TOPOLOGICAL PHASE TRANSITION IN THE SINE-GORDON MODEL

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The sine-Gordon model

$$\Gamma_k = \int_x \left[\frac{Z_k}{2} (\partial_\mu \varphi)^2 + V_k \right], \quad \tilde{V}_k = \tilde{u} \cos(\varphi)$$

The two phases are separated by the **Coleman fixed point** at

$$z^* = \frac{1}{8\pi}$$

- $z < z^*$: symmetric (*non-renormalizable*) phase
- $z > z^*$: broken symmetric (*renormalizable*) phase

The effective potential should be

$$\left. \begin{array}{l} \text{convex} \\ \text{periodic} \end{array} \right\} \Longrightarrow \textbf{flat}$$

2D Coulomb gas, XY model

RG equation

Functional RG equation for the effective action

$$k\partial_k \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{k\partial_k R_k}{R_k + \Gamma_k''}, \quad \text{with} \quad R_k = p^2 \left(\frac{k^2}{p^2}\right)^b, \; b \geq 1.$$

The evolution equations are

$$\partial_k V_k = \frac{1}{2} \int_p \frac{k\partial_k R_{k,p}}{zp^2 + R_{k,p} + V_k''}$$

$$\partial_k Z_k = \frac{1}{2} \int_p k\partial_k R_{k,p} \Bigg[-\frac{Z_k''}{[p^2 Z_k + R_{k,p} + V_k'']^2} + \frac{\frac{2}{d} Z_k'^2 p^2 + 4Z_k'(Z_k' p^2 + V_k''')}{(p^2 Z_k + R_{k,p} + V_k'')^3}$$

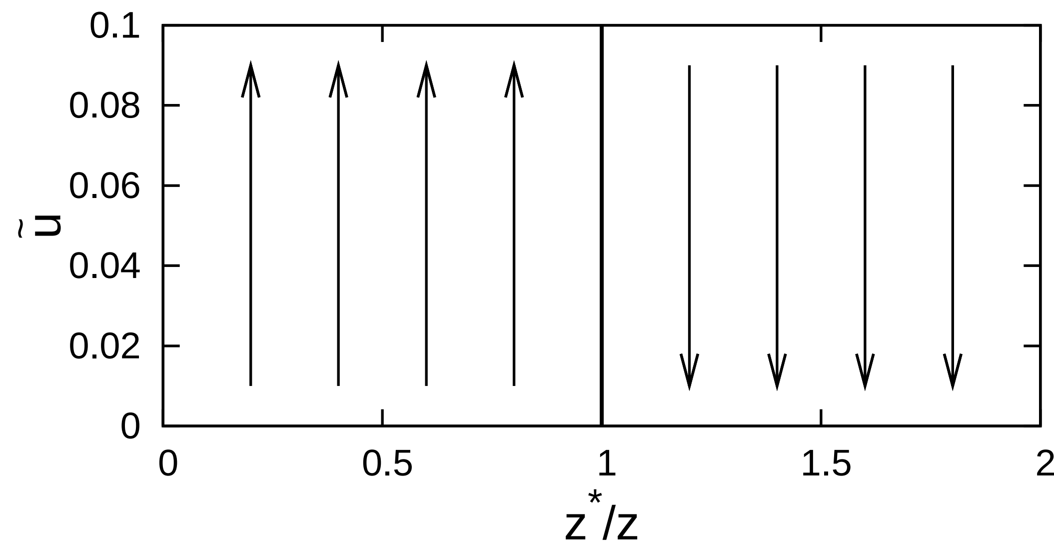
$$-2 \frac{(Z_k' p^2 + V_k''')^2 \left(Z_k + \partial_{p^2} R_{k,p} + \frac{4}{d} p^2 \partial_{p^2}^2 R_{k,p}\right)}{(p^2 Z_k + R_{k,p} + V_k'')^4}$$

$$\frac{+\frac{4}{d} Z_k' p^2 \left(Z_k' p^2 + V_k'''\right) \left(Z_k + p^2 \partial_{p^2} R_{k,p}\right)}{(p^2 Z_k + R_{k,p} + V_k'')^4} + \frac{\frac{8}{d} p^2 (Z_k' p^2 + V_k''')^2 \left(Z_k + \partial_{p^2} R_{k,p}\right)^2}{(p^2 Z_k + R_{k,p} + V_k'')^5}$$

