Renormalisation Group Flow of QCD in Coulomb Gauge in a Hamiltonian Formulation

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Outline

Introduction: QCD

- QCD: Description of the Strong Interaction
- Asymptotic freedom: Perturbation theory
- Confinement: Nonperturbative approaches to QCD necessary
  - Lattice gauge theory
  - Dyson-Schwinger equations (DSE)
  - Functional renormalisation group (FRG)

$\Rightarrow$ This work: FRG in Hamiltonian formulation of Yang-Mills theory in Coulomb gauge

$\Rightarrow$ Advantage: Access to potential between static colour charges possible

The QCD Lagrangian and Gauge Invariance

- QCD Lagrangian

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - m\right) q - \frac{1}{2}\,\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

$$D_\mu = \partial_\mu - igA_\mu^a t^a \,, \quad F_{\mu\nu}^a t^a = \frac{i}{g}\left[D_\mu, D_\nu\right]$$

- $\mathcal{L}$ invariant under gauge transformations

$$q \longrightarrow q^U = U\,q \,, \qquad U = e^{i\alpha^a t^a}$$

$$A_\mu \longrightarrow A_\mu^U = U\left(A_\mu^a t^a + \frac{i}{g}\partial_\mu\right)U^\dagger = \frac{i}{g} U D_\mu U^\dagger$$

Gauge Fixing: Weyl Gauge

- For canonical quantization, choose Weyl gauge $A_0^a(x) = 0$ for all x (no conjugate momentum to A_0)
- scalar product

$$\langle \phi | \psi \rangle = \int \mathcal{D}\mathbf{A} \phi^*(\mathbf{A}) \psi(\mathbf{A})$$

- States ψ still invariant under spatial gauge transformations $U(\mathbf{x})$: $\psi(\mathbf{A}^U) = \psi(\mathbf{A})$

Gauge Fixing: Coulomb Gauge

- Fix this residual freedom by choosing Coulomb gauge: $\nabla \cdot \mathbf{A}^a = 0$
- $$\int \mathcal{D}\mathbf{A} = \int \mathcal{D}U \int \mathcal{D}\mathbf{A}\ \delta(\nabla \cdot \mathbf{A}^a)\mathsf{FP}(\mathbf{A})$$
  $$\mathrm{Faddeev-Popov\ determinant}\ \mathsf{FP}(\mathbf{A}) = \mathsf{det}(-\nabla \cdot \mathbf{D}(\mathbf{A}))$$
- scalar product
  $$\langle\phi|\psi\rangle \propto \int \mathcal{D}\mathbf{A}\ \delta(\nabla \cdot \mathbf{A}^a)\mathsf{FP}(\mathbf{A})\phi^*(\mathbf{A})\psi(\mathbf{A})$$

Equal time correlation functions

Target of calculations: Equal time n-point correlation functions

$$\langle \mathbf{A}_{i_1}^{a_1}(\mathbf{p}_1, t=0) \cdots \mathbf{A}_{i_n}^{a_n}(\mathbf{p}_n, t=0)\rangle =$$
$$\int \mathcal{D}A\, \delta(\nabla \cdot \mathbf{A}^a) \mathsf{FP}(\mathbf{A}) \mathbf{A}_{i_1}^{a_1}(\mathbf{p}_1, t=0) \cdots \mathbf{A}_{i_n}^{a_n}(\mathbf{p}_n, t=0) |\psi_0(\mathbf{A})|^2$$

$\psi_0(\mathbf{A})$ is the vacuum wave functional.

$\Rightarrow$ representation of FP by means of ghost fields:

$$\mathsf{FP}(\mathbf{A}) = \int \mathcal{D}[\bar{c}, c] \exp\left(-\int \bar{c}(-\nabla \cdot \mathbf{D}(\mathbf{A}))c\right)$$

The Generating Functional Z

Representing

$$e^{-S[A,c,\bar{c}]} := |\psi_0(\mathbf{A})|^2 \exp\left(\int \bar{c}(-\nabla \cdot \mathbf{D}(\mathbf{A}))c\right)$$

gives the generating functional Z:

$$\begin{aligned} Z[J,\sigma,\bar{\sigma}] &= \langle\psi|e^{J\cdot A+\bar{\sigma}\cdot c+\bar{c}\cdot\sigma}|\psi\rangle \\ &= \int \mathcal{D}[A,c,\bar{c}] \exp(-S[A,c,\bar{c}] + J\cdot A + \bar{\sigma}\cdot c + \bar{c}\cdot\sigma) \end{aligned}$$

where

$$J \cdot A = \int \frac{d^3p}{(2\pi)^3} J^a(-\mathbf{p}) A^a(\mathbf{p}).$$

Evaluation by means of the Functional Renormalisation Group (FRG).

