Non-equilibrium current and relaxation dynamics of a charge-fluctuating quantum dot

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Motivation

Study of *non-equilibrium current and relaxation dynamics*

with two complementary RG approaches

$\longrightarrow$ *analytic solution* of flow equations

– identification of microscopic cutoff scales

– description of relaxation dynamics into steady state

$\longrightarrow$ access *all* parameter regimes

– interesting non-equilibrium effects away from p-h or left-right symmetry and resonances at $\epsilon = \pm V/2$

$\longrightarrow$ *time dependence*

Outline

→ Model

→ Methods: Real-time and functional RG

→ Results:

  – steady-state quantities

  – time evolution

→ Summary

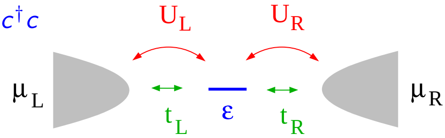
Interacting resonant level model

minimal model for charge-fluctuating quantum dot

$$H = \sum_{k\alpha} (\epsilon_{k\alpha} + \mu_\alpha)\, a^\dagger_{k\alpha}\, a_{k\alpha} + \epsilon\, c^\dagger c$$

$$+ \sum_{k\alpha} t_\alpha \left(a^\dagger_{k\alpha}\, c + \mathrm{h.c.} \right)$$

$$+ \left(\hat{n} - \tfrac{1}{2}\right) \sum_{kk'\alpha} U_\alpha :a^\dagger_{k\alpha}\, a_{k'\alpha}:$$



with $\alpha = L, R$, hybridization $\Gamma^0_\alpha = 2\pi t^2_\alpha$, and $\mu_{L/R} = \pm V/2$

scaling limit:

$t_L,\ t_R \to 0, \quad D \to \infty \quad$ and T_K constant

where $T_K = \sum_\alpha T^\alpha_K, \quad T^\alpha_K = \Gamma^0_\alpha \left(\frac{D}{T_K}\right)^{2U_\alpha}$

Previous approaches

equilibrium:

– relation to x-ray edge problem Nozière, De Dominicis '69

– mapping to Kondo model Wiegmann, Finkelstein '78

– Bethe Ansatz Filyov, Wiegmann '80

– perturbative RG Schlottmann '80

non-equilibrium:

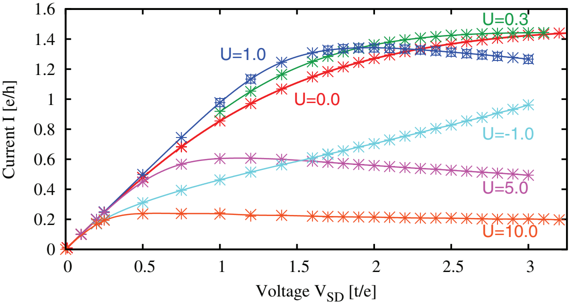
– Scattering BA Mehta, Andrei '06

– Scattering states, perturbation theory Doyon '07

– Perturbative RG, equilibrium NRG Borda *et al.* '07

– TD-DMRG, Field Theory Boulat *et al.* '08

Previous results



Boulat *et al.* '08

– negative differential conductance for large V

– power-law behavior of current in scaling limit

open questions:

→ non-equilibrium effects *beyond* V as additional cutoff ?

→ access all parameter regimes

→ time dependence

Functional RG and real-time RG in frequency space

both for small to intermediate Coulomb interactions

$\rightarrow$ Functional RG (FRG) captures steady state

for *arbitrary* system parameters

$\rightarrow$ Real-time RG in frequency space (RTRG-FS) allows for *analytic*

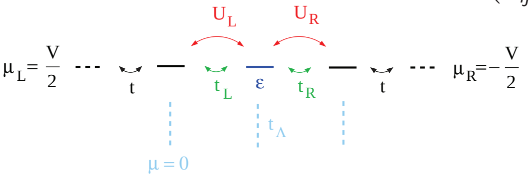
description of steady state *and* relaxation dynamics in scaling limit

$\Rightarrow$ combined use leads to reliable and comprehensive physical picture

Gezzi *et al.*, Jakobs *et al.* '07; Jakobs *et al.*, Schoeller '09

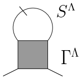
Method: functional renormalization group

– non-equilibrium Green's function $\mathcal{G}_{ij}(\omega) = \begin{pmatrix} G_{ij}^{--}(\omega) & G_{ij}^{-+}(\omega) \\ G_{ij}^{+-}(\omega) & G_{ij}^{++}(\omega) \end{pmatrix}$



