Noncommutative Field Theory and Gravity
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NC gauge theory and renormalisability

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UV/IR mixing in scalar NCQFT

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Oscillator model
$1/p^2$ model
joint work with Blaschke, Rofner, Sedmik, Schweda, 0912.2634[hep-th]

Remarks and outlook

Introduction

We consider 4D canonically deformed Euclidean space:

$$[x^i \stackrel{\star}{,} x^j] = \mathrm{i}\Theta^{ij} \, ,$$

$\Theta^{ij} = -\Theta^{ji} = \mathrm{const}$, with Weyl-Moyal $\star$-product

$$f \star g\,(x) = \mathrm{e}^{\frac{\mathrm{i}}{2}\Theta^{ij}\partial_i^x \partial_j^y} f(x)\, g(y)\Big|_{y\to x} \, .$$

Introduction

NC scalar ϕ^4

$$S = \int d^4x \left(\frac{1}{2} \partial_\mu \phi \, \partial_\mu \phi + \frac{m^2}{2} \phi^2 + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi \right)$$

Feynman rules:

- propagator $G(p) = \frac{1}{p^2+m^2}$
- vertex function $\Gamma(p_1, \ldots, p_4) = \lambda \delta^{(4)}(p_1 + p_2 + p_3 + p_4) \mathrm{e}^{-\mathrm{i} \sum_{i<j} p_i \Theta p_j}$

Introduction

2-point tadpole

$$\Pi(\Lambda, p) \propto \int d^4k \frac{2 + \cos k\tilde{p}}{k^2 + m^2} = \Pi^{UV}(\Lambda) + \Pi^{IR}(\Lambda, p)$$

with the IR-divergent non-planar part

$$\Pi^{IR} \sim \frac{1}{\tilde{p}^2}$$

$\tilde{p}_\mu = \Theta_{\mu\nu} p_\nu$;
not yet a problem: $\int d^4p\, \tilde{\phi}(p) \frac{1}{\tilde{p}^2} \tilde{\phi}(-p)$
but higher loop insertions yields: $\int d^4p\, \tilde{\phi}(p) \frac{1}{(\tilde{p}^2)^n} \tilde{\phi}(-p)$
UV/IR mixing destroys renormalizability.

Introduction

2 different strategies to cure UV/IR mixing:

1 - Adding an oscillator potential (Grosse, Wulkenhaar 03, 05):

$$S = \int d^D x \left(\frac{1}{2} \phi \star [\tilde{x}_\nu, [\tilde{x}^\nu, \phi]_\star]_\star + \frac{\Omega^2}{2} \phi \star \{\tilde{x}^\nu, \{\tilde{x}_\nu, \phi\}_\star\}_\star + \frac{\mu^2}{2} \phi \star \phi + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi \right)(x) ,$$

where $\tilde{x}_\nu = \theta^{-1}_{\nu\alpha} x^\alpha$ and $i\partial_\mu f = [\tilde{x}_\mu, f]_\star$

Introduction

2 - Adding a non-local term (Gurau, Magnen, Rivasseau, Tanasa 08): in momentum space:

$$S_{nl} = \int d^4p \, \frac{a}{2} \phi(p) \frac{1}{\tilde{p}^2} \phi(-p)$$

Also implements IR damping ($G(p) \to 0$, for $k \to 0$):

$$G(p) = \frac{1}{p^2 + m^2 + \frac{a^2}{p^2}}$$

$\Rightarrow$ perturbatively renormalizable to all orders.

NC gauge theory

Aim is to generalize both approaches to NC $U(1)$ gauge theory.

