi, \bar{\phi}) \right)$$

- $K_{i\bar{j}} = \partial_i \partial_{\bar{j}} K \Rightarrow$ Kähler metric determined by real $K(\phi, \bar{\phi})$.
- Usually $i > 1 \Rightarrow$ more than one field.
- $V = e^{K/M_{Pl}^2} \left(K^{i\bar{j}} D_i W \overline{D_j W} - \frac{3|W|^2}{M_{Pl}^2} \right) \Rightarrow$
  potential determined by $K(\phi, \bar{\phi})$ and holomorphic $W(\phi)$
- If $W \neq 0$, K, W can (unlike SUSY) be combined into one function
  $G(\phi, \bar{\phi}) = M_{Pl}^{-2} K(\phi, \bar{\phi}) + \log \left| \frac{W(\phi)}{M_{Pl}^3} \right|^2$

What the equations tell

- Supergravity is determined by only one function (plus one function for charged fields)
- Supergravity action generally has many fields

Multiple fields from $\mathcal{N}=1$ supergravity

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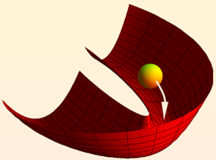

Figure: An example of a curved trajectory.

- Metric $K_{i\bar{j}}(\phi,\bar{\phi}) \Rightarrow$ *geometry*
- Potential $V(\phi,\bar{\phi}) \rightsquigarrow$ trajectory
- Usually trajectory $\neq$ geodesic[5], inflaton field mixes with other fields[6], even in the case of a large hierarchy of scales[7]
- Gives a (time-dependent) perpendicular acceleration $\eta_\perp = \sqrt{2E_{\mathrm{kin}}}/\kappa$
  - For slow roll $\epsilon, \eta_\parallel \ll 1$
  - $\eta_\perp$ not necessarily small!

[5] In case of "decoupling" the trajectory is a geodesic

[6] Known for multiple light fields, eg. Groot-Nibbelink, van Tent, 2000-2001 and many results thereafter

[7] Achúcarro, Gong, SH, Palma, Patil, 2010. Similar but incomplete observation made by Tolley and Wyman, 2009

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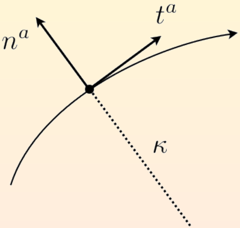

Figure: Curvature couples perpendicular direction

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Effective field theory with non-decoupled mass hierarchies

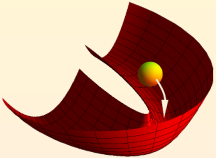

Figure: An example of a curved trajectory.

To summarize

- In supergravity many fields are expected
- In general QFTs no a priori reason to have just one light non-interacting field during inflation
- Effects of many light fields during inflation widely studied
- We need to
  - verify if and when one can safely truncate massive fields,
  - correctly integrate out heavy fields and find the corrections to the effective field theory.

Results: speed of sound

Action for an effective field theory for inflation is found to be[8]

$$S = \int \mathrm{d}\tau \mathrm{d}^3x \left[\dot{\varphi}^2 - e^{-\beta}(\nabla\varphi)^2 - M_L^2\varphi^2\right]$$

where

$$e^{\beta} = 1 + 4\left(\frac{\dot{\phi}}{\kappa M}\right)^2$$

Here, $\dot{\phi}$ is the turn rate, M the mass of the heavy direction.

Non-geodesic trajectories causes reduced speed of sound

From this we can learn that

- turns (deviations from geodesics) cause a reduced speed of sound,[9]
- non-Gaussianities can be expected.[10]

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[8] Achúcarro, Gong, SH, Palma, Patil, 2010 and in prep.
[9] A similar, but incomplete statement was made by Tolley and Wyman, 2009
[10] Chen, Wang, 2009

