Soft walls

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Outline

The outline of this talk is

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- Introduction
- The soft-wall model
- Graviton fluctuations
- Radion fluctuations
- The Higgs background
- EWSB
- Conclusion

Introduction

- Warped extra dimensions are useful to solve long-standing problems: hierarchy, flavor,...
- Also the AdS/CFT correspondence might deal with non-perturbative theories: technicolor, QCD,...
- We will concentrate on general 5D theories with a metric

Proper coodinates

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

$$\eta_{\mu\nu} = (-,+,+,+,+)$$

or in

Conformally flat coordinates

$$ds^2 = e^{-2A(z)}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right), \quad \frac{dz}{dy} = e^A$$

- The AdS metric $A_{AdS} = ky$ has conformal invariance
- The theory requires UV completion which translates into a UV brane at $y = 0$
- Conformal invariance has to be broken to generate a mass gap

IR brane

Conformal invariance is normally broken by an IR brane (RS1 [a]) at $y = y_c$. It can be stabilized by the GW mechanism [b]: it requires a stabilizing scalar in the gravitational background

[a] L. Randall and R. Sundrum, hep-ph/9905221
[b] W. Goldberger and M. Wise, hep-ph/9907447

- There is another way of breaking the conformal invariance if the scalar field has a singularity at $y = y_s$ which replaces the IR brane

- We call this a soft wall

Soft-walls

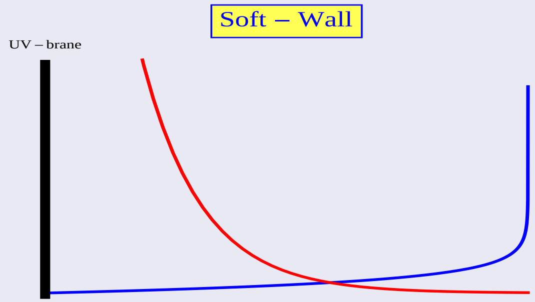
There is no IR brane and the extra dimension is non-compact but of finite length

$$\int e^{-A(z)} dz \equiv y_s = \int_0^{y_s} dy < \infty$$

- This implies that the IR brane is replaced by a

Naked curvature singularity at $y = y_s$ where $A(y_s) \to \infty$

- Stabilizing the distance y_s is similar to stabilizing the brane-to-brane distance y_c by the GW mechanism



- The stabilizing field has a divergence at $y = y_s$
- The metric backreacts and vanishes at the singularity
- No IR brane is required

The model

- We will introduce a scalar field ϕ with some boundary condition (BC) at the UV brane @ $y = 0$: ϕ_0
- We want naturalness to be fulfilled with an exponential relation between ky_s and ϕ_0 as

$$ky_s \sim e^{\nu\,\phi_0}$$

  In this way a hierarchy can be naturally generated with values $\nu, \phi_0 \simeq \mathcal{O}(1)$
- The presence of $\phi(y)$ will backreact on the AdS metric $A_{AdS} = ky$ providing a modification far from the UV brane. Solving the exact EOM is required
- We will solve the Einstein EOM in the bulk by [1]

"Superpotential" method (non supersymmetric models)

