Inflation, Gravity Waves and Dark Matter

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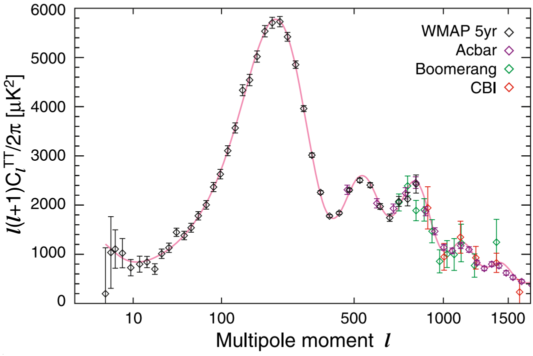
Outline

- Introduction
- Gauge Singlet Higgs Inflation
- Standard Model Higgs Inflation (& variants)
- Dark Matter Inflation
- MSSM Inflation
- Supersymmetric Higgs (Hybrid) Inflation
- Summary

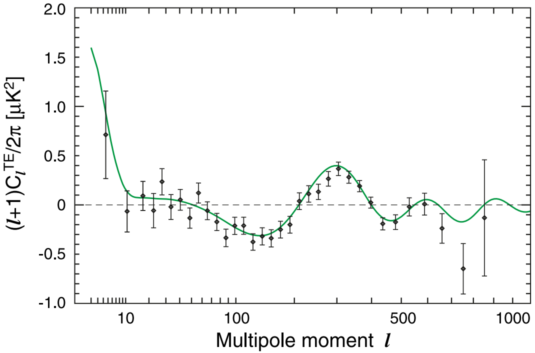
Evidence for Inflation

- TT power spectrum yields:
  - Scalar spectral index n_s
  - Baryonic and dark matter densities Ω_b, Ω_c
- Deviations from Gaussianity not reflected in power spectrum



Nolta et. al. (WMAP Collaboration), 2008

Temperature Anisotropy and CMB Polarization

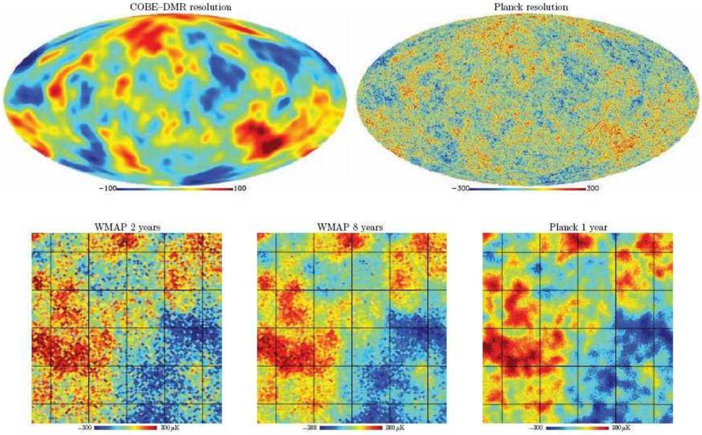


Nolta et. al. (WMAP Collaboration), 2008

In addition to temperature anisotropy, one expects the CMB to become polarized via Thompson scattering: Linear polarization results from the velocities of electrons and protons. With both the velocity field and temperature anisotropies created by primordial density fluctuations, one expects to see temperature-polarization correlations. ($\langle TT\rangle$, $\langle TE\rangle$, $\langle EE\rangle$, $\langle BB\rangle$)

Planck Resolution

Planck Blue Book



Planck and Tests of Inflation

[Efstathiou, Cambridge 2009]

- Capable of measuring large scale B-mode polarization in the CMB
  - Direct measurement of r (tensor-to-scalar ratio)
  - Can measure $r \gtrsim 0.03$, which can rule out many inflation models predicting $r \ll 1$
- Refine measurements of n_s, $\delta T/T$, $\Omega_{b,c,\Lambda}$, ...
- Measure or further restrict non-Gaussianity in CMB
  - Detection of non-Gaussian fluctuations can rule out large classes of inflation models
- Can probe low multipoles ($l < 10$) $\equiv$ large distances

Inflationary Cosmology

[Guth, Linde, Albrecht & Steinhardt, Starobinsky, Mukhanov, Hawking, ...]

Successful Primordial Inflation should:

- Explain flatness, isotropy;
- Provide origin of $\frac{\delta T}{T}$;
- Offer testable predictions for n_s, r, $dn_s/d\ln k$;
- Recover Hot Big Bang Cosmology;
- Explain the observed baryon asymmetry;
- Offer plausible CDM candidate;

Physics Beyond the SM?

Slow-roll Inflation

- Inflation is driven by some potential $V(\phi)$:
- Slow-roll parameters:

$$\epsilon = \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2, \ \eta = m_p^2\left(\frac{V''}{V}\right).$$

- The spectral index n_s and the tensor to scalar ratio r are given by

$$n_s - 1 \equiv \frac{d\ln\Delta_{\mathcal{R}}^2}{d\ln k}, \ r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2},$$

  where Δ_h^2 and $\Delta_{\mathcal{R}}^2$ are the spectra of primordial gravity waves and curvature perturbation respectively.
- Assuming slow-roll approximation (i.e. $(\epsilon, |\eta|) \ll 1$), the spectral index n_s and the tensor to scalar ratio r are given by

$$n_s \simeq 1 - 6\epsilon + 2\eta, \ r \simeq 16\epsilon.$$

Slow-roll Inflation

- The tensor to scalar ratio r can be related to the energy scale of inflation via

$$V(\phi_0)^{1/4} = 3.3 \times 10^{16}\, r^{1/4} \text{ GeV}.$$

- The amplitude of the curvature perturbation is given by

$$\Delta_{\mathcal{R}}^2 = \frac{1}{24\pi^2}\left(\frac{V/m_p^4}{\epsilon}\right)_{\phi=\phi_0} = 2.43 \times 10^{-9} \;(\text{WMAP7 normalization}).$$

- The spectrum of the tensor perturbation is given by

$$\Delta_h^2 = \frac{2}{3\,\pi^2}\left(\frac{V}{m_P^4}\right)_{\phi=\phi_0}.$$

- The number of e-folds after the comoving scale $l_0 = 2\,\pi/k_0$ has crossed the horizon is given by

$$N_0 = \frac{1}{m_p^2}\int_{\phi_e}^{\phi_0}\left(\frac{V}{V'}\right) d\phi.$$

Inflation ends when $\max[\epsilon(\phi_e), |\eta(\phi_e)|] = 1$.

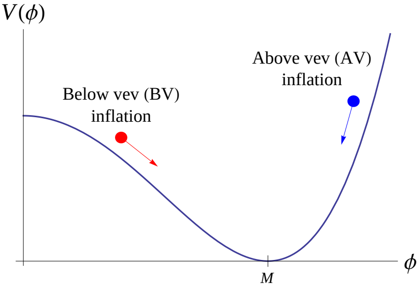
Tree Level Gauge Singlet Higgs Inflation

[Kallosh and Linde, 07; Rehman, Shafi and Wickman, 08]

- Consider the following Higgs Potential:

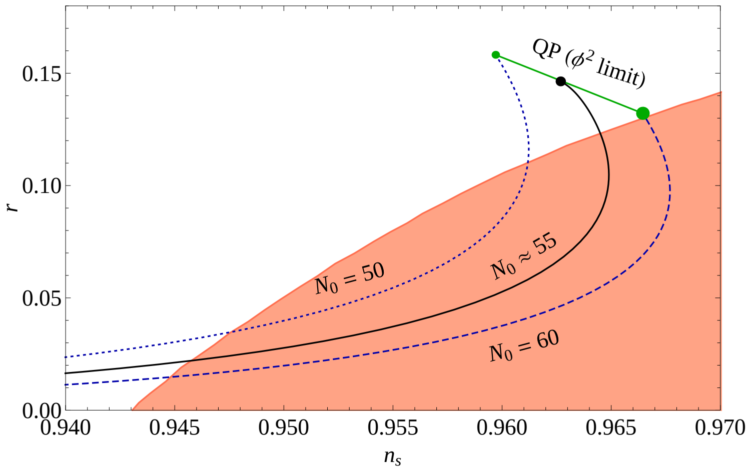
$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{M}\right)^2\right]^2 \quad \longleftarrow \text{(tree level)}$$

Here ϕ is a gauge singlet field.



