Recent news and results in the computation of NLO processes with new techniques

R. Pittau (U. of Granada)
Corfu, August 31, 2010

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2. New techniques
3. Issues on numerical stability

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1. Why (N)NLO calculations?
2. New techniques
3. Issues on numerical stability
4. Recent Results

Why (N)NLO QCD calculations?

- (N)NLO QCD calculations at Hadron Colliders are needed for:

  1. computing Backgrounds for New Physics Searches
  2. Measurements of fundamental quantities:

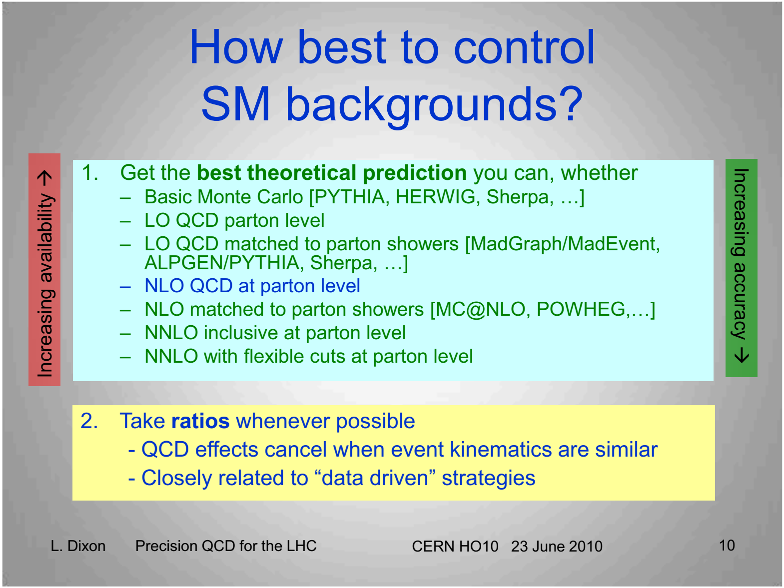
$$\begin{array}{ccc} \alpha_s & m_t & \\ M_W & M_H & \cdots \end{array}$$

- Heavy New Physics states undergo long chain decays
- SM Processes accompanied by multi-jet activity

$$\Longrightarrow$$

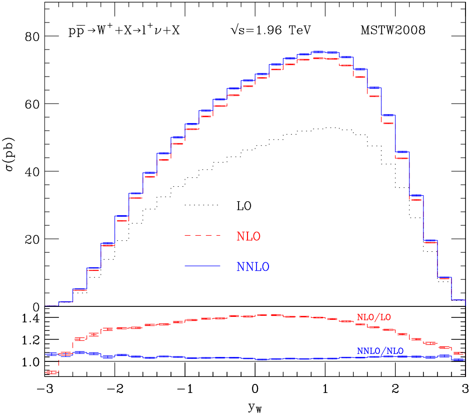
1. multileg (N)NLO calculations and MCs needed
2. for some processes the full set of the EW radiative corrections is also needed

