Multi - Higgs models and CP violation

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CP violation

- At Lagrangian level
- Spontaneous (T D Lee 1973)

Spontaneous CP violation requires that

- There is a transformation that may be physically interpreted as CP under which $\mathcal{L}$ is invariant
- There is no transformation that may be physically interpreted as CP under which both $\mathcal{L}$ and vacuum are invariant

Branco, Lavoura, Silva 1999

Gauge theories with fermions without scalar fields do not $\not{CP}$

Gumus, MNR 1995

Conditions for a given Higgs potential to violate CP (conserve)
at Lagrangian level

relevance emphasized Ginzburg, Krawczyk 2004

CP - odd Higgs basis invariants
that signal explicit CP violation
in an arbitrary Higgs basis
constructed in terms of the parameters of $\mathcal{L}$
independent of the gauge sym breaking

hard or soft symmetry breaking?
special cases of two and three Higgs doublets

CP - odd Higgs basis invariants involving Higgs vevs

Lavoura, Silva 1994
Botella, Silva 1994

x

Most general renormalizable polynomial, gauge invariance

Higgs pot. $\mathcal{L}_\phi = Y_{ab}\,\phi_a^\dagger \phi_b + Z_{abcd}\,(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$

n_d Higgs doublets

quadratic, and quartic terms only

Hermiticity of $\mathcal{L}_\phi$ implies

$Y_{ab}^* = Y_{ba}$ $\qquad Z_{abcd}^* = Z_{badc}$

Furthermore (by construction) $Z_{abcd} = Z_{cdab}$

Under Higgs basis transformations (HBT)

$\phi_a \xrightarrow{HBT} \phi_a' = V_{ai}\,\phi_i \qquad \phi_a^\dagger \xrightarrow{HBT} (\phi')_a^\dagger = V_{ai}^*\,\phi_i^\dagger$

the physics does not change

$Y_{ab} \xrightarrow{HBT} Y_{ab}' = V_{am}\,Y_{mn}\,V_{nb}^\dagger$

$Z_{abcd} \xrightarrow{HBT} Z_{abcd}' = V_{am}\,V_{cp}\,Z_{mnpq}\,V_{nb}^\dagger\,V_{qd}^\dagger$

Couplings that are complex may become real in another basis

Derivation of HB inv conditions, CP conservation

General method SM Bernabeu, Branco, Gronau 1986

$$\mathrm{Tr}\,[g_u g_u^\dagger, g_d g_d^\dagger]^3 = 0$$

g_u, g_d denote Yukawa couplings

necessary for an arbitrary number of generations

Most general CP transf of Higgs leaving kinetic term invariant

$$\phi_a \xrightarrow{CP} U_{ai}\,\phi_i^* \quad , \quad \phi_a^\dagger \xrightarrow{CP} U_{ai}^*\,\phi_i^T \qquad U \text{ unitary}$$

Necessary and sufficient condition for $\mathcal{L}_\phi$ to conserve CP

$$(Y^*)_{ab} = U_{am}^\dagger\, Y_{mn}\, U_{nb}$$

$$(Z^*)_{abcd} = U_{am}^\dagger\, U_{cp}^\dagger\, Z_{mnpq}\, U_{nb}\, U_{qd}$$

requirement existence of matrix U

Change of Higgs basis $U' = V U V^T$

x

We Want Higgs basis invariant conditions

Invariance of trace under similarity conditions

$$I_1 \equiv \mathrm{Tr}\left[Y Z_Y \hat{Z} - \hat{Z} Z_Y Y\right] = 0$$

$$I_2 \equiv \mathrm{Tr}\left[Y Z_2 \tilde{Z} - \tilde{Z} Z_2 Y\right] = 0$$

$n_d \times n_d$ matrices

$$\hat{Z}_{ab} \equiv Z_{abmm}, \quad \tilde{Z}_{ab} \equiv Z_{ammb}$$

$$(Z_Y)_{ab} \equiv Z_{abmn} Y_{mn}, \quad (Z_2)_{ab} \equiv Z_{apnm} Z_{mnpb}$$

Necessary and sufficient conditions for CP conservation for $n_d = 2$ barring special isolated points

