Asymptotic Safety in Scalar theories coupled to Gravity

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Asymptotic Safety Scenario - General Ideas

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- Theory is predictive if only finite number of parameters need to be fixed by experiments

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- If $\tilde{g}_i^* = 0$ for all $i \Rightarrow$ Asymptotically free (*e.g.* Yang-Mills theories), perturbation theory reliable in UV
- If some or all $\tilde{g}_i^* \neq 0 \Rightarrow$ Perturbation theory not reliable $\Rightarrow$ non-perturbative methods employed $\Rightarrow$ Asymptotically safe

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$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\left[\left(\frac{\delta^2 \Gamma_k}{\delta\Phi\,\delta\Phi} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k\right]$$

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$$t = \ln k$$
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- We use truncations to extract information from it.

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- Matter inclusions

Minimal: FP existence $\Rightarrow$ Constraints on number of matter fields (Percacci & Perini)

Nonminimal scalar $\Rightarrow$ FP existence and Finite UV critical surface (Percacci & Perini, Narain & Percacci, Narain & Rahmede)

Scalar coupled to EH-gravity

Truncation Considered (Narain & Percacci '09)

$$\Gamma_k[g,\phi] = \int d^dx\,\sqrt{g}\left(V(\phi^2) - F(\phi^2)R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right) + S_{GF} + S_{gh}$$

where

$$S_{GF} = \frac{1}{2\alpha}\int d^dx\,\sqrt{\bar{g}}\,F(\phi^2)\bar{g}^{\mu\nu}\,\chi_\mu\,\chi_\nu$$

$$\chi^\mu = \left(\bar{\nabla}_\nu h^{\nu\mu} - \frac{\beta+1}{d}\bar{\nabla}^\mu h\right) \quad .$$

$$S_{GH} = -\int d^dx\,\sqrt{\bar{g}}\,\bar{C}^\mu\left[\delta^\rho_\mu\bar{\Box} + \left(1 - \frac{2(1+\beta)}{d}\right)\bar{\nabla}_\mu\bar{\nabla}^\rho + \bar{R}^\rho_\mu\right]C_\rho \quad .$$

Background field method: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$. Background: d-sphere

EH + Scalar: Fixed Point (FP)

- FRGE $\Rightarrow \partial_t V$ and $\partial_t F$

$$\partial_t V(\phi^2) = \frac{1}{Vol}\,\partial_t \Gamma_k \Big|_{R=0} \,, \qquad \partial_t F(\phi^2) = -\frac{1}{Vol}\,\frac{\partial(\partial_t \Gamma_k)}{\partial R}\Big|_{R=0} \,.$$

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- Define dimensionless variables and dimensionless functions:

$$\begin{aligned}\tilde{\phi} &= k^{\frac{2-d}{2}}\,\phi \\ \tilde{R} &= k^{-2}\,R\end{aligned} \qquad\qquad \begin{aligned}\tilde{V}(\tilde{\phi}^2) &= k^{-d}\,V(\phi^2) \\ \tilde{F}(\tilde{\phi}^2) &= k^{2-d}\,F(\phi^2)\end{aligned}$$

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$$\begin{aligned} (\partial_t \tilde{V})[\tilde{\phi}^2] &= -d\,\tilde{V}(\tilde{\phi}^2) + (d-2)\,\tilde{\phi}^2\,\tilde{V}'(\tilde{\phi}^2) + k^{-d}\,(\partial_t V)[\phi^2] \\ (\partial_t \tilde{F})[\tilde{\phi}^2] &= -(d-2)\,\tilde{F}(\tilde{\phi}^2) + (d-2)\,\tilde{\phi}^2\,\tilde{F}'(\tilde{\phi}^2) + k^{-(d-2)}\,(\partial_t F)[\phi^2] \end{aligned}$$