- WMAP data favors BV inflation.

Tree Level Gauge Singlet Higgs Inflation



Radiative Corrections in Gauge Singlet Higgs Inflation

- Consider the following interaction of inflaton ϕ with some GUT symmetry breaking scalar boson Φ:

$$\mathcal{L}_{int} = \frac{\lambda_\Phi^2}{2}\,\phi^2\,\Phi^2$$

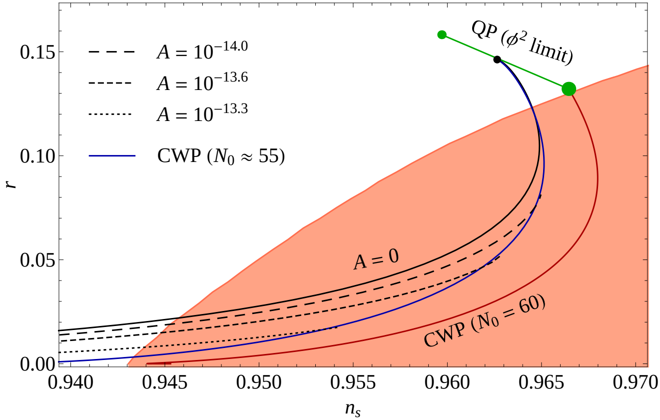
- Include Radiative Corrections (Quantum Smearing):

$$V = \left(\frac{m^2\,M^2}{4}\right)\,\left[1 - \left(\frac{\phi}{M}\right)^2\right]^2 + A\,\phi^4\,\left[\ln\left(\frac{\phi}{M}\right) - \frac{1}{4}\right] + \frac{A\,M^4}{4},$$

where $V(\phi = 0) \equiv V_0 = \frac{m^2\,M^2}{4} + \frac{A\,M^4}{4}$ and $A = \frac{\mathcal{N}\lambda_\Phi^4}{32\,\pi^2}$.

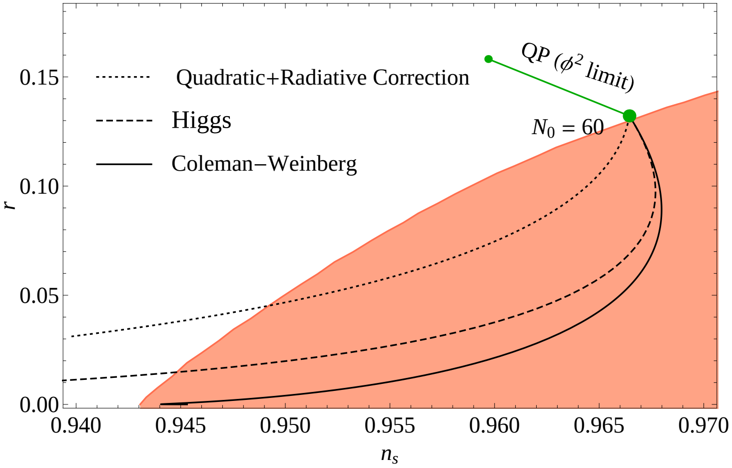
- Note that we can use 'Minkowski space' CW corrections provided the propagating fields have masses $\gg H$ (Hubble constant).

[Rehman and Shafi, 2010]

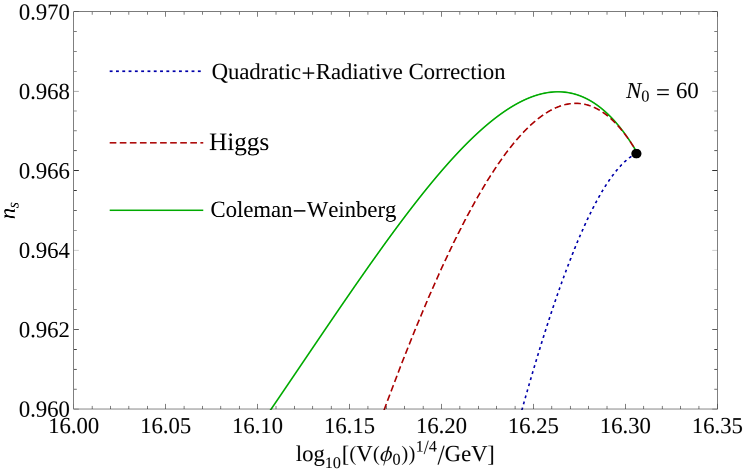


Note that $r \gtrsim 0.02$ if $n_s \gtrsim 0.96$. Thus, Planck will test Higgs inflation soon!

Quantum Smearing



Quantum Smearing



The vacuum energy scale during observable inflation is well below m_P. This implies that the quantum gravity effects are relatively unimportant here.

Standard Model Inflation?

[Bezrukov, Magnin and Shaposhnikov; De Simone, Hertzberg and Wilczek.]

[Barvinsky, Kamenshchik, Kiefer, Starobinsky and Steinwachs.]

[Okada, Rehman and Shafi.]

- Consider the following action with non-minimal coupling:

$$S_J = \int dx^4 \sqrt{-g}\{-|\partial H|^2 - \lambda \left(H^\dagger H - \frac{v^2}{2}\right)^2 + \frac{1}{2} m_P^2 \mathcal{R} + \xi H^\dagger H \mathcal{R}\}$$

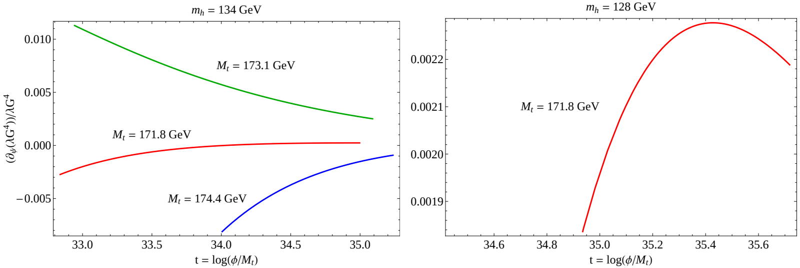
- In the Einstein frame the potential turns out to be:

$$V_E(\phi) \simeq \frac{\frac{\lambda G^4}{4}\phi^4}{\left(1+\frac{\xi G^2 \phi^2}{m_P^2}\right)^2}; \; G(t) = \exp[-\int_0^t dt' \gamma(t')/(1+\gamma(t'))]$$

where $H^T = \frac{1}{\sqrt{2}}(0, v+\phi)$, $t = \log[\frac{\phi}{M_t}]$ and $\gamma(t)$ is the anomalous dimension of the Higgs field.

- For large ϕ values, $V_E(\phi)$ gives rise to inflation.

Standard Model Inflation?



$\lambda(t)\, G(t)^4$ vs. t, between pivot-scale $t_0 = \log[\frac{\phi_0}{M_t}]$ and inflation end scale $t_e = \log[\frac{\phi_e}{M_t}]$

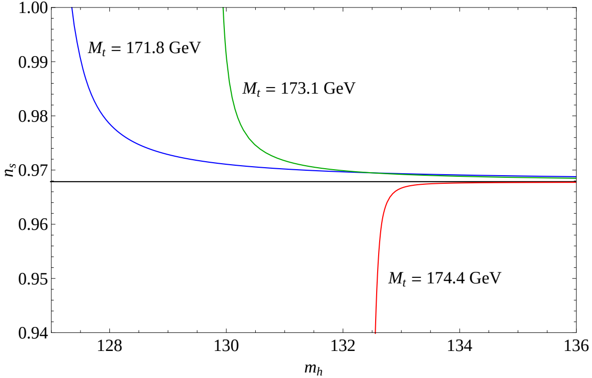
- The spectral index is

$$n_s \simeq 1 - \frac{2}{N_e} + \frac{\psi_0}{3}\left(\frac{\partial_{\psi_0}(\lambda\, G^4)}{(\lambda\, G^4)}\right); \qquad \psi_0 \equiv \frac{\sqrt{\xi}\,\phi_0}{m_P}$$

with

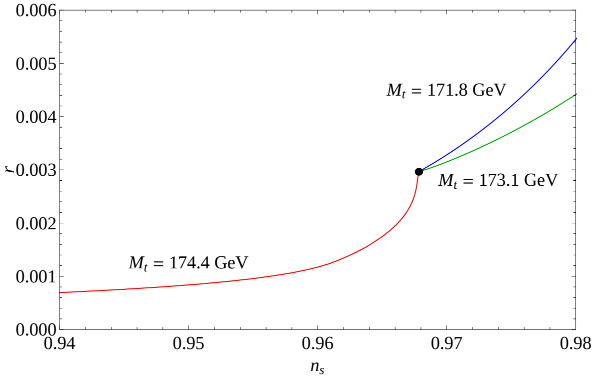
$$\Delta^2_{\mathcal{R}} \simeq \frac{\lambda}{\xi^2}\frac{N_e^2}{72\pi^2} \;\Rightarrow\; \xi \simeq \left(\frac{N_e}{6\sqrt{2}\,\pi\,\Delta_{\mathcal{R}}}\right)\sqrt{\lambda} \sim 10^4\ (!)$$

Standard Model Inflation?



