Critical Behaviour and Asymptotic Safety of Chiral Yukawa Systems

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Outline

1. Reminder
   - RG & fixed points
   - RG flow & β-functions
2. Critical behaviour: Chiral fermion models in d=3
   - Motivation: Quantitative control for correlated fermion systems
   - Four-fermion interactions and truncation
   - Fixed-point mechanisms and results
3. Asymptotic safety: Chiral Yukawa systems in d=4
   - Motivation: Triviality and Hierarchy in the standard model
   - Standard model and asymptotic safety
   - Truncation for chiral Yukawa systems
   - Fixed-points, critical exponents & predictivity

IR physics - Theory of critical phenomena

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- Non-perturbative renormalizability if FP is non-Gaußian

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- A fixed point is called *non-Gaußian*, if at least one fixed-point coupling is nonzero $g_j^* \neq 0$.

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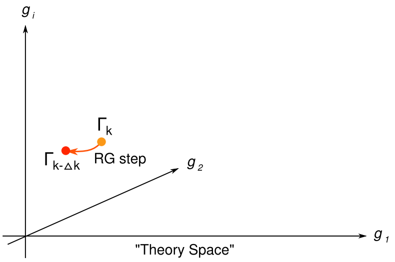

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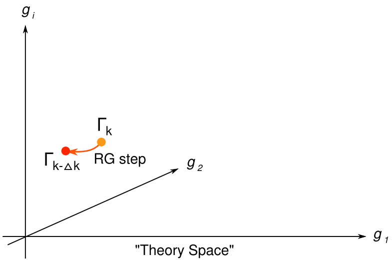

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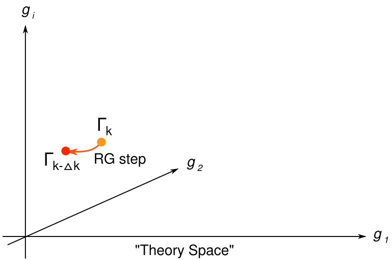

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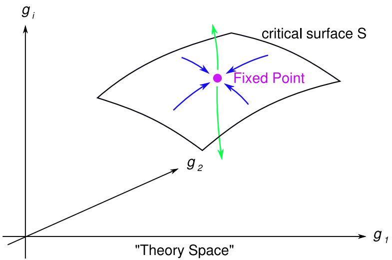

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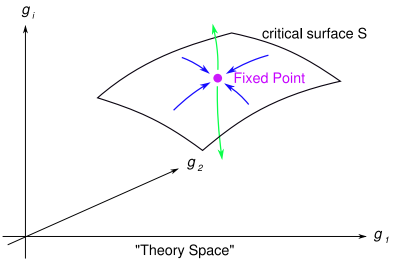

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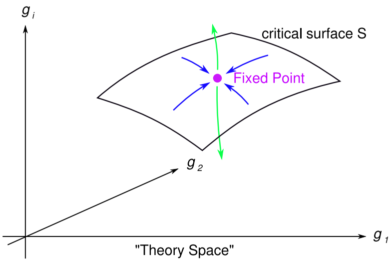

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- Impact of relevant/irrelevant couplings depends on physical context (IR or UV)

Phase transition and critical behaviour:

CHIRAL YUKAWA SYSTEMS IN d=3

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	η	ν	ω	ref. / year	
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- Also paradigm example for AS models → Renormalizing the nonrenormalizable
  (Gawedzki & Kupiainen, PRL 55, 363 (1985), Braun, D. Scherer, Gies arXiv:1009.xxxx)

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$+6$ terms with inverse flavour structure

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+6 \text{ terms with inverse flavour structure}
\end{gathered}$$

- terms with inverse flavour structure are not independent:
  $\rightarrow$ Fierz transformations $=$ 6 equations $\Rightarrow 12 - 6 = 6$ invariant terms