– *cutoff* via auxiliary reservoir $t_\Lambda = \sqrt{\Lambda}\, t$ with $\Lambda = \infty \to 0$

– hierarchy of flow equations truncated by $\Gamma^\Lambda = \Gamma^{\Lambda_0}$

$\frac{\partial}{\partial\Lambda}\Sigma^\Lambda =$ 

$$\mathcal{S}^\Lambda = -\mathcal{G}^\Lambda \left[\partial_\Lambda (\mathcal{G}_0^\Lambda)^{-1}\right] \mathcal{G}^\Lambda$$

→ transport by Landauer-Büttiker:

$$I_L = 2\pi i\, t^2 \int \rho_{\mathrm{bath}} \left[f_L(\omega) G_{ii}^{+-}(\omega) + (1 - f_L(\omega)) G_{ii}^{-+}(\omega) \right] d\omega$$

Wetterich '93; Morris '94; Metzner '99; Salmhofer and Honerkamp '01

Flow equations of rates Γ_α (scaling limit)

$$\frac{d\Gamma_\alpha}{d\Lambda} = -2U_\alpha \Gamma_\alpha \frac{\Lambda+\Gamma/2}{(\mu_\alpha-\epsilon)^2+(\Lambda+\Gamma/2)^2} \quad \rightarrow \quad \Gamma_\alpha = \Gamma_\alpha^0 \left(\frac{\Lambda_0}{\Lambda_{c,\alpha}}\right)^{2U_\alpha}$$

with $\Gamma = \sum_\alpha \Gamma_\alpha$, $\Lambda_0 \sim D$, and cutoff $\Lambda_{c,\alpha} = \max\{|\mu_\alpha - \epsilon|, \Gamma/2\}$

$V \gg \Gamma \;\Rightarrow\;$ *off-resonance* $|\frac{V}{2} - \epsilon| > \Gamma \;\rightarrow\; \Lambda_{c,\alpha} = V$

on-resonance $\frac{V}{2} = \epsilon \;\rightarrow\; \Lambda_{c,L} = \Gamma/2$ and $\Lambda_{c,R} = V$

scaling limit: $\Lambda_0 \rightarrow \infty$, $\Gamma_\alpha^0 \rightarrow 0$

$\rightarrow$ express in terms of invariant scale $T_K = \sum_\alpha \Gamma_\alpha^0 \left(\frac{2\Lambda_0}{T_K}\right)^{2U_\alpha}$

and asymmetry parameter $c^2 = T_K^L / T_K^R$

$$\Rightarrow \Gamma_L = c\, T_K \left(\frac{T_K}{2\Lambda_{L,\alpha}}\right)^{2U_L} \frac{c}{1+c^2}$$

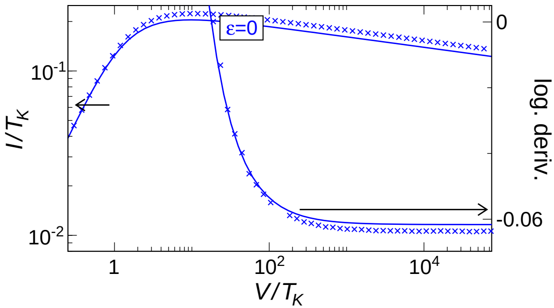
$$\Gamma_R = \frac{1}{c}\, T_K \left(\frac{T_K}{2\Lambda_{R,\alpha}}\right)^{2U_R} \frac{c}{1+c^2}$$

power laws with U-dependent exp.

Steady-state current $I(V)$

$$I = \frac{1}{\pi}\frac{\Gamma_L\Gamma_R}{\Gamma}\left[\arctan\frac{V-2\epsilon}{\Gamma} + \arctan\frac{V+2\epsilon}{\Gamma}\right]$$

off-resonance $\quad I \approx \frac{\Gamma_L\Gamma_R}{\Gamma} = T_K \frac{\left(\frac{T_K}{|V-2\epsilon|}\right)^{2U_L}\left(\frac{T_K}{|V+2\epsilon|}\right)^{2U_R}}{c\left(\frac{T_K}{|V-2\epsilon|}\right)^{2U_L} + \frac{1}{c}\left(\frac{T_K}{|V+2\epsilon|}\right)^{2U_R}} \frac{c}{1+c^2} \quad$ for $\quad V \gg \Gamma$



$U_L = U_R = 0.1/\pi$

RTRG-RS (lines)