The horizontal line is the classical tree level result which is independent of Higgs mass. All other curves include quantum corrections with different values of M_t.

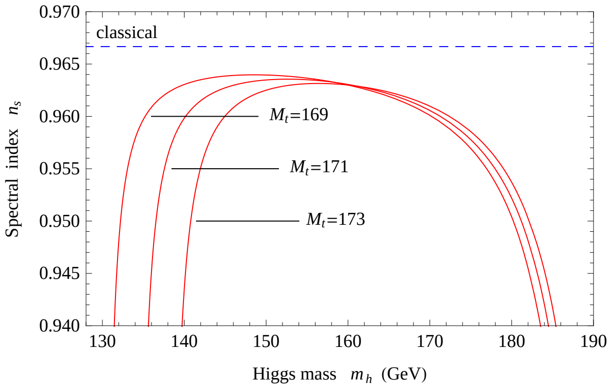
Standard Model Inflation?



r vs. n_s. The black circle corresponds to the classical tree level result. All other curves include quantum corrections with different values of M_t.

Standard Model Inflation?

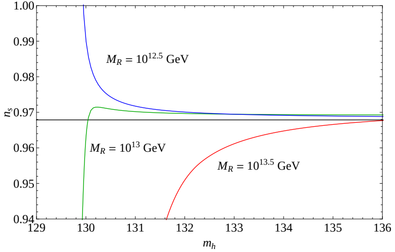
[Barvinsky, Kamenshchik, Kiefer, Starobinsky and Steinwachs]



The spectral index n_s as a function of the Higgs mass m_h for three values of the top quark mass.

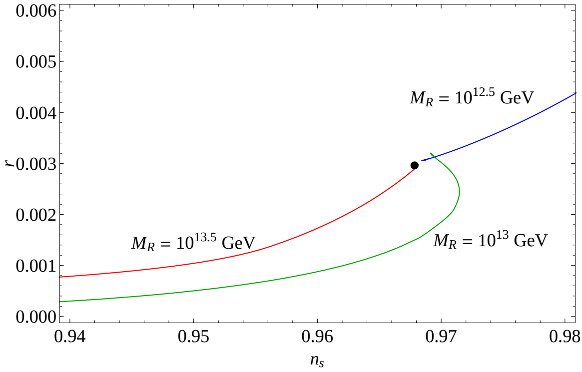
Type I Seesaw with Three Right Handed Neutrinos

[Okada, Rehman and Shafi, 09]



n_s vs. m_h for hierarchical (left panel) and inverted-hierarchical (right panel) neutrino mass spectrum for various values of M_R and $M_t = 173.1$ GeV.

Type I Seesaw with Three Right Handed Neutrinos



r vs. n_s for hierarchical neutrino mass spectrum for various values of M_R and $M_t = 173.1$ GeV. The black circle corresponds to the classical tree level result.

Challenges for SM inflation

[Burgess, Lee and Trott; Barbon and Espinosa; Hertzberg]

- Consider $g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{m_P}$, so that the term $\xi\,\phi^2\,\mathcal{R}$ yields

$$\frac{\xi}{m_P}\,\phi^2\,\eta^{\mu\nu}\,\partial^2\,h_{\mu\nu} + \cdots$$

  This suggests an effective cut-off scale $\Lambda \approx \frac{m_P}{\xi} \ll m_P$.

- The energy scale of inflation is estimated to be

$$E_i \simeq V_0^{1/4} = \frac{\lambda^{1/4} m_P}{4^{1/4}\sqrt{\xi}} \gg \Lambda = \frac{m_P}{\xi}.$$

- Thus it is not clear how reliable are the calculations.

- Indeed, SM inflation is mired in some controversy (see for instance, Lerner and McDonald [arXiv:0912.5463]).

Non-Minimal ϕ^4 Inflation

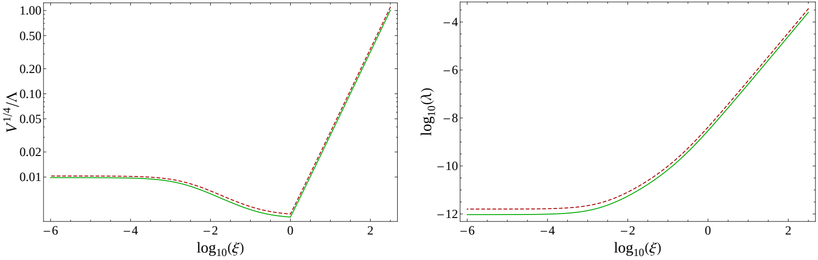
[Okada, Rehman, Shafi, 2010]

- Consider the following action in the Jordan frame:

$$S_J = \int d^4x\sqrt{-g}\left[\left(\frac{m_P^2+\xi\phi^2}{2}\right)\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4!}\,\phi^4\right],$$

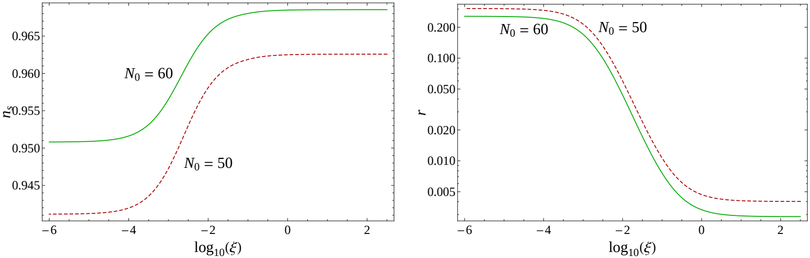
where ϕ is SM gauge singlet and we require that energy scale of inflation $\leq \Lambda \equiv m_P/\xi$.

Non-Minimal ϕ^4 Inflation



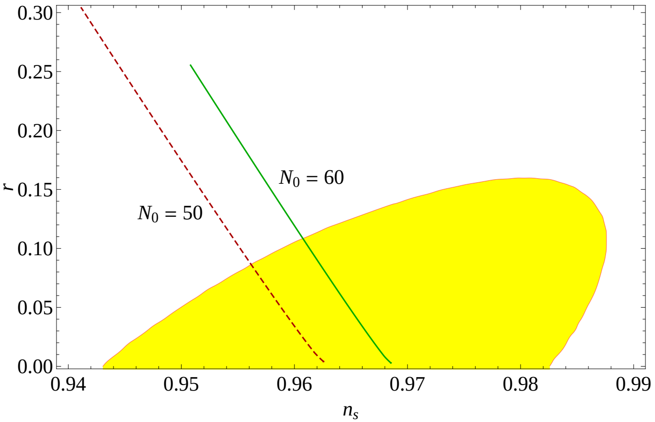
$V^{1/4}/\Lambda$ and $\log_{10}(\lambda)$ vs. $\log_{10}(\xi)$ for tree level non-minimal ϕ^4 inflation with the number of e-foldings $N_0 = 50$ (red dashed curve) and $N_0 = 60$ (green solid curve).

