Crumpling transition and flat phase in polymerized phantom membranes

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Outline

Motivations

- membranes: 2D-extended objects embedded in a d dimensional space and subject to fluctuations

- fluctuating membranes / surfaces occur in several domains:

- chemical physics - biology (of the cell):

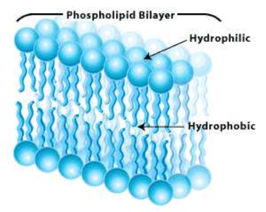
(Helfrich, Nelson-Peliti, David-Guitter, Aronovitz-Lubensky (70's - 80's))

main compound: amphiphiles molecules (ex: phospholipid)

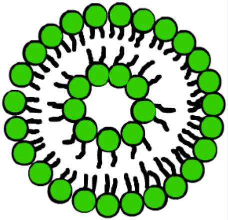
- one hydrophilic head
- one – or more – hydrophobic tails

bilayers:

- plane membrane ($L \simeq 1 - 110\ \mu$m and $\delta r \simeq 1$ nm)



- closed membranes (red blood cell, liposomes)



- High energy physics: (sum over) surfaces occurs within, e.g.:
  - strong coupling expansion of lattice gauge theory
  - discretization of Euclidean quantum gravity

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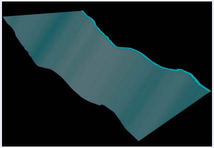
- string theory (Polyakov, David, Foerster (70's - 80's)))
  - a string sweeps out a surface (worldsheet)



  - action: $S = \tau \int d^2\boldsymbol{\sigma} \sqrt{g} \; \propto$ area

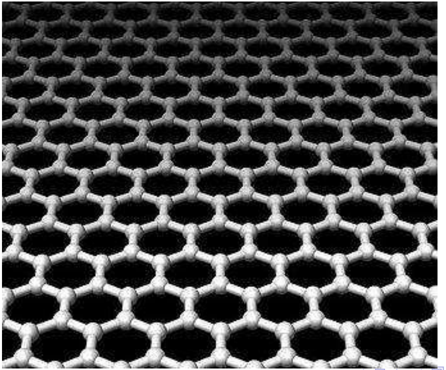
    $\Longrightarrow$ deep relations with fluid membranes: $\tau \equiv$ tension

(see also branes . . . (Polchinski (90's)))

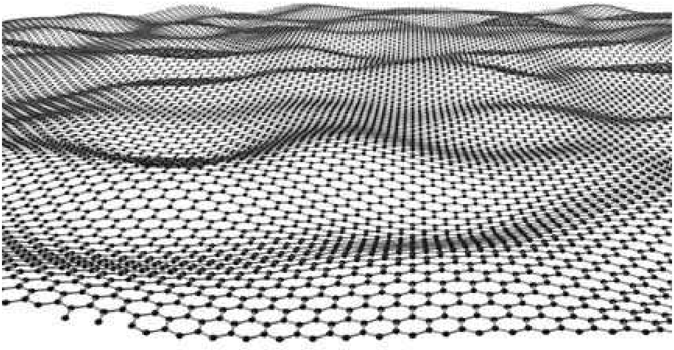
- condensed matter physics:

  graphene: carbon atoms on a honeycomb lattice

  - relativistic electrons $E = v|p|$ with $v \sim c/300$
  - extremely high mobility (propagation without scattering)

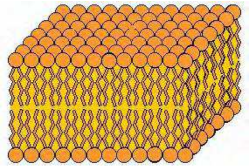


- mechanical properties:
  - extremely strong material
  - "soft" material: unique exemple of genuine 2D (metallic and polymerized) membrane

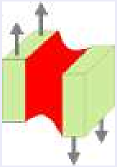


Fluid membranes vs polymerized membranes

Properties of fluid membranes



- very weak interaction between molecules
  $\Longrightarrow$ free diffusion inside the membrane plane (10^7 exchange/s)
  $\Longrightarrow$ no shear modulus



- very small compressibility: no stretching or contraction

$\Longrightarrow$ dominant contribution to the energy: bending energy

- coordinate transformation – reparametrization – invariance

  $\Longrightarrow$ free energy F written in terms of *geometrical* quantities

- parametrization:

  2D membranes: parametrized by the embedding:

$$\boldsymbol{\sigma} = (\sigma_1, \sigma_2) \rightarrow \mathbf{r}(\sigma_1, \sigma_2)$$