From Dixon's talk at HO-2010



W NNLO rapidity distribution at TEVATRON

Catani, Ferrera, Grazzini

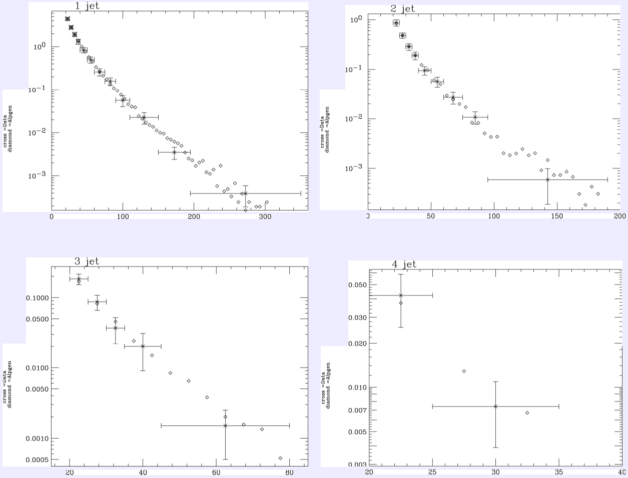


- Now the normalization is trustable

Tuning LO Monte Carlos with NLO calculations

Moretti, Piccinini, R. P., Treccani using MLM matching

ALPGEN vs Tevatron $W + j$ data



A typical $2 \to m$ process at NLO

$$\sigma^{NLO} = \int_m d\sigma^B + \int_m \left(d\sigma^V + \int_1 d\sigma^A\right) + \int_{m+1} \left(d\sigma^R - d\sigma^A\right)$$

1. $d\sigma^B$ is the Born cross section
2. $d\sigma^V$ is the Virtual correction (loop diagrams)
3. $d\sigma^R$ is the Real correction
4. $d\sigma^A$ and $\int_1 d\sigma^A$ are *unintegrated* and *integrated* counterterms (allowing to compute the Real part in 4 dimensions)

The Virtual corrections $d\sigma^V$

The decomposition of any 1-loop amplitude

$$
\begin{aligned}
A = & \sum_{i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}\bar{D}_{i_3}} \\
& + \sum_{i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}} \\
& + \sum_{i_0<i_1}^{m-1} b(i_0 i_1) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}} \\
& + \sum_{i_0}^{m-1} a(i_0) \int d^n\bar{q} \frac{1}{\bar{D}_{i_0}} + R
\end{aligned}
$$

The problem is getting the set $\mathcal{S} = \begin{cases} d(i_0 i_1 i_2 i_3), & c(i_0 i_1 i_2), \\ b(i_0 i_1), & a(i_0), \qquad R \end{cases}$

The OPP Method (Ossola, Papadopoulos, Pittau, 2007)

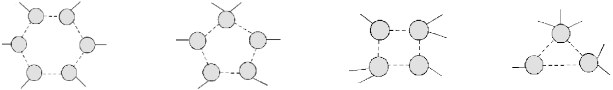
Working at the *integrand* level

$$A = \int d^n\bar{q} \; \left[\mathcal{A}(q) + \tilde{A}(q, \tilde{q}, \epsilon)\right]$$

$$\left(\begin{array}{c} \bar{q} = q + \tilde{q} \\ n = 4 + \epsilon \end{array} \right)$$

- For example, in the case of $pp \to t\bar{t}b\bar{b}$

$$\mathcal{A}(q) = \sum \underbrace{\frac{N_i^{(6)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_5}}} + \underbrace{\frac{N_i^{(5)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_4}}} + \underbrace{\frac{N_i^{(4)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_3}}} + \underbrace{\frac{N_i^{(3)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}}} + \cdots$$



The function to be sampled *numerically* to extract the coefficients

$$
\begin{aligned}
N_i^{(6)}(q) &= \sum_{i_0<i_1<i_2<i_3}^{5} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4} D_{i_5} \\
&+ \sum_{i_0<i_1<i_2}^{5} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] D_{i_3} D_{i_4} D_{i_5} \\
&+ \sum_{i_0<i_1}^{5} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] D_{i_2} D_{i_3} D_{i_4} D_{i_5} \\
&+ \sum_{i_0}^{5} \left[a(i_0) + \tilde{a}(q; i_0) \right] D_{i_1} D_{i_2} D_{i_3} D_{i_4} D_{i_5} \\
&+ \tilde{P}(q) D_{i_0} D_{i_1} D_{i_2} D_{i_3} D_{i_4} D_{i_5}
\end{aligned}
$$

Solving the OPP Equation 1

- The functional form of the *spurious* terms should be known

  Ossola, Papadopoulos, R. P., Nucl.Phys.B763:147-169,2007

  del Aguila, R. P., JHEP 0407:017,2004

Example ($p_0 = 0$)

$$\tilde{d}(q; 0123) = \tilde{d}(0123)\,\epsilon(q p_1 p_2 p_3)$$

$$\int d^n \bar{q} \frac{\tilde{d}(q; 0123)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} = \tilde{d}(0123) \int d^n \bar{q} \frac{\epsilon(q p_1 p_2 p_3)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} = 0$$

