Corfu September 2010

Fermion Mass Textures in F-theory SU(5)*

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GREECE

* *with G.G. Ross, to appear*

Fermion Mass Hierarchy

Standard lore in last ∼ 3 decades attempts (*GUTs, Strings,...*):
Fermion mass textures of **Froggat-Nielsen**-hierarchical structure

$$M_{f.} = \begin{pmatrix} \lambda_{11}\epsilon^a & \lambda_{12}\epsilon^b & \lambda_{13}\epsilon^c \\ \lambda_{21}\epsilon^b & \lambda_{22}\epsilon^d & \lambda_{23}\epsilon \\ \lambda_{31}\epsilon^c & \lambda_{32}\epsilon & 1 \end{pmatrix} \langle H \rangle$$

$\epsilon \sim 10^{-1}$ expansion parameter, a, b, c, d integer powers, and

$$\lambda_{ij} : \text{ unknown } \mathcal{O}(1) \text{ coefficients}$$

Purpose of this talk is to propose a way to...
... calculate these 'unknown' coefficients in **F-theory** GUTs

$$\lambda_{ij} \propto \int \psi_{f_i}\psi_{f_j}\phi_H$$

★ **F-theory** (*C. Vafa '96*)

- ▲ Defined on a background $\mathcal{R}^{3,1} \times \mathcal{X}$
- ▲ $\mathcal{X}$ elliptically fibered **CY** 4-fold over B_3
- ▲ B_3 complex 3-fold base.

Fibration is described by the Weierstrass model:

$$y^2 = x^3 + f\,x + g \tag{1}$$

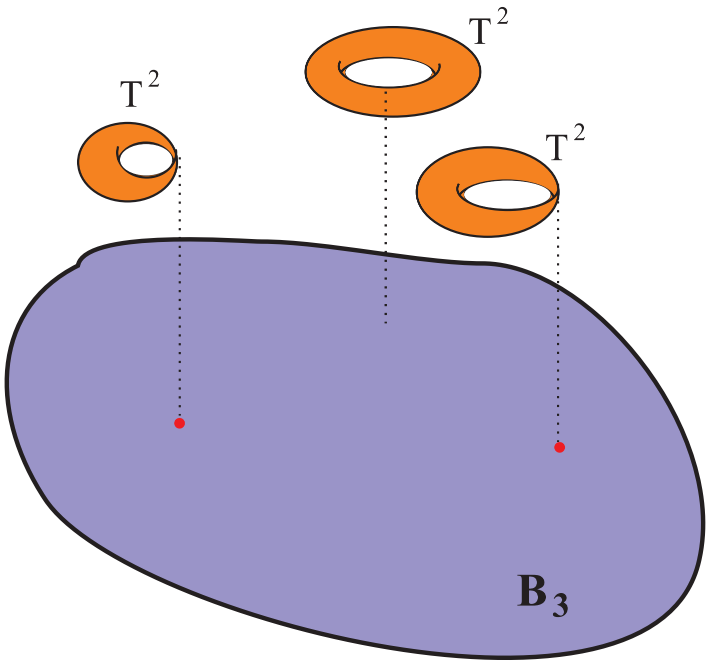
x, y parameters of the fibration

f, g functions of the base B_3, $\qquad (f \in K_{B_3}^{-4}, g \in K_{B_3}^{-6})$

The fiber degenerates at the zero loci of the discriminant

$$\Delta = 4\,f^3 + 27\,g^2 \tag{2}$$

The **singularities** of the manifold are coded in the polynomials f, g and determine the **gauge group** and **matter content** of F-theory compactification.



CY 4-fold over a base B_3 (only two dim/s shown). Every point of B_3 is represented by a torus with modulus $\tau = C_0 + \imath/g_s$.
Red points represent 7-branes, orthogonal to B_3. The torus degenerates at these 'points'.

SU(5)

Write Weierstrass equ. in 'expanded' form

$$y^2 + a_1 x\, y + a_3 y \quad = \quad x^3 + a_2\, x^2 + a_4 x + a_6 \tag{3}$$

▲ Let S a surface of singularities $S \in B_3$

z the coordinate on normal bundle to S, in B_3

S corresponds to $z = 0$ in B_3, $(B_3|_{z=0} \to S)$

▲ The order of vanishing of $a_i = b_i\, z^{n_i}$ characterizes the type of singularity

i.e, the **gauge group** supported by S.

Choice: $a_1 = -b_5, a_2 = b_4 z, a_3 = -b_3 z^2, a_4 = b_2 z^3, a_6 = z^5 b_0$

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 z^2 y + b_4 x^2 z + b_5 x y$$

$\Rightarrow$ **SU(5)** Singularity.

★ Gauge and Matter Fields in F-theory:

- **Gravity**: 10-d space
- **Gauge Fields:** 8d space on **seven-branes** (S) supporting gauge group G_S
- **Matter Fields:** 6d, on Riemann surfaces

$$\mathbf{\Sigma_i} = S \cap S_i, \quad i = 1, 2, ...$$

(i.e., on the intersections of the compact surface S with other surfaces S_i supporting some group G_i usually taken to be $G_i = U(1)_i$.)