Non-Minimal ϕ^4 Inflation



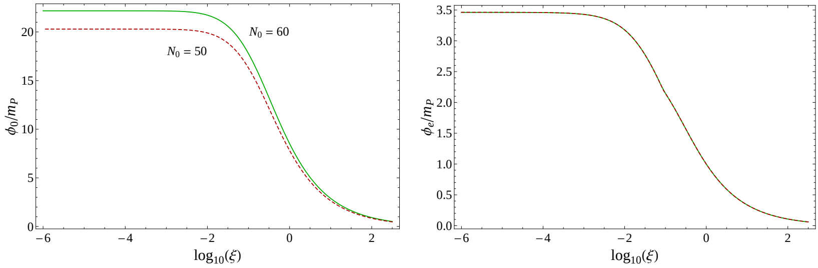
n_s and r vs. $\log_{10}(\xi)$ for tree level non-minimal ϕ^4 inflation with the number of e-foldings $N_0 = 50$ (red dashed curve) and $N_0 = 60$ (green solid curve).

Non-Minimal ϕ^4 Inflation



r and vs. n_s with $N_0 = 50$ and $N_0 = 60$. The WMAP 1-σ (68% confidence level) bounds are shown in yellow.

Non-Minimal ϕ^4 Inflation



ϕ_0/m_P and ϕ_e/m_P vs. $\log_{10}(\xi)$ with $N_e = 50$ and $N_e = 60$.

Non-Minimal ϕ^4 Inflation

[Okada, Rehman, Shafi, 2010]

- Consider the following interaction of inflaton ϕ with the right handed Majorana neutrino N:

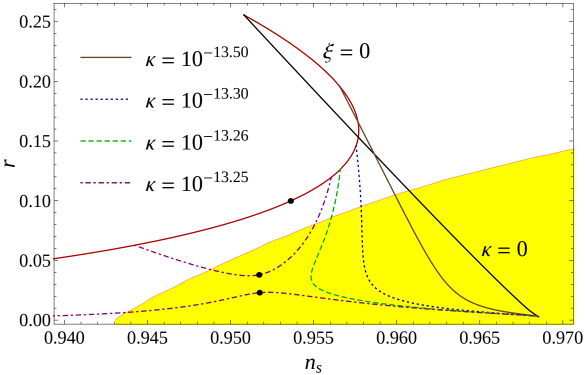
$$\mathcal{L}_{int} = \frac{1}{2}\, y_N\, \phi\, \overline{N}\, N$$

- Include Radiative Corrections (Quantum Smearing):

$$V_E(\phi) = \frac{\frac{1}{4!}\lambda\phi^4 - \kappa\phi^4 \ln(\phi/\mu)}{\left(1+\frac{\xi\,\phi^2}{m_P^2}\right)^2},$$

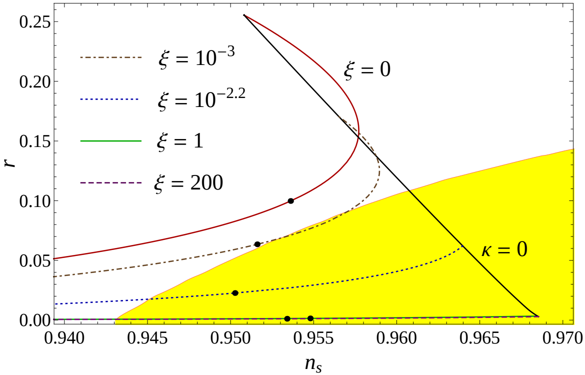
where $\kappa = y_N^4/(4\pi)^2$.

Non-Minimal ϕ^4 Inflation



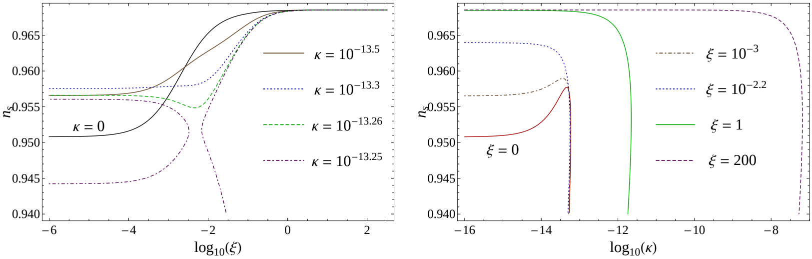
r vs. n_s for the radiatively corrected non-minimal ϕ^4 potential with the number of e-foldings $N_0 = 60$. The black dots represent the meeting points of the hilltop and the ϕ^4 solutions and correspond, for a given ξ, to the maximum value of κ.

Non-Minimal ϕ^4 Inflation



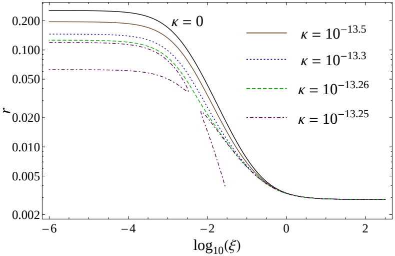
r vs. n_s for the radiatively corrected non-minimal ϕ^4 potential with the number of e-foldings $N_0 = 60$. The black dots represent the meeting points of the hilltop and the ϕ^4 solutions and correspond, for a given ξ, to the maximum value of κ.

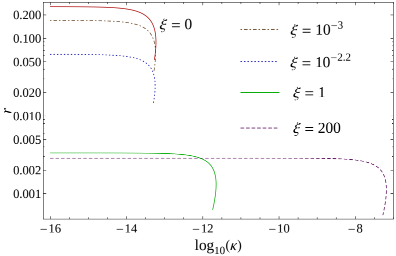
Non-Minimal ϕ^4 Inflation



n_s vs. $\log_{10}(\xi)$ and $\log_{10}(\kappa)$ with $N_0 = 60$.

Non-Minimal ϕ^4 Inflation

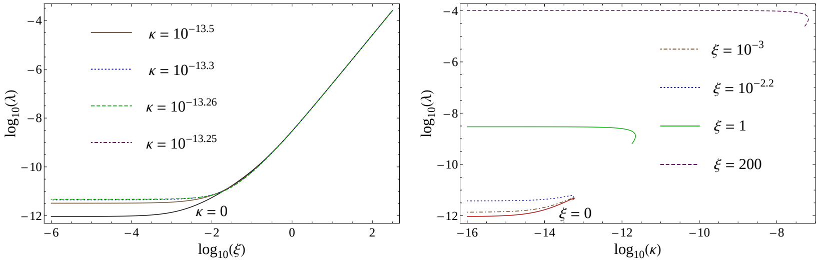




r vs. $\log_{10}(\xi)$ and $\log_{10}(\kappa)$ with $N_0 = 60$.

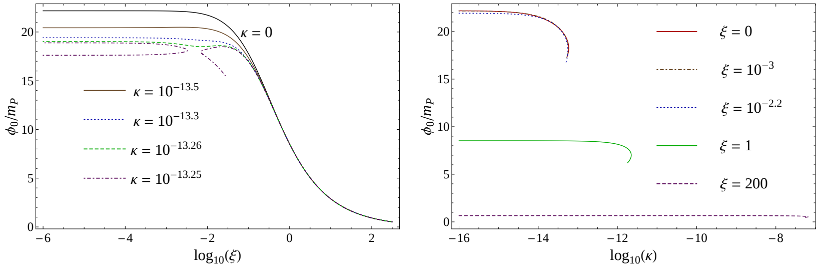
$r \gtrsim 0.002$, for $n_s \geq 0.96$

Non-Minimal ϕ^4 Inflation



$\log_{10}(\lambda)$ vs. $\log_{10}(\xi)$ and $\log_{10}(\kappa)$ with $N_0 = 60$.

Non-Minimal ϕ^4 Inflation



ϕ_0/m_P vs. $\log_{10}(\xi)$ and $\log_{10}(\kappa)$ with $N_0 = 60$.

(B-L) Inflation + ν_R Dark Matter

- Consider the gauge group $SM \times U(1)_{B-L}$ with a SM gauge singlet inflaton ϕ charged under $B - L$.
- This simple extension of SM naturally requires the presence of three right handed (RH) neutrinos due to gauge anomaly cancellations. With Z_2 parity one of the three RH neutrinos can be cold dark matter. [Okada, Seto, 2010]
- The origin of the baryon asymmetry can be explained through resonant leptogenisis with TeV scale RH neutrinos.