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- 1 scalar boson $\sim \bar{\psi}^a_{\mathrm{L}} \psi^b_{\mathrm{R}}$
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$\Rightarrow$ Yukawa action

$$S_{\mathrm{Yuk}} = \int \mathrm{d}^3 x \{ \frac{1}{2\lambda} \phi^{a\dagger} \phi^a + \bar{\psi}^a_{\mathrm{L}} \mathrm{i} \partial\!\!\!/ \psi^a_{\mathrm{L}} + \bar{\psi}_{\mathrm{R}} \mathrm{i} \partial\!\!\!/ \psi_{\mathrm{R}} + \phi^{a\dagger} \bar{\psi}_{\mathrm{R}} \psi^a_{\mathrm{L}} - \phi^a \bar{\psi}^a_{\mathrm{L}} \psi_{\mathrm{R}} \}$$

NLO derivative expansion

$$\Gamma_k = \int \mathrm{d}^3x \Big\{ Z_{\mathrm{L},k} \bar{\psi}_{\mathrm{L}}^a \mathrm{i}\partial\!\!\!/ \psi_{\mathrm{L}}^a + Z_{\mathrm{R},k} \bar{\psi}_{\mathrm{R}} \mathrm{i}\partial\!\!\!/ \psi_{\mathrm{R}} + Z_{\phi,k} \left(\partial_\mu \phi^{a\dagger} \right) (\partial^\mu \phi^a) + U_k(\rho) + \bar{h}_k \bar{\psi}_{\mathrm{R}} \phi^{a\dagger} \psi_{\mathrm{L}}^a - \bar{h}_k \bar{\psi}_{\mathrm{L}}^a \phi^a \psi_{\mathrm{R}} \Big\}, \quad \text{where } \rho = \phi^{a\dagger} \phi^a$$

- dimensionless quantities: $\tilde{\rho} = k^{2-d}\rho, \quad h^2 = k^{d-4}\bar{h}_k^2, \quad u(\tilde{\rho}) = k^{-d} U_k(\rho)|_{\rho = k^{d-2}\tilde{\rho}}$

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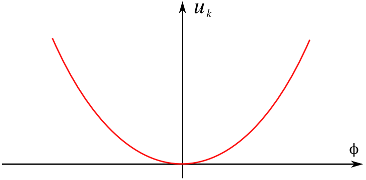

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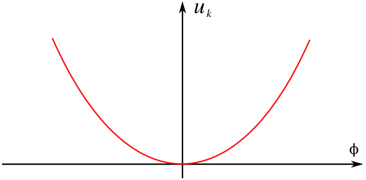

For SSB regime minimum is $\kappa_k := \tilde{\rho}_{\min} > 0$,

$$u_k = \frac{\lambda_{2,k}}{2!}(\tilde{\rho} - \kappa_k)^2 + \frac{\lambda_{3,k}}{3!}(\tilde{\rho} - \kappa_k)^3 + \ldots$$
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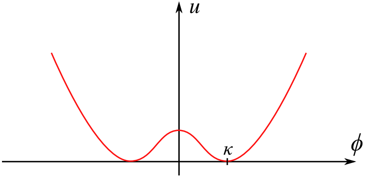

Fixed points & critical exponents in SYM and SSB

We find a NGFP for any N_{L}

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N_{L}	h_*^2	m_*^2	λ_2^*	η_ϕ^*	η_{L}^*	η_{R}^*	ν	ω
1	4.496	0.326	5.099	0.716	0.142	0.142	1.132	0.786
2	3.364	0.104	3.643	0.512	0.162	0.325	1.100	0.809