- The coefficients $\{d_i, c_i, b_i, a_i\}$ and $\{\tilde{d}_i, \tilde{c}_i, \tilde{b}_i, \tilde{a}_i\}$ are extracted by solving linear systems of equations

Solving the OPP Equation 2

The use of special values of q helps (Unitarity?)

$$D_0(q^\pm) = D_1(q^\pm) = D_2(q^\pm) = D_3(q^\pm) = 0$$

$$N^{(m-1)}(q^\pm) = \left[d(0123) + \tilde{d}(q^\pm; 0123)\right] \prod_{i \neq 0,1,2,3}^{m-1} D_i(q^\pm)$$

$$d(0123) = \frac{1}{2}\left[\frac{N^{(m-1)}(q^+)}{\prod_{i \neq 0,1,2,3}^{m-1} D_i(q^+)} + \frac{N^{(m-1)}(q^-)}{\prod_{i \neq 0,1,2,3}^{m-1} D_i(q^-)}\right]$$

What about R (= R_1 + R_2)?

The OPP Solution:

The origin of R_1

$$\frac{1}{\bar{D}_i} = \frac{1}{D_i}\left(1 - \frac{\tilde{q}^2}{\bar{D}_i}\right)$$ ⇒ predicted within OPP

The origin of R_2

$$R_2 = \int d^n\bar{q}\,\frac{\tilde{N}(q,\tilde{q},\epsilon)}{\bar{D}_0\cdots\bar{D}_{m-1}}$$ ⇒ effective tree-level Feynman Rules

up to 4 points *

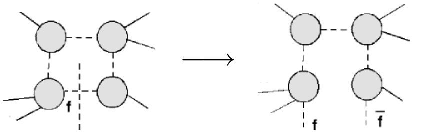
* QCD: Draggiotis, Garzelli, Papadopoulos, R. P., JHEP 0904:072,2009

EW: Garzelli, Malamos, R. P., JHEP 1001:040,2010

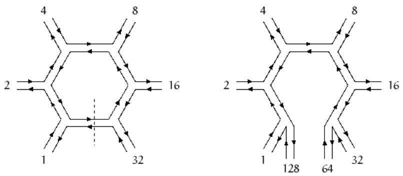
EW in the R_ξ and Unitary gauges: Garzelli, Malamos, R. P., soon

Recursion Relations at 1-loop (cutting 1 arbitrary leg)

- OPP + 1 hard-cut allow to use *the same tree-level Recursion Relations* for $m + 2$ tree-like structures



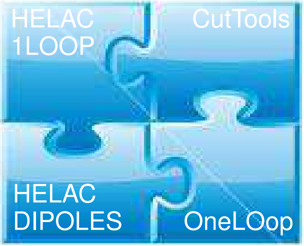
- The color can be treated *as at the tree level*



⇒ Tree level codes can be *transformed* into 1-loop ones

The Helac-NLO System

1. CutTools
   $\{d_i, c_i, b_i, a_i\}$ and R_1
2. HELAC-1LOOP
   $N(q)$ and R_2
3. OneLOop
   scalar 1-loop integrals
4. HELAC-DIPOLES
   Real correction and CS dipoles



(figure by G. Bevilacqua)

- Ossola, Papadopoulos, R. P., JHEP 0803 (2008) 042
- van Hameren, Papadopoulos, R. P., JHEP 0909 (2009) 106
- Czakon, Papadopoulos, Worek, JHEP 0908 (2009) 085

The HELAC-NLO group *

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Contributors

Caffarella Draggiotis Kanaki Ossola

Testing and improving the numerical accuracy of the NLO predictions (1006.3773 [hep-ph])

