Recent news and results in the computation of NLO processes with new techniques

R. Pittau (U. of Granada)
Corfu, August 31, 2010
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Why (N)NLO calculations?
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2. New techniques
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2. New techniques
3. Issues on numerical stability
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1. Why (N)NLO calculations?
2. New techniques
3. Issues on numerical stability
4. Recent Results
Why (N)NLO QCD calculations?

- (N)NLO QCD calculations at Hadron Colliders are needed for:
  1. computing **Backgrounds** for **New Physics** Searches
  2. **Measurements** of fundamental quantities:
     \[
     \alpha_s \quad m_t \\
     M_W \quad M_H \quad \cdots
     \]

- Heavy **New Physics** states undergo long chain decays
- **SM Processes** accompanied by multi-jet activity

⇒

1. **multileg** (N)NLO calculations and MCs needed
2. for some processes the **full set of the EW radiative corrections** is also needed
1. Get the **best theoretical prediction** you can, whether
   - Basic Monte Carlo [PYTHIA, HERWIG, Sherpa, ...]
   - LO QCD parton level
   - LO QCD matched to parton showers [MadGraph/MadEvent, ALPGEN/PYTHIA, Sherpa, ...]
   - NLO QCD at parton level
   - NLO matched to parton showers [MC@NLO, POWHEG, ...]
   - NNLO inclusive at parton level
   - NNLO with flexible cuts at parton level

2. Take **ratios** whenever possible
   - QCD effects cancel when event kinematics are similar
   - Closely related to “data driven” strategies
Why (N)NLO

New techniques

Accuracy issues

Results

$W$ NNLO rapidity distribution at TEVATRON

Catani, Ferrera, Grazzini

Now the normalization is trustable
Tuning LO Monte Carlos with NLO calculations

Moretti, Piccinini, R. P., Treccani using MLM matching

ALPGEN vs Tevatron $W + j$ data
A typical $2 \rightarrow m$ process at NLO

$$\sigma^{NLO} = \int_{m} d\sigma^B + \int_{m} \left( d\sigma^V + \int_{1} d\sigma^A \right) + \int_{m+1} \left( d\sigma^R - d\sigma^A \right)$$

1. $d\sigma^B$ is the Born cross section
2. $d\sigma^V$ is the Virtual correction (loop diagrams)
3. $d\sigma^R$ is the Real correction
4. $d\sigma^A$ and $\int_{1} d\sigma^A$ are *unintegrated* and *integrated* counterterms (allowing to compute the Real part in 4 dimensions)
The Virtual corrections $d\sigma^V$

The decomposition of any 1-loop amplitude

\[ A = \sum_{i_0<i_1<i_2<i_3}^{m-1} d(i_0i_1i_2i_3) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}\bar{D}_{i_3}} \]

\[ + \sum_{i_0<i_1<i_2}^{m-1} c(i_0i_1i_2) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}} \]

\[ + \sum_{i_0<i_1}^{m-1} b(i_0i_1) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}} \]

\[ + \sum_{i_0}^{m-1} a(i_0) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}} + R \]

The problem is getting the set $S = \{ d(i_0i_1i_2i_3), c(i_0i_1i_2), b(i_0i_1), a(i_0), R \}$
The OPP Method (Ossola, Papadopoulos, Pittau, 2007)

Working at the integrand level

\[ A = \int d^n \bar{q} \ [\mathcal{A}(q) + \tilde{A}(q, \tilde{q}, \epsilon)] \]

\[
\begin{pmatrix}
\tilde{q} = q + \tilde{q} \\
n = 4 + \epsilon
\end{pmatrix}
\]