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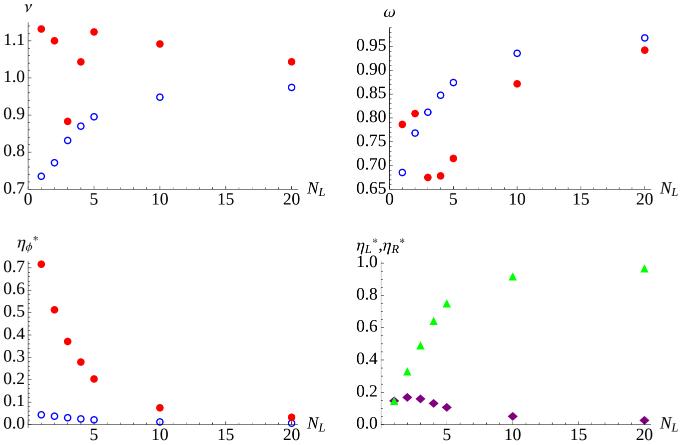
SSB: fixed-point needs balancing of fermion and boson fluctuations.

$N_{\rm L}$	h_*^2	κ_*	λ_2^*	η_ϕ^*	$\eta_{\rm L}^*$	$\eta_{\rm R}^*$	ν	ω
3	2.718	0.009	2.967	0.371	0.154	0.487	0.883	0.675
4	2.713	0.042	2.954	0.279	0.125	0.637	1.043	0.678
5	2.519	0.079	2.717	0.204	0.100	0.746	1.124	0.715
10	1.452	0.256	1.506	0.075	0.046	0.913	1.092	0.872
100	0.148	3.301	0.149	0.006	0.004	0.993	1.008	0.989

- Universal FP values and critical exponents as a function of N_{L}:



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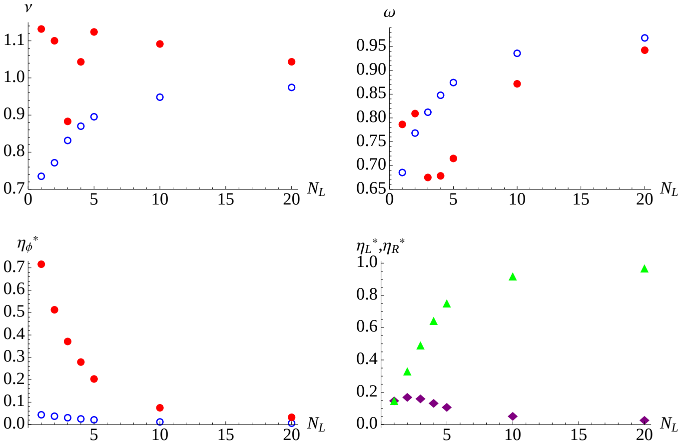

- Blue circles show values for Wilson-Fisher-FP of scalar $O(N = 2N_\mathrm{L})$-model
- Differences can be fully attributed to fermionic fluctuations near the phase transition

Asymptotic safety:

CHIRAL YUKAWA SYSTEMS IN d=4

- Higgs sector parametrizes masses of matter fields and weak gauge bosons
- will be directly tested at LHC
- Higgs sector plagued by two problems:

triviality & hierarchy problem

Motivation: Triviality & Hierarchy problem

- Parametrize Higgs field as

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{8}\phi^4 + \text{ i.a. with fermions}$$

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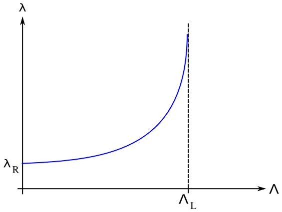

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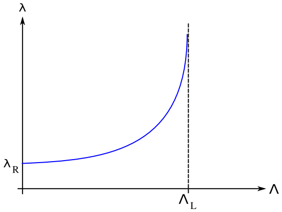

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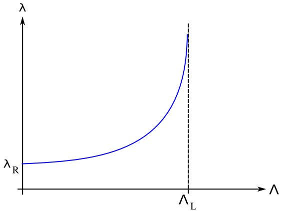

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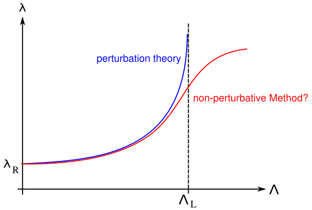