FRG (symbols)

power law *only* for left-right symmetry $U_L = U_R$: $I \sim V^{-2U}$

Steady-state current $I(V)$

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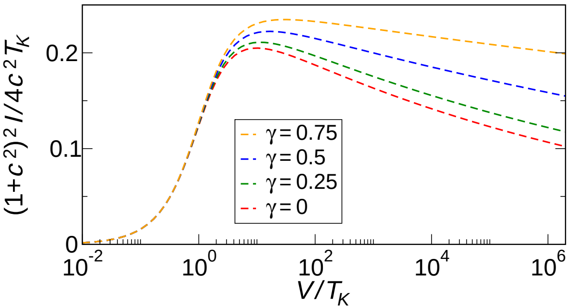

asymmetry effects

$U_{L/R} = (1 \pm \gamma)\, 0.1/\pi$

$\epsilon = 0$

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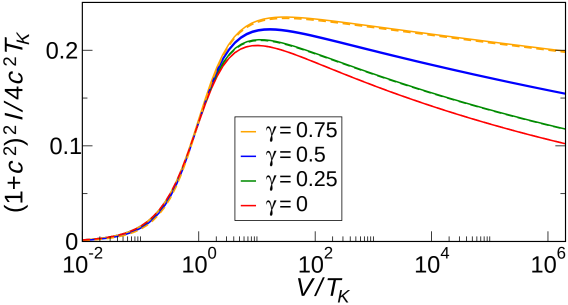


asymmetry effects

$U_{L/R} = (1 \pm \gamma)\, 0.1/\pi$

$\epsilon = 0$

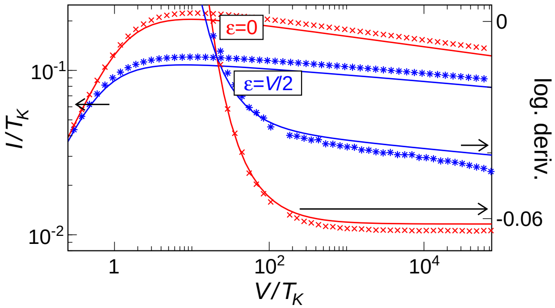
excellent agreement

to analytical results

Steady-state current $I(V)$

on-resonance $\quad \epsilon = V/2 \quad \rightarrow$ maximum of $G = dI/dV$

$$I \approx \frac{\Gamma_L \Gamma_R}{2\Gamma} = \frac{T_K}{2} \frac{\left(\frac{T_K}{\Gamma}\right)^{2U_L} \left(\frac{T_K}{2V}\right)^{2U_R}}{c\left(\frac{T_K}{\Gamma}\right)^{2U_L} + \frac{1}{c}\left(\frac{T_K}{2V}\right)^{2U_R}} \frac{c}{1+c^2}$$



$U_L = U_R = 0.1/\pi$

RTRG-RS (lines)

FRG (symbols)

NO power law !

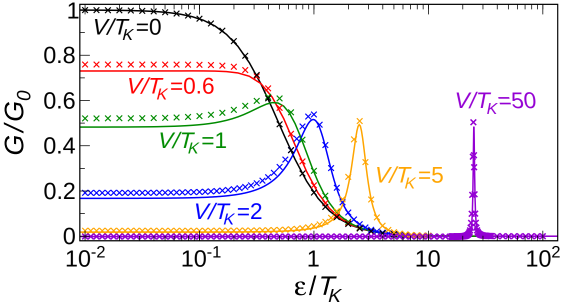
$\Rightarrow$ V *not* simple infrared cutoff for generic situation

physics in non-equilibrium more complex

Steady-state current $I(V)$

on-resonance $\quad \epsilon = V/2 \quad \rightarrow$ maximum of $G = dI/dV$

$$I \approx \frac{\Gamma_L \Gamma_R}{2\Gamma} = \frac{T_K}{2} \frac{\left(\frac{T_K}{\Gamma}\right)^{2U_L} \left(\frac{T_K}{2V}\right)^{2U_R}}{c\left(\frac{T_K}{\Gamma}\right)^{2U_L} + \frac{1}{c}\left(\frac{T_K}{2V}\right)^{2U_R}} \frac{c}{1+c^2}$$



$U_L = U_R = 0.1/\pi$

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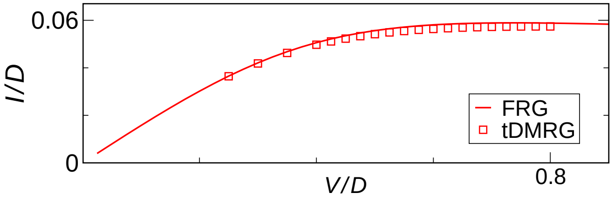
Comparison to exact results