Flow Equation for Effective Action Γ_k

- Introduction of a regulator term $\Delta S_k[A, c, \bar{c}]$:

$$\Delta S_k = \frac{1}{2} A \cdot R_{A,k} \cdot A + \bar{c} \cdot R_{c,k} \cdot c$$

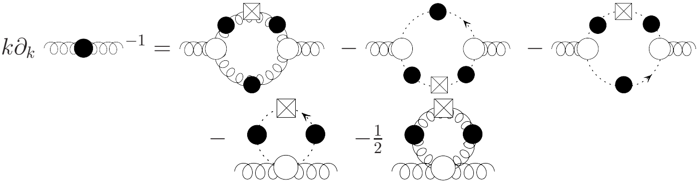
- Flow Equation for the Effective Action

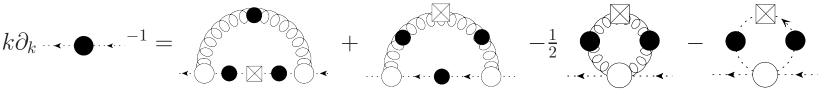
$$\partial_t \Gamma_k[A, c, \bar{c}] = \frac{1}{2} \mathrm{Tr} \left[\partial_t \mathcal{R}_k \left(\Gamma_k^{(2)}[A, c, \bar{c}] + \mathcal{R}_k \right)^{-1} \right]$$

C. Wetterich 1993

- Γ_k generates 1PI EQUAL time correlation functions

The Flows of Gluon and Ghost Propagator



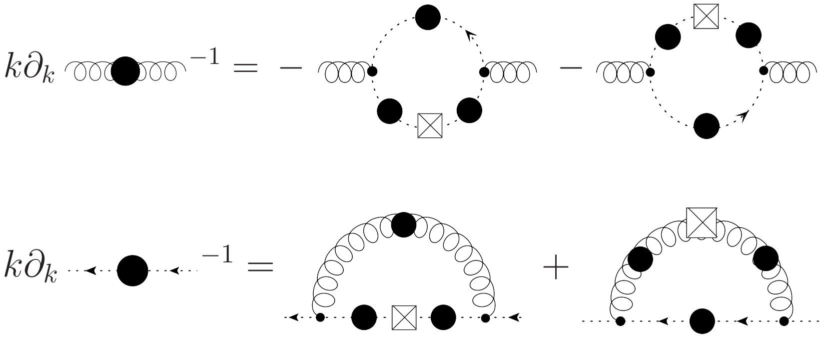


Truncation Scheme

- Assumption: IR scaling
  $\Rightarrow$ IR dominated by diagrams with largest number of ghost propagators ("ghost dominance")
  $\Rightarrow$ Dropping gluonic vertices
- Non-renormalization theorem for ghost-gluon vertex (Taylor, 1971). Also a consequence of IR scaling assumption
  $\Rightarrow$ Keeping the ghost-gluon vertex bare
- Drop tadpole diagrams

$\Rightarrow$ Flow equations for the propagators are now a decoupled system of equations.

Truncated Propagator Flows



Parametrization of the Propagators

- Gluon correlation function is parametrized in order to compare with the variational ansatz:

$$\langle AA \rangle_k(p) \sim \frac{1}{2\omega_k(p) + R_{A,k}} \stackrel{k\to 0}{\longrightarrow} \frac{1}{2\omega_0(p)}$$

- Ghost propagator is parametrized in the standard way:

$$\langle c\bar{c} \rangle_k(p) \sim \frac{1}{\frac{p^2}{d_k(p)} + R_{c,k}(p)} \stackrel{k\to 0}{\longrightarrow} \frac{d_0(p)}{p^2}$$

Initial Conditions

We require the propagators to fulfill:

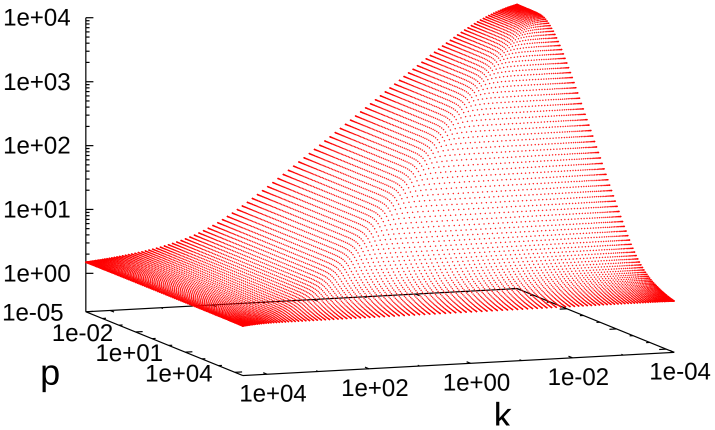
- Ghost: Scaling behaviour in IR: $d_{0,IR}(p) \sim p^{-\alpha_d}$
- Gluon: Asymptotic freedom: $\omega_{0,UV}(p) \sim p$

$\Rightarrow$ Initial conditions for ω_Λ and d_Λ^{-1}:

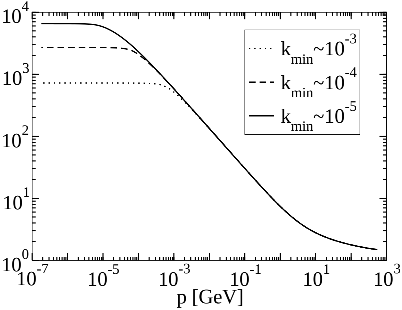
- $\omega_\Lambda(p) = p + a_\Lambda$
- $d_\Lambda^{-1}(p) = const._\Lambda$

Initial conditions have to be chosen such that the scaling behaviour condition for the ghost and the vanishing mass for the gluon are satisfied.

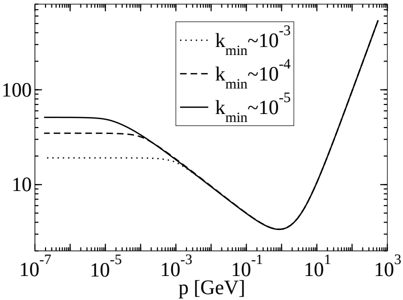
Flow of the Ghost Dressing Function $d_k(p)$



Flows of the Propagators



Ghost propagator dressing function $d_{k_{min}}(p)$



Gluon correlation function $\omega_{k_{min}}(p)$

$\Rightarrow$ IR scaling behaviour for both ghost and gluon propagator.

Infrared Exponents from the Flow Equation

- Ghost form factor

$$d_0(p)_{IR} \overset{p\to 0}{\longrightarrow} p^{-0.64}$$

- Gluon correlator

$$\omega_0(p)_{IR} \overset{p\to 0}{\longrightarrow} p^{-0.28}$$

$\Rightarrow$ These values of the exponents ($\alpha_\omega = 0.28,\ \alpha_d = 0.64$) satisfy the scaling relation

$$\alpha_\omega = 2\alpha_d - 1$$

obtained earlier by infrared analysis of the DSE.

Schleifenbaum, Leder, Reinhardt, 2006

Optimization

- Replace $\omega_k \to \omega_0 \ , \ d_k \to d_0$ in the loop integrals:

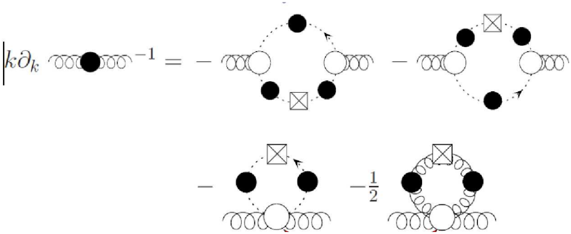
$\Rightarrow$ Analytical integration of flow integrals $\int_\Lambda^0 \frac{dk'}{k'}$ feasible

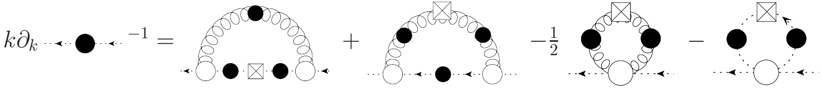
$\Rightarrow$ Integrated RG equations correspond to a Dyson-Schwinger equation (DSE).

$\Rightarrow$ Provides best approximation to the full theory in the IR.