1. To trust multi-leg NLO calculations one has to trust the numerical accuracy (especially for the Virtual Part)
2. To use multi-precision always is CPU-wise inviable
3. I present a new and reliable method to test the numerical accuracy of NLO calculations based on modern OPP/Generalized Unitarity techniques
4. A convenient solution to *rescue* most of the detected numerically inaccurate points is also proposed

Key point: These non standard techniques have the potential to self detect stability problems

The "N=N" test

Since a reconstruction of a function is involved in the OPP method

$$N(q') = N_{rec}(q')$$

at an *independent* value of q' allows (in principle) to test the goodness of the set of coefficients

Ossola, Papadopoulos, R. P. (2007)

Also the fact that combinations of coefficients should sum up to zero can be used

Mastrolia, Ossola, Reiter, Tramontano (2010)

1. The *arbitrariness* of q' introduces a unwanted, parameter upon which the check depends in an unpredictable way
2. Not *all* the reconstructed coefficients enter into the actual computation

If we could get independently the set of the non spurious coefficients

$$\mathcal{S}' = \left\{ \begin{array}{lll} d'(i_0 i_1 i_2 i_3)\,, & c'(i_0 i_1 i_2)\,, & \\ b'(i_0 i_1)\,, & a'(i_0)\,, & R'_1 \end{array} \right.$$

an independent determination would become possible

$$\begin{aligned} A' \quad = \quad & \sum_{i_0<i_1<i_2<i_3}^{m-1} d'(i_0 i_1 i_2 i_3) \int d^n \bar{q} \frac{1}{\bar{D}_{i_0} \bar{D}_{i_1} \bar{D}_{i_2} \bar{D}_{i_3}} \\ & + \sum_{i_0<i_1<i_2}^{m-1} c'(i_0 i_1 i_2) \int d^n \bar{q} \frac{1}{\bar{D}_{i_0} \bar{D}_{i_1} \bar{D}_{i_2}} \\ & + \sum_{i_0<i_1}^{m-1} b'(i_0 i_1) \int d^n \bar{q} \frac{1}{\bar{D}_{i_0} \bar{D}_{i_1}} \\ & + \sum_{i_0}^{m-1} a'(i_0) \int d^n \bar{q} \frac{1}{\bar{D}_{i_0}} + R'_1 + (R_2) \end{aligned}$$

⇒ a reliable estimator of the accuracy

$$E^A \equiv \frac{|A - A'|}{|A|}$$

1. The way to obtain $\mathcal{S}'$ is similar to the technique used to determine R_1
2. Under a shift $m_i^2 \to m_i^2 - \tilde{q}^2$ in the denominators of the OPP equation (testing the *same* $N(q)$ at shifted values)

$$\begin{aligned}\bar{c}(i_0 i_1 i_2) &= c(i_0 i_1 i_2) + \tilde{q}^2 c^{(2)}(i_0 i_1 i_2) \\ \bar{b}(i_0 i_1) &= b(i_0 i_1) + \tilde{q}^2 b^{(2)}(i_0 i_1) \\ \bar{a}(i_0) &= a(i_0)\end{aligned}$$

Incidentally

$$\begin{aligned}R_1 &= -\frac{i}{96\pi^2} d^{(2m-4)} - \frac{i}{32\pi^2} \sum_{i_0<i_1<i_2}^{m-1} c^{(2)}(i_0 i_1 i_2) \\ &- \frac{i}{32\pi^2} \sum_{i_0<i_1}^{m-1} b^{(2)}(i_0 i_1) \left(m_{i_0}^2 + m_{i_1}^2 - \frac{(p_{i_0} - p_{i_1})^2}{3} \right)\end{aligned}$$