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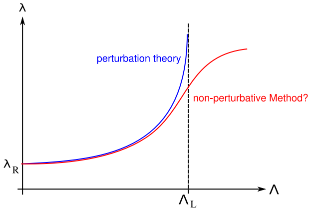

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- Hierarchy problem corresponds to existence of a large RG eigenvalues $\Theta_I > 0$ at a fixed point (e.g. in ϕ^4-theory at the GFP $\Theta = 2$)

Standard model and asymptotic safety

- If NGFP exists $\rightarrow$ Can render SM asymptotically safe!
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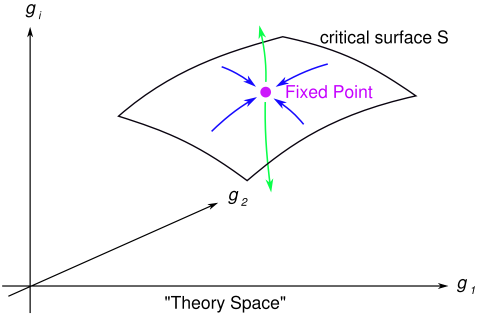
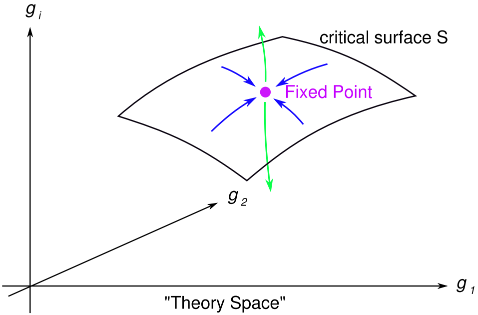
- As a toy model for the SM we will investigate a class of chiral Yukawa systems

Predictivity in the asymptotic safety scenario



- Dimension of the critical surface: $\Delta = \dim \mathcal{S} =$ Number of relevant directions



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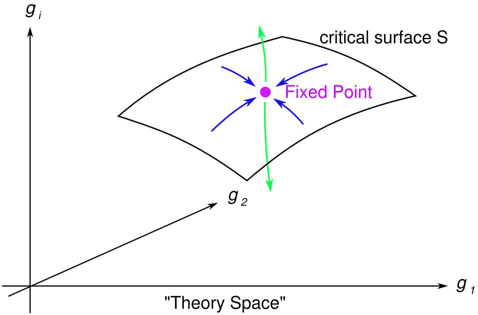

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- Nonperturbative RG computation $\Rightarrow$ Is there a NGFP? If yes, how large the Θ_I?

Toy model - chiral Yukawa system

Derivative expansion, leading-order truncation

$$\Gamma_k = \int d^4x \Big\{ i(\bar\psi_L^a \partial\!\!\!/ \psi_L^a + \bar\psi_R \partial\!\!\!/ \psi_R) + (\partial_\mu \phi^{a\dagger})(\partial^\mu \phi^a) + U_k(\rho) + \bar h_k \bar\psi_R \phi^{a\dagger} \psi_L^a - \bar h_k \bar\psi_L^a \phi^a \psi_R \Big\}$$

- N_{L} left-handed fermions ψ_{L}^a, 1 right-handed fermion ψ_{R}
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Expand effective potential about minimum $\kappa := \tilde\rho_{\rm min} > 0$ (SSB)

$$u = \frac{\lambda_2}{2!}(\tilde\rho - \kappa)^2 + \frac{\lambda_3}{3!}(\tilde\rho - \kappa)^3 + \dots$$
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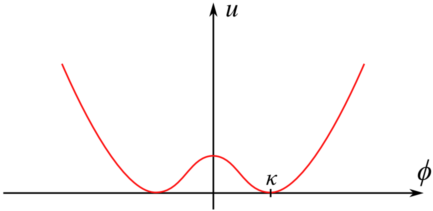