(B-L) Inflation + ν_R Dark Matter

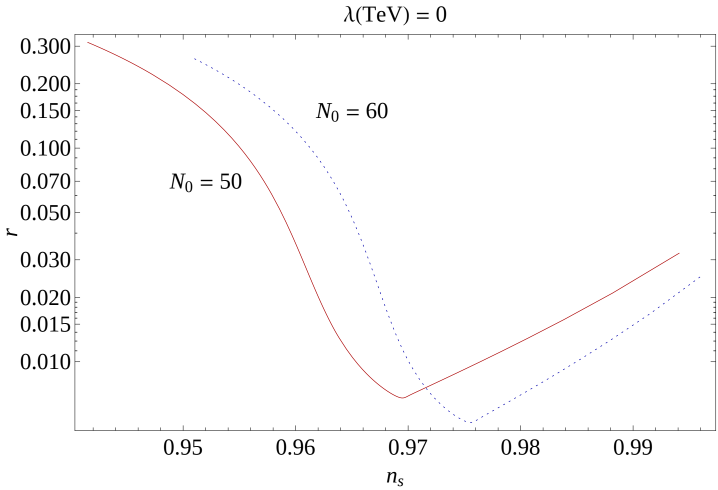
[Okada, Rehman and Shafi 2010]

- Using non-minimal ϕ^4 inflation discussed in previous slides, the inflationary effective potential, in the leading-log approximation, can be written as

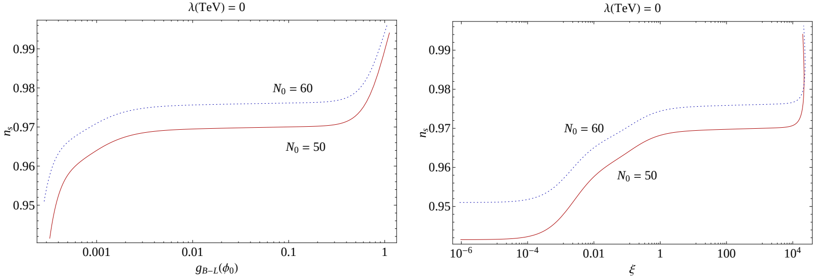
$$V(\phi) \simeq \left(\frac{\lambda(TeV)}{4} + \frac{96\, g_{B-L}^4}{16\, \pi^2} \ln\left[\frac{\phi}{TeV} \right] \right) \phi^4$$

where g_{B-L} is the value of the $B - L$ gauge coupling.

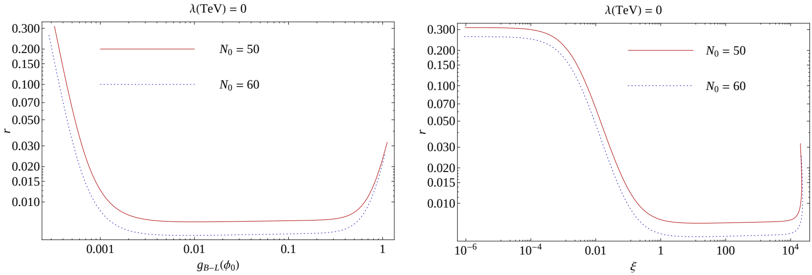
(B-L) Inflation + ν_R Dark Matter



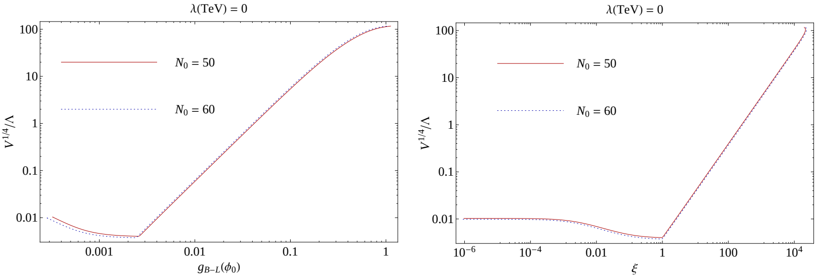
(B-L) Inflation + ν_R Dark Matter

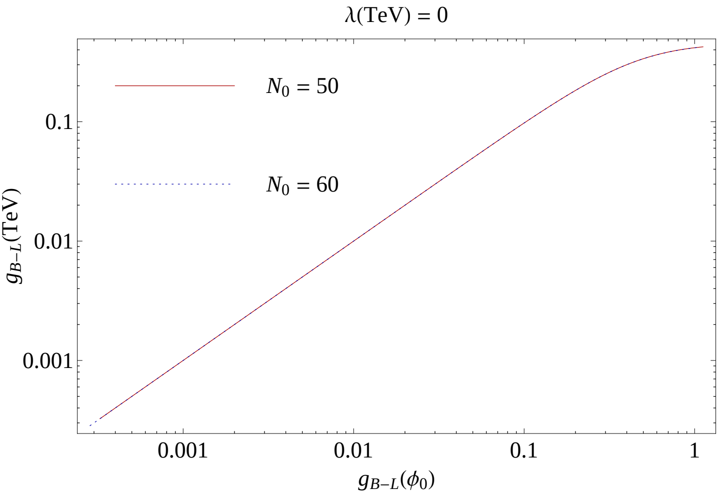


(B-L) Inflation + ν_R Dark Matter



(B-L) Inflation + ν_R Dark Matter




λ(TeV) = 0
N0 = 50
N0 = 60
0.1
0.01
0.001
gB−L(TeV)
0.001
0.01
0.1
1
gB−L(ϕ0)

Dark Matter Inflation

[Kofman, Linde, Starobinsky 94; Liddle, Lopez 09; Okada, Shafi 2010]

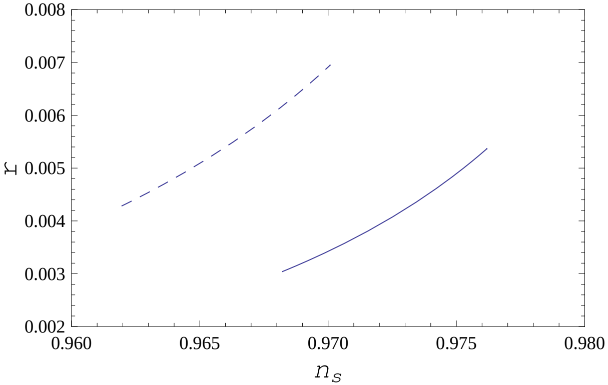
- Can the inflaton also be a dark matter particle?
- Looks possible if we augment the SM with a gauge singlet scalar field ϕ with mass $\lesssim$ 1 TeV, and use non-minimal ϕ^4 inflation discussed in previous slides (see also Lerner and McDonald arXiv:0909.0520v3 [hep-ph]).
- An important role in this story is played by the interaction

$$g^2|H|^2\phi^2,$$

where $g^2 \sim 0.1$ from dark matter considerations (Kanemura, Matsumoto, Nabeshima and Okada, arXiv:1005.5651 [hep-ph]).

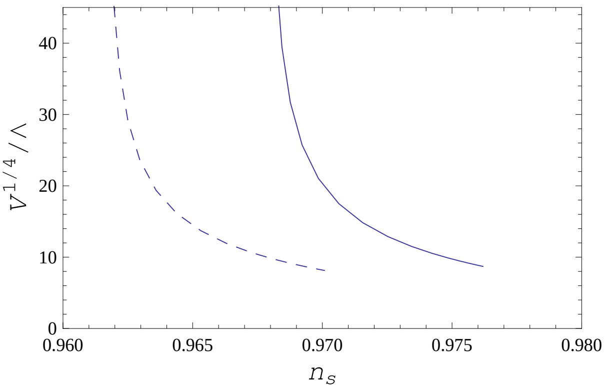
Dark Matter Inflation

- Thus, in this example of 'inflaton-dark matter unification', the ϕ quartic coupling cannot be much smaller than $\sim 10^{-3}$ or so. This allows one to make rather precise predictions for n_s and r.
- Dark matter ϕ particles compatible with WMAP bounds arise from preheating and subsequent $T_{reheat} \sim 10^7$ GeV. The energy density in the remnant ϕ oscillations turn out to be quite negligible.
- Stability of ϕ particle is ensured by an unbroken Z_2 symmetry.



r vs. n_s with $N_0 = 60$ (solid curve) and $N_0 = 50$ (dashed curve) e-foldings. Both curves lie within the WMAP 1-σ (68% confidence level) bounds.