  - $(\sigma_1, \sigma_2) \equiv$ local coordinates on the membrane
  - $\mathbf{r}$: d dimensional vector of bulk - or "target " - space

- Free energy:

$$F = r\int d^2\boldsymbol{\sigma}\,\sqrt{g} + \frac{\kappa}{2}\int d^2\boldsymbol{\sigma}\,\sqrt{g}\,\mathbf{K}^{\mu}_{\mu}.\mathbf{K}^{\nu}_{\nu} + \frac{\overline{\kappa}}{2}\int d^2\boldsymbol{\sigma}\,\sqrt{g}\,R$$

  - $g_{\mu\nu} = \partial_\mu \mathbf{r}.\partial_\nu \mathbf{r} \equiv$ metric induced by the embedding $\mathbf{r}(\boldsymbol{\sigma})$
  - $\mathbf{K}_{\mu\nu} = D_\mu\, D_\nu \mathbf{r} \equiv$ extrinsic – mean – curvature tensor
  - $R = \mathbf{K}^{\mu}_{\mu}.\mathbf{K}^{\nu}_{\nu} - \mathbf{K}^{\nu}_{\mu}.\mathbf{K}^{\mu}_{\nu} \equiv$ intrinsic scalar – Gaussian – curvature

- with:
  - r: surface tension (string tension)
  - κ: bending rigidity modulus (string curvature)
  - $\overline{\kappa}$: Gaussian rigidity modulus

- no fluctuation of area: tension term $\to 0$
- for a surface without boundary Gaussian curvature term is shape-independent due to Gauss-Bonnet theorem:

$$\int d^2\boldsymbol{\sigma}\,\sqrt{g}\;R\;=4\pi\chi=8\pi(1-h)$$

with

- χ : Euler characteristic
- h is the genus $\equiv$ number of "handles "

$\Longrightarrow$ for a fixed topology this term can be neglected

- Finally: bending free energy

$$F=\frac{\kappa}{2}\int d^2\boldsymbol{\sigma}\,\sqrt{g}\,\mathbf{K}_{\mu}^{\mu}.\mathbf{K}_{\nu}^{\nu}$$

Low-temperature fluctuations in fluid membranes

- Let us define $\hat{\mathbf{n}}$ by:

$$\partial_\mu \hat{\mathbf{n}} = K_{\mu\nu} \, \partial^\nu \mathbf{r}$$

where $\partial_\nu \mathbf{r}$ are tangent vectors

$\Longrightarrow$ $\hat{\mathbf{n}}$ is a unit vector normal to the surface

$$F = \frac{\kappa}{2} \int d^2\boldsymbol{\sigma} \, (\partial_\mu \hat{\mathbf{n}})^2 \qquad \Longrightarrow \qquad F = -\frac{\kappa'}{2} \sum_{\langle i,j \rangle} \hat{\mathbf{n}}_i . \hat{\mathbf{n}}_j$$

where $\hat{\mathbf{n}}_i$ is a unit normal vector on the plaquette i

- very close to a $O(d)$ nonlinear σ-model / Heisenberg spin system but with "spins" living on a surface

- Monge parametrization: $x = \sigma_1$, $y = \sigma_2$ and $z = h(x, y)$

$\Longrightarrow \mathbf{r}(x, y) = (x, y, h(x, y))$

thus: $$\hat{\mathbf{n}}(x, y) = \frac{(-\partial_x h, -\partial_y h, 1)}{\sqrt{1 + |\boldsymbol{\nabla} h|^2}}$$

- Free energy:

$$F \simeq \frac{\kappa}{2} \int d^2\mathbf{x} \; (\Delta h)^2 + \mathcal{O}(h^4)$$

- flat phase ? $\Longrightarrow \theta(x, y) \equiv$ angle between normal and axis $\mathbf{e_z}$:

$$\langle \theta(x, y)^2 \rangle = k_B T \int d^2q \; \frac{1}{\kappa \, q^2} \simeq \frac{k_B T}{\kappa} \ln \left(\frac{L}{a} \right)$$

$\Longrightarrow$ no long-range order in the normals

At higher order in h, κ is renormalized and decreased by fluctuations in the I.R.:

$$\kappa_R(q) = \kappa - \frac{3k_BT}{2\pi}\left(\frac{d}{2}\right)\ \ln\left(\frac{1}{qa}\right)$$