- For example, in the case of \( pp \rightarrow t\bar{t}b\bar{b} \)

\[
\mathcal{A}(q) = \sum \frac{N_i^{(6)}(q)}{\bar{D}_{i_0} \bar{D}_{i_1} \cdots \bar{D}_{i_5}} + \frac{N_i^{(5)}(q)}{\bar{D}_{i_0} \bar{D}_{i_1} \cdots \bar{D}_{i_4}} + \frac{N_i^{(4)}(q)}{\bar{D}_{i_0} \bar{D}_{i_1} \bar{D}_{i_2}} + \cdots
\]
The function to be sampled \textit{numerically} to extract the coefficients

\[ N^{(6)}_i(q) = \sum_{i_0 < i_1 < i_2 < i_3} \left[ d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4} D_{i_5} \]

\[ + \sum_{i_0 < i_1 < i_2} \left[ c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] D_{i_3} D_{i_4} D_{i_5} \]

\[ + \sum_{i_0 < i_1} \left[ b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] D_{i_2} D_{i_3} D_{i_4} D_{i_5} \]

\[ + \sum_{i_0} \left[ a(i_0) + \tilde{a}(q; i_0) \right] D_{i_1} D_{i_2} D_{i_3} D_{i_4} D_{i_5} \]

\[ + \tilde{P}(q) D_{i_0} D_{i_1} D_{i_2} D_{i_3} D_{i_4} D_{i_5} \]
Solving the OPP Equation 1

- The functional form of the *spurious* terms should be known
  del Aguila, R. P., JHEP 0407:017,2004

Example \((p_0 = 0)\)

\[
\tilde{d}(q;0123) = \tilde{d}(0123) \epsilon(qp_1p_2p_3)
\]

\[
\int d^n \bar{q} \frac{\tilde{d}(q;0123)}{D_0 D_1 D_2 D_3} = \tilde{d}(0123) \int d^n \bar{q} \frac{\epsilon(qp_1p_2p_3)}{D_0 D_1 D_2 D_3} = 0
\]

- The coefficients \(\{d_i, c_i, b_i, a_i\}\) and \(\{\tilde{d}_i, \tilde{c}_i, \tilde{b}_i, \tilde{a}_i\}\) are extracted by solving linear systems of equations
The use of special values of $q$ helps (Unitarity?)

$$D_0(q^\pm) = D_1(q^\pm) = D_2(q^\pm) = D_3(q^\pm) = 0$$

$$N^{(m-1)}(q^\pm) = \left[ d(0123) + \tilde{d}(q^\pm; 0123) \right] \prod_{i \neq 0,1,2,3}^{m-1} D_i(q^\pm)$$

$$d(0123) = \frac{1}{2} \left[ \frac{N^{(m-1)}(q^+)}{\prod_{i \neq 0,1,2,3}^{m-1} D_i(q^+)} + \frac{N^{(m-1)}(q^-)}{\prod_{i \neq 0,1,2,3}^{m-1} D_i(q^-)} \right]$$
What about $R \left(= R_1 + R_2 \right)$?

The OPP Solution:

**The origin of $R_1$**

\[
\frac{1}{\bar{D}_i} = \frac{1}{D_i} \left(1 - \frac{\bar{q}^2}{D_i}\right) \Rightarrow \text{predicted within OPP}
\]

**The origin of $R_2$**

\[
R_2 = \int d^n \bar{q} \frac{\bar{N}(q, \bar{q}, \epsilon)}{\bar{D}_0 \cdots \bar{D}_{m-1}} \Rightarrow \text{effective tree-level Feynman Rules up to 4 points *}
\]

* QCD: Draggiotis, Garzelli, Papadopoulos, R. P., JHEP 0904:072, 2009

EW: Garzelli, Malamos, R. P., JHEP 1001:040, 2010

EW in the $R_\xi$ and Unitary gauges: Garzelli, Malamos, R. P., soon
Recursion Relations at 1-loop (cutting 1 arbitrary leg)

- OPP + 1 hard-cut allow to use *the same tree-level Recursion Relations* for $m + 2$ tree-like structures