Effective field theory approach for speed of sound

Starting from a general effective single-field theory for inflation

$$S = \int \mathrm{d}\tau d^3x \left[\left(\frac{\mathrm{d}\varphi^T}{\mathrm{d}\tau} \right)^2 - \left(\nabla \varphi^T \right)^2 - \Omega_T(\tau) \varphi^T \varphi^T \right]$$
$$+ \frac{1}{2} \int \mathrm{d}\tau \mathrm{d}\tau' \mathrm{d}^3x \mathrm{d}^3x' \mathcal{O}(x,\tau) \varphi^T(x,\tau) G(x,x',\tau,\tau') \mathcal{O}(x',\tau') \varphi^T(x',\tau')$$

where
$$\mathcal{O}(x,\tau) := 2a^2 H^2 \eta_\perp \left(1 - \frac{1}{aH} \frac{d}{dt} \right)$$
$$G(x,x',\tau,\tau') := \left[\Box + \Omega_N(\tau) \right]^{-1}$$

and $\Omega_{T,N}$ the effective mass matrix entries in the direction $\{T, N\}$. One can see from this equation that the only way for having $c_s < 1$ is having a nonzero Ω_N and operator O, which requires a non-geodesic trajectory.[11]
With proper approximations reduces to effective field theory with $c_s < 1$

[11] Achúcarro, Gong, SH, Palma, Patil, in prep.

Numerical calculation of trajectories and powerspectra

The background trajectory is given by the differential equation

$$\frac{D\dot{\phi}^I}{DN} + (\epsilon - 3)\dot{\phi}^I + \frac{1}{H^2}V^I = 0$$

Using this equation, one can solve the equation for the perturbations

$$\frac{D^2{\varphi^I}^2}{DN} - (1-\epsilon)\frac{D\varphi^I}{DN} + \left[e^{2(1-\epsilon)(N-N_k)} - (2-\epsilon)\right]\varphi^I + C^I{}_J\varphi^J = 0$$

where $C^I{}_J = e^I{}_a e^b{}_J C^a{}_b$, and

$$C^a{}_b = \nabla_b V^a - R^a{}_{cdb}\dot{\phi}^c\dot{\phi}^d - \sqrt{2\epsilon}(\dot{\phi}^a V_b + \dot{\phi}_b V^a) + 2\epsilon(3-\epsilon)H^2\dot{\phi}^a\dot{\phi}_b$$

with H the Hubble acceleration.

Generating curved trajectories - numerical example

Example - double quadratic potential[12]

$$V = \frac{1}{2}m_1^2\phi_1^2 + \frac{1}{2}m_2^2\phi_2^2, \quad m_2/m_1 = 5$$

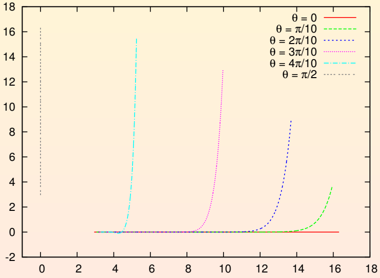

Figure: Trajectories for different initial conditions

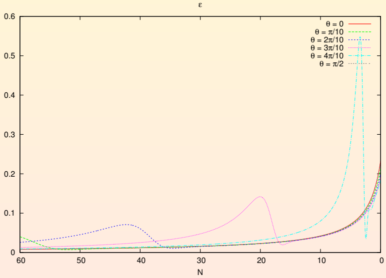

Figure: $\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2$ as function of N

[12] Agrees with results of approximate calculations by Peterson and Tegmark, 2010

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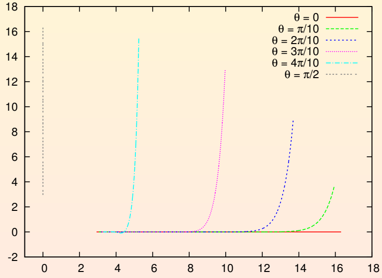

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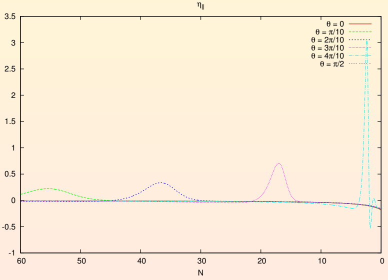

Figure: $\eta_\parallel = \frac{V''}{V}$ as function of N

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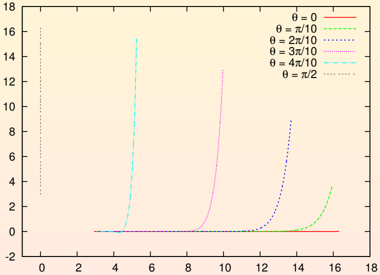

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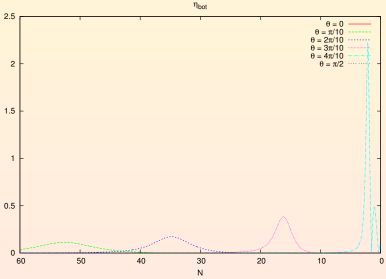