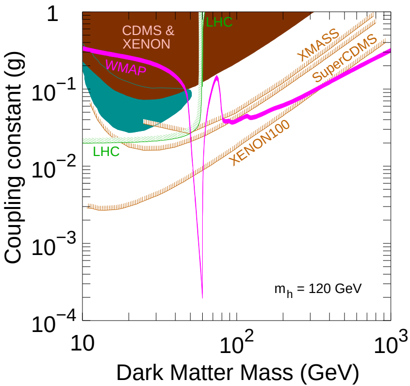
Dark Matter Inflation



$V^{1/4}/\Lambda$ vs. n_s with $N_0 = 60$ (solid curve) and $N_0 = 50$ (dashed curve) e-foldings.

SM Singlet Dark Matter

[Kanemura, Matsumoto, Nabeshima and Okada 2010]



Constraints on the nightmare scenario from WMAP, Xenon100 first data, and CDMS II experiments. Expected sensitivities to detect the signal of the dark matter at XMASS, SuperCDMS, Xenon100, and LHC experiments are also shown.

Why Supersymmetry?

- Resolution of the gauge hierarchy problem
- Unification of the SM gauge couplings at

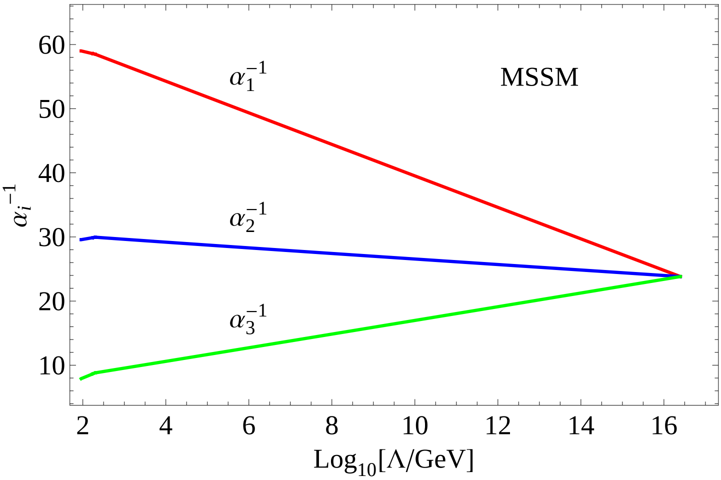
  $M_{GUT} \sim 2 \times 10^{16}$ GeV
- Cold dark matter candidate (LSP)

Other good reasons:

- Radiative electroweak breaking
- String theory requires susy

Leading candidate is the MSSM (Minimal Supersymmetric Standard Model)

Why Supersymmetry?



MSSM Inflation

[Allahverdi, Enqvist, Garcia-Bellido, Mazumdar]

- Numerous flat dimensions exist in MSSM.

  Utilize UDD and LLE.

  By suitable tuning of soft susy breaking parameters A and m_ϕ, an inflationary scenario may be realized.

- Flat directions lifted by higher dimensional operators

  $W = \frac{\lambda\,\Phi^n}{m_P^{n-3}}$, Φ is flat direction superfield.

  $$V = \frac{1}{2}m_\phi^2\phi^2 + A\cos(n\theta + \theta_A)\frac{\lambda_n\,\phi^n}{n m_P^{n-3}} + \frac{\lambda_n^2\,\phi^{2(n-1)}}{m_P^{2(n-3)}}$$

- For $A^2 \geq 8(n-1)m_\phi^2$, there is a secondary minimum at

  $\phi = \phi_0 \sim (m_\phi m_P^{n-3})^{\frac{1}{(n-2)}} \ll m_p$, with

  $V \sim m_\phi^2\phi_0^2 \sim m_\phi^2(m_\phi m_P^{n-3})^{\frac{2}{n-2}}$ (this can drive inflation)

MSSM Inflation

$$H_{inf} \sim \frac{(m_\phi \phi_0)}{m_P} \sim m_\phi (\frac{m_\phi}{m_P})^{\frac{1}{(n-2)}} \ll m_\phi$$

- To implement realistic inflation, one wants both the first and second derivatives of V to vanish at ϕ_0:

  Thus

$$V(\phi) \sim V(\phi_0) + \frac{1}{3!} V'''(\phi_0)(\phi - \phi_0)^3 + \cdots$$

  Using slow roll approximations (take $n = 6$, $\phi_0 \sim 10^{14}$ GeV, $m_\phi \sim 1 - 10$ TeV)

$$n_s \sim 1 - \frac{4}{N_0} \simeq 0.93$$

$$\frac{dn_s}{dlnk} \simeq -\frac{4}{N_0^2} \simeq -0.001$$

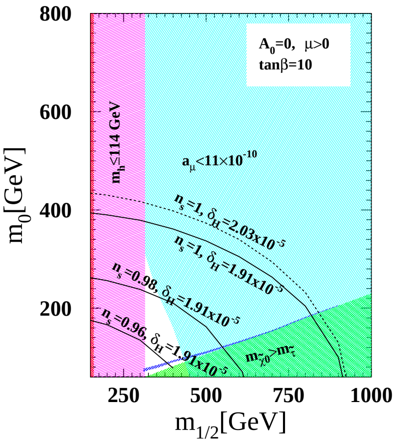
$$r \sim \frac{H}{m_P} \sim 10^{-16}$$

[Allahverdi, Mazumdar 2006]

- Now consider a small deviation from the saddle point parametrize by α:
$$\frac{A^2}{8(n-1)m_\phi^2} \equiv 1 + \left(\frac{n-2}{2}\right)^2 \alpha^2 \,.$$
- For $\alpha^2 \neq 0$, the saddle point becomes a point of inflection where $V''(\phi_0) = 0$, and the expressions for n_s is modified as
$$n_s = 1 - 4\sqrt{\Delta^2}\,\cot[N_0\sqrt{\Delta^2}],$$
where
$$\Delta^2 \equiv n^2(n-1)^2\alpha^2 N_0^2 \left(\frac{M_{\rm P}}{\phi_0}\right)^4 \,.$$
- For $\alpha^2 > 0$, we can obtain values of spectral index in the range
$$0.93 \leq n_s \leq 1,$$
for
$$0 \leq \Delta^2 \leq \frac{\pi^2}{4N_0^2} \,.$$

MSSM Inflation

[Allahverdi, Dutta, Mazumdar 2007]



We show the dark matter allowed region narrow blue corridor, (g-2)$_\mu$ region (light blue) for $a_\mu \leq 11 \times 10^{-8}$, Higgs mass ≤ 114 GeV (pink region) and LEPII bounds on SUSY masses (red). We also show the the dark matter detection rate by vertical blue lines.

CMSSM and Inflation

- CMSSM (Constrained MSSM) is a special case of MSSM in which we assume that SUSY is spontaneously broken in some ‘hidden’ sector and this information is transmitted to our (visible) sector via gravity.

- One employs universal soft SUSY breaking terms (say at M_{GUT}); these include scalar masses, gaugino masses, trilinear couplings, etc.

- In our discussion of SUSY hybrid inflation we will follow this minimal SUGRA scenario for the soft terms; note, however, that their magnitudes can be $\gg$ TeV.

Supersymmetric Higgs (Hybrid) Inflation

[Dvali, Shafi, Schaefer; Copeland, Liddle, Lyth, Stewart, Wands '94]

[Lazarides, Schaefer, Shafi '97][Senoguz, Shafi '04; Linde, Riotto '97]

- Attractive scenario in which inflation can be associated with symmetry breaking $G \longrightarrow H$
- Simplest inflation model is based on

$$W = \kappa\, S\, (\Phi\, \overline{\Phi} - M^2)$$

S = gauge singlet superfield, $(\Phi\,, \overline{\Phi})$ belong to suitable representation of G

- Need $\Phi\,, \overline{\Phi}$ pair in order to preserve susy while breaking $G \longrightarrow H$ at scale $M \gg$ TeV, susy breaking scale.
- R-symmetry

$$\Phi\, \overline{\Phi} \rightarrow \Phi\, \overline{\Phi}, \;\; S \rightarrow e^{i\alpha}\, S, \;\; W \rightarrow e^{i\alpha}\, W$$