Local potential approximation

Linearized RG equation $(b = 1)$

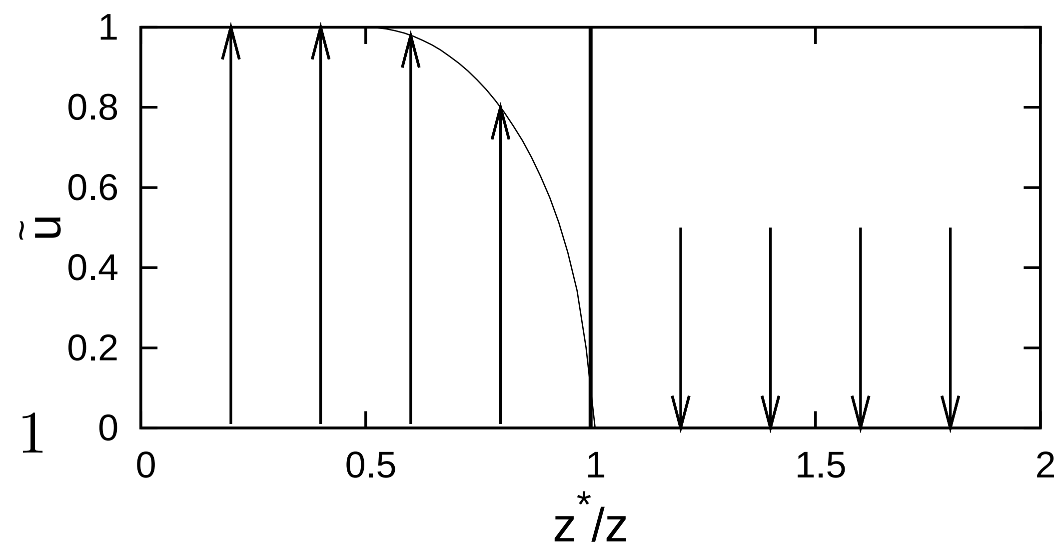
$$(2 + k\partial_k)\tilde{u} \quad = \quad \frac{\tilde{u}}{4\pi z}$$



RG equation exact in $\tilde{u}$ $(b = 1)$

$$(2 + k\partial_k)\tilde{u} = \frac{1}{2\pi\tilde{u}z}\left[1 - \sqrt{1 - \tilde{u}^2}\right]$$

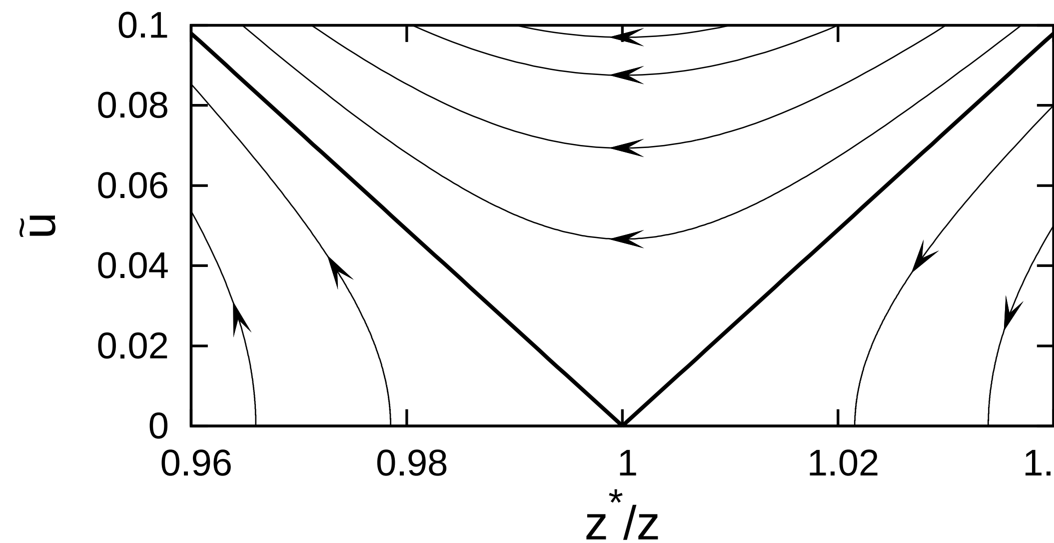
$$\tilde{u}^* = \begin{cases} 1 \text{ for } z^*/z < 0.5 \\ \sqrt{\frac{4z^*}{z} - \frac{4z^{*2}}{z^2}} \text{ for } 0.5 < z^*/z < 1 \\ 0 \text{ for } 1 < z^*/z \end{cases}$$