$\Longrightarrow$ makes the divergence of $\langle\theta(x,y)^2\rangle$ worse

rem

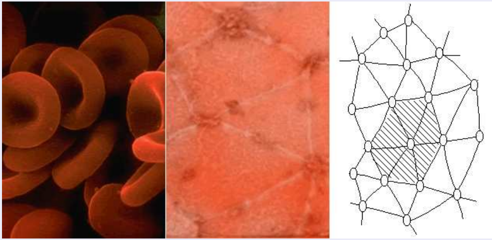
- lattice spins vs surface spins: $d-2 \Longrightarrow d/2$
- correlations: $\langle\hat{\mathbf{n}}(\mathbf{r}).\hat{\mathbf{n}}(\mathbf{0})\rangle \sim e^{-r/\xi}$
- correlation length – mass gap: $\xi \simeq a\, e^{(4\pi\kappa/3k_BTd)}$

Polymerized membranes

- 2D membranes made of linked molecules

ex:

- red blood cell
- inorganic membranes: graphene



- lattice free energy:

$$F = -\frac{\kappa'}{2}\sum_{\langle i,j\rangle}\hat{\mathbf{n}}_i.\hat{\mathbf{n}}_j + \sum_{\langle a,b\rangle} V(|\mathbf{r}_a - \mathbf{r}_b|)$$

where $\mathbf{r}_a$ is the position of the a-th vertex

- no diffusion $\Longrightarrow$ $\exists$ shear modulus

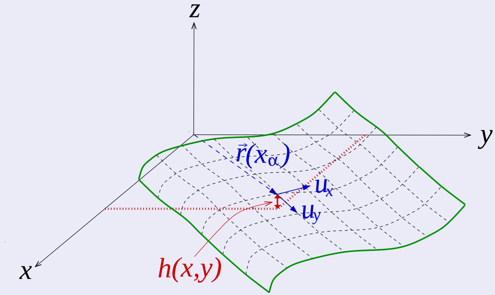
  $\Longrightarrow$ free energy made of bending and elastic energy

- $\exists$ prefered metric $\Longrightarrow$ reparametrization invariance broken

  $\Longrightarrow$ free energy: no longer "*geometric*"

Low-temperature fluctuations in polymerized membranes

- reference configuration: $\mathbf{r}_0(x,y) = (x, y, z = 0)$
- fluctuations: $\mathbf{r}(x,y) = \mathbf{r}_0 + (u_1(x,y), u_2(x,y), h(x,y))$



- stress tensor:

$$u_{\mu\nu} = \frac{1}{2}\left[\partial_\mu u_\nu + \partial_\mu u_\nu + \partial_\mu \mathbf{u}.\partial_\nu \mathbf{u} + \partial_\mu h\, \partial_\nu h\right]$$

  - u_ν describes the longitudinal – phonon-like – degrees of freedom
  - h describes height, capillary – Goldstone - like – fluctuations

- free energy

$$F \simeq \int d^2\mathbf{x}\ \left[\frac{\kappa}{2}(\Delta h)^2 + \mu(u_{\mu\nu})^2 + \frac{\lambda}{2}(u_{\mu\mu})^2\right]$$

$\kappa \equiv$ bending rigidity $\quad\quad \lambda,\ \mu \equiv$ elastic constants

- non-trivial coupling between longitudinal - in plane - and height fluctuations

Gaussian approximation:

$$u_{\mu\nu} \simeq \frac{1}{2}\left[\partial_\mu u_\nu + \partial_\mu u_\nu + \partial_\mu h\, \partial_\nu h\right]$$

integrate over u:

$$F_{eff} = \frac{\kappa}{2}\int d^2\mathbf{x}\ (\Delta h)^2 + \frac{K}{8}\int d^2\mathbf{x}\left(P^T_{\mu\nu}\partial_\mu h\, \partial_\nu h\right)^2$$

- $P^T_{\mu\nu} = \delta_{\mu\nu} - \partial_\mu\partial_\nu/\nabla^2$
- $K = 4\mu(\lambda+\mu)/(2\mu+\lambda)$.

- Effective rigidity: $\kappa_{eff}^{-1}(\mathbf{q}) \equiv q^4 \langle |h(\mathbf{q})|^2 \rangle$

with: $\kappa_{eff}(\mathbf{q}) = \kappa + k_B T K \int d^2k \frac{\left[\hat{q}_\mu P^T_{\mu\nu} \hat{q}_\nu\right]^2}{\kappa_{eff}(\mathbf{q}+\mathbf{k})|\mathbf{q}+\mathbf{k}|^4}$

$\Longrightarrow \kappa_{eff}(\mathbf{q}) \sim \sqrt{k_B T K}/q$ increased by fluctuations !