- **Interactions:** 4d, triple intersections

$$\mathbf{W_Y} \ : \ S \cap S_i \cap S_j \to \text{point}$$

At the intersections symmetry is enhanced!

$$G_{\Sigma_i} \supset G_S \times G_{S_i}$$

$G_S = SU(5)$: Singularity enhancement:
Matter curves accommodating $\bar{\mathbf{5}}$ are associated with $SU(6)$
Matter curves accommodating $\mathbf{10}$ are associated with $SO(10)$

$$\Sigma_{\bar{5}} = S \cap S_{\bar{5}} \quad \Rightarrow \quad SU(5) \to SU(6)$$

$$\mathrm{ad}_{SU_6} = 35 \quad \to \quad 24_0 + 1_0 + 5_6 + \bar{5}_{-6} \tag{4}$$

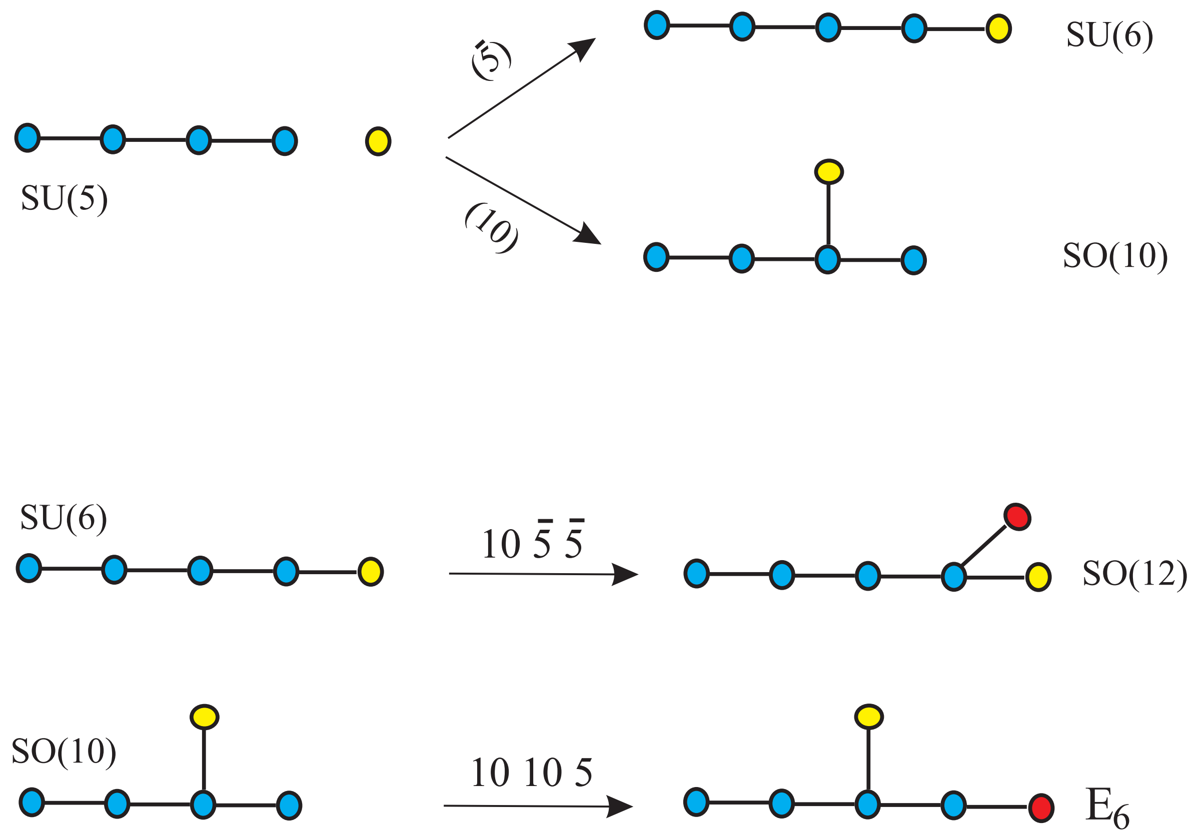
$$\Sigma_{10} = S \cap S_{10} \quad \Rightarrow \quad SU(5) \to SO(10)$$

$$\mathrm{ad}_{SO_{10}} = 45 \quad \to \quad 24_0 + 1_0 + 10_4 + \overline{10}_{-4} \tag{5}$$

Further enhancement in triple intersections→ **Yukawas**:

$$SO(10) \equiv E_5 \quad \Rrightarrow \quad E_6 \to \mathbf{10} \cdot \mathbf{10} \cdot \mathbf{5}$$

$$SU(6) \quad \Rrightarrow \quad SO(12) \to \mathbf{10} \cdot \bar{\mathbf{5}} \cdot \bar{\mathbf{5}}$$


SU(5)
($\bar{5}$)
(10)
SU(6)
SO(10)
SU(6)
10 $\bar{5}$ $\bar{5}$
SO(12)
SO(10)
10 10 5
E_6

★ Twisted YM and F-Spectrum

10-d Super YM theory :

$$\begin{cases} 10dim \text{ Gauge Field } A \\ \text{Adjoint fermions in } 16_+ \text{ of } SO(9,1) \end{cases}$$

Under Reduction $SO(9,1) \to SO(7,1)$ fields decompose to

$$\begin{cases} 8dim \text{ Gauge Field } A \\ \text{scalars } \varphi, \bar{\varphi} = \Phi_8 \pm \imath\Phi_9 \\ \text{fermions } \Psi_\pm \end{cases}$$

F-theory described by 8-d YM on $R^{7,1} = R^{3,1} \times S$.