Two Higgs doublets. The General case

$$V_{2HD} = m_1 \phi_1^\dagger \phi_1 + p e^{i\varphi} \phi_1^\dagger \phi_2 + p e^{-i\varphi} \phi_2^\dagger \phi_1 + m_2 \phi_2^\dagger \phi_2 + a_1 (\phi_1^\dagger \phi_1)^2 +$$
$$+ a_2 (\phi_2^\dagger \phi_2)^2 + b (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + b' (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) + c_1 e^{i\theta_1} (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_1) +$$
$$+ c_1 e^{-i\theta_1} (\phi_1^\dagger \phi_1)(\phi_1^\dagger \phi_2) + c_2 e^{i\theta_2} (\phi_2^\dagger \phi_2)(\phi_2^\dagger \phi_1) + c_2 e^{-i\theta_2} (\phi_2^\dagger \phi_2)(\phi_1^\dagger \phi_2) +$$
$$+ d e^{i\delta} (\phi_1^\dagger \phi_2)^2 + d e^{-i\delta} (\phi_2^\dagger \phi_1)^2$$

phase dependence explicit

excess of parameters

Choice of HB, no loss of generality

- quadratic terms diagonal
- elimination of one of $\theta_1, \theta_2, \delta$ (rephasing a Higgs field)

11 independent parameters

only two independent CP violating phases

Therefore we expect two conditions

Necessary and <u>Sufficient</u> conditions for CP invariance, in $d=2$

(barring the consideration of special isolated points)

in the basis $pe^{i\varphi}=0$

$$I_1 \equiv \frac{i}{2} c_1 c_2 (m_1-m_2)^2 \sin(\theta_2-\theta_1) = 0$$

$$I_2 \equiv \frac{i}{2}(m_1-m_2)\Big[d c_1^2 \sin(\delta+2\theta_1) + d c_2^2 \sin(\delta+2\theta_2) + 2d c_1 c_2 \sin(\delta+\theta_1+\theta_2) + c_1 c_2 (a_2-a_1)\sin(\theta_1-\theta_2)\Big] = 0$$

Notice $m_1=m_2$ both I_i, $i=1,2$ vanish

Likewise for $c_1=c_2$, $\theta_2=\theta_1+\pi$

Choice of $\delta=0$, no loss of generality

$I_1=0 \quad \Rightarrow \quad \theta_2=\theta_1 \; , \; \theta_2=\theta_1+\pi$

Case $\theta_2=\theta_1$

$$I_2 = \frac{i}{2}(m_1-m_2)\, d\,(c_1+c_2)^2 \sin(2\theta_1) = 0$$

two solutions $\quad \theta_1=\theta_2=0 \; , \; \theta_1=\theta_2=\frac{\pi}{2}$

Case $\theta_2 = \theta_1 + \pi$

$I_2 = \frac{i}{2}(m_1 - m_2)\, d\, (c_1 - c_2)^2 \sin(2\theta_1) = 0$

same two solutions for θ_1 : $\theta_1 = 0$ or $\theta_1 = \pi/2$

- $\rho e^{i\varphi} = 0$, $\delta = 0$, $\theta_1 = \theta_2 = 0$

  $\mathcal{L}_\phi$ is CP invariant (no phases)

  ϕ_1 and ϕ_2 transform with U given by $\quad U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

- $\rho e^{i\varphi} = 0$, $\delta = 0$, $\theta_1 = \theta_2 = \pi/2$

  $\mathcal{L}_\phi$ is CP invariant

  ϕ_1 and ϕ_2 transform with U given by $\quad U = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Special case $m_1 = m_2$ $\quad$ $\not{C}\not{P}$ is possible