1. Under a new mass shift $m_i^2 \to m_i^2 - \tilde{q}_1^2$

$$\begin{aligned}
\bar{c}_1(i_0 i_1 i_2) &= c(i_0 i_1 i_2) + \tilde{q}_1^2 c^{(2)}(i_0 i_1 i_2) \\
\bar{b}_1(i_0 i_1) &= b(i_0 i_1) + \tilde{q}_1^2 b^{(2)}(i_0 i_1) \\
\bar{a}_1(i_0) &= a(i_0)
\end{aligned}$$

$$\Longrightarrow$$

$$\begin{aligned}
a'(i_0) &= \bar{a}_1(i_0) \\
b'(i_0 i_1) &= \frac{\bar{b}(i_0 i_1) + \bar{b}_1(i_0 i_1)}{2} - \frac{\tilde{q}^2 + \tilde{q}_1^2}{2}\, b^{(2)}(i_0 i_1) \\
c'(i_0 i_1 i_2) &= \frac{\bar{c}(i_0 i_1 i_2) + \bar{c}_1(i_0 i_1 i_2)}{2} - \frac{\tilde{q}^2 + \tilde{q}_1^2}{2}\, c^{(2)}(i_0 i_1 i_2)
\end{aligned}$$

2. Analogously one obtains independent determinations of box coefficients and R_1, namely the whole set $\mathcal{S}'$

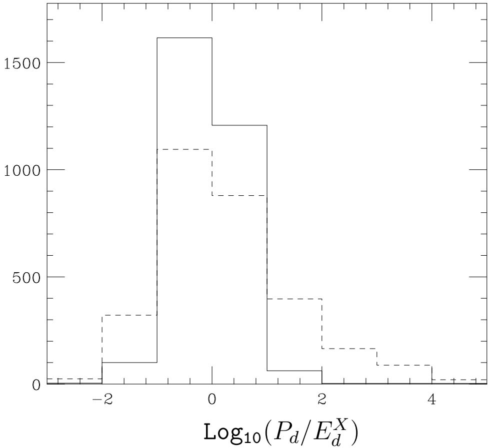
Important

One can fit $N_{rec}(q)$ *instead of* $N(q)$

$\Rightarrow$ *very moderate CPU cost*

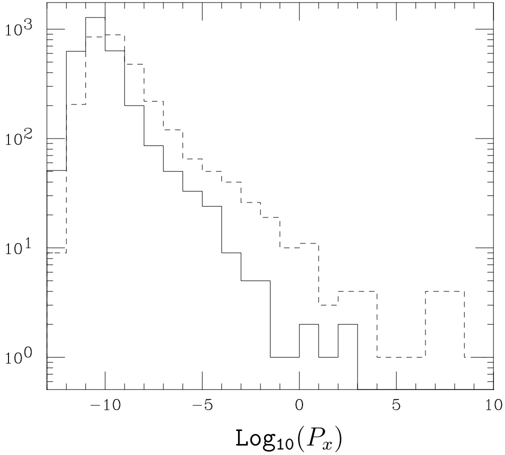
Testing the Estimators $E^A = \frac{|A-A'|}{|A|}$ and $E^N = \frac{|N-N_{rec}|}{|N|}$

1. 3000 P.S. Points for 1 FD of $\gamma\gamma \to 4\gamma$ with `CutTools`
2. Ratio of True Precision/Estimator (NO CUTS):



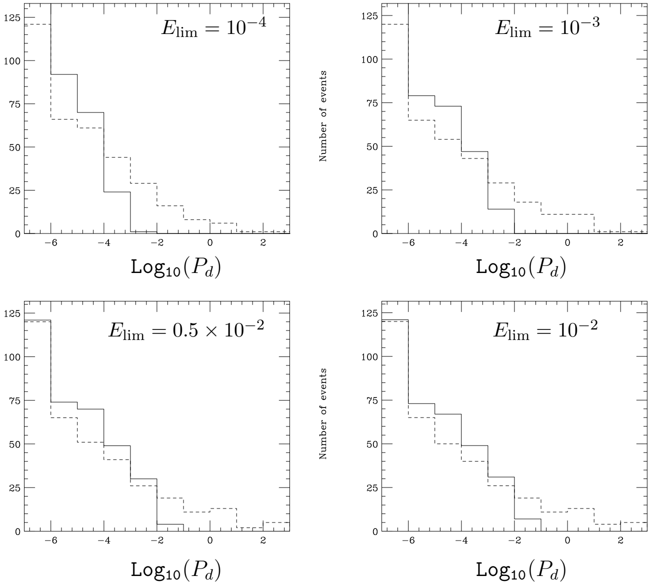
Rescuing the inaccurate points

1. Fitting the set $\mathcal{S}$ in multi-precision *while keeping $N(q)$ in double precision* (important for interfacing)



