NONCOMMUTATIVE SUPERGRAVITY

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1. INTRODUCTION

- WHY NC AT SMALL SCALES $\sqrt{\frac{\hbar G}{c^3}}$
- $[x^\mu, x^\nu] = i\theta^{\mu\nu}$
- KNOWN (OLD) EXAMPLES :
  - PHASE SPACE OF QM : $[x^i, p^j] = i\hbar\delta^{ij}$
  - ELECTRONS IN 2-DIM + STRONG MAGNETIC FIELD $\vec{B}$

    $\{x^\mu, x^\nu\} = \frac{c}{e|B|}\varepsilon^{\mu\nu}$. QUANTIZATION $\longrightarrow$ NC POSITION OP.
- SNYDER (1947)
- DEFORMATION (OF PRODUCT DUE TO) QUANTIZATION

  ARISES IN THE NC STRUCTURE OF QM

- WEYL QUANTIZATION RULE

CLASSICAL PHASE SPACE FUNCTIONS $\xrightarrow[W]{}$ QUANTUM OPERATORS

$$q^m p^n \rightarrow W(q^m p^n) = \frac{1}{2^n} \sum_{k=0}^{n} \binom{n}{k} \hat{p}^{n-k} \hat{q}^m \hat{p}^k$$

ex: $W(qp^2) = \frac{1}{4}(\hat{p}^2\hat{q} + 2\hat{p}\hat{q}\hat{p} + \hat{q}\hat{p}^2)$

- YIELDS HERMITIAN OPERATOR
- CAN BE RESTATED AS $W(q^m p^n) = :e^{-\frac{i\hbar}{2}\frac{\partial^2}{\partial q \partial p}} q^m p^n:$

$: :$ MEANS $q \to \hat{q}$, $p \to \hat{p}$ AND $\hat{q}$ ORDERED TO LEFT

- W IS INVERTIBLE $\Rightarrow$ 1-1 CORRESP. BETWEEN OP. AND PHASE SPACE FUNCTIONS

- MOYAL PRODUCT

$$A * B \equiv W^{-1}\left(W(A)\, W(B) \right)$$

VON NEUMANN (1931), GROENEWOLD (1946), MOYAL (1949) ...

- EXPLICITLY

$$A * B = A(q,p)\, e^{\frac{i\hbar}{2}\Delta}\, B(q,p)$$

$$\Delta \equiv \frac{\overleftarrow{\partial}}{\partial q}\frac{\overrightarrow{\partial}}{\partial p} - \frac{\overleftarrow{\partial}}{\partial p}\frac{\overrightarrow{\partial}}{\partial q} \qquad A\,\Delta\, B = \{A,B\}_{PB}$$

- INHERITS THE PROPERTIES OF THE OP. PRODUCT:

ASSOCIATIVE , NONCOMMUTATIVE

KONTSEVICH, FEDOSOV : GIVEN A POISSON STRUCTURE

$\{A,B\} = \theta^{ij}(x)\,\partial_i A\,\partial_j B$ ON A MANIFOLD $\mathcal{M}$

THERE IS ESSENTIALLY ONE $*$ PRODUCT

$$A * B = AB + \frac{i\hbar}{2}\{A,B\} + O(\hbar^2)$$

UP TO LINEAR REDEFINITIONS OF A, B

$$A \to D(\hbar)A \equiv A + \hbar D_1(A) + \hbar^2 D_2(A) + \cdots$$

D_i : $\mathrm{Fun}\,\mathcal{M} \to \mathrm{Fun}\,\mathcal{M}$ diff op.

