On Models of Quantum Spacetime and Covariance

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Outline

Doubly or Singly?
- Planck Length and Lorentz-Fitzgerald (LF) contraction
- LF, UR and covariance on quantum spacetime

Canonical Quantum Spacetime and (Twisted) Covariance
- Canonical Quantum Spacetime (reminder)
- “Older” Approaches to Covariance
- Twisted Covariance (reminder)
- Back to Groupoids

Enough of θ! What about κ? (joint work with L. Dabrowski)
- Another (undeformed) Poincaré covariant model
- A general situation

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Planck Length and Lorentz-Fitzgerald (LF) contraction

Commutation relations among coordinates are imposed to define a noncommutative replacement of the classical localisation algebra $\mathscr{C}_\infty(\mathbb{R}^{d+1})$=complex continuous functions of $\mathbb{R}^d$, vanishing at infinity.

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Commutations relations typically involve one parameter, assumed to be of order of Planck length $\lambda_P \sim 10^{-33}$cm (reason: with $a(m)$=Compton wavelength and $b(m)$=Schwarzschild radius, $a(m) \sim b(m)$ has solution m=Planck mass, in which case $a \sim b \sim$Planck length.)

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- true for lattice models
- for non commuting coordinates, it depends on the model; there are counterexamples (e.g. the seminal DFR model, cf Doplicher's talk).

LF, UR and covariance on quantum spacetime

In DFR model (1994), coordinates $\boldsymbol{q}^\mu$ (s.a. ops) are covariant under a unitary representation of the Poincaré group, which means

$$\boldsymbol{U}(\Lambda, a)^{-1}\boldsymbol{q}^\mu \boldsymbol{U}(\Lambda, a) = \Lambda^\mu{}_\nu \boldsymbol{q}^\nu + a^\mu.$$

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This is not in contrast with the CR being driven by a dimensionful parameter. Hence, **Singly Special Relativity** may well be already **Doubly Special**! It depends on the model. DSR does not force us into the realm of modified/broken/violated covariance.

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In general, the uncertainty

$$\boldsymbol{A} \mapsto \Delta(\boldsymbol{A}) := \sqrt{\langle \boldsymbol{A}^2 \rangle - \langle \boldsymbol{A} \rangle^2}$$

is not a linear functional ($\boldsymbol{A}$ a generic operator). Hence, despite the notations, $\Delta(\boldsymbol{q}^\mu)$ is not a covariant 4-vector: $\Delta(\Lambda^\mu{}_\nu \boldsymbol{q}^\nu) \neq \Lambda^\mu{}_\nu \Delta(\boldsymbol{q}^\nu)$.
No necessary contradiction between Uncertainty Relations (UR) driven by dimensionful parameters and LF.

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Existence of a minimal length too, is not in contradiction with LF in covariant models (cf Doplicher's talk).

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Weyl quantisation (from ordinary functions to ops):

$$f(\boldsymbol{x}) := \int dk\, \check{f}(k) e^{ik_\mu \boldsymbol{q}^\mu}, \quad \text{where} \quad \check{f}(k) := \int \frac{dx}{(2\pi)^4}\, f(x) e^{-ik_\mu x^\mu}.$$

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Classical functions equipped with non local, noncommutative Star-product defined by:

$$f(\boldsymbol{q}) g(\boldsymbol{q}) = (f \star_\sigma g)(\boldsymbol{q}),$$

viz.

$$(f \star_\sigma g)(x) = \int dk\, e^{ik_\mu x^\mu} \int dh\, \check{f}(h) \check{g}(k-h) e^{-\frac{i}{2} h_\mu \theta^{\mu\nu} k_\nu},$$

which equals the usual Moyal expansion on analytic symbols. This gives a noncommutative algebra $\mathscr{A}_\sigma$.

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2. we fix a choice of σ_0 in a given reference frame (“privileged”), and in any other frame we take the corresponding tensor transform σ of σ_0.
   We thus get a collection of algebras $\{\mathscr{A}_\sigma\}$ labeled by the matrices σ in the orbit Σ of the initial σ_0.
   The action of the Lorentz transformations can be seen as a groupoid, connecting pairs of algebras: if Λ sends σ_1 to σ_2, we have a corresponding isomorphism from $\mathscr{A}_{\sigma_1} \to \mathscr{A}_{\sigma_2}$. Two different such arrows can be combined if the tip of the first is the same as the tail of the second. Observers are not equivalent.

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3. We consider $\{\mathscr{A}_\sigma\}$ as a bundle of algebras; the star product is defined fibrewise on sections:

$$(F \star G)(\sigma; \cdot) = F(\sigma; \cdot) \star_\sigma G(\sigma; \cdot)$$

   For appropriate choice of the orbit Σ, this is DFR. Action of Poincaré group by automorphism of the algebra of sections is NOT fibrewise. Observers are equivalent.

