The generalized Ginsparg-Wilson relation for lattice supersymmetry and its consequences

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Outline

Motivation

Why is lattice supersymmetry interesting?

- SUSY is a way to extend Poincaré symmetry.
- Part of the AdS/CFT correspondence.
- May be an ingredient of a GUT.
- Lattice yields results for analytically non-accessible aspects.
- Way to go beyond perturbation theory.

$\Longrightarrow$ Fully supersymmetric lattice theory needed.

A fundamental problem

Two SUSY generators anti-commute to derivative operator.
$\Longrightarrow$ Successive SUSY transformations yield translation.
$\Longrightarrow$ SUSY action invariant under (infinitesimal) translations.

Invariance in the continuum. Consider an action

$$S = \int \mathrm{d}x \, \alpha(x)\beta(x)\gamma(x).$$

Small translation of the fields α, β, γ by δx yields

$$\delta S = \delta x \int \mathrm{d}x \, [(\partial_x \alpha)\beta\gamma + \alpha(\partial_x \beta)\gamma + \alpha\beta(\partial_x \gamma)] = \delta x \int \mathrm{d}x \partial_x \, (\alpha\beta\gamma) = 0.$$

In general, no Leibniz rule for lattice difference operator ∇:

$$\nabla(\alpha_m\beta_m\gamma_m) \neq (\nabla\alpha)_m\beta_m\gamma_m + \alpha_m(\nabla\beta)_m\gamma_m + \alpha_m\beta_m(\nabla\gamma)_m;$$

Typical lattice action: no invariance under small translation.
$\Longrightarrow$ Not possible to obtain total derivatives like in continuum.
$\Longrightarrow$ How to construct invariant actions?

No-Go-Theorem: Leibniz rule only recovered by both non-local action and lattice derivative operator (Bergner; 2010).

$\Longrightarrow$ numerically expensive, ansatz not applicable to any theory;

State of the art: SUSY theories with one conserved supercharge.

A way out

Similar problem in lattice QCD with chiral symmetry:
Chiral lattice action has 'doublers' (two zero eigenvalues of difference operator, doubling of fermion species).

Introduction of modified symmetry relation (Ginsparg, Wilson; 1982):

$$\{D, \gamma_5\} = 2aD\gamma_5 D, \text{ lattice spacing } a.$$

Relation for fermionic action obtained by block-spin-transformation.

Solution: overlap operator (Neuberger; 1998),

$$D_{overlap} = 1 + \gamma_5 \mathrm{sgn}(\gamma_5 D).$$

GW-relation can be generalized for any linear symmetry.
Consider lattice action $S[\phi]$ with fields ϕ_i, and infinitesimal symmetry operation

$$\phi_i \rightarrow \phi_i + \epsilon M_{ij}\phi_j,$$

with generator M.

Generalized GW-relation (Bergner, Bruckmann, Pawlowski; 2008):

$$\delta S \sim M_{ij}\phi_j \frac{\partial S}{\partial \phi_i} = \left(M\alpha^{-1}\right)_{ij} \left[\frac{\partial S}{\partial \phi_j}\frac{\partial S}{\partial \phi_i} - \frac{\partial^2 S}{\partial \phi_j \partial \phi_i}\right],$$

with SUSY-breaking blocking matrix α.
Derived from a Wilsonian effective action ansatz:

$$e^{-S[\phi]} = \mathrm{Sdet}^{1/2}\alpha \int \mathrm{d}\varphi\, e^{-\frac{1}{2}(\phi-\phi_f[\varphi])\alpha(\phi-\phi_f[\varphi])-S_{cl}[\varphi]};$$

Application to lattice SUSY

Non-linear equation; mixes different orders of field products.

- For interacting SUSY action, not solved yet.
- Non-polynomial solutions likely.

Consider lattice supersymmetry operators M, $\bar{M}$.

$$\{M, \bar{M}\} \sim \nabla, \text{ with lattice derivative } \nabla.$$

One can show in general (Bergner, Bruckmann, Echigo, Igarashi, Pawlowski, Schierenberg: work in progress):
Lattice action fulfills relation for M and $\bar{M}$;
$\Longrightarrow$ Relation is also fulfilled for anti-commutator ∇.

$$\delta S \sim \nabla_{ij}\phi_j \frac{\partial S}{\partial \phi_i} = \left(\nabla\alpha^{-1}\right)_{ij}\left[\frac{\partial S}{\partial \phi_j}\frac{\partial S}{\partial \phi_i} - \frac{\partial^2 S}{\partial \phi_j \partial \phi_i}\right];$$

$$\delta S \sim \nabla_{ij}\phi_j \frac{\partial S}{\partial \phi_i} = \left(\nabla\alpha^{-1}\right)_{ij}\left[\frac{\partial S}{\partial \phi_j}\frac{\partial S}{\partial \phi_i} - \frac{\partial^2 S}{\partial \phi_j \partial \phi_i}\right];$$

Properties of α (and α^{-1}):

$$\alpha_{ij} = (-1)^{|i||j|}\alpha_{ji};\ |i| = \begin{cases} 0 \text{ for } i \text{ bosonic,} \\ 1 \text{ for } i \text{ fermionic.} \end{cases}$$

Bosonic and fermionic refers to product $\phi_i\,\alpha_{ij}\,\phi_j$.
Symmetrization:

$$\left(\nabla\alpha^{-1}\right)_{ij} \to \frac{1}{2}\left(\nabla\alpha^{-1}\right)_{ij} + \frac{1}{2}(-1)^{|i||j|}\left(\nabla\alpha^{-1}\right)_{ji};$$

Natural assumptions:

- translational invariant operator ∇ and blocking α (commuting with each other);
- antisymmetric ∇;

Then:

$$\left(\nabla\alpha^{-1}\right)_{ij} + (-1)^{|i||j|}\left(\nabla\alpha^{-1}\right)_{ji} = \nabla_{ik}\alpha^{-1}_{kj} + (-1)^{|i||j|}\alpha^{-1}_{ki}\nabla_{jk}$$
$$= \nabla_{ik}\alpha^{-1}_{kj} - (-1)^{|i||j|+|i||k|}\alpha^{-1}_{ik}\nabla_{kj} = \nabla_{ik}\alpha^{-1}_{kj} - \nabla_{ik}\alpha^{-1}_{kj} = 0.$$

Generalized GW-relation becomes

$$\delta S \sim \nabla_{ij}\phi_j \frac{\partial S}{\partial \phi_i} = 0.$$

$\Longrightarrow$ Naive relation for translational invariance, same problems like before.

Constraints for lattice actions

Consequences for supersymmetric lattice actions?

- With typical assumptions, generalized GW-relation yields no improvement.
- Relax some of our assumptions.

Possibilities:

- Choose α and/or ∇ translational non-invariant.
- Give ∇ a symmetric part.

1. Drop the translational invariance of α.

$\Longrightarrow$ Resulting action not invariant under shifts of a lattice spacing.

2. Drop translational invariance of ∇.
3. Give ∇ a symmetric part.

$\Longrightarrow$ Violates a condition arising from blocking procedure.
$\Longrightarrow$ Action very likely to be non-invariant like above.

Translational non-invariant action seems inevitable.

Summary

No-Go-Theorem: Local lattice action with SUSY in conventional way is impossible.
Way out: Generalized Ginsparg-Wilson-relation for SUSY.

However, relation also has to hold for anti-commutator of two supercharges

$$\{M, \bar{M}\} \sim \nabla.$$

Need to relax some natural assumptions to solve it.

$\Longrightarrow$ Translational non-invariant actions have to be taken into account.

Thanks for the attention.