$$A'(y) = W(\phi), \quad \phi'(y) = \partial W/\partial\phi$$

$$V(\phi) = 3(\partial W/\partial\phi)^2 - 12W^2$$

[1] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, hep-th/9909134

- The model [2] is defined by the

Superpotential

$$W(\phi) = k(1 + e^{-\nu\,\phi})$$

- It has an analytical solution provided by the

Gravitational background

$$A(y) = ky - \frac{1}{\nu^2}\log\left(1 - \frac{ky}{ky_s}\right)$$

$$\phi(y) = \frac{1}{\nu}\log[\nu^2(ky_s - ky)]$$

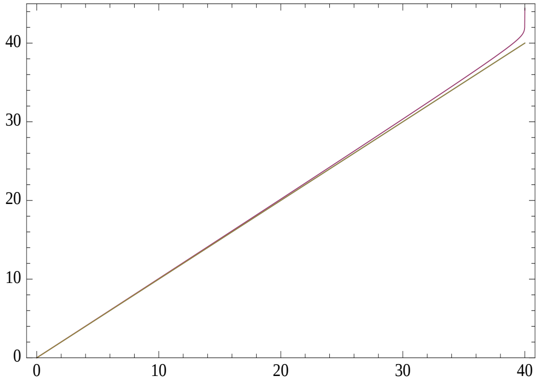
- The length y_s is related to the boundary field ϕ_0

Singularity location

$$ky_s = \frac{1}{\nu^2}e^{-\nu\,\phi_0}$$

[2] J. A. Cabrer, G. von Gersdorff and M. Quiros, arXiv:0907.5361

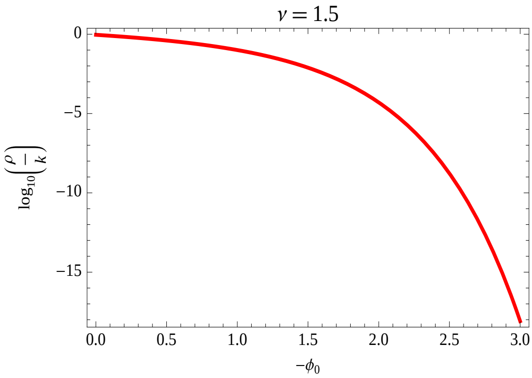
- The metric $A(y)$ separates from the AdS metric only near the singularity. E.g. for $\nu = 2$, $ky_s = 40$



- The low scale ρ can be related to ϕ_0 as a double exponential

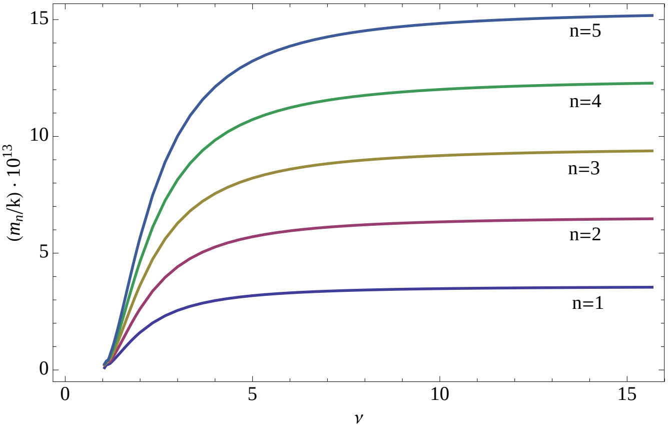
$$\rho = k\,\exp\left\{-\frac{1}{\nu^2}\left(e^{-\nu\,\phi_0} - \nu\phi_0 - \log\nu^2\right)\right\}$$

- A big hierarchy can be naturally obtained with $|\phi_0| \simeq \mathcal{O}(1)$



Graviton fluctuations

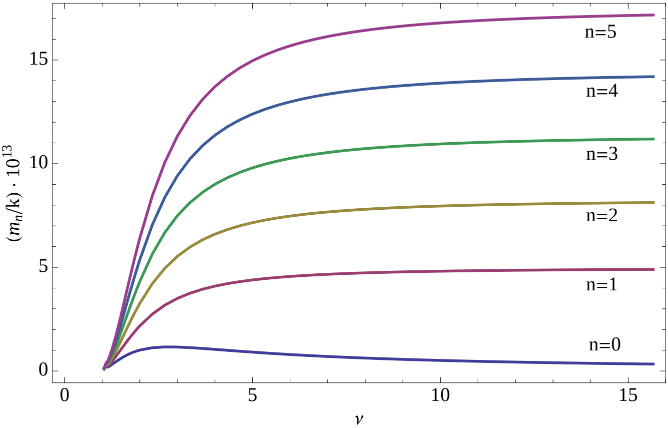
- We have solved numerically the EOM
- For $ky_s = 30$ the first levels mass spectra :