S has a Kähler structure $\Rightarrow$
8-dYM: admits unique topological twist preserving $\mathcal{N} = 1$ SUSY.
(*Beasley, Heckmann, Vafa, 0802.3391*)

- Under twisting, scalars & fermions become **forms:**

$$\text{scalars} : \varphi \quad = \quad \varphi_{mn}\, dz^m \wedge dz^n$$

$$\text{fermions} : \quad = \quad \begin{cases} \eta_\alpha & (0,0) \\ \psi_{\dot\alpha} = \psi_{\dot\alpha m}\, dz^m & (1,0) \\ \chi_\alpha = \chi_{\dot\alpha mn}\, dz^m \wedge dz^n & (2,0) \end{cases}$$

The above can be organised in $\mathcal{N} = 1$ multiplets

$$(A_\mu, \eta),\ (A_{\bar m}, \psi_{\bar m}),\ (\phi_{12}, \chi_{12})$$

★ Zero-modes Equations

Recall that **S** Kähler spanned by $z_{1,2}$, with form given by

$$\omega = \frac{\imath}{2}\left(dz^1 \wedge d\bar{z}^1 + dz^2 \wedge d\bar{z}^2\right)$$

Assume a background for the adjoint scalar φ

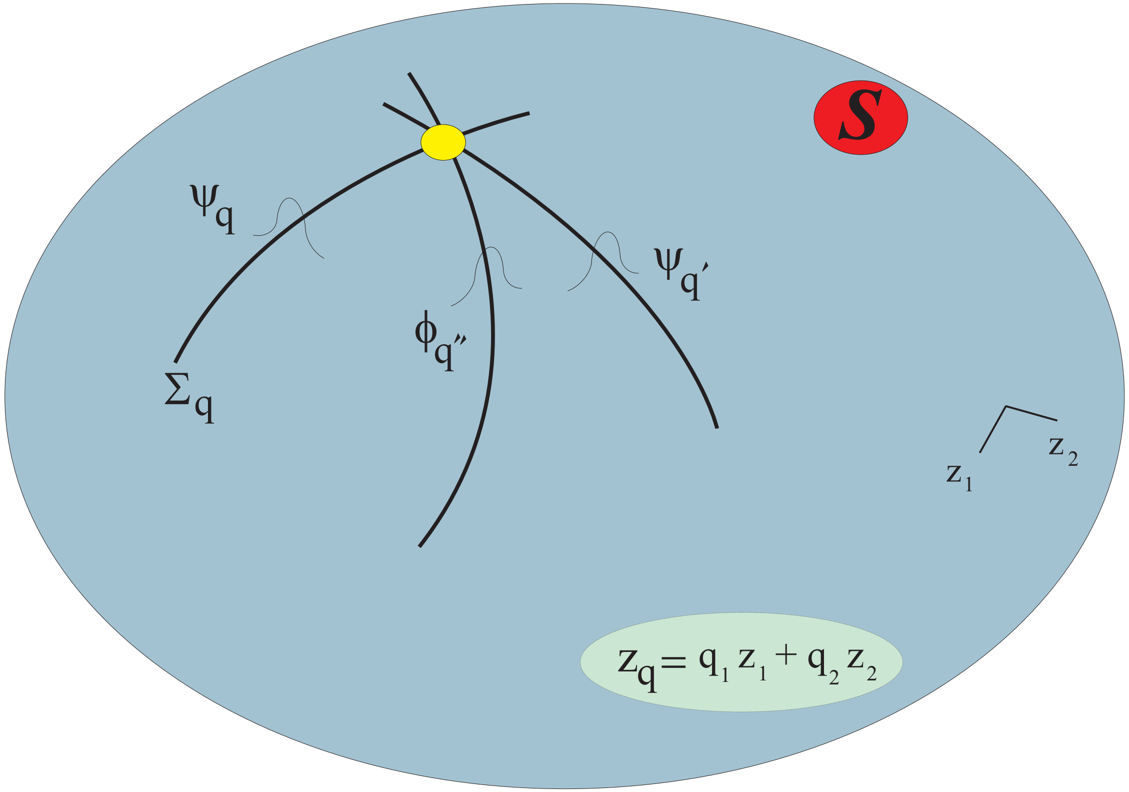
$$\langle\varphi\rangle = m^2(z_1 Q_1 + z_2 Q_2)$$

Variation of Action gives the D.Equs for **zero modes**:

$$\begin{aligned}
\partial_1\psi_1 + \partial_2\psi_2 - m^2(q_1\bar{z}_1 + q_2\bar{z}_2)\chi &= 0 \\
\bar{\partial}_1\chi - m^2(q_1\, z_1 + q_2\, z_2)\psi_1 &= 0 \\
\bar{\partial}_2\chi - m^2(q_1\, z_1 + q_2\, z_2)\psi_2 &= 0
\end{aligned}$$

- Solution: Gaussian profile for zero mode wavefunction:

$$\psi \propto exp\{-m^2 q\, |\cos\theta z_1 + \sin\theta z_2|^2\},\;\; q = \sqrt{q_1^2 + q_2^2}, \tan\theta = q_2/q_1$$



Trilinear coupling of two fermion fields ψ_i and a Higgs ϕ

Trilinear Yukawa coupling Integral:

- Computation in terms of overlapping wavefunction integrals

$$\lambda_{ij} = \frac{M_*^4}{(2\pi)^2} \int_S \psi_i \cdot \psi_j \cdot \phi \, dz_1 \wedge d\bar{z}_1 \wedge dz_2 \wedge d\bar{z}_2$$

- ψ normalization : (*Font,Ibanez 2009*)

$$\mathcal{C} = M_*^4 \int_S |\psi|^2 dz \wedge d\bar{z} = \frac{\pi}{q} \frac{M_*^4}{m^2} R^2 = \frac{\pi}{q} \frac{1}{\sqrt{a_G}}$$

(*assuming* $M_* \sim m$ *and* $M_*^2 R^2 \sim a_G$, $R \to$ *curvature*)

$$\lambda_{ij} = \frac{4\sqrt{\pi \, a_G}^{3/2}}{q + q' + q''} \frac{(q q' q'')^{3/2}}{(q_1 q_2' - q_1' q_2)^2} .$$