Twisted Covariance (reminder)

An apparently different approach is provided by twisted covariance. Pick the privileged frame corresponding to σ_0; the star product can be written as

$$f \star_{\sigma_0} g = m_{\sigma_0}(f \otimes g) = (m \circ \mathcal{F}_{\sigma_0})(f \otimes g)$$

for suitable linear operator $\mathcal{F}_{\sigma_0}$ on $\mathscr{A}_{\sigma_0} \otimes \mathscr{A}_{\sigma_0}$.

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Define usual action on functions

$$(\alpha(\Lambda, a)f)(x) = f(\Lambda^{-1}(x - a)), \quad \alpha^{(2)}(\Lambda, a) = \alpha_{(\Lambda,a)} \otimes \alpha_{(\Lambda,a)}$$

and twist the action $\alpha^{(2)}$ as:

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Then by direct check we have twisted covariance of the product

$$\alpha(\Lambda, a) \circ m_{\sigma_0} = m_{\sigma_0} \circ \alpha^{(2)}_{\sigma_0}(\Lambda, a),$$

which may be regarded as deformation of usual covariance $\alpha(\Lambda, a) \circ m = m \circ \alpha^{(2)}(\Lambda, a)$.

Back to Groupoids

However, we have the simple relation

$$\alpha^{(2)}(\Lambda, a) \circ \mathcal{F}_{\sigma_0} = \mathcal{F}_{\sigma} \circ \alpha^{(2)}(\Lambda, a), \qquad \sigma^{\mu\nu} = \Lambda^{\mu}{}_{\mu'} \Lambda^{\nu}{}_{\nu'} \sigma_0^{\mu'\nu'}.$$

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It follows that l.h.s. of twisted covariance is the same as

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Yet another equivalent description of the same situation (which I regard as the most transparent) is: take the full DFR model, with an additional rule for selecting the admissible localisation states: those which, restricted to the σ dependence only, appear as delta measures around σ_0 to the privileged observer.

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The natural question then is: why should we reject those states?

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Consider the relations

$$[\boldsymbol{X}^\mu, \boldsymbol{X}^\nu] = i(\boldsymbol{V}^\mu(\boldsymbol{X} - \boldsymbol{A})^\nu - \boldsymbol{V}^\nu(\boldsymbol{X} - \boldsymbol{A})^\mu),$$
$$[\boldsymbol{X}^\mu, \boldsymbol{V}^\nu] = [\boldsymbol{X}^\mu, \boldsymbol{A}^\nu] = [\boldsymbol{A}^\mu, \boldsymbol{V}^\nu] = 0,$$

complemented with the constraint $\boldsymbol{V}_\mu \boldsymbol{V}^\mu = \boldsymbol{I}$.

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Statement: there exist a universal representation of the above relations by selfadjoint operators, which is covariant under a unitary representation of the Poincaré group, namely

$$\boldsymbol{U}(\Lambda, a)^{-1} \boldsymbol{X}^\mu \boldsymbol{U}(\Lambda, a) = \Lambda^\mu{}_\nu \boldsymbol{X}^\nu + a^\mu,$$
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We look for irreducible representations; by Schur's lemma, $\boldsymbol{V} = v\boldsymbol{I}, \boldsymbol{A} = a\boldsymbol{I}$. In particular there is the solution $v_0 = (1, 0, 0, 0), a_0 = 0$, which gives the well known κ-Minkowski relations

$$[\boldsymbol{X}^0_{(0)}, \boldsymbol{X}^j_{(0)}] = i\boldsymbol{X}^j_{(0)}.$$

Fix an irrep $\boldsymbol{X}_{(0)}$of κ-Minkowski. For each $L = (\Lambda_L, a_L)$,

$$\boldsymbol{X}_{(L)} = L\boldsymbol{X}_{(0)}, \quad \boldsymbol{V}_{(L)} = \Lambda_L v_{(0)} \boldsymbol{I}, \quad A_{(L)} = L a_{(0)} \boldsymbol{I},$$

is an irrep of our new model. This gives a covariant family of irreps, labeled by the orbit

$$\Xi = \{v \in \mathbb{R}^4 : v^\mu v_\mu = 1\} \times \mathbb{R}^4$$

under the action

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This construction goes along the same idea underlying crossed products (also known as covariance algebras).

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4. These comments are not meant to defend "Wigner orthodoxy" on symmetries in the noncommutative setting. New ideas may be necessary to find out The Way. But there is much to understand about motivations and interpretations.

References

1. **[arXiv:0901.3109]** Short paper on relations DFR/Tw. Cov.
2. **[arXiv:0902.0575]** Technical paper on relations DFR/Tw. Cov.
3. **[arXiv:1004.5261]** Review on DFR model + general comment on Tw.Cov
4. **[arXiv:1004.5091]** Canonical κ-Minkowski **(with L. Dąbrowski)**
5. **[arXiv:0912.5451]** Lorentz Covariant κ-Minkowski + Deformed Lorentz covariance **(with L. Dąbrowski and M. Godliński)**
6. **[arXiv:1006.5658]** Poincaré Covariant κ-Minkowski **(with L. Dąbrowski)**