LC (1978) : RELATION TO OTHER QUANTIZATION RULES

NB isomorphic algebras, but **inequivalen**t hamiltonians
cf Vassilevich

2. $\ast$ FIELD THEORIES

- POINT OF VIEW : USUAL FUNCTIONS OF $\mathbb{R}^4$ WITH DEFORMED PRODUCT (MOYAL)

$$A(x) * B(x) \equiv A(x)\, e^{\frac{i}{2} \overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu} B(x)$$

$$= AB + \frac{i}{2}\theta^{\mu\nu}\partial_\mu A \partial_\nu B + \frac{1}{2!}\left(\frac{i}{2}\right)^2 \theta^{\mu\nu}\theta^{\rho\sigma}(\partial_\mu\partial_\rho A)(\partial_\nu\partial_\sigma B)$$

$$+ \cdots$$

- IN PARTICULAR $\quad x^\mu * x^\nu - x^\nu * x^\mu = i\theta^{\mu\nu}$

- STUDY OF $\ast$ DEFORMED FIELD THEORIES

$$S = \int d^D x\, \mathcal{L}(\phi)_*$$

- CYCLICITY OF $\int \phi * \chi * \cdots \, d^D x$

NEXT : * GRAVITY AND SUPERGRAVITY

- FORMS, d, $\wedge$, $\mathcal{L}_v$ ... $\to$ * DIFF CALCULUS
- MORE GENERAL: TWISTED DIFF CALCULUS

$\to$ DEFORM PRODUCTS OF FORMS

3. TWISTED DIFF CALCULUS

WORONOVICZ, ... WESS GROUP

i) $g * h \equiv \mu \circ F^{-1}(g \otimes h)$, $\quad F$: TWIST

$F \in U\Xi \otimes U\Xi$, $\quad \Xi$: LINEAR SPACE OF SMOOTH VECTOR FIELDS ON SMOOTH MANIFOLD $\mathcal{M}$

- Example : MOYAL TWIST (A PARTICULAR ABELIAN TWIST)

$$F = \exp\left(-\frac{i}{2}\theta^{\mu\nu}\partial_\mu \otimes \partial_\nu\right)$$

- NOTATION : $\quad F = f^\alpha \otimes f_\alpha$, $\quad F^{-1} = \bar{f}^\alpha \otimes \bar{f}_\alpha$

THUS : $f * g = \bar{f}^{\alpha}(f)\, \bar{f}_{\alpha}(g)$

- $*$- EXTERIOR PRODUCT $\wedge_*$

$$\tau \wedge_* \tau' \equiv \bar{f}^{\alpha}(\tau) \wedge \bar{f}_{\alpha}(\tau')$$

$$\bar{f}^{\alpha}(\tau) \equiv \mathcal{L}_{\bar{f}^{\alpha}} \tau$$

ASSOCIATIVE

- EXTERIOR DERIVATIVE d

$$d(f * g) = df * g + f * dg$$

$$d(\tau \wedge_* \tau') = d\tau \wedge_* \tau' + (-1)^{\deg(\tau)}\, \tau \wedge_* d\tau'$$

USUAL (GRADED) LEIBNIZ RULE

- INTEGRATION: GRADED CYCLICITY

FOR ABELIAN TWISTS:

$$\int \tau \wedge_\star \tau' = (-1)^{\deg(\tau)\deg(\tau')} \int \tau' \wedge_\star \tau$$

UP TO BDY TERMS

- COMPLEX CONJUGATION

CHOOSING REAL FIELDS IN THE DEF. OF TWIST $\Rightarrow$

$$(f \star g)^* = g^* \star f^*$$

$$(\tau \wedge_\star \tau')^* = (-1)^{\deg(\tau)\deg(\tau')} \tau'^* \wedge_\star \tau^*$$

4. * GRAVITY

i) CLASSICAL

$$S = \int R^{ab} \wedge V^c \wedge V^d \epsilon_{abcd} = -4 \int R \sqrt{-g}\, d^4x$$

WITH $R^{ab} \equiv d\omega^{ab} - \omega^{ac} \wedge \omega_c{}^b$

- INDEX-FREE :

$$S = i \int \mathrm{Tr} \left(R \wedge V \wedge V \gamma_5 \right)$$

$V = V^a \gamma_a$, $\Omega = \frac{1}{4} \omega^{ab} \gamma_{ab}$, $R = d\Omega - \Omega \wedge \Omega$

THEN $R = \frac{1}{4}\left(d\omega^{ab} - \omega^{ac}\wedge\omega_c{}^b\right)\gamma_{ab} = \frac{1}{4}R^{ab}\gamma_{ab}$