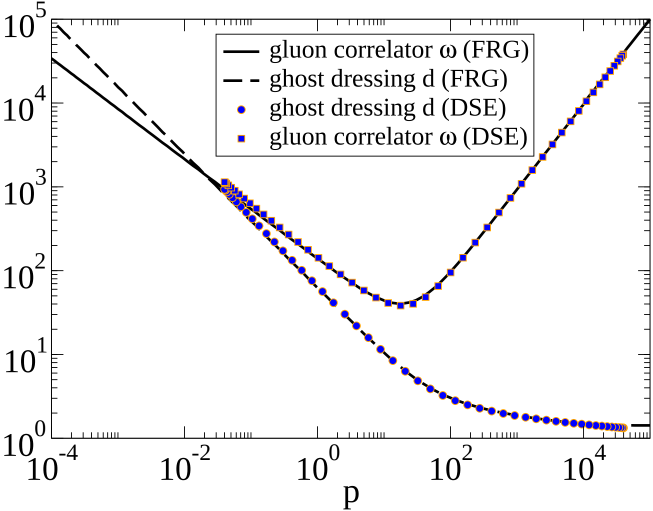
Pawlowski, Litim, Nedelko, von Smekal, 2004

Optimization





Optimized FRG and the DSE



Optimized FRG vs. FRG without Tadpoles: IR Exponents

- FRG without tadpoles

$$\boxed{\alpha_\omega = 0.28,\ \alpha_d = 0.64}$$

Leder, Pawlowski, Reinhardt, Weber, arXiv:1006.5710

- Optimized FRG and DSE

$$\boxed{\alpha_\omega = 0.60,\ \alpha_d = 0.80}$$

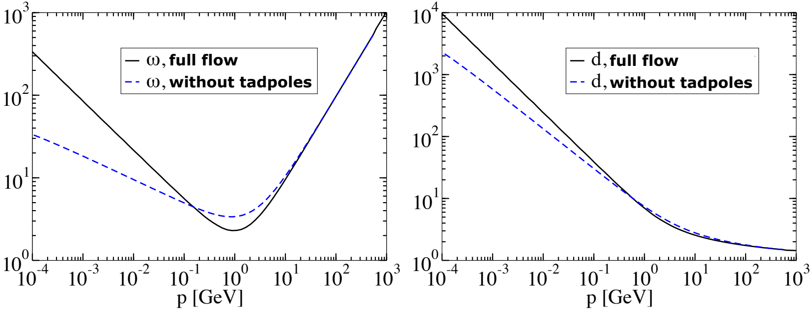
Leder, Pawlowski, Reinhardt, Weber, arXiv:1006.5710

Feuchter, Reinhardt, 2004

$\Rightarrow$ They satisfy the scaling relation

$$\boxed{\alpha_\omega = 2\alpha_d - 1}$$

Optimized FRG vs. FRG without Tadpoles: Propagators

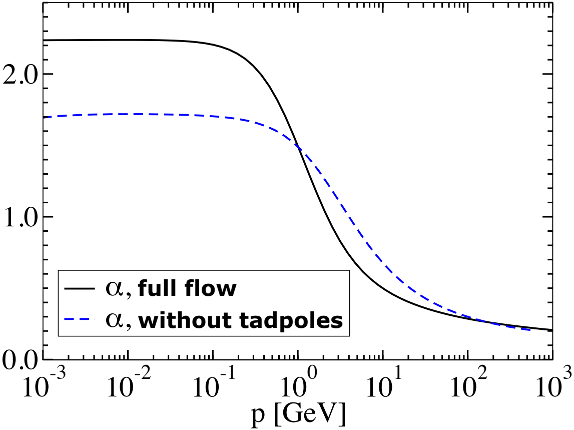


Gluon correlator $\omega_0(p)$

Ghost propagator dressing function $d_0(p)$

Optimized FRG vs. FRG without Tadpoles: Coupling

$$\alpha(p) \sim d^2(p)\omega^{-1}(p)p$$



Summary and Outlook

Summary

- Ghost and gluon propagators have been calculated, using as condition a power law behaviour in the IR for the ghost form factor.
- IR enhancement of the ghost dressing function meets the Gribov-Zwanziger confinement condition.
- Quantitative behaviour of the propagators in the DSE approach is reproduced.