- The color can be treated *as at the tree level*

⇒ Tree level codes can be *transformed* into 1-loop ones
The Helac-NLO System

1. CutTools
   \( \{d_i, c_i, b_i, a_i\} \) and \( R_1 \)

2. HELAC-1LOOP
   \( N(q) \) and \( R_2 \)

3. OneLOop
   scalar 1-loop integrals

4. HELAC-DIPOLES
   Real correction and CS dipoles

- van Hameren, Papadopoulos, R. P., JHEP 0909 (2009) 106
- Czakon, Papadopoulos, Worek, JHEP 0908 (2009) 085
The HELAC-NLO group

G. Bevilacqua  M. Czakon  M. Garzelli
A. van Hameren  A. Kardos  A. Lazopoulos
J. Malamos  C.G. Papadopoulos  R. P.
M. Worek

Contributors

Caffarella  Draggiotis  Kanaki  Ossola
Testing and improving the numerical accuracy of the NLO predictions (1006.3773 [hep-ph])

1. To trust multi-leg NLO calculations one has to trust the numerical accuracy (especially for the Virtual Part)
2. To use multi-precision always is CPU-wise inviable
3. I present a new and reliable method to test the numerical accuracy of NLO calculations based on modern OPP/Generalized Unitarity techniques
4. A convenient solution to rescue most of the detected numerically inaccurate points is also proposed

Key point: These non standard techniques have the potential to self detect stability problems
The “N=N” test

Since a reconstruction of a function is involved in the OPP method

\[ N(q') = N_{\text{rec}}(q') \]

at an *independent* value of \( q' \) allows (in principle) to test the goodness of the set of coefficients


Also the fact that combinations of coefficients should sum up to zero can be used

Mastrolia, Ossola, Reiter, Tramontano (2010)

1. The *arbitrariness* of \( q' \) introduces a unwanted, parameter upon which the check depends in an unpredictable way

2. Not *all* the reconstructed coefficients enter into the actual computation
If we could get \textbf{independently} the set of the \underline{non spurious} coefficients

$$S' = \left\{ d'(i_0i_1i_2i_3), c'(i_0i_1i_2), b'(i_0i_1), a'(i_0), R'_1 \right\}$$

an \underline{independent} determination would become possible

$$A' = \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} d'(i_0i_1i_2i_3) \int d^n \bar{q} \frac{1}{\bar{D}_{i_0} \bar{D}_{i_1} \bar{D}_{i_2} \bar{D}_{i_3}}$$

$$+ \sum_{i_0 < i_1 < i_2}^{m-1} c'(i_0i_1i_2) \int d^n \bar{q} \frac{1}{\bar{D}_{i_0} \bar{D}_{i_1} \bar{D}_{i_2}}$$

$$+ \sum_{i_0 < i_1}^{m-1} b'(i_0i_1) \int d^n \bar{q} \frac{1}{\bar{D}_{i_0} \bar{D}_{i_1}}$$

$$+ \sum_{i_0}^{m-1} a'(i_0) \int d^n \bar{q} \frac{1}{\bar{D}_{i_0}} + R'_1 + (R_2)$$

\Rightarrow \text{a reliable estimator of the accuracy}

$$E^A \equiv \frac{|A - A'|}{|A|}$$
1. The way to obtain $S'$ is similar to the technique used to determine $R_1$

2. Under a shift $m_i^2 \to m_i^2 - \tilde{q}^2$ in the denominators of the OPP equation (testing the same $N(q)$ at shifted values)

\[
\begin{align*}
\bar{c}(i_0i_1i_2) &= c(i_0i_1i_2) + \tilde{q}^2 c^{(2)}(i_0i_1i_2) \\
\bar{b}(i_0i_1) &= b(i_0i_1) + \tilde{q}^2 b^{(2)}(i_0i_1) \\
\bar{a}(i_0) &= a(i_0)
\end{align*}
\]

Incidentally

\[
R_1 = -\frac{i}{96\pi^2} d^{(2m-4)} - \frac{i}{32\pi^2} \sum_{i_0 < i_1 < i_2} c^{(2)}(i_0i_1i_2) - \frac{i}{32\pi^2} \sum_{i_0 < i_1} b^{(2)}(i_0i_1) \left( m_{i_0}^2 + m_{i_1}^2 - \frac{(p_{i_0} - p_{i_1})^2}{3} \right)
\]
Why (N)NLO