AND $i\int \mathrm{Tr}\left(R\wedge V\wedge V\gamma_5\right) =$

$$\frac{i}{4}\int R^{ab}\wedge V^c\wedge V^d \underbrace{\mathrm{Tr}\left(\gamma_{ab}\gamma_c\gamma_d\gamma_5\right)}_{-4i\epsilon_{abcd}}$$

IS THE USUAL EINSTEIN-HILBERT ACTION

- INVARIANCES :
  - DIFF $\mathcal{L}_\varepsilon = i_\varepsilon d + d i_\varepsilon$ $\qquad \varepsilon = \varepsilon^\mu\partial_\mu$
  - LOCAL LORENTZ $\delta_\varepsilon V = -V\varepsilon + \varepsilon V$ $\qquad \varepsilon = \frac{1}{4}\varepsilon^{ab}\gamma_{ab}$

$\delta_\varepsilon\Omega = d\varepsilon - \Omega\varepsilon + \varepsilon\Omega$

• UNDER LORENTZ : $\delta_\varepsilon R = \varepsilon R - R\varepsilon$

THUS $\delta_\varepsilon S = -i\,\delta_\varepsilon \int \mathrm{Tr}(R \wedge V \wedge V \gamma_5) = 0$

BY CYCLICITY OF Tr, AND BECAUSE $[\gamma_5, \varepsilon] = 0$

HERMITICITY

$$\gamma_0 V \gamma_0 = V^\dagger, \quad -\gamma_0 \Omega \gamma_0 = \Omega^\dagger, \quad -\gamma_0 \varepsilon \gamma_0 = \varepsilon^\dagger$$

CAN BE USED TO CHECK REALITY OF ACTION

CHARGE CONJUGATION

$$CVC = V^T, \quad C\Omega C = \Omega^T, \quad C\varepsilon C = \varepsilon^T$$

ii) * GRAVITY

$$S = -i \int \mathrm{Tr}\left(R \wedge_* V \wedge_* V \gamma_5 \right)$$ (CHAMSEDDINE 2003)

WITH :

$$V = V^a \gamma_a + \tilde{V}^a \gamma_a \gamma_5$$

$$\Omega = \frac{1}{4} \omega^{ab} \gamma_{ab} + i \omega \mathbb{1} + \tilde{\omega} \gamma_5$$

* INVARIANCES :

- DIFF $\mathcal{L}_\varepsilon = i_\varepsilon d + d i_\varepsilon$
- LOCAL $SO(1,3) \times U(1) \times U(1)$ $(SL(2,\mathbb{C}) \rightarrow GL(2,\mathbb{C}))$

$$\delta_\varepsilon V = -V * \varepsilon + \varepsilon * V \,, \quad \delta_\varepsilon \Omega = d\varepsilon - \Omega * \varepsilon + \varepsilon * \Omega$$

$$\varepsilon = \frac{1}{4} \varepsilon^{ab} \gamma_{ab} + i \varepsilon^{\cdot} \mathbb{1} + \tilde{\varepsilon}^{\cdot} \gamma_5$$

$\star$- GAUGE INVARIANCE DUE TO

$$\delta_\varepsilon R = -R \star \varepsilon + \varepsilon \star R$$

CYCLICITY OF Tr, GRADED CYCLICITY OF $\int$

ε STILL COMMUTES WITH γ_5

- FIELD EQS (INDEX-FREE)

EINSTEIN: $Tr\left[\Gamma_{a,a5}\left(iV \wedge_\star R + iR \wedge_\star V\right)\right] = 0$

TORSION: $Tr\left[\Gamma_{ab,1,5}\left(iT \wedge_\star V - iV \wedge_\star T\right)\right] = 0$

WITH $T \equiv dV - \Omega \wedge_\star V - V \wedge_\star \Omega \;=\; T^a \gamma_a + \tilde{T}^a \gamma_a \gamma_5$

SOLUTION: $T = 0$

- FIELDS : V^a $\tilde{V}^a$
  ω^{ab} $\omega, \tilde{\omega}$

- HOW TO GET RID OF EXTRA FIELDS IN $\theta \to 0$ LIM.?

- ANSWER : CHARGE CONJUGATION CONDITIONS ON V, ω (EXPLOITING θ-DEPENDANCE) COMPATIBLE WITH $*$-TRANSF.