Outlook

- Calculation of the Coulomb potential (String tension)
- Inclusion of quark fields

The Generating Functional Z_k

$$Z[j] = \int \mathcal{D}\chi \, e^{-S[\chi]+j\cdot\chi}$$

The Generating Functional Z_k

$$Z_k[j] = \int \mathcal{D}_\Lambda \chi \, e^{-S[\chi] - \Delta S_k[\chi] + j \cdot \chi}$$

The Generating Functional Z_k

$$Z_{k}[j] = \int \mathcal{D}_{\Lambda}\chi\, e^{-S[\chi]-\Delta S_k[\chi]+j\cdot\chi}$$

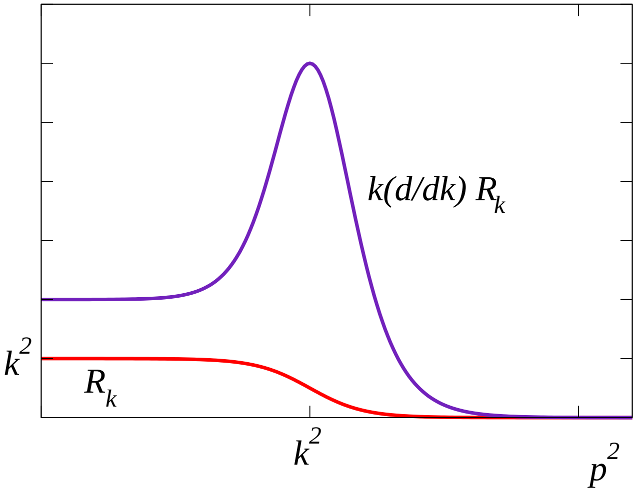
$$\Delta S_k[\chi] = \frac{1}{2}\chi \cdot R_k \cdot \chi := \frac{1}{2}\int \frac{d^d p}{(2\pi)^d}\chi(-\mathbf{p})R_k(\mathbf{p})\chi(\mathbf{p})$$

IR-regularisation (mass term): $\lim_{p^2/k^2\to 0} R_k(p) > 0$

recovering full theory: $\lim_{k^2/p^2\to 0} R_k(p) = 0$

bare action S for $k\to\infty$: $\lim_{k^2/p^2\to \infty} R_k(p) \to \infty$

A Typical Regulator $R_k(p)$



$Z_k \rightarrow W_k \rightarrow \Gamma_k$: The Effective Action Γ_k

$$Z[j] = e^{W[j]}$$

$$\Gamma[\phi] = -W[j] + j \cdot \phi$$

with j such that $\phi = \frac{\delta W[j]}{\delta j}$

$Z_k \to W_k \to \Gamma_k$: The Effective Action Γ_k

$$Z_k[j] = e^{W_k[j]}$$

$$\Gamma_k[\phi] = -W_k[j_k] + j_k \cdot \phi - \Delta S_k[\phi]$$

with j_k such that $\phi = \frac{\delta W_k[j_k]}{\delta j}$

$Z_k \to W_k \to \Gamma_k$: The FRG for Γ_k

The Functional Renormalisation Group Equation

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \left[\partial_t R_k \left(\Gamma_k^{(2)}[\phi] + R_k \right)^{-1} \right]$$

C. Wetterich 1993

$$\text{where } \Gamma_k^{(2)}[\phi] = \frac{\delta^2 \Gamma_k[\phi]}{\delta\phi\,\delta\phi} \quad \text{and} \quad k\frac{d}{dk} =: \partial_t$$

- Γ_k interpolates between $\Gamma_{k=\Lambda} = S_{bare}$ and $\Gamma_{k=0} = \Gamma$.
- Physics for $k > \Lambda$ is regarded to be included already in Γ_Λ.
- Therefore, cutoff Λ is NOT taken to ∞, Λ is large but finite.

Approximation schemes

Truncations should be systematic and consistent.