Wave function renormalization

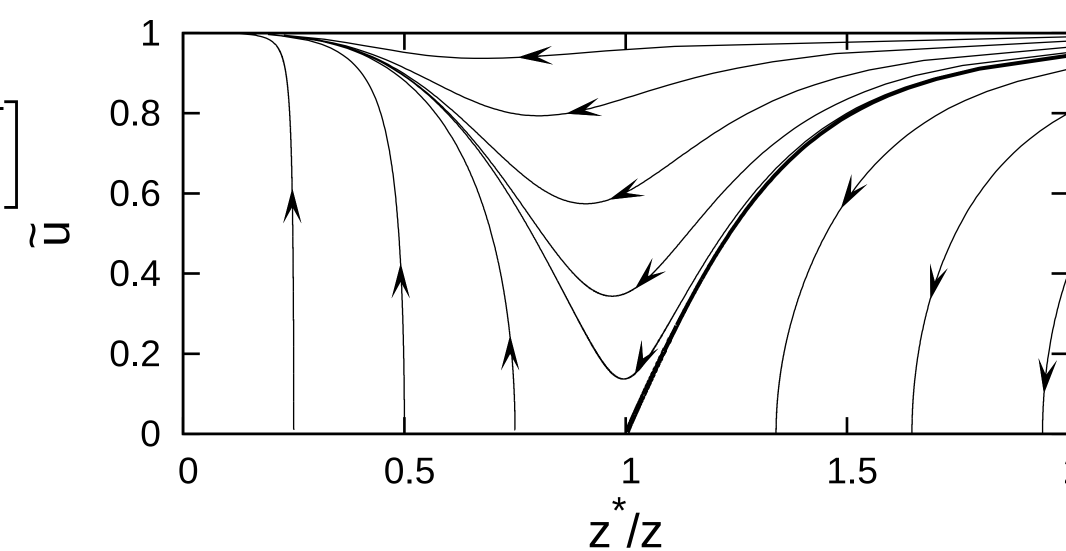
Linearized RG equations $(b = 1)$

$$\begin{aligned}(2 + k\partial_k)\tilde{u} &= \frac{\tilde{u}}{4\pi z} \\ k\partial_k z &= -\frac{\tilde{u}^2}{24\pi}\end{aligned}$$



RG equations exact in $\tilde{u}$ $(b = 1)$

$$\begin{aligned}(2 + k\partial_k)\tilde{u} &= \frac{1}{2\pi\tilde{u}z}\left[1 - \sqrt{1 - \tilde{u}^2}\right] \\ k\partial_k z &= -\frac{1}{24\pi}\frac{\tilde{u}^2}{(1 - \tilde{u}^2)^{3/2}}\end{aligned}$$

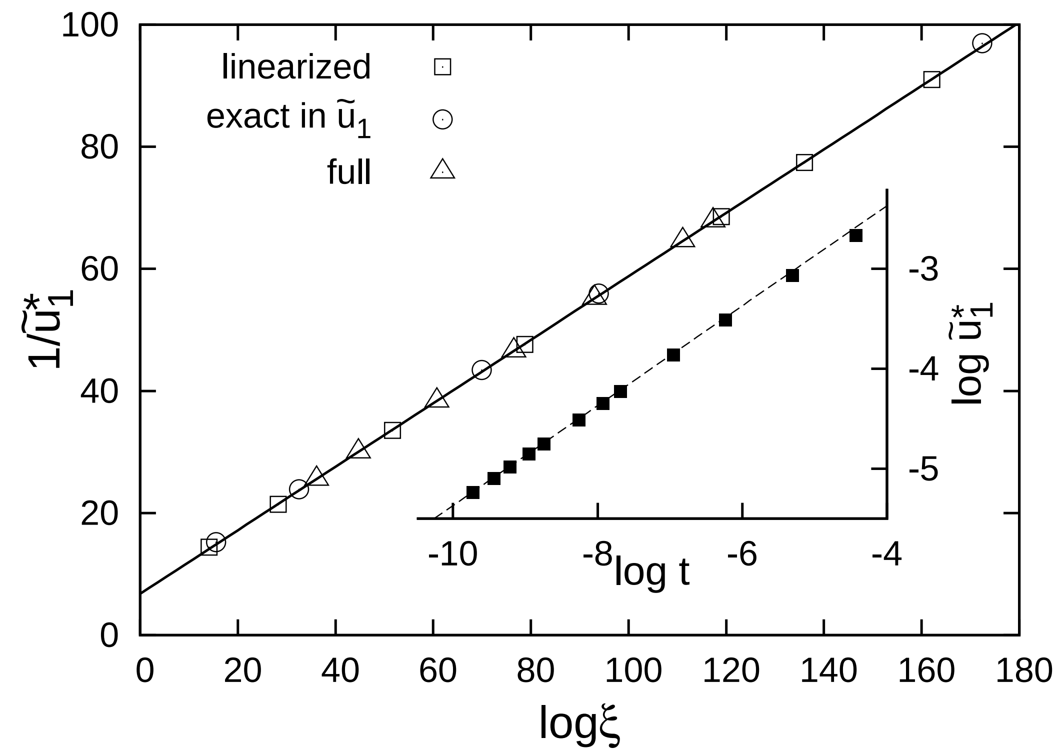


Topological (KT) phase transition

Linearized RG equations (for any b)