Figure: $\eta_\perp = \sqrt{2E_{\rm kin}}/\kappa$ as function of N

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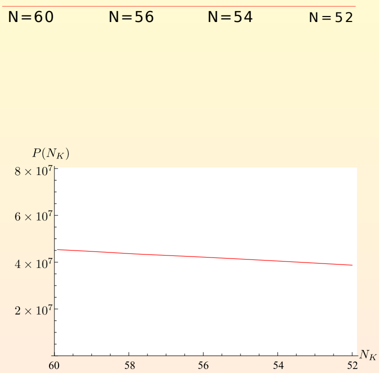
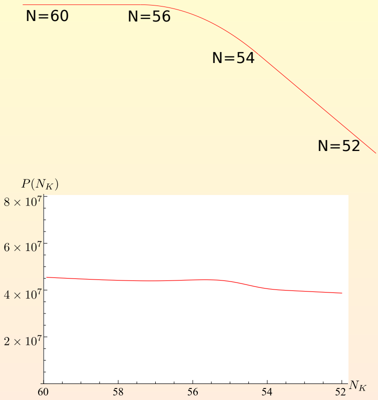
Power spectrum of a curved inflaton trajectory



(a) Without bend



(b) With bend, $\kappa^{-1} = \kappa(N)^{-1}$, with $\kappa_{\max}^{-1} = 0.76$

Figure: Power spectra of the same (real) scalar field model ($\epsilon = 0.00875, \eta = 0.0075$, corresponding to $n_s = 0.963, r = 0.14$)[13]

[13] Achúcarro, Gong, SH, Palma, Patil, in prep.

Power spectra of a curved inflaton trajectories

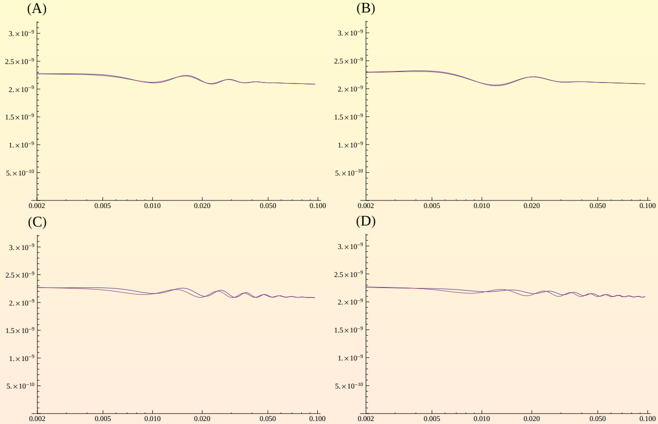


Figure: The primordial power spectrum $P(k)$ obtained for four dierent choices of the parameters ΔN, $\eta_{\perp,\max}$ and M^2. The scale k appears in units of Mpc^{-1}. A: $\Delta N = 0:25$, $\eta_{\perp,\max} = 1$, $M^2/H^2 = 80$. B: $\Delta N = 0.5$, $\eta_{\perp,\max} = 1$, $M^2/H^2 = 50$. C: $\Delta N = 0.1$, $\eta_{\perp,\max} = 1$, $M^2/H^2 = 50$. D: $\Delta N = 0.01$, $\eta_{\perp,\max} = 1$, $M^2/H^2 = 30$.[14]

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[14] Achúcarro, Gong, SH, Palma, Patil, in prep.

Concluding remarks

Conclusions

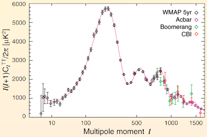
On general grounds a curved field space is expected

- Might leave observable imprints in the CMB, even in the power spectrum.
- *Curved trajectories* cause a reduced speed of sound
  - non-Gaussianities
  - bumps in the power spectrum
- Future CMB observations will thus allow us to study supergravity from the CMB!

Work in progress...

- Understand if there is a degeneracy between the effects of a turn and other sources of CMB modifications
- Study other observable effects of curved trajectories, such as non-Gaussianities
- Understand multifield inflation in the context of supergravity

Goal...



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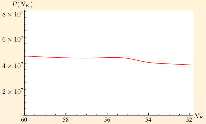

$\Longrightarrow$