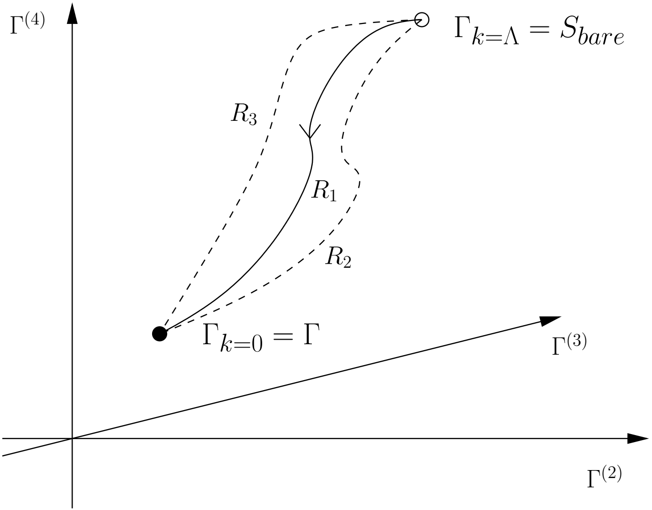
- Derivative expansion

$$\Gamma_k[\phi] = \int d^d x \left\{ U_k(\phi) + \frac{1}{2} Z_k(\phi)(\partial_\mu \phi)^2 + \mathcal{O}(\partial^4) \right\}$$

- Vertex expansion

$$\Gamma_k[\phi] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^d x_1 \ldots d^d x_n \Gamma_k^{(n)}(x_1, \ldots, x_n) \phi(x_1) \ldots \phi(x_n)$$

Theory Space



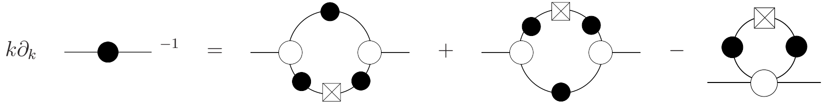
Example: Propagator flow

$$\frac{\delta^2}{\delta\phi\,\delta\phi}\bigg|_{\phi=0} \partial_t \Gamma_k[\phi] = \frac{\delta^2}{\delta\phi\,\delta\phi}\bigg|_{\phi=0} \frac{1}{2} Tr\left[(\partial_t R_k) \left(\Gamma_k^{(2)}[\phi] + R_k \right)^{-1} \right]$$

Example: Propagator flow

$$\left.\frac{\delta^2}{\delta\phi\,\delta\phi}\right|_{\phi=0}\partial_t\Gamma_k[\phi] = \left.\frac{\delta^2}{\delta\phi\,\delta\phi}\right|_{\phi=0}\frac{1}{2}Tr\left[(\partial_t R_k)\left(\Gamma_k^{(2)}[\phi]+R_k\right)^{-1}\right]$$

$$\Rightarrow \partial_t\Gamma_k^{(2)} = Tr\left\{(\partial_t R_k)[\Gamma_k^{(2)}+R_k]^{-1}\Gamma_k^{(3)}[\Gamma_k^{(2)}+R_k]^{-1}\Gamma_k^{(3)}[\Gamma_k^{(2)}+R_k]^{-1}\right\} - \frac{1}{2}Tr\left\{(\partial_t R_k)[\Gamma_k^{(2)}+R_k]^{-1}\Gamma_k^{(4)}[\Gamma_k^{(2)}+R_k]^{-1}\right\}$$



Definition of Z_k

Introduction of a regulator term $\Delta S_k[A, c, \bar{c}]$:

$$\Delta S_k = \frac{1}{2} A \cdot R_{A,k} \cdot A + \bar{c} \cdot R_{c,k} \cdot c$$

k-dependent generating functional Z_k:

$$Z_k[J, \sigma, \bar{\sigma}] = \langle \psi | \exp[-\Delta S_k + J \cdot A + \bar{\sigma} \cdot c + \bar{c} \cdot \sigma] | \psi \rangle$$

Path integral representation:

$$Z_k[J, \sigma, \bar{\sigma}] = \int \mathcal{D}[A, \bar{c}, c] \, \exp(-S - \Delta S_k + J \cdot A + \bar{\sigma} \cdot c + \bar{c} \cdot \sigma)$$

The flow equation for Γ_k

- k-dependent Schwinger functional W_k:

$$Z_k = e^{W_k}$$

- Modified Legendre transform $\rightarrow$ effective action Γ_k:

$$\Gamma_k = -W_k + J_k \cdot A + \bar{\sigma}_k \cdot c + \bar{c} \cdot \sigma_k - \frac{1}{2} A \cdot R_{A,k} \cdot A - \bar{c} \cdot R_{c,k} \cdot c$$

$$A = \frac{\delta W_k}{\delta j} \qquad c = \frac{\delta W_k}{\delta \bar{\sigma}} \qquad \bar{c} = -\frac{\delta W_k}{\delta \sigma}$$