Special case $c_1 = c_2$ $\quad \theta_2 = \theta_1 + \pi$ $\quad$ $\not{C}\not{P}$ is possible

in both cases $\quad I_1 = 0$, $I_2 = 0$ automatically

$I_3 \equiv \mathrm{Tr}\,[Z_2 Z_3 \hat{Z} - \hat{Z} Z_3 Z_2] = 0$, $Z_3 = Z_{amrp} Z_{mrns} Z_{pnsb}$

barring special isolated point $a_1 = a_2$, $c_1 = c_2$, $\theta_2 = \theta_1 + \pi$

(no Y's only Z's)

I_3 is built exclusively from quartic couplings

$I_3 \neq 0$ implies hard CP violation

Yet, choice $pe^{i\varphi} = 0$ (HB) implies all CP violating phases appear in quartic couplings

In this HB soft CP violation will be "hidden"

$$I_3 = c_1 \sin(\theta_1 - \theta_2) + c_2 \sin(2(\theta_1 - \theta_2)) + c_3 \sin(\delta + 2\theta_2) + c_4 \sin(\delta + 2\theta_1) + c_5 \sin(\delta + \theta_1 + \theta_2) + c_6 \sin(\delta + 3\theta_1 - \theta_2) + c_7 \sin(\delta + 3\theta_2 - \theta_1)$$

c are polynomials of degree 6 in a's, b's, c's and d

$I_3 = 0$ automatically for $a_1 = a_2$, $c_1 = c_2$, $\theta_2 = \theta_1 + \pi$

however there is the possibility of CP violation in this case

Special isolated point $\quad a_1 = a_2 \,,\ c_1 = c_2 \,,\ \theta_2 = \theta_1 + \pi$

$$V' = m_1 \phi_1^\dagger \phi_1 + m_2 \phi_2^\dagger \phi_2 + a\left[(\phi_1^\dagger \phi_1)^2 + (\phi_2^\dagger \phi_2)^2\right] +$$
$$+ b\,(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + b'(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) +$$
$$+ c\left[e^{i\theta}(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_1) - e^{i\theta}(\phi_2^\dagger \phi_2)(\phi_2^\dagger \phi_1) + h.c.\right] +$$
$$+ d\left[e^{i\delta}(\phi_1^\dagger \phi_2)^2 + h.c\right]$$

$$a \equiv a_1 = a_2 \qquad c \equiv c_1 = c_2 \qquad \theta \equiv \theta_1$$

$$I_1 = 0 \,,\quad I_2 = 0 \,,\quad I_3 = 0$$

Quartic couplings conserve CP provided $\quad U = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

However this symmetry is broken by quadratic terms for $m_1 \neq m_2$

CP is violated softly, need fourth condition

$$I_4 = \mathrm{Im}\,(Z_{acbd}\, Z_{cedg}\, Z_{ehfq}\, Y_{ga}\, Y_{hb}\, Y_{qf}) = 0$$

Gunion, Haber 2005

Under HBT relations for a's, c's and θ's are invariant

Davidson, Haber 2005

Absence of the 3-plet in the decomposition of the quartic Higgs potential into irreducible reps of the SU(2) HBT for 2HD

Ivanov 2005

Another interesting example of soft CP breaking

$$a_1 = a_2 = a \quad , \quad c_1 = c_2 = c \quad , \quad \theta_1 = \theta_2 = \theta$$

$$V'' = m_1 \phi_1^\dagger \phi_2 + m_2 \phi_2^\dagger \phi_2 + a\left[(\phi_1^\dagger \phi_1)^2 + (\phi_2^\dagger \phi_2)^2\right] +$$
$$+ b\,(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + b'(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) +$$
$$+ c\left[e^{i\theta}(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_1) + e^{i\theta}(\phi_2^\dagger \phi_2)(\phi_2^\dagger \phi_1) + h.c.\right] +$$
$$+ d\left[e^{i\delta}(\phi_1^\dagger \phi_2)^2 + h.c.\right]$$

Quartic couplings conserve CP for $U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Symmetry broken for $m_1 \neq m_2$

$I_3 = 0$ as it should

$I_1 = 0$

Yet $I_2 = 2c^2 d\,(m_1 - m_2)\sin(\delta + 2\theta)$

again "hidden" soft CP breaking

In another HB : $U' = V U V^T$

Choosing $V = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ one obtains $U' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