ϑ-DEPENDENT FIELDS

- MOYAL TWIST $F^{-1} = e^{\frac{i}{2}\vartheta\,\Theta^{\rho\sigma}\partial_\rho\otimes\partial_\sigma}$ $\qquad$ ϑ DIMENSIONFUL

- $\phi_\vartheta(x) = \phi_0(x) + \vartheta\,\phi_1(x) + \vartheta^2\phi_2(x) + \cdots$

  $\varepsilon_\vartheta(x) = \varepsilon_0(x) + \vartheta\,\varepsilon_1(x) + \vartheta^2\varepsilon_2(x) + \cdots$

  → INFINITE TOWER OF x-DEPENDENT FIELDS

  FINITE NUMBER AT EACH ORDER IN ϑ

  AT 0-TH ORDER ONLY CLASSICAL FIELDS CONTRIBUTE

- GAUGE TRANSF OF $\phi_i(x)$ DEDUCED BY EXPANDING IN ϑ

$$\delta_\varepsilon V = -V\star\varepsilon + \varepsilon\star V \,, \quad \delta_\varepsilon\Omega = d\varepsilon - \Omega\star\varepsilon + \varepsilon\star\Omega$$

NB: ϕ_0 TRANSFORMS WITH CLASSICAL δ^0_ε

- THE SEIBERG-WITTEN MAP CAN BE USED TO RELATE HIGHER-ORDER FIELDS TO THE CLASSICAL FIELDS IN A WAY CONSISTENT WITH THE * GAUGE VARIATIONS

$$\delta_\varepsilon \phi(\phi_0) = \phi(\delta^0_\varepsilon \phi_0)$$

THEN THE *-DEFORMED THEORY CONTAINS ONLY THE ϕ_0 FIELDS

HERMITICITY AND CHARGE CONJUGATION

$$\gamma_0 V \gamma_0 = V^\dagger, \quad -\gamma_0 \Omega \gamma_0 = \Omega^\dagger, \quad -\gamma_0 \varepsilon \gamma_0 = \varepsilon^\dagger$$

$\rightarrow$ ACTION IS REAL

$$C V_\theta C = V^T_{-\theta}, \quad C \Omega_\theta C = \Omega^T_{-\theta}, \quad C \varepsilon_\theta C = \varepsilon^T_{-\theta}$$

$$\rightarrow \quad V^a_\theta = V^a_{-\theta}, \quad \tilde{V}^a_\theta = -\tilde{V}^a_{-\theta}$$

$$\omega^{ab}_\theta = \omega^{ab}_{-\theta}, \quad \omega_\theta = -\omega_{-\theta}, \quad \tilde{\omega}_\theta = -\tilde{\omega}_{-\theta}$$

$\rightarrow$ FOR $\theta \rightarrow 0$ ONLY V^a_0, ω^{ab}_0 SURVIVE

THE COMM. LIMIT IS THE USUAL FIRST-ORDER GRAVITY.

6. * GRAVITY + FERMIONS (SPIN 1/2)

i) CLASSICAL

$$S = i\int \mathrm{Tr}(R\wedge V\wedge V\gamma_5) - \int \mathrm{Tr}((D\psi)\bar{\psi}\wedge V\wedge V\wedge V\gamma_5)$$

ψ : DIRAC SPINOR, $D\psi \equiv d\psi - \Omega\psi$

$$\mathrm{Tr}((D\psi)\bar{\psi}\wedge V\wedge V\wedge V\gamma_5) = \mathrm{Tr}((D\psi)\bar{\psi}\wedge V^a\wedge V^b\wedge V^c \underbrace{\gamma_a\gamma_b\gamma_c\gamma_5}_{\epsilon_{abcd}\gamma^d})$$

$$= \bar{\psi}\gamma^a D\psi\wedge V^b\wedge V^c\wedge V^d\,\epsilon_{abcd}$$

$$= \bar{\psi}\not{D}\psi \,\det V\, d^4x$$

- INVARIANCES
  - DIFF
  - LOCAL LORENTZ $\delta_\varepsilon \psi = \varepsilon \psi \to \delta_\varepsilon \bar{\psi} = -\bar{\psi}\varepsilon$