Flow Equation for the Effective Action

$$\partial_t \Gamma_k[A, c, \bar{c}] = \frac{1}{2} \mathrm{Tr} \left[\partial_t \mathcal{R}_k \left(\Gamma_k^{(2)}[A, c, \bar{c}] + \mathcal{R}_k \right)^{-1} \right]$$

C. Wetterich 1993

Flow of Γ_k: Explicit Form

$$\partial_t \Gamma_k = \frac{1}{2}\, Tr \left[\begin{pmatrix} \dot{R}_{A,k} & & \\ & -\dot{R}_{c,k} & \\ & & -\dot{R}^T_{c,k} \end{pmatrix} \cdot \right.$$

$$\left. \cdot \begin{pmatrix} \frac{\delta^2 \Gamma_k}{\delta A \delta A} + R_{A,k} & \frac{\delta^2 \Gamma_k}{\delta A \delta c} & \frac{\delta^2 \Gamma_k}{\delta A \delta \bar{c}} \\ -\frac{\delta^2 \Gamma_k}{\delta \bar{c} \delta A} & -\frac{\delta^2 \Gamma_k}{\delta \bar{c} \delta c} + R_{c,k} & -\frac{\delta^2 \Gamma_k}{\delta \bar{c} \delta \bar{c}} \\ \frac{\delta^2 \Gamma_k}{\delta c \delta A} & \frac{\delta^2 \Gamma_k}{\delta c \delta c} & \frac{\delta^2 \Gamma_k}{\delta c \delta \bar{c}} + R^T_{c,k} \end{pmatrix}^{-1} \right]$$

Parametrization of the Propagators

- Weyl gauge ⇒ only spatial fields $A^a_{x,y,z}$ occur, $A^a_0 = 0$
- Coulomb gauge ⇒ transversality of the gluon propagator: the transversal projector $t_{ij}(\mathbf{p}) = \delta_{ij} - \hat{p}_i \hat{p}_j$ enters.

$$\frac{\delta^2 \Gamma_k[0]}{\delta A^a_i(\mathbf{p})\ \delta A^b_j(\mathbf{q})} = \delta^{ab} t_{ij}(\mathbf{p}) 2\omega_k(p) (2\pi)^3 \delta^3(\mathbf{p}+\mathbf{q})$$

$$\frac{\delta^2 \Gamma_k[0]}{\delta \bar{c}^a(\mathbf{p})\ \delta c^b(\mathbf{q})} = \delta^{ab} g \frac{p^2}{d_k(p)} (2\pi)^3 \delta^3(\mathbf{p}+\mathbf{q})$$

$$\frac{\delta^3 \Gamma_k}{\delta \bar{c}^a(\mathbf{p}_1)\ \delta c^b(\mathbf{p}_2)\ \delta A^c_i(\mathbf{p}_3)} = i g f^{abc} p_{1,j} t_{ij}(\mathbf{p}_3) (2\pi)^3 \delta^3(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3)$$

Fine Tuning - Gluon

$$p \sim \Lambda : \; \omega_0(p) - \omega_\Lambda(p) = \underbrace{\int_\Lambda^0 \frac{dk'}{k'} \int \frac{d^3\ell}{(2\pi)^3} I^\omega_{k'}[d_{k'}](\boldsymbol{\ell}, \mathbf{p})}_{\text{fit } p-a \text{ for } p\sim\Lambda}$$

With $\omega_\Lambda(p) = p + a \Rightarrow \omega_0(p) \sim p$ for $p \sim \Lambda$

Fine Tuning - Ghost

$$f(p) \propto p^A \Rightarrow \frac{d}{d\ln p} \ln f(p) = A$$

To obtain a power law, add d_Λ^{-1} to the integrated flow:

$$\frac{d}{dp}\left(\frac{d}{d\ln p} \ln\left(\int_\Lambda^0 \frac{dk'}{k'} \ldots + d_\Lambda^{-1}\right)\right) \stackrel{!}{=} 0$$

⇒ Determine d_Λ^{-1}.

⇒ Only IR scaling is put in, the horizon condition ($\alpha_d > 0$) will be a result of the calculation.

Solving Differential Equations ?