- normal fluctuations:

$$\langle \theta(x,y)^2 \rangle = k_B T \int d^2q \, \frac{1}{\kappa_{eff}(q) q^2} < \infty!$$

  - renormalization of $K \Longrightarrow \kappa_{eff}(q) \sim q^{-\eta}$ with $0 < \eta < 1$

- $\Longrightarrow$ Long-range order between normals even in $D = 2$!

- no trouble with Mermin-Wagner Theorem

Gaussian curvature R:

$$R(\mathbf{x}) = -\Delta(\partial_\mu h \partial_\mu h) + \partial_\mu \partial_\nu (\partial_\mu h \partial_\nu h)$$

and:

$$F_{eff} = \frac{\kappa}{2} \int d^2\mathbf{x}\ (\Delta h)^2 + K \int d^2\mathbf{x} \int d^2\mathbf{y}\ R(\mathbf{x}) G(\mathbf{x}-\mathbf{y}) R(\mathbf{y})$$

where:

$$G(\mathbf{x}-\mathbf{y}) \propto |\mathbf{x}-\mathbf{y}|^2 \ln |\mathbf{x}-\mathbf{y}|$$

$\Longrightarrow$ Long-range interaction between curvatures

physics of polymerized membranes:

- spontaneous symmetry breaking in $D = 2$ and even in $D < 2$

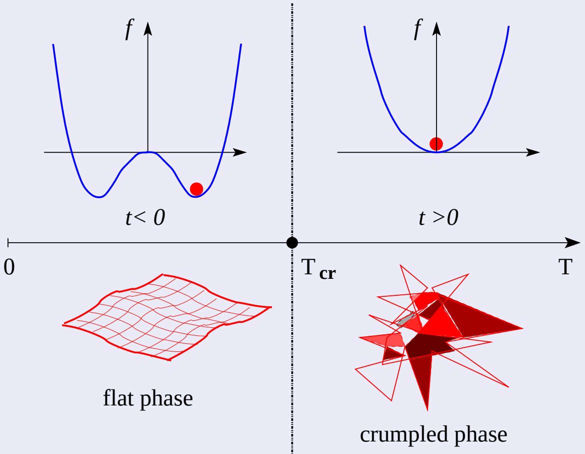
$\Longrightarrow$ low-temperature - flat - phase with non-trivial correlations in the I.R.

$$\begin{cases} G_{hh}(\mathbf{q}) \sim q^{-(4-\eta)} \\ G_{uu}(\mathbf{q}) \sim q^{-(6-D-2\eta)} \end{cases}$$

with $\quad \eta \neq 0$

$D_{low-crit}$: $G_{hh}(\mathbf{q}) \sim G_{uu}(\mathbf{q}) \to \eta(D_{low-crit}) = 2 - D_{low-crit}$

- crumpling-to-flat phase transition when T is varied
  (Aronovitz, David-Guitter, Leibler, Nelson, Peliti, Radzihovsky 80's, 90's)



Crumpling-to-flat phase transition

- Landau-Ginzburg free energy:
  - order parameter: tangents $t_\alpha = \partial_\alpha \mathbf{r}$
  - $O(d)$ and translation invariance

$$F = \int d^D\boldsymbol{\sigma}\ \frac{\kappa}{2}\left(\partial_\alpha\partial_\alpha \mathbf{r}\right)^2 + \frac{t}{2}(\partial_\alpha \mathbf{r})^2 + \lambda\left(\partial_\alpha \mathbf{r}.\partial_\beta \mathbf{r}\right)^2 + \mu\left(\partial_\alpha \mathbf{r}.\partial_\alpha \mathbf{r}\right)^2$$

$$+ \frac{b}{2}\int d^D\boldsymbol{\sigma}\ \int d^D\boldsymbol{\sigma}'\ \delta^d(\mathbf{r}(\boldsymbol{\sigma}) - \mathbf{r}(\boldsymbol{\sigma}'))$$

rem:

- self-avoidance term: neglected
- F describes $O(d) \to O(D) \times O(d-D)$
  Goldstone modes expected: $D(d-D)$ but only $d-D$!

