Beta Functions of the (Gauged) Nonlinear Sigma Model

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$$S = \frac{\zeta}{2} \int d^4x \, h_{\alpha\beta}(\phi) \partial_\mu \phi^\alpha \partial^\mu \phi^\beta$$

$$h_{\alpha\beta}(\phi) = h_{\alpha\beta}(0) + \partial_\gamma h_{\alpha\beta}(0) \phi^\gamma + \frac{1}{2} \partial_\gamma \partial_\sigma h_{\alpha\beta}(0) \phi^\gamma \phi^\sigma + \cdots$$

$$\begin{aligned} \phi^{\alpha\prime} &= \phi^{\alpha\prime}(\phi) \\ h'_{\alpha\beta}(\phi') &= \frac{\partial \phi^\gamma}{\partial \phi^{\alpha\prime}} \frac{\partial \phi^\sigma}{\partial \phi^{\beta\prime}} h_{\gamma\sigma}(\phi) \end{aligned}$$

Symmetries:

$$\delta\phi^\alpha = K^\alpha(\phi)$$

$$\nabla_\alpha K_\beta + \nabla_\beta K_\alpha = 0$$

K^I generates $\mathrm{Iso}\,\mathcal{M}$:

$$\left[K^I, K^J\right] = -f_{IJL} K^L$$

Chiral model:

$$\mathcal{M} = SU(N)$$
$$\text{Iso}\,\mathcal{M} = SU(N)_L \otimes SU(N)_R$$

$$K^I = \left(R^i, L^i\right)$$

Right-invariant vector fields R^i generate left-multiplication,

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$O(N)$ model:

$$\mathcal{M} = S^{N-1} = O(N)/O(N-1)$$
$$\text{Iso}\,\mathcal{M} = O(N)$$

Background field expansion:

$$\phi^\alpha = \bar{\phi}^\alpha + \pi^\alpha$$

$$\xi^\alpha \in \mathcal{TM} \text{ s.t. } \mathrm{Exp}_{\bar{\phi}(x)}\xi^\alpha(x) = \bar{\phi}^\alpha(x) + \pi^\alpha(x)$$

yields the functional

$$S[\phi] = S[\bar{\phi}, \xi]$$

Codello&Percacci [arXiv:0810.0715]
$SU(N)$-model:

$$\Gamma_k = \frac{\zeta}{2}\int d^4x\, h_{\alpha\beta}(\phi)\partial_\mu\phi^\alpha\partial^\mu\phi^\beta$$

$$\zeta = k^2\bar{\zeta}$$

$$\beta_{\bar{\zeta}} = -2\bar{\zeta} + \frac{N}{32\pi^2}$$

Percacci&Z.[arXiv:0910.0851]
Higher derivative in $d = 4$:

$$\Gamma_k = \frac{1}{2}\int d^4x \Big[\zeta\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta h_{\alpha\beta}(\varphi) + \lambda^{-1}\Box\varphi^\alpha\Box\varphi^\beta h_{\alpha\beta}(\varphi) + \partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta\partial_\nu\varphi^\gamma\partial^\nu\varphi^\delta\lambda^{-1}T_{\alpha\beta\gamma\delta}(\varphi)\Big]$$

$\mathcal{M} = S^D$; at 1-loop $\nu = 1/2$;

$$T_{\alpha\beta\gamma\delta} = f_1 \delta_{\alpha(\gamma}\delta_{\beta)\delta} + f_2 \delta_{\alpha\beta}\delta_{\gamma\delta}$$

For $D > 2$:

NFP1, NFP2 ($1/\zeta^\star \neq 0; \lambda = 0; f_i \neq 0$)

GFP1, GFP2 ($1/\zeta^\star = 0; \lambda = 0; f_i \neq 0$)

$\mathcal{M} = SU(N)$; at 1-loop $\nu = 1/2$;
$T_{\alpha\beta\gamma\delta} = f_1 \delta_{\alpha(\gamma}\delta_{\beta)\delta} + f_2 \delta_{\alpha\beta}\delta_{\gamma\delta} + f_3 f_{\alpha(\gamma}{}^{\epsilon} f_{\beta)\delta\epsilon} + f_4 d_{\alpha(\gamma}{}^{\epsilon} d_{\beta)\delta\epsilon}$

For $N = 2, 3$:
NFP1, NFP2, GFP1, GFP2

For $N > 3$:
No FPs!