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$$(\partial_t \tilde{F})[\tilde{\phi}^2] = -(d-2)\,\tilde{F}(\tilde{\phi}^2) + (d-2)\,\tilde{\phi}^2\,\tilde{F}'(\tilde{\phi}^2) + k^{-(d-2)}\,(\partial_t F)[\phi^2]$$

At FP: $\partial_t \tilde{V} = 0$ and $\partial_t \tilde{F} = 0$

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Make Ansatz:

$$
\begin{aligned}
V &= k^d\, \tilde{\lambda}_0 && \text{Equation } n \geq 1 \Rightarrow \text{Identically Satisfied} \\
F &= k^{d-2}\, \tilde{\xi}_0 && n = 0 \text{ only nontrivial equation left} \Rightarrow \text{GMFP}
\end{aligned}
$$

$$
\tilde{V}^{(i)} = 0, \qquad \tilde{F}^{(i)} = 0, \; \forall\, i > 0
$$

Gravity is minimally coupled at GMFP.

EH+Scalar: Linearized Flow around GMFP

Taylor expand V and F around $\phi^2 = 0$

$$V(\phi^2) = \sum_{n=0}^{\infty} \lambda_{2n}\, \phi^{2n} \;\; ; \qquad F(\phi^2) = \sum_{n=0}^{\infty} \xi_{2n}\, \phi^{2n} \,.$$

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Introduce notation: $V_0 = V$ and $V_1 = -F$. Dimensionless potentials: $\tilde{V}_a = k^{d-2a}\, V_a$, where a is either 0 or 1.

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$$(M_{ij})_{ab} = \left.\frac{\delta\left(\frac{1}{i!}\partial_t \tilde{V}_a^{(i)}(0)\right)}{\delta\left(\frac{1}{j!}\tilde{V}_b^{(j)}(0)\right)}\right|_{FP} \;\; ; \;\; M = \begin{pmatrix} M_{00} & M_{01} & 0 & 0 & \cdots \\ 0 & M_{11} & M_{12} & 0 & \cdots \\ 0 & 0 & M_{22} & M_{23} & \cdots \\ 0 & 0 & 0 & M_{33} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

EH+Scalar: Stability Matrix Properties

Each M_{ij} is a 2×2 matrix. Nonzero entries are related:

$$M_{ii} = (d-2)\, i + M_{00} \;\; ; \qquad M_{i,i+1} = (i+1)\,(2i+1)\, M_{01}$$

(Observed numerically, proved mathematically at one-loop)

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M_{00} and M_{01} knowledge at GMFP $\Rightarrow$ Full stability matrix is known at GMFP. Eigenvalues of M are eigenvalues of M_{ii}

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$$e_i^{(a)} = (d-2)\, i + e_0^{(a)} \tag{1}$$

where $e_i^{(a)}$ is the a-th eigenvalue of M_{ii}

EH + Scalar, Numerical results: GMFP in $d = 4$

In $d = 4$ for De-donder gauge ($\alpha = 0$, $\beta = \frac{d}{2} - 1$)
The FP Solution

$$\tilde{\lambda}_0^* = 0.00862 \;\; ; \qquad \tilde{\xi}_0^* = 0.02375 \;\; .$$

critical exponents ($\theta_i^{(a)} = -e_i^{(a)}$) are,

$$2.143 \pm 2.879i \;\; , \qquad 0.143 \pm 2.879i$$

The eigenvectors in this truncation are

$$\begin{pmatrix} 0.3557 \pm 0.3776i \\ 0.8549 \\ 0 \\ 0 \end{pmatrix} \;\; , \qquad \begin{pmatrix} (-18.059 \pm 7.310i) \times 10^{-4} \\ (-30.723 \pm 10.763i) \times 10^{-4} \\ 0.3557 \pm 0.3776i \\ 0.8549 \end{pmatrix}$$