$$\Rightarrow \delta_\varepsilon D\psi = \varepsilon (D\psi)$$

$$\delta_\varepsilon ((D\psi)\bar{\psi}) = \varepsilon ((D\psi)\bar{\psi}) - ((D\psi)\bar{\psi})\varepsilon$$

$$S = i\int Tr(R \wedge V \wedge V \gamma_5) - \int Tr((D\psi)\bar{\psi} \wedge V \wedge V \wedge V \gamma_5)$$

$$\delta_\varepsilon S = 0$$

- $\bar{\psi} \equiv \psi^\dagger \gamma_0$

ii) * GRAVITY + FERMIONS

$$S = -i\int \mathrm{Tr}\left(R \wedge_* V \wedge_* V \gamma_5\right) + \int \mathrm{Tr}\left((D\psi)\bar{\psi} \wedge_* V \wedge_* V \wedge_* V \gamma_5\right)$$

- INVARIANT UNDER DIFF AND
  * GAUGE $SO(1,3) \times U(1) \times U(1)$ $\quad$ ($SL(2,\mathbb{C}) \Rightarrow GL(2,\mathbb{C})$)

- CHARGE CONJUGATE $\psi^C \equiv C(\bar{\psi})^T$
  TRANSFORMS UNDER *-GAUGE AS $\psi_{-\theta}$

- $\Rightarrow$ MAJORANA CONDITION
  $\psi^C_\theta = \psi_{-\theta}$ $\qquad$ ($\Rightarrow \psi^\dagger_\theta \gamma_0 = \psi^T_{-\theta} C$)

7. * SUPERGRAVITY

i) CLASSICAL

$$S = i\int \mathrm{Tr}(R\wedge V\wedge V\gamma_5) - 2\int \mathrm{Tr}\left[\left((D\psi)\wedge\bar{\psi} + \psi\wedge D\bar{\psi}\right)\wedge V\gamma_5\right]$$

- $$\mathrm{Tr}\left[(D\psi)\wedge\bar{\psi}\wedge V\gamma_5\right] = \bar{\psi}\gamma^5\wedge V\wedge D\psi = \bar{\psi}\gamma^5\gamma_a\wedge D\psi\wedge V^a$$

$$= \epsilon^{\mu\nu\rho\sigma}\,\bar{\psi}_\mu\gamma^5\gamma^a D_\rho\psi_\sigma\, V^a_\nu\, d^4x$$

RARITA - SCHWINGER (SPIN 3/2)

- $$D\psi = d\psi - \Omega\psi$$
  $$D\bar{\psi} = d\bar{\psi} - \bar{\psi}\Omega$$

- MAJORANA CONDITION
  $$\psi^\dagger\gamma_0 = \psi^T C$$

- TORSION EQ : $\delta\Omega$ VARIATION

$$\delta S = \int \mathrm{Tr}\left[\delta\Omega\left[-i\,(TV - VT)\gamma_5 - 2\psi\wedge\bar{\psi}\wedge V\gamma_5 - 2V\gamma_5\wedge\psi\wedge\bar{\psi}\right]\right]$$

WITH $T \equiv dV - \Omega\wedge V - V\wedge\Omega$

- SOLUTION : $T = i\,(\psi\wedge\bar{\psi} - \gamma_5\,\psi\wedge\bar{\psi}\,\gamma_5) = i[\psi\wedge\bar{\psi}, \gamma_5]\gamma_5$

REPRODUCES $\mathcal{D}V^c = \frac{i}{2}\bar{\psi}\gamma^c\psi$

USING FIERZ IDENTITY

$$\psi_A\wedge\bar{\psi}_B = \frac{1}{4}\Big(\mathbb{1}\,\bar{\psi}_B\psi_A + \gamma_5\,\bar{\psi}_B\gamma_5\psi_A + \gamma_5\gamma_a\,\bar{\psi}_B\gamma^5\gamma^a\psi_A + \gamma_a\bar{\psi}_B\gamma^a\psi_A$$
$$- \frac{1}{8}\gamma^{ab}\,\bar{\psi}_B\gamma_{ab}\psi_A\Big)$$