New techniques

Accuracy issues

Results

1. Under a new mass shift $m_i^2 \rightarrow m_i^2 - \tilde{q}_1^2$

\[
\bar{c}_1(i_0i_1i_2) = c(i_0i_1i_2) + \tilde{q}_1^2 c^{(2)}(i_0i_1i_2)
\]

\[
\bar{b}_1(i_0i_1) = b(i_0i_1) + \tilde{q}_1^2 b^{(2)}(i_0i_1)
\]

\[
\bar{a}_1(i_0) = a(i_0)
\]

\[\Rightarrow\]

\[
a'(i_0) = \bar{a}_1(i_0)
\]

\[
b'(i_0i_1) = \frac{\bar{b}(i_0i_1) + \bar{b}_1(i_0i_1)}{2} - \frac{\tilde{q}^2 + \tilde{q}_1^2}{2} b^{(2)}(i_0i_1)
\]

\[
c'(i_0i_1i_2) = \frac{\bar{c}(i_0i_1i_2) + \bar{c}_1(i_0i_1i_2)}{2} - \frac{\tilde{q}^2 + \tilde{q}_1^2}{2} c^{(2)}(i_0i_1i_2)
\]

2. Analogously one obtains independent determinations of box coefficients and $R_1$, namely the whole set $S'$
One can fit $N_{rec}(q)$ instead of $N(q)$

$\Rightarrow$ very moderate CPU cost
Testing the Estimators $E^A \equiv \frac{|A-A'|}{|A|}$ and $E^N \equiv \frac{|N-N_{rec}|}{|N|}$

1. 3000 P.S. Points for 1 FD of $\gamma\gamma \rightarrow 4\gamma$ with CutTools
2. Ratio of True Precision/Estimator (NO CUTS):

\[
\text{Log}_{10} \left( \frac{P_d}{E_d^X} \right)
\]
Rescuing the inaccurate points

Fitting the set $S$ in multi-precision *while keeping* $N(q)$ *in double precision* (important for interfacing)
Using $E^A$ to rescue only the inaccurate points

$E_{\text{lim}}$ is a threshold value of $E^A$ above which the MP routines activate for the fit only while keeping $N(q)$ in double precision. A new $E^{A'}$ is then computed that way and the event is discarded if $E^{A'} > E_{\text{lim}}$. 

\[ E_{\text{lim}} = 10^{-4} \]

\[ E_{\text{lim}} = 10^{-3} \]

\[ E_{\text{lim}} = 0.5 \times 10^{-2} \]

\[ E_{\text{lim}} = 10^{-2} \]
The number of recomputed and discarded points

<table>
<thead>
<tr>
<th>$E_{\text{lim}}$</th>
<th>$N_{mp}$</th>
<th>$N_{dis}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-4}$</td>
<td>90</td>
<td>14</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>62</td>
<td>8</td>
</tr>
<tr>
<td>$.5 \times 10^{-2}$</td>
<td>44</td>
<td>6</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>40</td>
<td>6</td>
</tr>
</tbody>
</table>

Over a total of 3000 points

Concluding remarks

1. The rescue procedure is able to recover most of the inaccurate points
2. The estimator $E^A$ efficiently detects and discards the unrecoverable points
A NLO analysis of $ttH$ production vs $ttbb$ and $ttjj$ backgrounds at 14 TeV with HELAC-NLO

Cross sections at NLO

\[ pp \rightarrow t\bar{t}b\bar{b} + X \]