Fixed-point mechanism

Loop contributions to the running of κ

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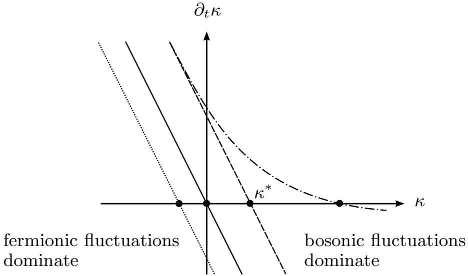

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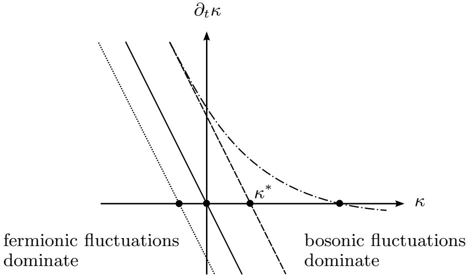

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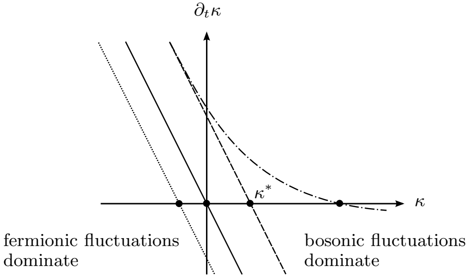

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- near FP the vev exhibits a conformal behaviour vev $\sim k$

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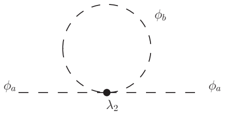
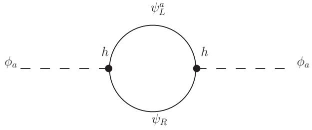
$\beta_\kappa = -2\kappa +$ 

$-$ 

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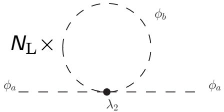

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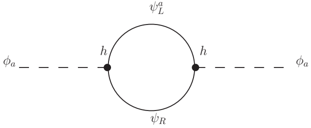
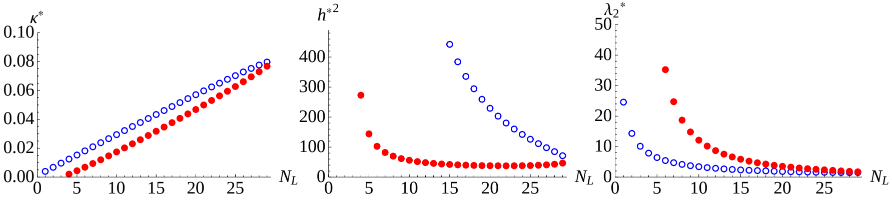


Ask not what a fixed point can do for you, ask what you can do for a fixed point! Increase N_{L}*!*

Fixed-points and critical exponents

We find NGFPs for $1 \leq N_L \leq 29$



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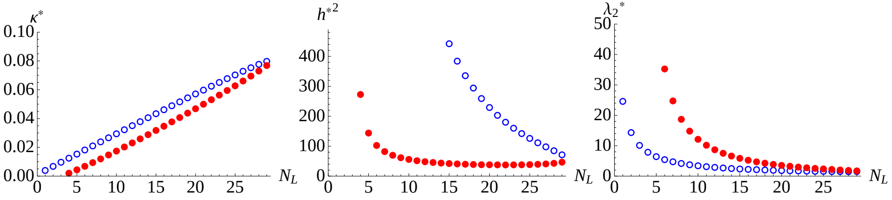
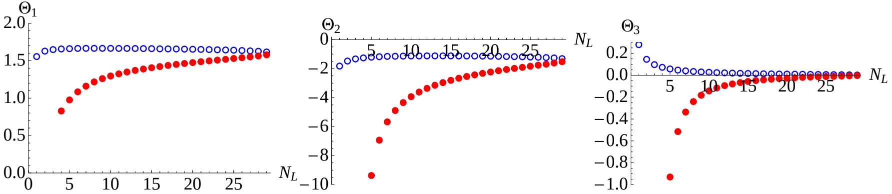


and the RG eigenvalues



Red FPs: 1 relevant direction $\rightarrow$ 1 physical parameter to be fixed

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- Choosing $v = 246\mathrm{GeV}$ and $N_{\mathrm{L}} = 10$ as an example, we find

$$m_{\text{Higgs}} = 0.97 v = 239\mathrm{GeV}, \quad m_{\text{top}} = 5.56 v = 1422\mathrm{GeV}.$$