$$(2 + k\partial_k)\tilde{u} = \frac{1}{4\pi z}\tilde{u},$$

$$k\partial_k z = -\frac{\tilde{u}^2}{z^{2-2/b}}c_b$$



Scaling of the correlation length ($c_b = b\Gamma(3 - 2/b)\Gamma(1 + 1/b)/48\pi$)

$$\xi \sim e^{\sqrt{\pi}/(\tilde{u}^* 8\sqrt{c_b}) + \tilde{u}^*(b-1)(2b-1)\sqrt{c_b}2^{1-6/b}\pi^{5/2-2/b}b^{-2} + \mathcal{O}(\tilde{u}^{*2})}.$$

Field dependent Z_k

Let

$$Z_k = z + z_1 \cos\varphi$$

The RG equations

$$\begin{aligned}
(2 + k\partial_k)\tilde{u} &= \frac{\tilde{u}}{4\pi z} + \frac{3\tilde{u}z_1^2}{16\pi z^3} \\
k\partial_k z &= -\frac{\tilde{u}^2}{24\pi} - \frac{\tilde{u}z_1}{24\pi z} + \frac{z_1^2}{16\pi z^2} + \frac{7\tilde{u}^2 z_1^2}{192\pi z^2} \\
k\partial_k z_1 &= \frac{z_1}{4\pi z} + \frac{\tilde{u}^2 z_1}{48\pi z} - \frac{\tilde{u}z_1^2}{96\pi z^2}
\end{aligned}$$

It gives the same KT transition.

Coupled sine-Gordon model

Effective action

$$\Gamma = \int_x \left[\sum_{n=1}^{N} \frac{Z_k}{2}(\partial_\mu \varphi_n)^2 + V_k\right], \quad \tilde{V}_k = \frac{1}{2}\tilde{G}\left(\sum_{n=1}^{N}\varphi_n\right)^2 + \tilde{u}\sum_{n=1}^{N}\cos\varphi_n$$

RG equations ($b = 1$), $\tilde{G}_k \sim k^{-2}$

$$\begin{aligned}(2 + k\partial_k)\tilde{V}_k &= \frac{1}{4\pi z}\log[\det(1 + \tilde{V}_k'')] \\ k\partial_k z &= -\frac{1}{12\pi N}\mathrm{Tr}\frac{(\sum_k \partial_{\varphi_k}\tilde{V}_k'')^2}{(1+\tilde{V}_k'')^3}\end{aligned}$$

Linearized RG equations

$$\begin{aligned}(2 + k\partial_k)\tilde{u} &= \frac{1}{4\pi z}\tilde{u}\frac{1 + (N-1)\tilde{G}}{1 + N\tilde{G}} \to \frac{1}{4\pi z}\tilde{u}\left(1 - \frac{1}{N}\right) \\ k\partial_k z &= -\frac{1}{24\pi}\tilde{u}^2\frac{(1 + (N-1)\tilde{G})^3}{(1 + N\tilde{G})^3} \to -\frac{1}{24\pi}\tilde{u}^2\left(1 - \frac{3}{N}\right)\end{aligned}$$

Finite order or topological phase transition?

In LPA approximation one gets scaling for the correlation length

$$\xi = \sqrt{\frac{1-8\pi z_N^*}{16\pi G}} \frac{1}{\sqrt{z - z_N^*}} \sim t^{-1/2}$$

where $z_N^* = N8\pi(N-1)$.

$O(N)$ rotation to diagonalize the mass matrix $\phi_i = \mathcal{R}_{ij}\varphi_j$ ($N=2$)

$$\Gamma_{rot} = \int_x \left[\frac{Z_k}{2}(\partial_\mu \phi_1)^2 + \frac{Z_k}{2}(\partial_\mu \phi_2)^2 + \frac{1}{2} 2\tilde{G}\phi_2^2 + 2\tilde{u}\cos\frac{\phi_1}{\sqrt{2}} \cos\frac{\phi_2}{\sqrt{2}} \right]$$

Perturbation in the massive field component

$$\Gamma_{rot}^{pert} \approx \int_x \left[\frac{Z_k}{2}(\partial_\mu \phi_1)^2 \left(1 + \frac{\tilde{u}^2}{384\tilde{G}^4\pi} \sin^2\frac{\phi_1}{\sqrt{2}} \right) + 2\tilde{u}\cos\frac{\phi_1}{\sqrt{2}} \right]$$

- field dependent Z_k appears
- KT transition remains unchanged