<table>
<thead>
<tr>
<th>( \sigma^B_{LO} ) [fb]</th>
<th>( \sigma^B_{NLO} ) [fb]</th>
<th>( K )-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1489.2 ± 0.9</td>
<td>2642 ± 3</td>
<td>1.77</td>
</tr>
</tbody>
</table>

\[ \mu_R = \mu_F = \mu_0 = m_t \text{ (CTEQ6)} \]

\[ pp \rightarrow t\bar{t}H + X \rightarrow t\bar{t}b\bar{b} + X \]

<table>
<thead>
<tr>
<th>( \sigma^S_{LO} ) [fb]</th>
<th>( \sigma^S_{NLO} ) [fb]</th>
<th>( K )-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>150.375 ± 0.077</td>
<td>207.268 ± 0.150</td>
<td>1.38</td>
</tr>
</tbody>
</table>

\[ \mu_R = \mu_F = \mu_0 = m_t + m_H/2 \text{ (CTEQ6)} \]

- \( p_T(b) > 20 \text{ GeV} \), \( \Delta R(b, \bar{b}) > 0.8 \), \( |\eta_b| < 2.5 \)
Distributions at NLO

**Why (N)NLO**
- New techniques
- Accuracy issues

**Results**

**Distributions at NLO**

**NLO Signal**

**LO Signal**

**NLO ttbb Background**

**LO ttbb Background**

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Recent news and results in the computation of NLO processes
Scale dependence of the $ttbb$ Background
Scale dependence of the Signal

- NLO
- LO
- NLO (jet veto)
The effect of a jet veto on the Signal/Background ratio

The extra radiation is mainly at low $p_T$ and in the central region.

- With $p_T(j) < 50$ GeV:
  
  \[
  (S/B)_{LO} = 0.10 \quad (S/B)_{NLO-veto} = 0.064 \quad (S/B)_{NLO} = 0.079
  \]
Scale dependence of the $ttjj$ Background

\[
\sigma(ttjj)_{LO} = 120.17 (8) \text{ pb} \quad \mu_R = \mu_F = \mu_0 = m_t \quad \text{(CTEQ6)}
\]

\[
\sigma(ttjj)_{NLO} = 106.97(17) \text{ pb}
\]
$m_{jj}$ distribution of the $ttjj$ Background

Recent news and results in the computation of NLO processes
Hardest jet $p_T$ distribution of the $ttjj$ Background

$\frac{d\sigma}{dp_{T,j}}$ [pb/GeV] vs $p_{T,j}$ [GeV]

NLO vs LO
NLO QCD corrections to $pp \rightarrow W^+ \rightarrow e^+\nu_e$ at the 7 TeV with HELAC-NLO

Parameters

$$\sqrt{s} = 7 \text{ TeV} \quad p_T(\ell^\pm) > 1 \text{ GeV}$$

$$|\eta(\ell^\pm)| < 5 \quad \mu_F = \mu_R = M_W$$

Results cross-checked with MCFM
The Cross section

\[ \sigma_{LO} = 4737.7(1.0) ^{-492.2 (10\%)} \]
\[ +426.9 (9\%) \text{ pb} \]

\[ \sigma_{NLO} = 5670.6(1.6) ^{-85.8 (1.5\%)} \]
\[ +107.5 (1.9\%) \text{ pb} \]

The \( m_{e\nu} \) and \( y_{e\nu} \) distributions

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Recent news and results in the computation of NLO processes
The $p_t(e^+)$ and $y(e^+)$ distributions

The $p_{t,miss}$ distribution

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Recent news and results in the computation of NLO processes
Conclusions

1. **New techniques and ideas** allowed an impressive progress in the field of NLO calculations
   - I discussed the OPP method
   - I discussed a way to test/improve the numerical stability of OPP/Generalized Unitarity based computations

2. I presented results obtained for LHC physics in the framework of the HELAC-NLO system
   - An NLO analysis of $ttbb$ Higgs signal vs $ttbb$ and $ttjj$ background at 14 TeV
   - $pp \rightarrow W^+ \rightarrow e^+\nu_e$ at 7 TeV

3. The final goal is delivering **public NLO codes** (matched with Parton shower) useful for analyzing the data