- Level spacing shrinks in the conformal limit $\nu \to 1$ (RS is $\nu \to \infty$)

Radion fluctuations

- For $ky_s = 30$ the first levels mass spectra:



The radion/graviton ratio

$$m_{radion}/m_{grav}^{(1)} \ll 1$$

The Higgs background

- In soft-walls there is no IR brane and thus the Higgs has to propagate in the bulk
- The Higgs doublet in the Standard Model (SM) can be described by

$$H(x,y) = \frac{1}{\sqrt{2}} \left[\begin{array}{c} 0 \\ h(y) + \xi(x,y) \end{array} \right] e^{i\vec{\chi}(x,y)\,\vec{\sigma}}$$

- We will assume the Higgs does not perturb the previous mechanism for fixing the radion mass.
- We then impose the

Regularity condition

$h(y)$ is regular at y_s

- In that case the Higgs does not backreact on the gravitational metrics

- The Higgs background satisfies the equation of motion

Bulk EOM

$$h'' - 4A'h' = \frac{\partial V}{\partial h}, \quad V(h) = \text{bulk potential}$$

and

Boundary conditions

$$h'(0) = \left.\frac{\partial \lambda_0}{\partial h}\right|_{y=0}, \quad \lambda_0(h) = \text{UV brane potential}$$

- I will present a model where EW symmetry is broken at the UV brane

EWSB

- EWSB is triggered by the potentials

UV breaking

$$V(H) = a(a-4)|H|^2$$

and

$$\lambda_0(H) = M_0|H|^2 + \gamma_0|H|^4$$

- The bulk EOM is solved by a linear combination of Whittaker functions

$$h(y) = c_W\, e^{2A} W_{\frac{-4}{(a-2)\nu^2},\frac{4-\nu^2}{2\nu^2}}[2(a-2)(y_s-y)]$$
$$+c_M\, e^{2A} M_{\frac{-4}{(a-2)\nu^2},\frac{4-\nu^2}{2\nu^2}}[2(a-2)(y_s-y)]$$

- Regularity and BC at UV imply (for $a \geq 2$)

BC at UV (for $a \geq 2$)

$$\text{Regularity} \Rightarrow c_W = 0$$

and for $h_0 = c_M M_{\frac{-4}{(a-2)\nu^2}, \frac{4-\nu^2}{2\nu^2}}[2(a-2)y_s]$

$$h_0^2 \simeq \frac{4-a-M_0}{\gamma_0}$$

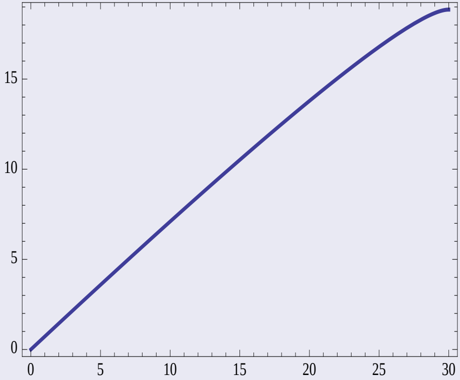
- h_0 (γ_0) is fixed by the EW condition

EWSB condition

$$v_{SM}^2 = \int_0^{y_s} h^2(y) e^{-2A(y)} dy$$

The Higgs background looks like:

$log_{10}\, h(y)/h(0)$ as a function of ky for $a = 2.1$, $\nu = 1.5$, $y_s = 30/k$