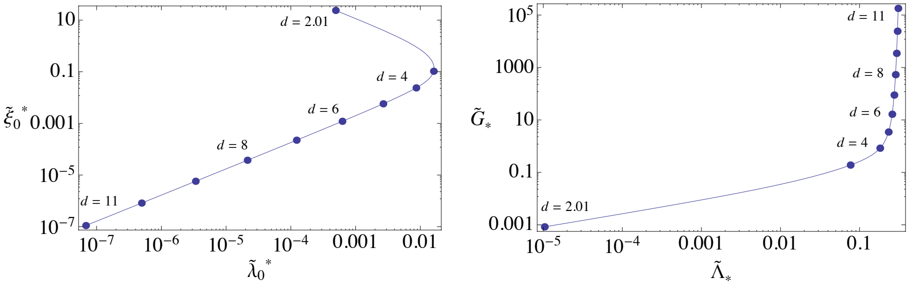
EH + Scalar, Numerical results: GMFP in any d

FP in various dimensions in De-donder gauge

d	$\tilde{\lambda}_0^*$	$\tilde{\xi}_0^*$	$\tilde{\Lambda}^*$	$\tilde{G}^*$	$\theta_1,\ \theta_2$
2.001	4.968×10^{-5}	2.386×10^{2}	1.041×10^{-7}	8.339×10^{-5}	2.001, 0.001
3	1.605×10^{-2}	1.047×10^{-1}	7.666×10^{-2}	1.900×10^{-1}	$1.627 \pm 0.754\ i$
4	8.620×10^{-3}	2.375×10^{-2}	1.814×10^{-1}	8.375×10^{-1}	$2.143 \pm 2.879\ i$
5	2.669×10^{-3}	5.744×10^{-3}	2.323×10^{-1}	3.463	$3.236 \pm 4.996\ i$
6	6.230×10^{-4}	1.207×10^{-3}	2.581×10^{-1}	1.648×10	$4.818 \pm 7.039\ i$
7	1.225×10^{-4}	2.235×10^{-4}	2.740×10^{-1}	8.900×10	$6.744 \pm 9.004\ i$
8	2.133×10^{-5}	3.738×10^{-5}	2.853×10^{-1}	5.322×10^{2}	$8.945 \pm 10.904\ i$
9	3.380×10^{-6}	5.747×10^{-6}	2.941×10^{-1}	3.462×10^{3}	$11.396 \pm 12.748\ i$
10	4.960×10^{-7}	8.228×10^{-7}	3.014×10^{-1}	2.418×10^{4}	14.089 ± 14.537
11	6.817×10^{-8}	1.107×10^{-7}	3.079×10^{-1}	1.797×10^{5}	$17.025 \pm 16.261\ i$

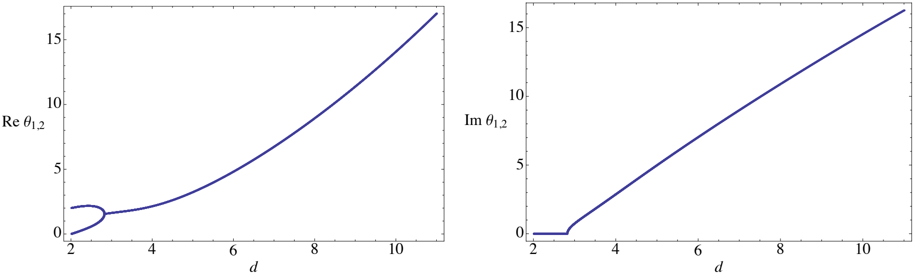
EH+Scalar, Numerical results: GMFP in any d

Graphical representation of FP in various dimensions



EH+Scalar, Numerical results: GMFP in any d

The critical exponents in various dimensions



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- A new FP where both gravity and matter non-trivial

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- FP with matter trivial and gravity non-trivial (GMFP)
- FP with matter non-trivial and gravity trivial (Wilson Fisher)
- A new FP where both gravity and matter non-trivial
- This third FP is systematically studied in the following way

$$V(\phi^2) = \sum_{n=0}^{a} \lambda_{2n} \phi^{2n} \quad ; \qquad F(\phi^2) = \sum_{n=0}^{b} \xi_{2n} \phi^{2n} \,.$$