Example: **F-$SU(5)$**[★]

... obtained from... E_8-breaking chain... via maximal subgroup:

$$E_8 \supset SU(5) \times U(5)_{\perp} \supset SU(5) \times \mathbf{U(1)^4}$$

$\mathbf{U(1)}$'s related by monodromies ...identifying directions t_i in the $SU(5)_{\perp}$ Cartan subalgebra:

$$Q_t = \mathrm{diag}\{t_1, t_2, t_3, t_4, t_5\}$$

t_i subject to traceless condition: $t_1 + t_2 + t_3 + t_4 + t_5 = 0$.
Imposing Z_2 monodromy ($t_1 \leftrightarrow t_2$) gauge symmetry reduces to:

$$\mathbf{SU(5)} \times \mathbf{U(1)^3}$$

★ *Dudas-Palti: JHEP* **1001** *(2010) 127;arXiv:0912.0853*
King, GKL, Ross, Nucl. Phys. B **838** *(2010) 119; arXiv:1005.1025.*

Field	$SU(5)\times SU(5)_\perp$ **Rep.**	$\mathbf{t_i}$ direction	R-parity
Q_3, U_3^c, l_3^c	$(10,5)$	$t_{1,2}$	$-$
Q_2, U_2^c, l_2^c	$(10,5)$	t_4	$-$
Q_1, U_1^c, l_1^c	$(10,5)$	t_3	$-$
D_3^c, L_3	$(\overline{5},10)$	$t_{1,2}+t_4$	$-$
D_2^c, L_2	$(\overline{5},10)$	$t_{1,2}+t_3$	$-$
D_1^c, L_1	$(\overline{5},10)$	t_3+t_4	$-$
H_u	$\left(5,\overline{10}\right)$	$-t_1-t_2$	$+$
H_d	$\left(\overline{5},10\right)$	t_3+t_5	$+$
θ_{ij}	$(1,24)$	t_i-t_j	$+$
θ'_{ij}	$(1,24)$	t_i-t_j	$-$

$$\langle\theta_{14}\rangle \cdot \langle\theta_{43}\rangle \neq 0$$

- **Rank one** Quark mass matrices (*Dudas-Palti, arXiv:0912.0853*)

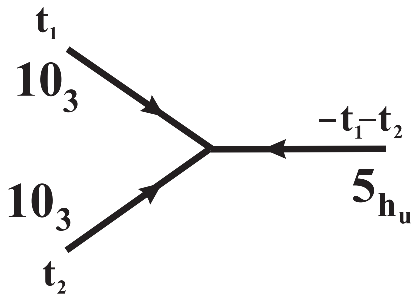
$$M_d = \begin{pmatrix} \lambda^d_{11}\theta^2_{14}\theta^2_{43} & \lambda^d_{12}\theta_{14}\theta^2_{43} & \lambda^d_{13}\theta_{14}\theta_{43} \\ \lambda^d_{21}\theta^2_{14}\theta_{43} & \lambda^d_{22}\theta_{14}\theta_{43} & \lambda^d_{23}\theta_{14} \\ \lambda^d_{31}\theta_{14}\theta_{43} & \lambda^d_{32}\theta_{43} & 1\times\lambda^d_{33} \end{pmatrix} v_b, \tag{6}$$

$$M^u = \begin{pmatrix} \lambda^u_{11}\theta^2_{14}\theta^2_{43} & \lambda^u_{12}\theta^2_{14}\theta_{43} & \lambda^u_{13}\theta_{14}\theta_{43} \\ \lambda^u_{21}\theta^2_{14}\theta_{43} & \lambda^u_{22}\theta^2_{14} & \lambda^u_{23}\theta_{14} \\ \lambda^u_{31}\theta_{14}\theta_{43} & \lambda^u_{32}\theta_{14} & 1\times\lambda^u_{33} \end{pmatrix} v_u \tag{7}$$

- λ_{ij} computed from overlapping integrals ... *expected of* $\mathcal{O}(1)$.

Are λ_{ij} really $\sim \mathcal{O}(1)$???

Computing $\lambda_{33} \equiv \lambda_{top}$**:** Define 'vector basis': $|t_i>_j = \delta_{ij}$
$(i.e. < t_1| = \{1, 0, 0, 0, 0\},\ etc)$



Define Locally the set of orthonormal operators Q_i:

$$Q_1 = \frac{1}{\sqrt{30}}\{3, 3, -2, -2, -2\}$$

$$Q_2 = \frac{1}{\sqrt{2}}\{1, -1, 0, 0, 0\},$$

$$Q_3 = \frac{1}{\sqrt{2}}\{0, 0, 1, 0, -1\}$$

$$Q_4 = \frac{1}{2}\{0, 0, 1, -2, 1\}$$

Vertex states $|t_{1,2}>, |-t_1-t_2>$ of top coupling are annihilated by:

$$Q_{3,4}\,|t_{1,2}> = 0$$

while, acting by $Q_{1,2}$:

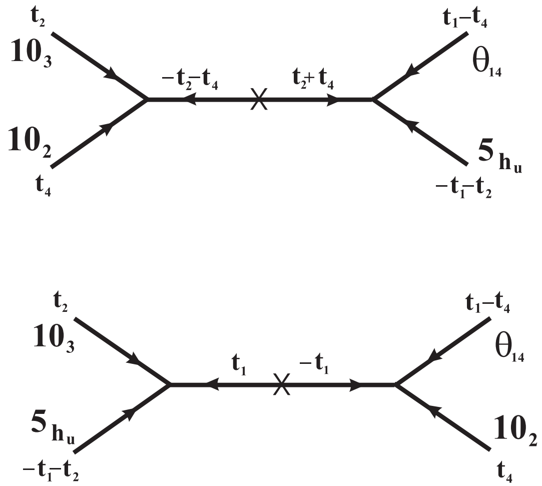
$$\{q_1, q_2\} = \left\{\sqrt{\frac{3}{10}}, \frac{1}{\sqrt{2}}\right\},\ \{q'_1, q'_2\} = \left\{\sqrt{\frac{3}{10}}, -\frac{1}{\sqrt{2}}\right\}$$