- ad 1 - oscillator approach
  Induced gauge action (de Goursac, Wallet, Wulkenhaar 07; Grosse, MW 07)

$$S = \int d^4x \left\{ \frac{3}{\theta}(1-\rho^2)(\tilde{\mu}^2-\rho^2)(\tilde{X}_\nu \star \tilde{X}^\nu - \tilde{x}^2) + \frac{3}{2}(1-\rho^2)^2\big((\tilde{X}_\mu \star \tilde{X}_\mu)^{\star 2} - (\tilde{x}^2)^2\big) - \frac{\rho^4}{4} F_{\mu\nu}F_{\mu\nu} \right\},$$

where $F_{\mu\nu} = -i[\tilde{x}_\mu, A_\nu]_\star + i[\tilde{x}_\nu, A_\mu]_\star - i[A_\mu, A_\nu]_\star$
$\tilde{X}_\mu = \tilde{x}_\mu + A_\mu$, $\rho = \frac{1-\Omega^2}{1+\Omega^2}$, $\tilde{\mu}^2 = \frac{m^2\theta}{1+\Omega^2}$

NC gauge theory

Action proposed by Buric, Grosse, Madore 10

$$S = \int d^2x \Big((1-\alpha^2) F_{12}^{*2} - 2(1-\alpha^2)\mu\, F_{12} \star \phi + (5-\alpha^2)\mu^2\phi^2$$
$$+4i\alpha F_{12} \star \phi^{\star 2} + (D_i\phi)^2 - \alpha^2\{p_i + A_i \overset{\star}{,} \phi\}^2 \Big)$$

NC gauge theory

- ad 2 - $1/p^2$ model

  IR divergent contribution: $\Pi_{\mu\nu} \propto \frac{\tilde{p}_\mu \tilde{p}_\nu}{(\tilde{p}^2)^2}$

  Gauge invariant implementation: additional term

$$F_{\mu\nu} \frac{1}{\tilde{D}^2 D^2} F_{\mu\nu}$$

  proposed by Blaschke, Gieres, Kronberger, Schweda, MW 08;
  when expanded in the gauge field, vertices with arbitrary number of photon legs occur $\Rightarrow$ localisation

NC gauge theory

two different ways to implement the localization

Blaschke, Rofner, Schweda, Sedmik 08:

$$\int d^4x F_{\mu\nu} \frac{a^2}{\tilde{D}^2 D^2} F_{\mu\nu} \to \int d^4x \left(aB_{\mu\nu} F_{\mu\nu} - B_{\mu\nu} \star \tilde{D}^2 D^2 B_{\mu\nu} \right)$$

additional degrees of freedom introduced

Vilar, Ventura, Tedesco, Lemes 09:
used BRST doublet structure in order to avoid introduction of additional degrees of freedom

not renormalizable

NC gauge theory

Virtue of the approach: IR damping is implemented in the "soft breaking" part

Gribov-Zwanziger approach to QCD (Zwanziger 89, 93), where an IR modification of the propagator is suggested to cure the Gribov ambiguities without changing UV renormalizability.

NC gauge theory

$$
\begin{aligned}
S &= S_{inv} + S_{gf} + S_{aux} + S_{break} + S_{ext} \,, \\
S_{inv} &= \int d^4x \, \frac{1}{4} F_{\mu\nu} F_{\mu\nu} \,, \\
S_{gf} &= \int d^4x \, s(\bar{c}\partial_\mu A_\mu) \,, \\
S_{aux} &= \int d^4x \, s \, \left(\bar{\psi}_{\mu\nu} B_{\mu\nu} \right) \,, \\
S_{break} &= \int d^4x \, s \left((\bar{Q}_{\mu\nu\alpha\beta} B_{\mu\nu} + Q_{\mu\nu\alpha\beta} \bar{B}_{\mu\nu}) \frac{1}{\tilde{\Box}} (f_{\alpha\beta} + \sigma \frac{\theta_{\alpha\beta}}{2} \tilde{f}) \right) \,, \\
S_{ext} &= \int d^4x \, (\Omega^A_\mu s A_\mu + \Omega^c s c) \,,
\end{aligned}
$$

where $f_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$, $\Theta_{\alpha\beta} = \epsilon \, \theta_{\alpha\beta}$ and $\tilde{f} = \theta_{\alpha\beta} f_{\alpha\beta}$.