- RG-equations are two coupled 1^{st} order ODE in k.
- Possible solution: Integrating numerically from initial conditions $[\omega_\Lambda(p), d_\Lambda(p)]$ down to $[\omega_0(p), d_0(p)]$.
- Initial conditions: $[\omega_\Lambda(p) = p, d_\Lambda(p) = 1]$

Approximate Solution: Propagators

- Gluon equation

$$\omega_0(p) - \omega_\Lambda(p) = \frac{N_c}{4} \left[\int \frac{d^3q}{(2\pi)^3} \left(\frac{q^2}{d_0(q)} + R_{c,k}(q) \right)^{-1} \cdot \cdot \left(\frac{|\mathbf{q}+\mathbf{p}|^2}{d_0(|\mathbf{q}+\mathbf{p}|)} + R_{c,k}(|\mathbf{q}+\mathbf{p}|) \right)^{-1} \cdot \ldots \right]_{k=\Lambda}^{k=0}$$

- Ghost equation

$$d_0^{-1}(p) - d_\Lambda^{-1}(p) = N_c \left[\int \frac{d^3r}{(2\pi)^3} (2\omega_0(q) + R_{A,k}(q))^{-1} \cdot \cdot \left(\frac{|\mathbf{q}+\mathbf{p}|^2}{d_0(|\mathbf{q}+\mathbf{p}|)} + R_{c,k}(|\mathbf{q}+\mathbf{p}|) \right)^{-1} \cdot \ldots \right]_{k=\Lambda}^{k=0}$$

Flow Equation for ω_k and d_k

$$\partial_t \omega_k(p) = -\frac{N_c}{2}\int \frac{d^3q}{(2\pi)^3}\partial_t R_{c,k}(q)\left(\frac{q^2}{d_k(q)}+R_{c,k}(q)\right)^{-2} \cdot \cdot \left(\frac{|\mathbf{q}+\mathbf{p}|^2}{d_k(|\mathbf{q}+\mathbf{p}|)}+R_{c,k}(|\mathbf{q}+\mathbf{p}|)\right)^{-1} q^2(1-(\hat{\mathbf{p}}\cdot\hat{\mathbf{q}})^2)$$

$$\partial_t d_k^{-1}(p) = N_c\int \frac{d^3q}{(2\pi)^3}(1-(\hat{\mathbf{p}}\cdot\hat{\mathbf{q}})^2) \cdot \Bigg[\partial_t R_{A,k}(q)[2\omega_k(q)+R_{A,k}(q)]^{-2}\left(\frac{|\mathbf{q}+\mathbf{p}|^2}{d_k(|\mathbf{q}+\mathbf{p}|)}+R_{c,k}(|\mathbf{q}+\mathbf{p}|)\right)^{-1} + \partial_t R_{c,k}(q)\left(\frac{q^2}{d_k(q)}+R_{c,k}(q)\right)^{-2}[2\omega_k(|\mathbf{q}+\mathbf{p}|)+R_{A,k}(|\mathbf{q}+\mathbf{p}|)]^{-1} \cdot \ldots$$

Iterative Solution

To incorporate the appropriate initial conditions, cast the differential flow equations into an integral form:

$$\omega_k(p) - \omega_\Lambda(p) = \int_\Lambda^k \frac{dk'}{k'} \int \frac{d^3\ell}{(2\pi)^3} I^\omega_{k'}[d_{k'}](\boldsymbol{\ell}, \mathbf{p})$$

$$d_k^{-1}(p) - d_\Lambda^{-1}(p) = \int_\Lambda^k \frac{dk'}{k'} \int \frac{d^3\ell}{(2\pi)^3} I^d_{k'}[\omega_{k'}, d_{k'}](\boldsymbol{\ell}, \mathbf{p})$$

$\Rightarrow$ Iteration of $\omega_k(p)$ and $d_k(p)$ with simultaneous determination of the initial conditions $\omega_\Lambda(p)$ and $d_\Lambda(p)$.

Infrared Exponents from the Approximate Flow Equation

$$\Rightarrow \qquad \omega_0(p)_{IR} \sim p^{-0.60} \quad d_0(p)_{IR} \sim p^{-0.80}$$

One of two possible solutions previously found by IR-analysis of DSE and a numerical calculation

Schleifenbaum, Leder, Reinhardt, 2006;
Feuchter, Reinhardt, 2004

$$\Rightarrow \qquad \omega_0(p)_{IR} \sim p^{-1} \quad d_0(p)_{IR} \sim p^{-1}$$

The second possible solution found by IR-analysis of DSE and a numerical calculation has not been found yet in this approximation.

Schleifenbaum, Leder, Reinhardt, 2006;
Epple, Reinhardt, Schleifenbaum, 2007