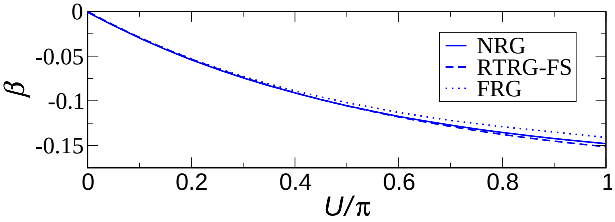
DMRG results

$U_L = U_R = 0.3\,\pi/4$

$t_L = t_R = 0.5\,\pi/4$

$\epsilon = 0$

Schmitteckert '08



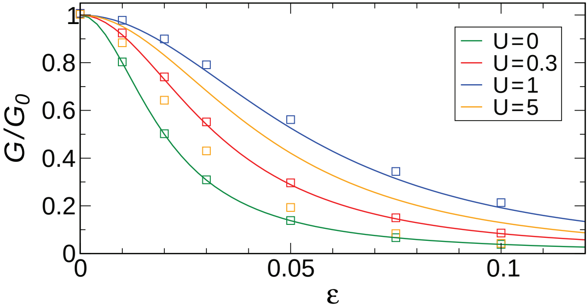
NRG data

$\beta = -U/(1+2U)$

charge susceptibility $\qquad \chi = -\frac{\partial \langle \hat{n} \rangle}{\partial \epsilon}\Big|_{\epsilon=0} \sim \Gamma_0^{\beta}$

Linear Conductance

V=0



$t_L = t_R = 0.1$

FRG (lines)

DMRG (symbols)

Schmitteckert '08

– interaction induces enhancement

– *non-monotonic* behavior

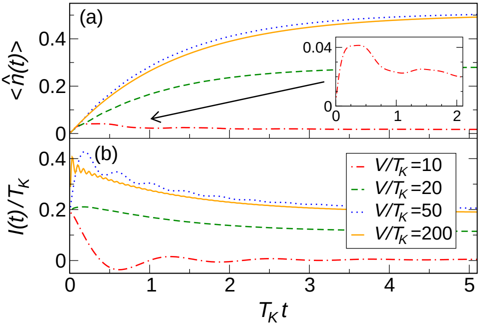
→ more accurate results by including two-particle vertex renormalization

Time evolution

→ Talk by M. Pletukhov

long-time behavior off-resonance ($\epsilon, V, |\epsilon - V/2| \gg T_K, 1/t$)

$$\langle \hat{n}(t)\rangle \approx \left(1 - e^{-\Gamma_1 t}\right)\langle \hat{n}\rangle - \frac{1}{2\pi}\, e^{-\Gamma_2 t}\, (T_K t)^{1+2U}$$
$$\times \left[\frac{\sin\left((\epsilon + \frac{V}{2})t\right)}{(\epsilon + \frac{V}{2})^2\, t^2} - \frac{\pi U}{4} \frac{\cos\left((\epsilon + \frac{V}{2})t\right)}{(\epsilon + \frac{V}{2})^2\, t^2} + (V \to -V)\right]$$



$\Gamma_1 \sim \Gamma$ (charge relaxation)

$\Gamma_2 \sim \Gamma/2$ (level broadening)

V relevant energy scale

setting oscillation frequency

($\epsilon = 10\ T_K$)

→ exponential *and* power-law decay $\sim t^{2U-1}$

Summary

promising RG approaches for non-equilibrium local quantum system

coupled to environment in weak-coupling regime

– provides description for all parameter regimes

– steady-state $I(V)$ exhibits power law only in specific cases

– new scaling behavior on resonance

– analytic description of relaxation dynamics: exponential decay

supplemented by characteristic oscillations and power-law behavior

Refs.: - Karrasch, Andergassen, Pletyukhov, Schuricht, Borda, Meden, Schoeller, EPL **90**, 30003 (2010);

- Karrasch, Pletyukhov, Borda, Meden, PRB **81**, 125122 (2010)

Method: real-time RG in frequency space (RTRG-FS)

$\longrightarrow$ new aspects of non-eq. RG in Liouville space

- evolution of density matrix
- provides dephasing rates as cutoff

$\longrightarrow$ Keldysh indices can be avoided

- effort comparable to equilibrium formalism

$\longrightarrow$ expansion in coupling between quantum dot and reservoirs

$\longrightarrow$ description of dynamics at finite bias

- closed analytic form
- beyond Markovian theories