NC gauge theory

Sources $\bar{Q}, Q, \bar{J}, J$ ensure BRST invariance; in the IR they take their physical values:

$$\begin{aligned}\bar{Q}_{\mu\nu\alpha\beta}|_{phys} &= 0, \qquad \bar{J}_{\mu\nu\alpha\beta}|_{phys} = \frac{\gamma^2}{4}(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha}), \\ Q_{\mu\nu\alpha\beta}|_{phys} &= 0, \qquad J_{\mu\nu\alpha\beta}|_{phys} = \frac{\gamma^2}{4}(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha})\,.\end{aligned}$$

$$\begin{gathered} sA_\mu = D_\mu c\,, \quad sc = igcc\,, \quad s\bar{c} = b\,, \quad sb = 0\,, \\ s\bar{\psi}_{\mu\nu} = \bar{B}_{\mu\nu}\,, \quad s\bar{B}_{\mu\nu} = 0\,, \quad sB_{\mu\nu} = \psi_{\mu\nu}\,, \quad s\psi_{\mu\nu} = 0\,, \\ s\bar{Q} = \bar{J}\,, \quad s\bar{J} = 0\,, \quad sQ = J\,, \quad sJ = 0\,. \end{gathered}$$

NC gauge theory

Feynman rules:

vertex functions are the usual ones, as in "naive" NCQED;
propagators are complicated:

$$G^A_{\mu\nu}(k) = \left(k^2 + \frac{\gamma^4}{\tilde{k}^2}\right)^{-1} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} - \frac{\bar{\sigma}^4}{(k^2 + (\bar{\sigma}^4 + \gamma^4)\frac{1}{\tilde{k}^2})} \frac{\tilde{k}_\mu \tilde{k}_\nu}{(\tilde{k}^2)^2}\right)$$

but for loop-calculation can be approximated:

$$G^A_{\mu\nu} \sim \frac{1}{k^2}(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}), \; k^2 >> 1 \, .$$

NC gauge theory

Power counting formula:

$$d_G = 4 - E_A - E_{c\bar{c}} \,.$$

1-loop results: Λ momentum cut-off

Vacuum polarization:

$$\Pi_{\mu\nu} = \frac{2g^2}{\epsilon^2\pi^2}\frac{\tilde{p}_\mu\tilde{p}_\nu}{(\tilde{p}^2)^2} + \frac{13g^2}{3(4\pi)^2}(p^2\delta_{\mu\nu} - p_\mu p_\nu)\ln\Lambda \,,$$

transversal.

NC gauge theory

Vertex corrections:

$$\Gamma^{3A,IR}_{\mu\nu\rho} = -\frac{2ig^3}{\pi^2}\cos\frac{\epsilon p_1\tilde{p}_2}{2}\sum_{j=1,2,3}\frac{\tilde{p}_{j,\mu}\tilde{p}_{j,\nu}\tilde{p}_{j,\rho}}{\epsilon(\tilde{p}_j^2)^2},$$

$$\Gamma^{3A,UV}_{\mu\nu\rho} = -\frac{17g^2}{6(4\pi)^2}\ln\Lambda\,\tilde{V}^{3A,tree}_{\mu\nu\rho}(p_1,p_2,p_3)\,,$$

$$\Gamma^{4A}_{\mu\nu\rho\sigma} = -\frac{5}{8\pi^2}\ln\Lambda\;\tilde{V}^{4A,tree}_{\mu\nu\rho\sigma}\,.$$

β-function negative:

$$\beta = -\frac{7g^3}{12\pi^2}\,.$$

Remarks and outlook

- 1-loop corrections reduce to the ones known from "naive" NCQED (e.g. Hayakawa 99; Matusis, Susskind, Toumbas 00).
- Divergences can be absorbed in the tree level action.
- Modifications at higher loops expected.
- Renormalization (dis-)proof of the latest $1/p^2$-model applying a some renormalization scheme - such as multi-scale analysis or flow equations.