Substitution to the overlapping integral:

$$\lambda_{top} = 0.31 \times \left(\frac{a_G}{a_{G_0}}\right)^{\frac{3}{4}}, \quad a_{G_0} = \frac{1}{24}$$

(... presence of exotic matter: $a_G > a_{G_0} \Rightarrow \lambda_{top} > 0.31$)

Computing higher dimensional couplings: Example: λ_{23}.



$10_3\, 10_2\, 5_{h_u}\theta_{14}$ with exchange of massive messenger states

KK-mode wavefunction: Obeys modified DE

$$(\partial_1 - \imath A_1)\psi_1 - m^2 q_1 \bar{z}_1 \chi = 0$$

Solution: $\psi_1 \sim e^{-q_1 m^2 \xi\, |z_1|^2}$, $\xi < 1$.

Left vertex of U_{23}-graphs:

$$10_i 10_j 5_{KK} \,:\; \{t_i\}_{0-mode} + \{t_j\}_{0-mode} \to \{-t_i - t_j\}_{KK}$$

Calculation of the overlapping integral:

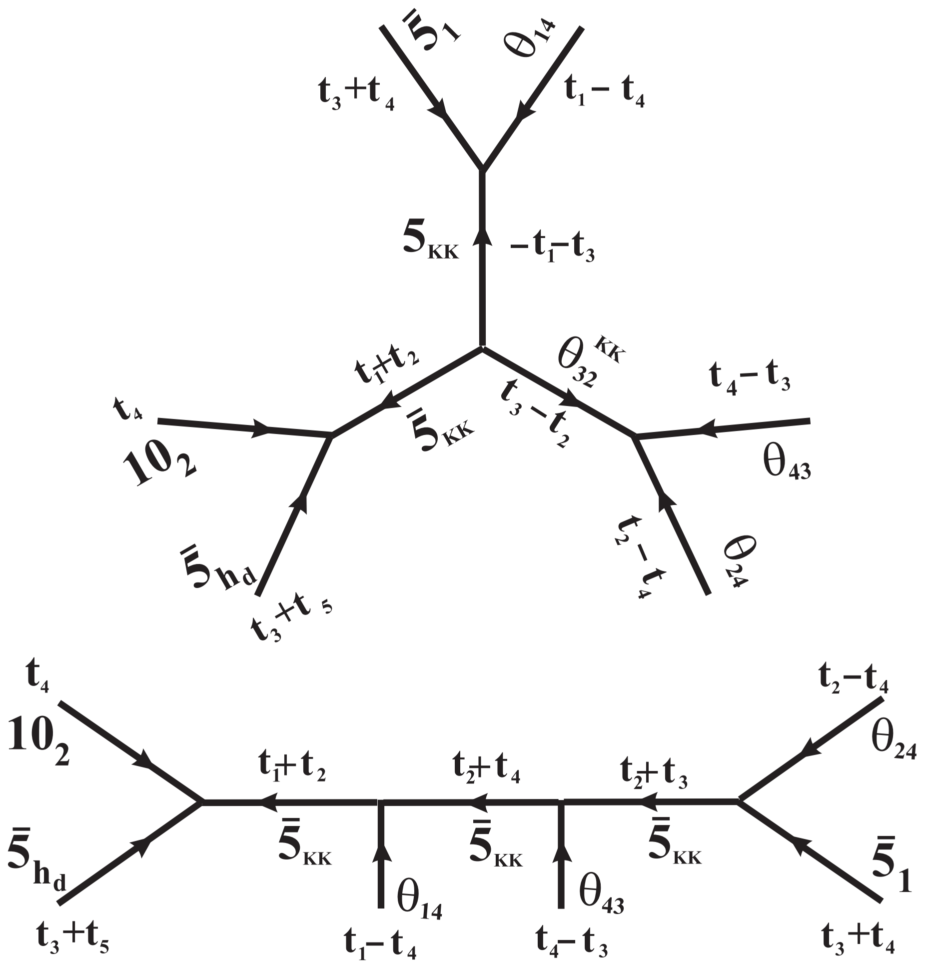
$$I_a(\xi) = \frac{8\sqrt{2\pi\,\xi}}{3\cdot 5^{3/4}\left(4\xi + \sqrt{6}\right)}$$

Right vertices ($\overline{10}\cdot 10\cdot 1$ and $\overline{5}\cdot 5\cdot 1$) :

$$I_x(\xi) = \frac{8\sqrt{10\pi\,\xi}}{3\cdot 3^{3/4}\left(4\sqrt{5}\xi + 5\sqrt{2}\right)},\; I_y = \frac{2\cdot 3^{3/4}\sqrt{10\pi}\sqrt{\xi}}{7\left(\left(5+\sqrt{15}\right)\xi + \sqrt{15}\right)}$$

U_{23} - **Yukawa Coupling:**

$$\lambda^u_{23}(\xi) = I_a(\xi)\cdot\left(I_x(\xi) + I_y(\xi)\right)$$



Representative graphs for λ_{21}^{b} Yukawa coupling.