$\Rightarrow$ W is a unique renormalizable superpotential

Susy Higgs (Hybrid) Inflation

- Some examples of gauge groups:

$$G = U(1)_{B-L},\ \text{(Supersymmetric superconductor)}$$

$$G = SU(5) \times U(1),\quad (\Phi = 10),\quad \text{(Flipped } SU(5)\text{)}$$

$$G = 3_c \times 2_L \times 2_R \times 1_{B-L},\ (\Phi = (1, 1, 2, +1))$$

$$G = 4_c \times 2_L \times 2_R,\ (\Phi = (\overline{4}, 1, 2)),$$

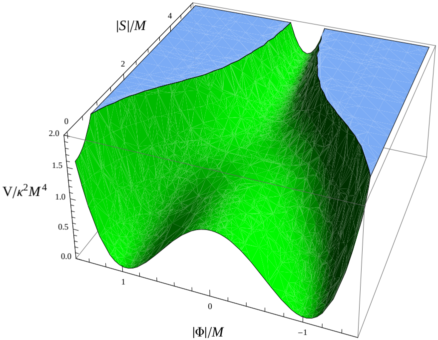
$$G = SO(10),\ (\Phi = 16)$$

- Tree Level Potential

$$V_F = \kappa^2 \left(M^2 - |\Phi^2|\right)^2 + 2\kappa^2 |S|^2 |\Phi|^2$$

- Susy vacua

$$|\langle\overline{\Phi}\rangle| = |\langle\Phi\rangle| = M, \ \langle S\rangle = 0$$



Susy Higgs (Hybrid) Inflation

Take into account radiative corrections (because during inflation $V \neq 0$ and susy is broken by $F_S = -\kappa\, M^2$)

- Mass splitting in $\Phi - \overline{\Phi}$

$$m_{\pm}^2 = \kappa^2\, S^2 \pm \kappa^2\, M^2, \quad m_F^2 = \kappa^2\, S^2$$

- One-loop radiative corrections

$$\Delta V_{\text{1loop}} = \frac{1}{64\pi^2} \text{Str}[\mathcal{M}^4(S)(\ln \frac{\mathcal{M}^2(S)}{Q^2} - \frac{3}{2})]$$

- In the inflationary valley ($\Phi = 0$)

$$V \simeq \kappa^2\, M^4 \left(1 + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x)\right)$$

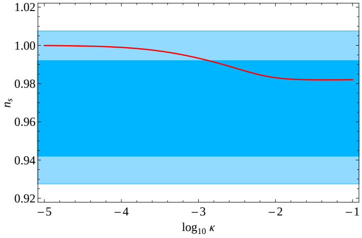
where $x = |S|/M$ and

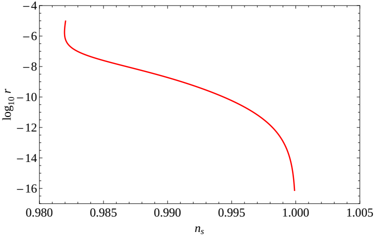
$$F(x) = \frac{1}{4}\left(\left(x^4 + 1\right) \ln \frac{(x^4 - 1)}{x^4} + 2x^2 \ln \frac{x^2+1}{x^2-1} + 2 \ln \frac{\kappa^2 M^2 x^2}{Q^2} - 3 \right)$$

Susy Higgs (Hybrid) Inflation

[Dvali, Shafi, Schaefer '94]

Tree Level plus radiative corrections:





$$n_s \approx 1 - \frac{1}{N_0} \approx 0.98$$

$\delta T/T \propto (M/M_P)^2 \sim 10^{-5} \longrightarrow$ attractive scenario $(M \sim M_G)$

Full Story

Also include supergravity corrections + soft susy breaking terms

- The minimal Kähler potential can be expanded as

$$K = |S|^2 + |\Phi|^2 + |\overline{\Phi}|^2$$

- The Sugra scalar potential is given by

$$V_F = e^{K/m_p^2}\left(K_{ij}^{-1} D_{z_i} W D_{z_j^*} W^* - 3m_p^{-2}|W|^2\right)$$

where we have defined

$$D_{z_i} W \equiv \frac{\partial W}{\partial z_i} + m_p^{-2} \frac{\partial K}{\partial z_i} W;\; K_{ij} \equiv \frac{\partial^2 K}{\partial z_i \partial z_j^*}$$

and $z_i \in \{\Phi, \overline{\Phi}, S, ...\}$

Full Story

[Senoguz, Shafi '04; Jeannerot, Postma '05]

- Take into account sugra corrections, radiative corrections and soft susy breaking terms:

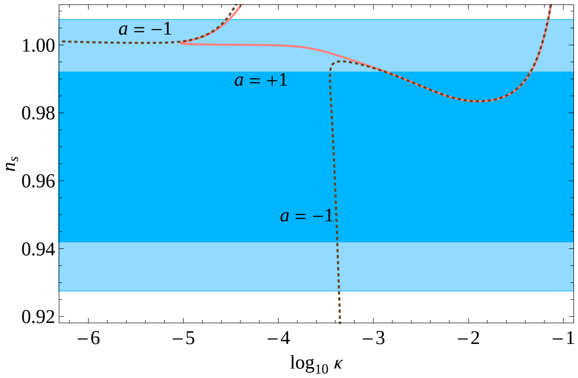
$$V \simeq \kappa^2 M^4 \left(1 + \left(\frac{M}{m_p}\right)^4 \frac{x^4}{2} + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x) + a \left(\frac{m_{3/2} x}{\kappa M}\right) + \left(\frac{m_{3/2} x}{\kappa M}\right)^2\right)$$

where $a = 2\,|2-A|\cos[\arg S + \arg(2-A)]$, $x = |S|/M$ and $S \ll m_P$.

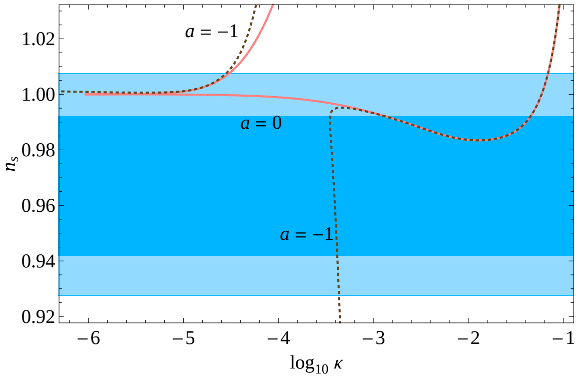
Note: No 'η problem' with minimal (canonical) Kähler potential !

Results

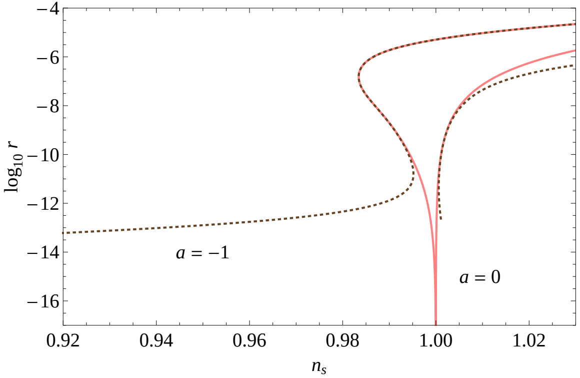
[Rehman, Shafi, Wickman, 2009]



Results



Results



$r \lesssim 10^{-4}$ within 2-σ bounds of WMAP data

Supersymmetric Flipped SU(5)

[Antoniadis, Ellis, Hagelin, Nanopoulos, 1987]

[For recent discussion and additional references see Nanopoulos, hep-ph/0211128; Li, Nanopoulos and Walker, arXiv:0910.0860 [hep-ph]]

Flipped SU(5) $\equiv$ SU(5)$\times$U(1)$_X$

- Chiral superfields are arranged as

$$\mathbf{10}_1 = \begin{pmatrix} d^c & Q \\ & \nu^c \end{pmatrix}, \quad \overline{\mathbf{5}}_{-3} = \begin{pmatrix} u^c \\ \overline{L} \end{pmatrix}, \quad \mathbf{1}_5 = e^c$$