Using E^A to rescue *only* the inaccurate points

E_{lim} is a threshold value of E^A above which the MP routines activate for the fit only while keeping $N(q)$ in double precision.
A new $E^{A\prime}$ is then computed that way and the event is discarded if $E^{A\prime} > E_{lim}$



The number of recomputed and discarded points

E_{lim}	N_{mp}	N_{dis}
10^{-4}	90	14
10^{-3}	62	8
$.5 \times 10^{-2}$	44	6
10^{-2}	40	6

Over a total of 3000 points

Concluding remarks

1. The rescue procedure is able to recover most of the inaccurate points
2. The estimator E^A efficiently detects and discards the unrecoverable points

A NLO analysis of ttH production vs $ttbb$ and $ttjj$ backgrounds at 14 TeV with HELAC-NLO

Based on arXiv:1003.1241 [hep-ph],
Phys.Rev.Lett.104:162002,2010 and JHEP 0909:109,2009

Cross sections at NLO

$pp \to t\bar{t}b\bar{b} + X$

σ^B_{LO} [fb]	σ^B_{NLO} [fb]	K-factor
1489.2 ± 0.9	2642 ± 3	1.77

$\mu_R = \mu_F = \mu_0 = m_t$ (CTEQ6)

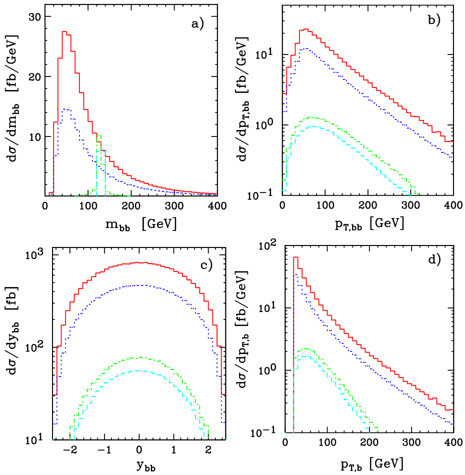
$pp \to t\bar{t}H + X \to t\bar{t}b\bar{b} + X$

σ^S_{LO} [fb]	σ^S_{NLO} [fb]	K-factor
150.375 ± 0.077	207.268 ± 0.150	1.38

$\mu_R = \mu_F = \mu_0 = m_t + m_H/2$ (CTEQ6)

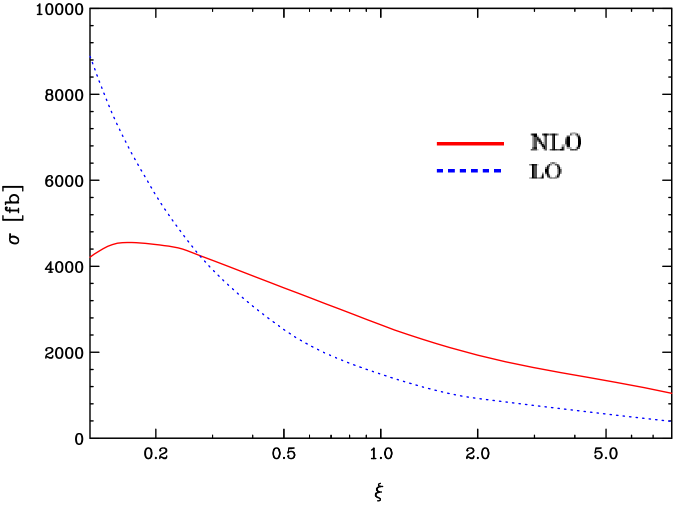
- $p_T(b) > 20$ GeV, $\Delta R(b, \bar{b}) > 0.8$, $|\eta_b| < 2.5$