EH+Scalar: Other Non-trivial FP

(a, b)	$\tilde\lambda_0^*$	$\tilde\lambda_2^*$	$\tilde\lambda_4^*$	$\tilde\lambda_6^*$	$\tilde\lambda_8^*$	$\tilde\xi_0^*$	$\tilde\xi_2^*$	$\tilde\xi_4^*$	$\tilde\xi_6^*$	$\tilde\xi_8^*$
(2,1)	0.0196	-0.1646	-0.1595			0.1088	-0.03108			
(3,1)	0.01994	-0.1758	-0.1958	-0.2796		0.1096	-0.03810			
(4,1)	0.02002	-0.1783	-0.2041	-0.3466	-0.5579	0.1098	-0.03969			
(2,2)	0.01894	-0.1408	-0.1241			0.1071	-0.01122	0.04297		
(3,2)	0.01971	-0.1680	-0.1848	-0.2879		0.1089	-0.03131	0.01731		
(4,2)	0.01988	-0.1735	-0.1975	-0.3544	-0.5687	0.1093	-0.03542	0.01121		
(3,3)	0.01911	-0.1469	-0.1469	-0.1935		0.1074	-0.01420	0.05017	0.1617	
(4,3)	0.01953	-0.1618	-0.1768	-0.3083	-0.6569	0.1084	-0.02571	0.03197	0.1102	
(4,4)	0.01923	-0.1512	-0.1572	-0.2496	-0.4911	0.1077	-0.01728	0.04732	0.1765	0.3868

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(2,2)	0.01894	-0.1408	-0.1241			0.1071	-0.01122	0.04297		
(3,2)	0.01971	-0.1680	-0.1848	-0.2879		0.1089	-0.03131	0.01731		
(4,2)	0.01988	-0.1735	-0.1975	-0.3544	-0.5687	0.1093	-0.03542	0.01121		
(3,3)	0.01911	-0.1469	-0.1469	-0.1935		0.1074	-0.01420	0.05017	0.1617	
(4,3)	0.01953	-0.1618	-0.1768	-0.3083	-0.6569	0.1084	-0.02571	0.03197	0.1102	
(4,4)	0.01923	-0.1512	-0.1572	-0.2496	-0.4911	0.1077	-0.01728	0.04732	0.1765	0.3868

(a, b)	θ_1'	θ_1''	θ_3	θ_4	θ_5	θ_6	θ_7			
(2,1)	1.648	0.592	-0.956	-3.902	-13.46					
(3,1)	1.650	0.554	-1.079	-3.776	-11.20	-29.397				
(4,1)	1.650	0.543	-1.105	-3.673	-10.02	-24.01	-49.31			
(a, b)	θ_1'	θ_1''	θ_2'	θ_2''	θ_5	θ_6	θ_7	θ_8		
(2,2)	1.649	0.656	-7.979	1.261	-0.559	-3.192				
(3,2)	1.652	0.589	-7.933	3.909	-0.835	-3.578	-27.67			
(4,2)	1.652	0.570	-7.635	4.083	-0.898	-3.626	-22.64	- 47.78		
(a, b)	θ_1'	θ_1''	θ_2'	θ_2''	θ_3'	θ_3''	θ_7	θ_8	θ_9	
(3,3)	1.649	0.641	-6.703	2.097	-14.12	8.990	-0.512	-2.991		
(4,3)	1.651	0.603	-6.448	3.343	-13.94	10.91	-0.657	-3.287	-42.28	
(a, b)	θ_1'	θ_1''	θ_2'	θ_2''	θ_3'	θ_3''	θ_3'	θ_3''	θ_9	θ_{10}
(4,4)	1.650	0.630	-5.958	2.008	-12.88	7.966	-20.07	19.03	-0.513	-2.977