- Compared to standard SU(5), these multiplets correspond to the interchange

$$u^c \longleftrightarrow d^c, \quad e^c \longleftrightarrow \nu^c$$

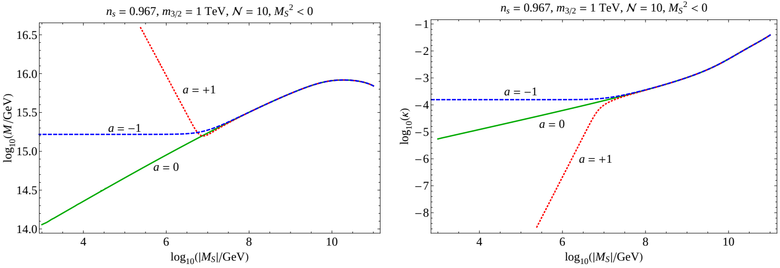
Properties of Flipped vs. Standard SU(5)

	Flipped SU(5) (Minimal)	SU(5) (Minimal)
Low scale susy	Yes	Yes
Doublet-triplet splitting	Yes	Fine tuning
Dimension 5 proton decay	Eliminated!	Challenging
μ problem	No	Fine tuning
Inflation	Easy	Difficult
Dimension 6 proton decay	$\tau_p \sim 10^{34}$–10^{36} yrs	$\tau_p \sim 10^{35}$–10^{36} yrs
Monopole problem	No	Yes
Seesaw mechanism	Automatic	No
Charge quantization	No	Yes
Unification of gauge couplings	Can be arranged	Yes
CDM	Yes	Yes

Minimal Hybrid Inflation and Flipped SU(5)

[Rehman, Shafi, Wickman, 2009]

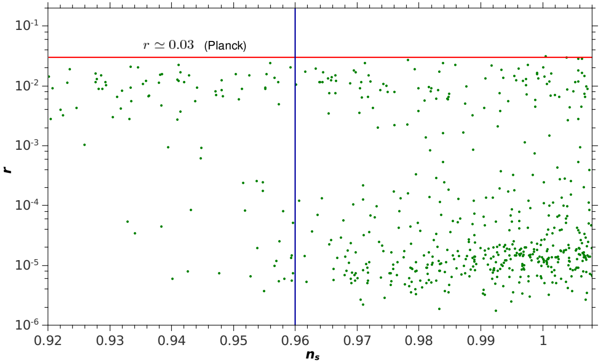
- Consider susy hybrid inflation in flipped SU(5), where Φ is a 10-plet
- Allowing the soft mass squared to vary, the potential appears as
$$V \simeq \kappa^2 M^4 \left(1 + \left(\frac{M}{m_p}\right)^4 \frac{x^4}{2} + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x) + a\left(\frac{m_{3/2} x}{\kappa M}\right) + \left(\frac{M_S\, x}{\kappa M}\right)^2\right)$$
- If $M_S^2 < 0$ the soft susy breaking mass squared term drives the spectral index toward red-tilted values
- The minimal model consistent with $n_s = 0.96 - 0.97$ leads to predictions of the proton lifetime of order 10^{34}–10^{36} years



$\log_{10}(M/\text{GeV})$ and $\log_{10}(\kappa)$ vs. $\log_{10}(|MS|/\text{GeV})$ in the flipped $SU(5)$ model ($\mathcal{N} = 10$), with n_s fixed at the central value 0.967.

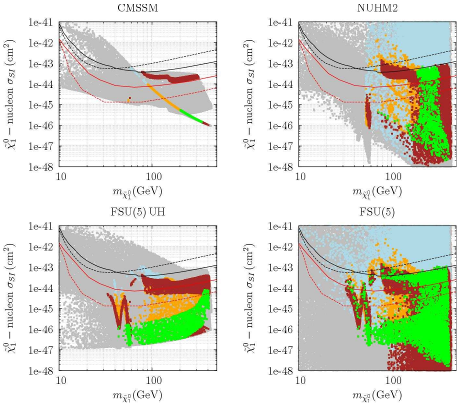
Non-Minimal SUSY Hybrid Inflation and Tensor Modes

- As we have seen, the minimal SUSY hybrid inflation model yields r values $\lesssim 10^{-4}$
- A more general analysis with a non-minimal Kähler potential can lead to larger r-values



Dark Matter and Flipped SU(5)

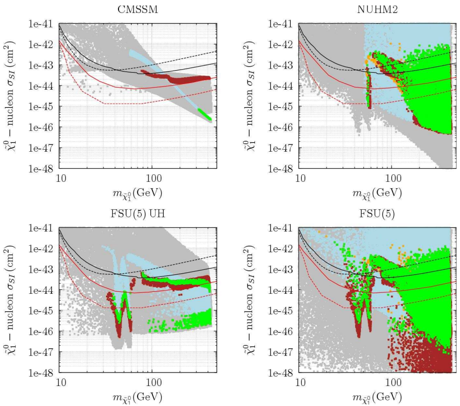
[Gogoladze, Khalid, Raza, Shafi, 2009]



Plots of the $\tilde{\chi}_1^0$-nucleon spin-independent cross-section versus $m_{\tilde{\chi}_1^0}$ for the CMSSM, FSU(5)-UH, NUHM2 and FSU(5)models for $\tan\beta = 10$. Also shown are current limits from CDMS II (solid black line), XENON10 (dashed black) and projected reach of SuperCDMS (solid red) and XENON100 (dashed red).

Dark Matter and Flipped SU(5)

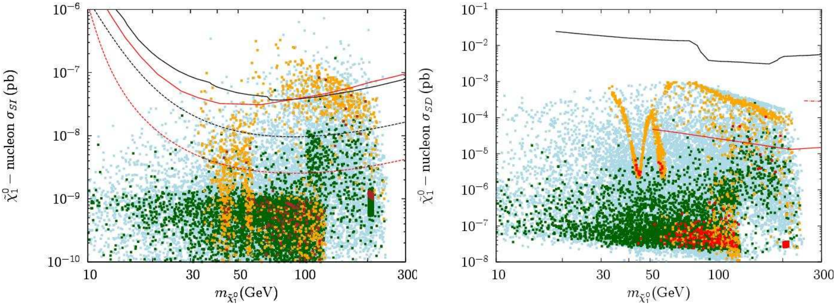
[Gogoladze, Khalid, Raza, Shafi, 2009]



Plots of the $\tilde{\chi}_1^0$-nucleon spin-independent cross-section versus $m_{\tilde{\chi}_1^0}$ for the CMSSM, FSU(5)-UH, NUHM2 and FSU(5)models for $\tan\beta = 50$. Also shown are current limits from CDMS II (solid black line), XENON10 (dashed black) and projected reach of SuperCDMS (solid red) and XENON100 (dashed red).

$SU(4)_c \times SU(2)_L \times SU(2)_R$ DM with Yukawa Unification

[Gogoladze, Khalid, Raza, Shafi, 2010]



Plots in the $\sigma_{\rm SI}$ - $m_{\tilde{\chi}_1^0}$ and $\sigma_{\rm SD}$- $m_{\tilde{\chi}_1^0}$ planes. In the $\sigma_{\rm SI}$ - $m_{\tilde{\chi}_1^0}$ plane we show the current bounds (black lines) and future reaches (red lines) of the CDMS (solid lines) and Xenon (dotted lines) experiments. In the $\sigma_{\rm SD}$ - $m_{\tilde{\chi}_1^0}$ plane we show the current bounds from Super K (black line) and IceCube (dotted red line) and future reach of IceCuce DeepCore (red solid line).

Summary

- The predictions of r (primordial gravity waves) for various models of inflation are as follows:
  - Gauge Singlet Higgs Inflation:
  $$r \gtrsim 0.02 \quad \text{for} \quad n_s \geq 0.96$$
  - SM Higgs Inflation:
  $$r \sim 0.003 \ (n_s \sim 0.968)$$
  - Non-Minimal ϕ^4 Inflation:
  $$r \gtrsim 0.002 \quad \text{for} \quad n_s \geq 0.96$$
  - Dark Matter Inflation:
  $$0.003 \lesssim r \lesssim 0.007$$
  - MSSM Inflation:
  $$r \sim 10^{-16} \quad \text{with} \quad 0.93 \lesssim n_s \lesssim 1$$
  - Susy Higgs (Hybrid) Inflation:
  $$r \lesssim 10^{-4} \ \text{(minimal)}, \qquad r \lesssim 0.05 \ \text{(non-minimal)}$$
- Results from PLANCK are eagerly awaited!

Summary

- In addition, we expect the LHC to provide answers to the following long-standing and very basic questions:
  - Is there a 'SM Higgs boson' and what is its mass?
  - Is there low scale supersymmetry? LSP dark matter?