EWSB IN THE GAUGE SECTOR

We will illustrate the mechanism with an abelian example

- The Higgs is defined as

$$H(x,y) = \frac{1}{\sqrt{2}}[h(y) + \xi(x,y)]e^{ig_5\chi(x,y)}$$

- The action is invariant under 5D gauge transformations

5D gauge transformations

$$A_M(x,y) \to A_M(x,y) + \frac{1}{g_5}\partial_M\alpha(x,y)$$

$$\chi(x,y) \to \chi(x,y) + \frac{1}{g_5}\alpha(x,y)$$

- We will take the 5D gauge condition

$$\partial^\mu A_\mu - m_A^2\chi + (e^{-2A}A_5)' = 0, \quad m_A(y) = g_5 h(y) e^{-A(y)}$$

- The 4D theory is invariant under $\alpha(x) = \alpha(x,y)/f(y)$ gauge transformations and contains:
  - $$A_\mu(x,y) = \frac{a_\mu(x)\cdot f(y)}{\sqrt{y_s}}$$
  - $$G(x,y) = m_A^2\chi - \left(e^{-2A}A_5\right)' = \frac{m_f\, G(x)\cdot f(y)}{\sqrt{y_s}}$$
  - $$K(x,y) = \chi' - A_5 = \frac{K(x)\cdot \eta(y)}{\sqrt{y_s}}$$
- With profiles

Profiles

$$m_f^2 f + (e^{-2A} f')' - m_A^2 f = 0, \quad \text{Neumann BC}$$

$$m_\eta^2\eta + \left[m_A^{-2}\left(e^{-2A}m_A^2\eta\right)'\right]' - m_A^2\eta = 0, \quad \text{Dirichlet BC}$$

- We can find an approximation for the light gauge boson mode in the limit where the breaking is small and thus there is a light mode with almost constant profile

Analytical approximation

$$f_A^0(y) = 1 - \delta_A + \delta f_A(y)$$

$$\delta f_A(y) = \int_0^y dy'\, e^{2A(y')} \int_0^{y'} dy'' \left[m_A^2(y'') - m_{f_A^0}^2 \right]$$

$$\delta_A = \frac{1}{y_s} \int_0^{y_s} dy\, \delta f_A(y)$$

- The light mode mass

Mass of light mode

$$m_{f_A^0}^2 = \frac{1}{y_s} \int_0^{y_s} m_A^2(y) dy$$

ELECTROWEAK CONSTRAINTS

- In our 5D model (for fixed values of the parameters $\nu,\ y_s,\dots$) we have the free parameters (g_5, g_5', h_0, a) which fix the physical spectrum of zero mode masses
- Once we have fixed the condition $m_{f_Z} = m_Z^{(ph)}$ the eigenvalue m_{f_W} is a prediction of the theory
- The parameter

$$\rho_0 = \frac{m_{f_W}^2}{c_W^2 m_{f_Z}^2} \equiv 1 - \Delta\rho = 1 - s_W^2 \tilde{\delta}_Z$$

can deviate from unity which amounts to a violation of the custodial symmetry (CS)

$$\tilde{\delta}_V = \frac{m_V^2}{k^2} y_s \int_0^{y_s} \left(\Omega - \frac{y}{y_s}\right)^2 e^{2A} dy$$

$$\Omega(y) = \frac{U(y)}{U(y_s)}, \quad U'(y) = h^2(y) e^{-2A(y)}$$

- We will be assuming here (not necessarily an assumption) that fermions are localized on the UV brane in which case

$$g_V = g_V^{SM} f_V(0) \equiv g_V[1-\delta_V]$$

- The latter changes the definition of the Fermi constant measured in the μ-decay and the Z widths which constrain the

EWPT Parameters

$$\delta_Z = \frac{m_Z^2}{k^2} y_s \int_0^{y_s} dy\, e^{2A(y)} \left(1 - \frac{y}{y_s}\right) \left(\Omega - \frac{y}{y_s}\right)$$

$$\delta_W = c_W^2\, \delta_Z$$

through the observables $\bar{s}_\ell^2$, $\Gamma_{\ell^+\ell^-}, \ldots$

- We can also express the departure with respect to the SM predictions in the language of the usual parameters (S, T, U)
- It turns out that