Results: (simplified case $\xi = 1$)

$$M_d = \begin{pmatrix} 0.12\,\theta_{14}^2\theta_{43}^2 & 0.11\,\theta_{14}\theta_{43}^2 & 0.18\,\theta_{14}\,\theta_{43} \\ 0.14\,\theta_{14}^2\theta_{43} & 0.16\,\theta_{14}\theta_{43} & 0.20\,\theta_{14} \\ 0.09\,\theta_{14}\theta_{43} & 0.17\,\theta_{43} & 0.29 \end{pmatrix}$$

$$M_u = \begin{pmatrix} 0.09\,\theta_{14}^2\theta_{43}^2 & 0.22\,\theta_{14}^2\theta_{43} & 0.16\,\theta_{14}\theta_{43} \\ 0.22\,\theta_{14}^2\theta_{43} & 0.18\,\theta_{14}^2 & 0.22\,\theta_{14} \\ 0.16\,\theta_{14}\theta_{43} & 0.22\,\theta_{14} & 0.31 \end{pmatrix}$$

For $\langle\theta_{43}\rangle \sim \frac{1}{2}, \;\; \langle\theta_{14}\rangle \sim \frac{1}{10} \rightarrow$

Reasonable hierarchy and CKM-mixing

▲▼ $\mathcal{C}$harged $\mathcal{L}$eptons ▲▼

Major problem in $\mathbf{SU(5)}$:

$$m_s = m_\mu \;\&\; m_d = m_e \quad \text{at } M_{GUT}$$

SOLUTION:

▲▼**Splitting of $SU(5)$-reps** via the **FLUX mechanism**

Two types of fluxes:

▲M_{10}, M_5: connected to $U(1)$'s$\in SU(5)_\perp$: determine the chirality of complete $10, 5 \in SU(5)$.

▲N_Y: related to Cartan generators of $SU(5)_{GUT}$. They are taken along $U(1)_Y \in SU(5)_{GUT}$ and **split** $SU(5)$-reps.

$$matter\, curve\, 10^{(3)}: \quad \rightarrow \quad Q_2 + u_2^c \equiv \begin{pmatrix} c \\ s \end{pmatrix} + c^c$$

$$matter\, curve\, 10^{(4)}: \quad \rightarrow \quad e_2^c \equiv \mu^c$$

Flux-splitting of **10**'s:

$$n_{(3,2)_{\frac{1}{6}}} - n_{(\bar{3},2)_{-\frac{1}{6}}} = M_{10} \quad (8)$$

$$n_{(\bar{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} = M_{10} - N_Y$$

$$n_{(1,1)_1} - n_{(1,1)_{-1}} = M_{10} + N_Y \quad (9)$$

Flux-splitting of **5**'s:

$$n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} = M_5 \quad (10)$$

$$n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_5 + N_Y$$

$\Rightarrow$ Resulting $M_{\ell epton} \neq M_{down}$

$$M_\ell = \begin{pmatrix} \theta_{14}^2\theta_{43}^2 & \theta_{15}\theta_{14}\theta_{43} & \theta_{14}\,\theta_{43} \\ \theta_{14}^2\theta_{43} & \theta_{15}\theta_{43} & \theta_{14} \\ \theta_{14}\theta_{43} & \theta_{15} & 0.295 \end{pmatrix}$$

Neutrinos

Z_2-monodromy $\theta'_{12} \leftrightarrow \theta'_{21} \Rightarrow$

$\theta'_{12} =$ **Right-Handed Majorana Neutrino**

(*Bouchard et al, 0904.1419*)

Parameters can be adjusted to give bi-tri maximal mixing.

Features of F-Models

1. Unified and in particular Exceptional **Groups** are back!
2. **Matter** is deeply and intimately connected to the **Internal Geometry**
3. **Chirality** related to topology and to the internal flux
4. Higher unification scale, **safe proton**
5. Calculable Yukawas and **acceptable fermion mass hierarchy**

ADDITIONAL MATERIAL

MONODROMIES

Recall coefficients b_i from $E_8 \to SU(5)$ deformed singularity.
A parameter s can be defined locally on **S**

$$f(s) = b_0 s^5 + b_2 s^3 + b_3 s^2 + b_4 s + b_5 = 0$$

whose roots are t_i. Thus $b_j = b_j(t_i)$. $\quad (b_1 = \sum_i t_i = 0)$
Factorization of $f(s)$ implies symmetries (monodromies for t_i

Monodromies for $SU(5) \to U(1)^4$, $(t_1 + t_2 + t_3 + t_4 + t_5 = 0)$:

$$\begin{aligned} \mathcal{S}_4 &: \{t_1, t_2, t_3, t_4\},\ \{t_5\} \\ \mathcal{Z}_2 \times \mathcal{Z}_2 &: \{t_1, t_2\},\ \{t_3, t_4\}, \{t_5\} \\ \mathcal{Z}_2 &: \{t_1, t_2\},\ \{t_3\}, \{t_4\}, \{t_5\} \\ \cdots & \quad \cdots \end{aligned} \tag{11}$$

Gauge Couplings and the GUT scale

In F-theory gauge coupling unification is distorted by

I: Y-Flux breaking mechanism

&

II: Threshold effects from heavy modes

$SU(5)$ Fields expected to contribute:

$$\Sigma_{5/\bar{5}} \quad \to \quad D + \ell \quad + c.c.$$

$$\Sigma_{10,\overline{10}} \quad \to \quad Q + u^c + e^c \ + c.c.$$

$$24_{(Bulk)} \quad \to \quad (8,1)_0 + (1,3)_0 + (1,1)_0 + Q' = (3,\bar{2})_{-5} + \bar{Q}' = (\bar{3},2)_5$$