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- Include $SU(N_{\mathrm{L}})$ gauge bosons - *work in progress*

- Promote global $SU(N_{\mathrm{L}})_{\mathrm{L}}$ symmetry to be local: $U(x) = e^{-ig\omega^i(x)T^i}$, $(i = 1, \ldots, N_{\mathrm{L}}^2 - 1)$
- g is gauge coupling constant (leave as a free parameter), T^i generators of gauge group
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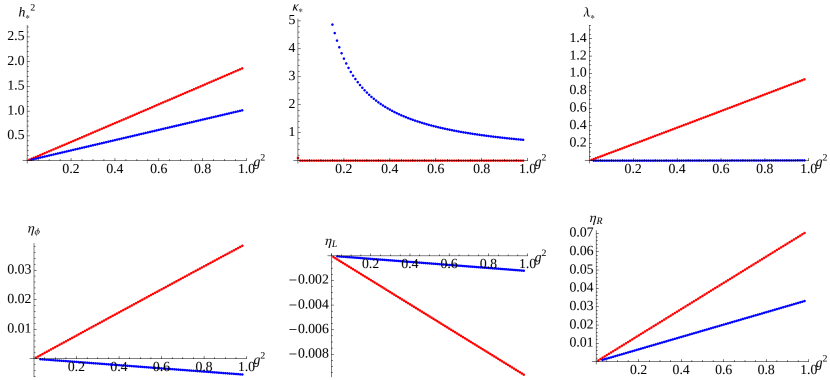
Inclusion of gauge bosons - First results

- NLO derivative expansion with effective potential up to 12th order in ϕ
- Second fixed point (blue circles), does not emerge from the GFP in the limit of vanishing g^2
- Anomalous dimensions at FP are $\lesssim 1$
- NGFP has real parts of RG eigenvalues that run towards 1.5 for $\theta_{1,2}$ for $g^2 \to 0$ (not towards its canonical power counting values 2 and 0, see blue circles)
- Products $\kappa_* h_*^2$, $\kappa_* g^2$, $\kappa_* \lambda_*$ represent masses of the top quark, gauge boson and Higgs $\to$ Remain constant as $g^2 \to 0$



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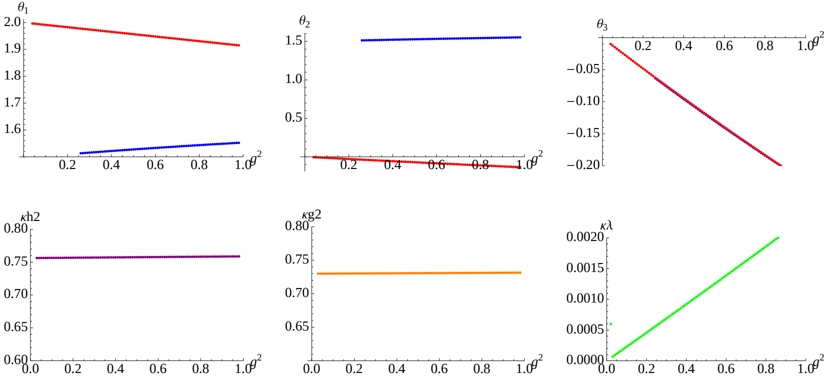

Conclusions

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- Benchmark values for critical exponents in new universality classes
- Take into account other condensation channels - competing order parameters

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Thank you for your attention.