Perturbatively

For D-dimensional membranes embedded in a d-dimensional space:

- pertubatively around $D_{upp.crit.} = 4$ $\exists\ d_{cr}$ such that:
  - $d > d_{cr}$: second order phase transition
  - $d < d_{cr}$: first order phase transition

  <u>however</u>:

- d_{cr} only known at first order in $\epsilon = 4 - D$ with $d_{cr}(D = 4) \simeq 219 \Longrightarrow$ what about $D = 2$ and $d = 3$?
- flat phase properties: also poorly determined ($\epsilon = 4 - D$)
- no lower-critical-dimension approach

Nonperturbative approach to polymerized membranes

(J.P. Kownacki and D.M., 08)

- Effective action: $\Gamma_k[\partial_\mu \mathbf{r}]$
  - $O(d)$ and translation invariant
  - expanded around a flat state:

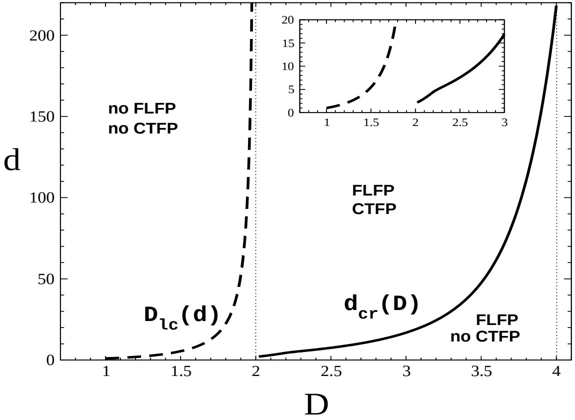
$$\mathbf{r} = \zeta_k \sum_{\alpha=1}^{D} x_\alpha \, \mathbf{e}_\alpha$$

with: $\zeta_k \sim$ flatness factor $\sim$ magnetization

$$\Longrightarrow \Gamma_k\left[\partial_\mu \mathbf{r}\right] = \int d^D\boldsymbol{\sigma} \, \frac{Z_k}{2}\left(\partial_\alpha\partial_\alpha \mathbf{r}\right)^2 \quad + \tilde{\lambda}_k\left(\partial_\alpha \mathbf{r}.\partial_\beta \mathbf{r} - \zeta_k^2 \, \delta_{\alpha\beta}\right)^2 + \tilde{\mu}_k\left(\partial_\alpha \mathbf{r}.\partial_\alpha \mathbf{r} - D\,\zeta_k^2\right)^2$$

$\Longrightarrow$ crumpling-to-flat transition and flat phase $\quad (\overline{\zeta}_k^2 \to \infty)$

- physics everywhere between $D = 4$ and $D = D_{lc}(d)$



- FLFP: Flat Phase Fixed Point
- CTFP: Crumpling Transition Fixed Point

- $d_{cr}(D=2) \simeq 2 \Longrightarrow$ the crumpled-to-flat transition in $d=3$ of 2^{nd} order with $\eta = 0.627$

  MC data: $\eta = 0.71(5)$ (Bowick et al. (96))

  $\Longrightarrow$ higher orders needed to stabilize the results

  (K. Essafi, J.P. Kownacki, D.M.)

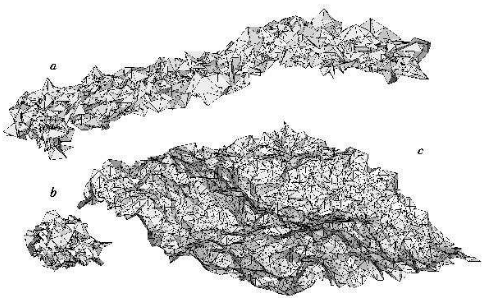
- Flat phase: $\eta_{FP} = 0.849$

  MC computation with a realistic interatomic potential for graphene: $\eta = 0.850$!
  (Los, Katsnelson, Yazyev, Zakharchenko and Fasolino (09))

- η_{FP} :
  - no correction beyond the order r^4 !
    (K. Essafi, J.P. Kownacki and D.M.)
  - almost no correction beyond the order ∂^4 !
    (Braghin and Hasselmann (10))

Prospects

- extension to systems with different kind of internal orders :
  - in-plane anisotropy $\Longrightarrow$ tubular membranes



(Radzihovski - Toner (98), Essafi - Kownacki - D.M.)

Prospects

- disorder

  (Radzihovski - Toner (95), Essafi, Kownacki, D.M.)

  challenging problems:

- self-avoidance
- graphene-like systems: interaction between electronic and membranes degrees of freedom

  $\Longrightarrow$ condensed matter realization of matter coupled to "curve space"

- ...