• SUPERSYMMETRY

$$\delta_\varepsilon V = i(\bar{\varepsilon}\gamma^a\psi)\gamma_a = -i\,[\psi\bar{\varepsilon} - \varepsilon\bar{\psi}, \gamma_5]\gamma_5$$

$$\delta_\varepsilon \psi = d\varepsilon - \Omega\varepsilon$$

$$\delta_\varepsilon \bar{\psi} = d\bar{\varepsilon} + \bar{\varepsilon}\Omega$$

$$\Rightarrow \quad \delta_\varepsilon D\psi = -R\varepsilon$$

$$\delta_\varepsilon D\bar{\psi} = \bar{\varepsilon}R$$

$$\delta_\varepsilon S = 0$$

IF ψ, ε MAJORANA SPINORS

($\Rightarrow$ $\bar{\psi}\varepsilon = \bar{\varepsilon}\psi$, $\bar{\psi}\gamma_5\varepsilon = \bar{\varepsilon}\gamma_5\psi$ etc...)

ii) * SUPERGRAVITY

$$S = i\int \mathrm{Tr}\left(R\wedge_* V\wedge_* V\gamma_5\right) - 2\int \mathrm{Tr}\left(\left(D\psi\wedge_*\bar{\psi} + \psi\wedge_* D\psi\right)\wedge_* V\gamma_5\right)$$

INVARIANT UNDER

- DIFF $\mathcal{L}_\varepsilon = i_\varepsilon d + d i_\varepsilon$ $\qquad \varepsilon = \varepsilon^\mu \partial_\mu$
- GAUGE $SO(1,3)\times U(1)\times U(1)$ $\qquad \varepsilon = \frac{1}{4}\varepsilon^{ab}\gamma_{ab} + \varepsilon^{\cdot}\mathbb{1} + \tilde{\varepsilon}^{\cdot}\gamma_5$
- SUSY $\delta_\varepsilon V = -i\left[\psi * \bar{\varepsilon} - \varepsilon * \bar{\psi}, \gamma_5\right]\gamma_5$

  $\delta_\varepsilon \psi = d\varepsilon - \Omega * \varepsilon$

  - BROKEN FOR MAJORANA GRAVITINO !
  - UNBROKEN FOR WEYL GRAVITINO ($\rightarrow$ COMPLEX VIELBEIN)

$$V = V^a \gamma_a + i v \mathbb{1}$$

$D=3$, $N=1$ * SUPERGRAVITY

$$\Omega = \frac{1}{4}\omega^{ab}\gamma_{ab} + i\omega \mathbb{1}$$

$$S = \int \mathrm{Tr}(R \wedge_* V) + i \int \mathrm{Tr}(D\psi \wedge_* \bar{\psi})$$

(CACCIATORI, MARTUCCI by different method)

- INVARIANT UNDER
  - DIFF
  - LOCAL $SO(1,2) \times U(1) \approx U(1,1)$ * GAUGE
  - $N=1$ * SUPERSYMMETRY

$$\delta_\varepsilon V = i(\varepsilon * \bar{\psi} - \psi * \bar{\varepsilon}), \quad \delta_\varepsilon \psi = d\varepsilon - \Omega * \varepsilon$$

7. SOLUTIONS

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- SOLUTIONS OF CLASSICAL GRAVITY AND THEIR KILLING VECTORS
- USE A COMMUTING SUBSET K OF THESE KILLING VECTORS TO DEFINE A $\star$ PRODUCT (ABELIAN TWIST)
- CHOOSE VIELBEIN V SUCH THAT $\mathcal{L}_K V = 0$
- THEN ALL $\star$ PRODUCTS INVOLVING V's REDUCE TO ORDINARY PRODUCTS !

$\Rightarrow$ $\boxed{V \text{ IS A SOLUTION ALSO OF } \star\text{-EQ. OF MOTION}}$

8. CONCLUSIONS

- D=3 *SUPERGRAVITY : *SUPERSYMMETRIC
  USUAL COMM. LIMIT

  D=4 *SUPERGRAVITY : 1) MAJORANA ψ → NO *SUSY
  USUAL COMM. LIMIT
  2) WEYL ψ → *SUSY
  CHIRAL LIMIT (COMPLEX V)