DISCRETE SYM IMPORTANT TO SUPPRESS HFCNC NATURALLY

NFC Glashow, Weinberg 1977

For three or more ϕ doublets V invariant under separate reflexions ($Z_2 \times Z_2 \times Z_2 \times \cdots$ sym) need not conserve CP symmetry Weinberg 1976

Furthermore, three Higgs doublets may violate CP spontaneously while having NFC Branco 1980, Branco, Buras, Gerard 1985

General three Higgs potential has

$N_{phases} = \frac{1}{4}\left[n_d^2 (n_d^2 - 1)\right] - (n_d - 1)$; 16 indep phases, $n_d = 3$

Simpler CP-odd invariants can be built for $n_d = 3$

$I_S = \mathrm{Tr}\left([Y, \tilde{Z}]^3\right)$, also $\tilde{Z} \to \hat{Z}$

HB Y diagonal

$I_S = 6i (Y_{22} - Y_{11})(Y_{33} - Y_{11})(Y_{33} - Y_{22}) \, \mathrm{Im}\left(\tilde{Z}_{12} \tilde{Z}_{23} \tilde{Z}_{31}\right)$

WEINBERG THREE HIGGS DOUBLET MODEL

$Z_2 \times Z_2 \times Z_2$ symmetry under separate reflection $\phi_1 \to -\phi_1$ (together with appropriately chosen transf for the quarks)

$$V = \sum_{i=1,2,3} m_i \phi_i^\dagger \phi_i + a_{ii} (\phi_i^\dagger \phi_i)^2 + \sum_{i<j} 2 b_{ij} (\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) +$$
$$+ 2 c_{ij} (\phi_i^\dagger \phi_j)(\phi_j^\dagger \phi_i) + \left[d_{ij} e^{i\theta_{ij}} (\phi_i^\dagger \phi_j)^2 + h.c. \right]$$

three different $d_{ij} e^{i\theta_{ij}}$ terms, only three can be complex

$(n_d - 1)$ phase redefinitions are possible

$Y, \tilde{Z}, \hat{Z}$ are diagonal matrices, hence $I_5 = 0$ automatically

a relevant CP-odd invariant is given by

$$I_2^W = \operatorname{Im} \left[Z_{ankq} Y_{nm} Z_{mrqs} \hat{Z}_{rt} Z_{tksa} \right]$$

Explicit form

$$I_2^W = d_{12} d_{13} d_{23} \left[(m_3 - m_2)(a_{11} - b_{23}) - (m_3 - m_1)(a_{22} - b_{13}) + (m_2 - m_1)(a_{33} - b_{12}) \right] \sin(\theta_{12} - \theta_{13} + \theta_{23})$$

non degenerate values for the m_i, a non-vanishing I_2^W indicates $(\theta_{12} - \theta_{13} + \theta_{23}) \neq 0$

THREE HIGGS DOUBLETS WITH NFC and SOFT CP BREAKING

$Z_2 \times Z_2 \times Z_2$ symmetry softly broken by quadratic terms in V

CP can be violated even if all quartic terms are real

Y in this case is no longer diagonal

$I_5 = \mathrm{Tr}\left([Y, \tilde{Z}]^3\right)$ no longer vanishes

$$I_5 = 6\,(a_{22} - a_{33} + c_{12} - c_{13})(a_{33} - a_{11} + c_{23} - c_{12}) \times$$
$$\times\,(a_{22} - a_{11} + c_{23} - c_{13})\, y_{12}\, y_{13}\, y_{23} \times \sin\left[\varphi_{12} - \varphi_{13} + \varphi_{23}\right]$$

where $Y_{ij} = y_{ij}\, e^{i\varphi_{ij}}$

$I_2^W = 0$ automatically in this case

it signals difference between hard and soft CP breaking

Conclusions

Multi-Higgs models are very interesting candidates for NP

Possibility of new sources of CP violation from Higgs sector

LHC results may bring surprises for the Higgs sector