EH+Scalar: Other Non-trivial FP

(a, b)	$\tilde{\lambda}_0^*$	$\tilde{\lambda}_2^*$	$\tilde{\lambda}_4^*$	$\tilde{\lambda}_6^*$	$\tilde{\lambda}_8^*$	$\tilde{\lambda}_{10}^*$	$\tilde{\lambda}_{12}^*$	$\tilde{\lambda}_{14}^*$	$\tilde{\lambda}_{16}^*$	$\tilde{\xi}_0^*$
(1,0)	0.01813	-0.1088								0.1060
(2,0)	0.01880	-0.1343	-0.1561							0.1065
(3,0)	0.01894	-0.1395	-0.1942	-0.2633						0.1066
(4,0)	0.01898	-0.1407	-0.2032	-0.3284	-0.4998					0.1066
(5,0)	0.01899	-0.1410	-0.2053	-0.3437	-0.6182	-0.9604				0.1066
(6,0)	0.01899	-0.1411	-0.2058	-0.3472	-0.6452	-1.180	-1.826			0.1066
(7,0)	0.01899	-0.1411	-0.2059	-0.3479	-0.6511	-1.228	-2.229	-3.380		0.1066
(8,0)	0.01899	-0.1411	-0.2059	-0.3481	-0.6524	-1.238	-2.313	-4.091	-5.977	0.1066

EH+Scalar: Other Non-trivial FP

(a, b)	$\tilde{\lambda}_0^*$	$\tilde{\lambda}_2^*$	$\tilde{\lambda}_4^*$	$\tilde{\lambda}_6^*$	$\tilde{\lambda}_8^*$	$\tilde{\lambda}_{10}^*$	$\tilde{\lambda}_{12}^*$	$\tilde{\lambda}_{14}^*$	$\tilde{\lambda}_{16}^*$	$\tilde{\xi}_0^*$
(1,0)	0.01813	-0.1088								0.1060
(2,0)	0.01880	-0.1343	-0.1561							0.1065
(3,0)	0.01894	-0.1395	-0.1942	-0.2633						0.1066
(4,0)	0.01898	-0.1407	-0.2032	-0.3284	-0.4998					0.1066
(5,0)	0.01899	-0.1410	-0.2053	-0.3437	-0.6182	-0.9604				0.1066
(6,0)	0.01899	-0.1411	-0.2058	-0.3472	-0.6452	-1.180	-1.826			0.1066
(7,0)	0.01899	-0.1411	-0.2059	-0.3479	-0.6511	-1.228	-2.229	-3.380		0.1066
(8,0)	0.01899	-0.1411	-0.2059	-0.3481	-0.6524	-1.238	-2.313	-4.091	-5.977	0.1066

(a, b)	θ_1'	θ_1''	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}
(1,0)	1.659	0.753	-1.699							
(2,0)	1.675	0.745	-1.594	-10.59						
(3,0)	1.679	0.742	-1.485	-8.384	-22.41					
(4,0)	1.68	0.741	-1.434	-7.341	-18.19	-36.96				
(5,0)	1.68	0.741	-1.414	-6.84	-15.93	-31.12	-54.10			
(6,0)	1.68	0.741	-1.407	-6.609	-14.70	-27.47	-47.25	-73.60		
(7,0)	1.68	0.740	-1.405	-6.509	-14.02	-25.287	-42.062	-66.663	-95.172	
(8,0)	1.68	0.740	-1.405	-6.469	-13.67	-23.94	-38.77	-59.72	-89.58	-118.4

Summary and Conclusion

- A FP found in all dimensions at which all matter couplings vanish (GMFP)
- The GMFP in $d = 4$ is continuously connected to GMFP in $d = 2 + \epsilon$
- At GMFP critical exponents obey recursive relations. This can be used to simplify computation by considering smaller truncations
- Critical exponents are complex for $3 \leq d \leq 11$ and real for $d \lesssim 2.8$
- At GMFP the dimension of the critical suface is four for $3 \leq d \leq 11$
- In $d = 3$ a "new" FP found with both matter and gravity non-trivial but has unphysical properties