Gauging:

$$\begin{aligned} \delta\phi^\alpha &= -\epsilon^I K_I^\alpha(\phi) \\ \delta A_\mu^I &= \partial_\mu \epsilon^I + f_{JL}{}^I A_\mu^J \epsilon^L \end{aligned}$$

$$D_\mu \phi^\alpha = \partial_\mu \phi^\alpha + A_\mu^I K_I^\alpha(\phi)$$

$$F_{\mu\nu}^I = \partial_\mu A_\nu^I - \partial_\nu A_\mu^I + f^I{}_{JL} A_\mu^J A_\nu^L$$

Background field expansion:

$$A_\mu^I = \bar{A}_\mu^I + a_\mu^I$$

$$\mathrm{Exp}_{\bar{\phi}(x)} \xi^\alpha(x) = \bar{\phi}^\alpha(x) + \pi^\alpha(x)$$

yields the functional

$$S[\phi, A] = S[\bar{\phi}, \xi, \bar{A}, a]$$

$$\begin{aligned} D_\mu \xi^\alpha &= \partial_\mu \xi^\alpha + \partial_\mu \phi^\beta \Gamma_\beta{}^\alpha{}_\delta \xi^\delta + A_\mu^I \nabla_\beta K_I^\alpha \xi^\beta \\ D_\mu a_\nu^I &= \partial_\mu a_\nu^I + f^I{}_{JL} A_\mu^J a_\nu^L \end{aligned}$$

Fabbrichesi,Percacci,Tonero&Z.
$SU(N)_L$-gauged model:

$$\Gamma_k = \frac{\zeta}{2}\int d^4x\, h_{\alpha\beta}(\phi) D_\mu\phi^\alpha D^\mu\phi^\beta + \frac{1}{4g^2}\int d^4x\, F^I_{\mu\nu}F^{I\mu\nu}$$

$$D_\mu\phi^\alpha = \partial_\mu\phi^\alpha + A^i_\mu R^\alpha_i(\phi)$$

$$S_{g.f.} = \frac{1}{2\alpha g^2}\int d^4x\, f^i f^i$$

$$f_i = D_\mu a^i_\mu + \alpha g^2 \zeta R^i_\beta \xi^\beta$$

$$S_{ghost} = \int d^4x\, \bar{c}(-D^2 + \alpha g^2\zeta)c + \dots$$

At 1-loop:

$$\begin{aligned}
\beta_{\bar\zeta} &= -2\bar\zeta + \frac{N}{16\pi^2}\Big[\frac{1}{2}\frac{1}{1+\alpha\bar\zeta g^2} + \zeta g^2\Big(\frac{3}{4}\frac{1}{(1+\bar\zeta g^2)(1+\alpha\bar\zeta g^2)^2} \\
&\quad + \frac{3}{4}\frac{1}{(1+\bar\zeta g^2)^2(1+\alpha\bar\zeta g^2)} + \frac{\alpha}{2}\frac{1}{(1+\alpha\bar\zeta g^2)^3}\Big)\Big] \\
\beta_g &= -\frac{Ng^3}{32\pi^2}\Big[\frac{20}{3}\frac{1}{1+\bar\zeta g^2} + \frac{1}{3}\left(2-\frac{1}{4}\right)\frac{1}{1+\alpha\bar\zeta g^2}\Big]
\end{aligned}$$

Work in progress
$SU(2)_L \times U(1)_R$ model (χEW model):

$$D_\mu \phi^\alpha = \partial_\mu \phi^\alpha + A^i_\mu R^\alpha_i(\phi) - B_\mu L^\alpha_3(\phi)$$

$$\begin{aligned}\Gamma_k &= \frac{1}{2}\zeta \int d^4x \, h_{\alpha\beta} D_\mu \phi^\alpha D^\mu \phi^\beta \\ &+ \frac{1}{4g^2}\int d^4x \, F^i_{\mu\nu} F^{\mu\nu}_i + \frac{1}{4g'^2}\int d^4x \, B_{\mu\nu} B^{\mu\nu} \\ &+ \frac{S}{32\pi^2}\int d^4x \, B^{\mu\nu} W^i_{\mu\nu} R^\alpha_i L_{3\alpha} - \frac{e^2 T}{8\pi^2}\int d^4x \, D_\mu \phi^\alpha D_\mu \phi^\beta L^3_\alpha L^3_\beta\end{aligned}$$

S and T fixed points

$$T^\star = -\frac{3g'^2}{16\pi e^2} \qquad S^\star = \frac{1}{24\pi}$$

are repulsive.

Summary:

- one loop beta functions of NLSM in d=4 in two- and four-derivative truncations
- nontrivial fixed points
- one loop beta functions of left gauged chiral model
- one loop beta functions of (bosonic sector of) χEWPT

Ongoing work

- inclusion of fermions
- unitarity bounds
- phenomenological applications