Distributions at NLO

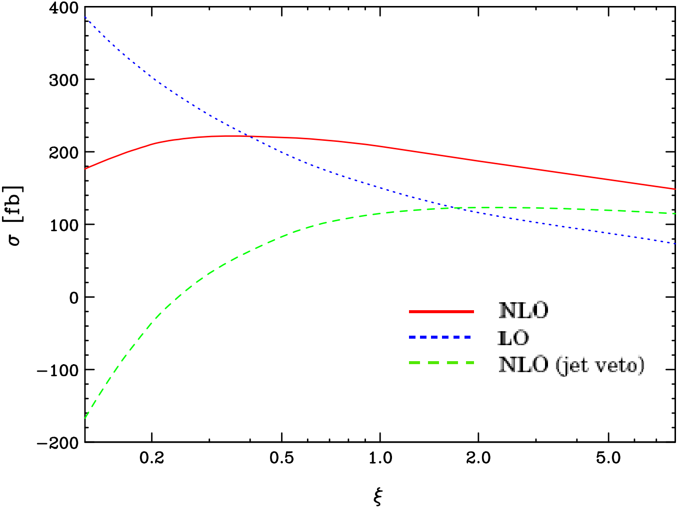


_____ NLO Signal _____ NLO $ttbb$ Background
_____ LO Signal _____ LO $ttbb$ Background

Scale dependence of the $ttbb$ Background



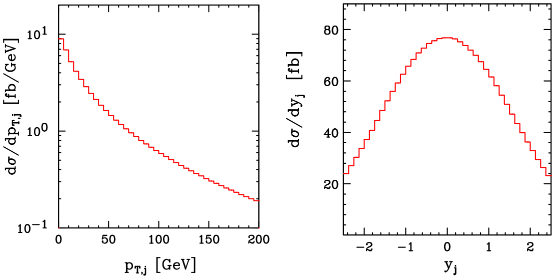
Scale dependence of the Signal



The effect of a jet veto on the Signal/Background ratio

The extra radiation is mainly at low p_T and in the central region

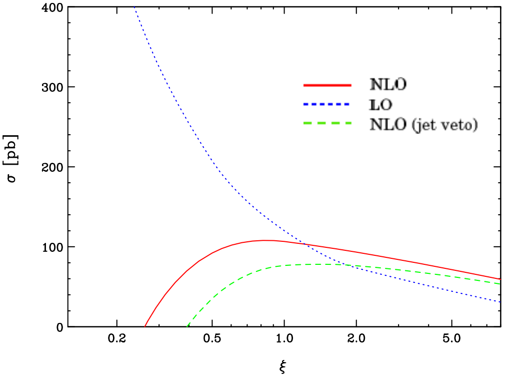
Signal



- With $p_T(j) < 50$ GeV:

$$(S/B)_{LO} = 0.10 \qquad (S/B)_{NLO-veto} = 0.064$$
$$(S/B)_{NLO} = 0.079$$

Scale dependence of the $ttjj$ Background

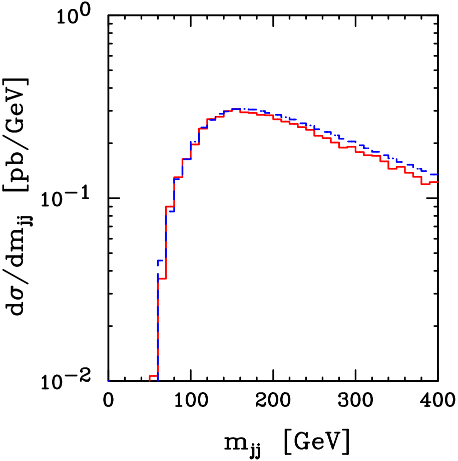


$$\sigma(ttjj)_{LO} = 120.17\ (8)\ \text{pb}$$

$$\sigma(ttjj)_{NLO} = 106.97(17)\ \text{pb}$$

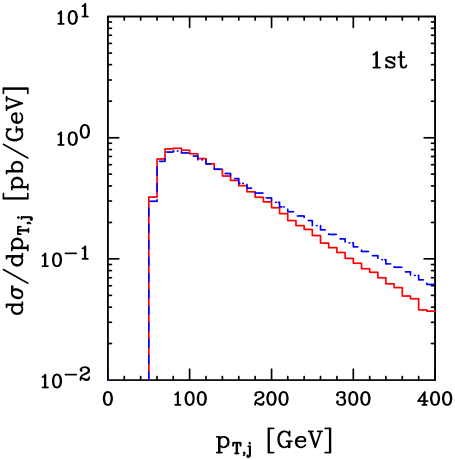
$\mu_R = \mu_F = \mu_0 = m_t$ (CTEQ6)

m_{jj} distribution of the $ttjj$ Background



_____ NLO _____ LO

Hardest jet p_T distribution of the $ttjj$ Background



_____ NLO _____ LO

NLO QCD corrections to $pp \to W^+ \to e^+\nu_e$ at the 7 TeV with HELAC-NLO

Parameters

$$\sqrt{s} = 7 \text{ TeV} \quad p_T(\ell^\pm) > 1 \text{ GeV}$$
$$|\eta(\ell^\pm)| < 5 \quad \mu_F = \mu_R = M_W$$

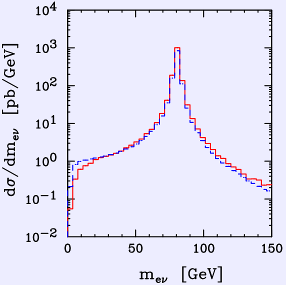
Results cross-checked with MCFM

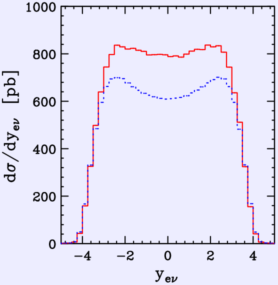
The Cross section

$$\sigma_{LO} = 4737.7(1.0) \begin{array}{c} -492.2\ (10\%) \\ +426.9\ (9\%) \end{array} \text{pb}$$

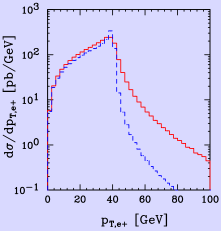
$$\sigma_{NLO} = 5670.6(1.6) \begin{array}{c} -85.8\ (1.5\%) \\ +107.5\ (1.9\%) \end{array} \text{pb}$$

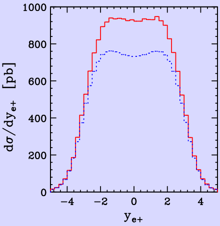
The $m_{e\nu}$ and $y_{e\nu}$ distributions



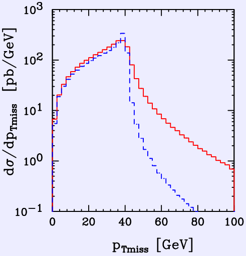


The $p_t(e^+)$ and $y(e^+)$ distributions





The $p_{t,miss}$ distribution



Conclusions

1. New techniques and ideas allowed an impressive progress in the field of NLO calculations
   - I discussed the OPP method
   - I discussed a way to test/improve the numerical stability of OPP/Generalized Unitarity based computations
2. I presented results obtained for LHC physics in the framework of the HELAC-NLO system
   - An NLO analysis of $ttbb$ Higgs signal vs $ttbb$ and $ttjj$ background at 14 TeV
   - $pp \to W^+ \to e^+ \nu_e$ at 7 TeV
3. The final goal is delivering public NLO codes (matched with Parton shower) useful for analyzing the data