$\cdots + \cdots$ states decoupling at M_{CFT} (see *Vafa*, 2010)

I. Flux thresholds

$$\frac{1}{a_3(M_G)} = \frac{1}{a_G} - y, \ \frac{1}{a_2(M_G)} = \frac{1}{a_G} - y + x, \ \frac{1}{a_1(M_G)} = \frac{1}{a_G} - y + \frac{3}{5}x$$

(*Blumenhagen, Phys.Rev.Lett.102:071601,2009.*)

$$x = -\frac{1}{2}S\int c_1^2(\mathcal{L}_Y), \quad y = \frac{1}{2}S\int c_1^2(\mathcal{L}_a)$$

$\mathcal{L}_a$ non-trivial line bundle, $S = e^{-\phi} + \imath C_0$ the axion-dilaton field.

elimination of exotics $Q', \bar{Q}'$,

$$\chi(S, \mathcal{L}_{\mathcal{Y}}) = 0 \ \Rightarrow \int c_1^2(\mathcal{L}_Y) = -2$$

$$\Rightarrow x \geq 0 \ \& \ a_3 \geq a_1 \geq a_2 \tag{12}$$

At M_{GUT} :

$$\frac{5}{3}\frac{1}{a_1(M_G)} = \frac{1}{a_2(M_G)} + \frac{2}{3}\frac{1}{a_3(M_G)} \tag{13}$$

★ $a_i(M_G)$ relation weaker than unification $a_i(M_G) = a_G$. Use it to:
1) **raise** M_G to make d-5 proton decay operators harmless.
2) increase a_G-value to improve top-quark mass predictions

II: Threshold effects from heavy modes

M_X : Decoupling scale of extra matter

$$[5(b_1^x - b_1) - 2(b_3^x - b_3)] \ln\left(\frac{M_G}{M_X}\right) = 0 \tag{14}$$

a: Minimal case: If extra matter **only** *color* triplets, $[\cdots] \equiv 0 \Rightarrow$

$$\forall\, x \quad \exists\ M_X$$

$\Rightarrow M_G \sim 2 \times 10^{16}$GeV is found as the scale where (13) holds.

b: General case:

Extra matter (in vector like pairs) n_u, n_L, n_Q, n_{e^c}.

Assume decoupling at scale M_X.

Inequalities (12) for $a_i(M_U)$ are converted to extra matter constraints

$$(n_d - n_L - 2n_e - n_u + 3n_Q)\ \ln\frac{M_U}{M_X} > 32\ \ln\frac{M_U}{M_Z} - 4\pi\left(\frac{\cos^2\theta_W}{a_{em}} - \frac{5}{3a_3}\right)$$

$$(n_d - n_L + 3n_e + 4n_u - 7n_Q)\ \ln\frac{M_U}{M_X} > -28\ \ln\frac{M_U}{M_Z} + 2\pi\frac{3 - 8\sin^2\theta_W}{a_{em}}$$

Constraints can be converted to $M_X - M_U$ plots for values of

$$\beta_x = b^x_Y - b^x_2 - (2/3)b^x_3$$

(For example $\beta_x = 10$ for $\{2Q, 2u\}$, $\{4Q, 6u\}$, or $\{6Q, 8u, 2e\}$, ... etc.)