$(S,\ T,\ U)$ parameters

$$\alpha(m_Z)T = s_W^2 \tilde{\delta}_Z$$

$$\frac{\alpha(m_Z)}{4 s_W^2 c_W^2} S = -2\delta_Z$$

$$\frac{\alpha(m_Z)}{4 s_W^2} (S + U) = -2\delta_W$$

- Or using the relation $\delta_W = c_W^2 \delta_Z$

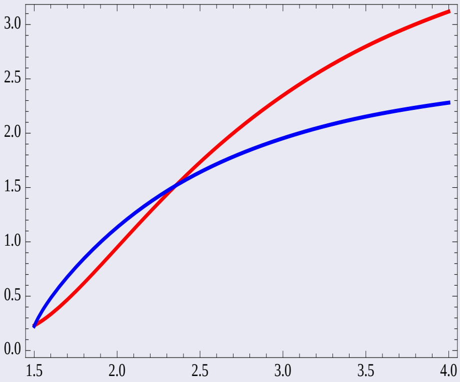
$$\alpha(m_Z)T = s_W^2 \tilde{\delta}_Z \ , \ \alpha(m_Z)S = -8 s_W^2 c_W^2 \delta_Z \ , \ \alpha(m_Z)U \simeq 0$$

- The strongest constraint is on T

NUMERICAL RESULTS

We can obtain a lower bound on m_{KK} from the bounds on T and S

$\alpha T, \alpha S < 10^{-3}$, $y_s = 35/k$, $a \simeq 2$: m_{KK}/TeV Vs ν



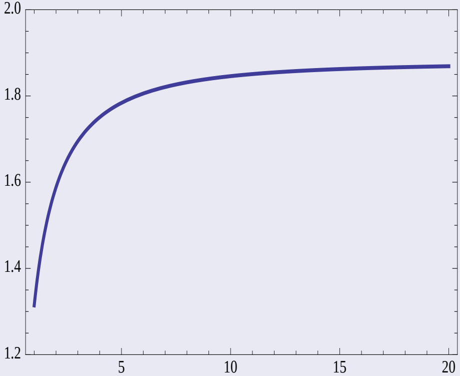
No explicit CS is required!

One of the reasons why the bounds go down when $\nu \longrightarrow 1$ is

$$h(y) \sim e^{a_{eff}(\nu,a)y}$$

and [for RS $a_{eff}(\infty, a) = a$]

a_{eff} Vs ν for $a = 2$



and the Higgs profile is much less IR localized than in the RS case

Conclusion

- We have proposed a set of soft-wall models with AdS geometry near the UV brane
- The large hierarchy is generated without fine-tuning by a background scalar field
- The limit $\nu \to \infty$ is RS
- For $\nu = 1$ the spectrum is continuum above a mass gap ρ and it can model unparticles
- We propose models of EWSB with a bulk Higgs
- Electroweak constraints can be satisfied in a way similar to the SM: no extra custodial symmetry has to be introduced
- Indirect and direct constraints can be satisfied for KK-masses of $\mathcal{O}(1)$ TeV
- The model is Higgsless and KK modes unitarize *WW* scattering (Pokorski's talk)
- One can also break EWS in the bulk with a light Higgs at the price of some fine tuning

Backup slides

- Expression for S, T simplify in the RS case as

$(S,\ T,\ U)$ parameters

$$\alpha(m_Z)T^{RS} = s_W^2 \frac{m_Z^2}{\rho^2}(ky_s)\frac{(a-1)^2}{a(2a-1)} + \dots$$

$$\alpha(m_Z)S^{RS} = 2s_W^2 \frac{m_W^2}{\rho^2}\frac{a^2-1}{a^2} + \dots$$

- T is volume enhanced while S is not
- No analytical expression exists for soft-wall metric