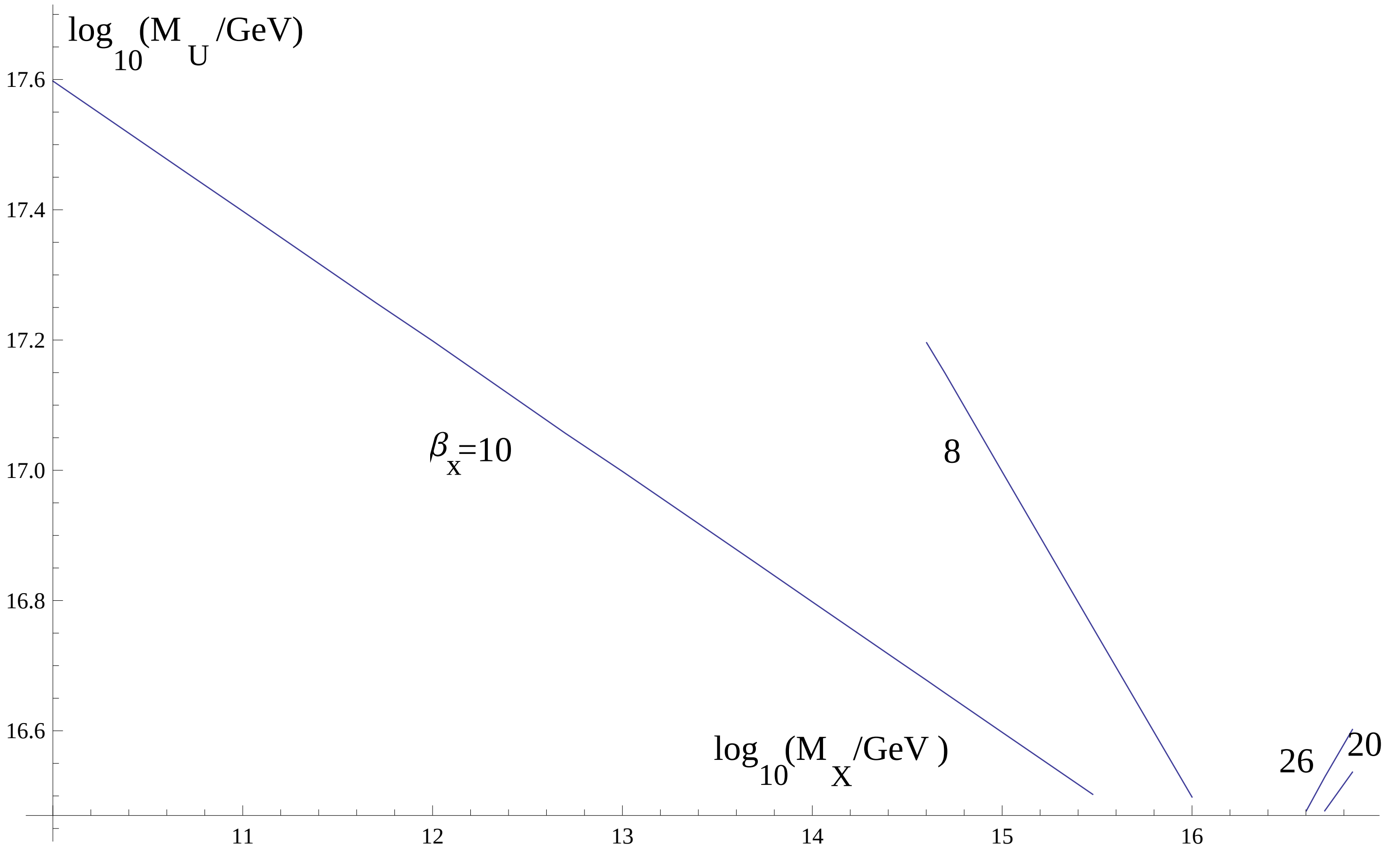


Figure 1: M_U as a function of the decoupling scale M_X for extra matter context $\beta_x = 10, 8$, and $\beta_x = 20, 26$.

M_X(GeV)	M_U(GeV)	$a_3(M_U)$	$(5/3)a_1(M_U)$	$a_2(M_U)$	x
10^{11}	6.23×10^{17}	0.1568	0.1510	0.1473	0.4103
10^{12}	2.90×10^{17}	0.0993	0.0973	0.0961	0.3335
10^{13}	1.51×10^{17}	0.0743	0.0735	0.0730	0.2308
10^{14}	8.22×10^{16}	0.0599	0.0596	0.5947	0.1154

Table 1: Two-loop results for the $SU(5)$ GUT scale M_U and the 'shifted' gauge coupling values $a_i(M_U)$ in the case of two vector-like $Q + \bar{Q}$ quark pairs and three $u^c + \bar{u}^c$ pairs. The corresponding decoupling scale M_X is shown in the first column.

Gaugino masses

Departure from Standard gaugino scenario due to gauge coupling splitting at M_{GUT} ⋆:

$$\mathbf{2\,\frac{M_3}{a_3} + 3\,\frac{M_2}{a_2} - 5\,\frac{M_1}{a_1} = 0} \tag{15}$$

GKL & N.D. Tracas, arXiv:0912.1557, E.P.J.C67:489-498,2010;

Sum rule (15) holds irrespectively of the initial $M_i(M_G)$ (universal or not) and leads to interesting phenomenological implications. (see also *T. Li et al, arXiv:1002.1031*)

⋆ *The same sum rule has been obtained in the context of no-scale SUGRA. See J. Ellis et al, Phys.Lett.B155:381,1985.*

III: Threshold effects from KK-modes:

Decoupling limit: light KK-modes from SU(5) gauge fields and $\Sigma_{10}, \Sigma_{\bar{5}}$ matter curves induce corrections to a_i^{-1} (Donagi, Wijnholt 08):

$$\delta_i = \delta_i^g + \delta_i^c + \delta_i^h$$

Modifications of M_U at one loop

$$M_U' = e^{-\frac{2\pi}{\beta_x}\delta} M_U \quad \text{with}\ \delta = \frac{5}{3}\delta_1 - \delta_2 - \frac{2}{3}\delta_3 \tag{16}$$

Corrections can be expressed in terms of **Ray-Singer Torsion** (Donagi,Wijnholt '08)

Estimates

Calculation of thresholds for choices of particular line bundles (*GKL and N.D. Tracas*, arXiv:0912.1557)

Gauge thresholds:

$$\delta_i^g = \frac{b_i^g}{4\pi}\log\frac{M^2}{\mu^2} + \tilde{\delta}_i^g$$

$$\tilde{\delta}_i^g = \frac{b_i^{5/6}}{2\pi}(T_{5/6} - T_0) = -\frac{1}{\pi} < 0$$

Matter thresholds:

$$\Sigma_{\bar{5}} : \delta^{\bar{5}} = 0$$

$$\Sigma_{10} : \delta^{10} = \frac{1}{\pi}(T_{\mathcal{O}} - T_{L_z}) > 0$$

...*work in progress...*

Conclusion

1. **Matter** is deeply and intimately connected to the **Internal Geometry**

2. **Yukawas and Fermion mass Hierarchy** are related to topology and possibly to the internal flux

3. Unified and in particular Exceptional **Groups** are back!

4. Higher unification scale, **safe proton**

5. New sum rule **for gaugino masses**

Tate's Algorithm

Group	a_1	a_2	a_3	a_4	a_6	Δ
...	...	...	...	...	...	...
$SU(2n)$	0	1	n	n	$2n$	$2n$
$SU(2n+1)$	0	1	n	$n+1$	$2n+1$	$2n+1$
$SO(10)$	1	1	2	3	5	7
E_6	1	2	3	3	5	8
E_7	1	2	3	3	5	9
E_8	1	2	3	4	5	10

Table 2: (*Tate, 1975, Bershadsky et.al. hep-th/9605200.*) The order of vanishing of the coefficients $a_i \sim z^{n_i}$ and the corresponding gauge group. The highest singularity allowed in the elliptic fibration is E_8.