Yang-Mills Propagators in Landau Gauge at Non-Vanishing Temperature

Leonard Fister, Jan M. Pawlowski,
Universität Heidelberg

. . . work in progress

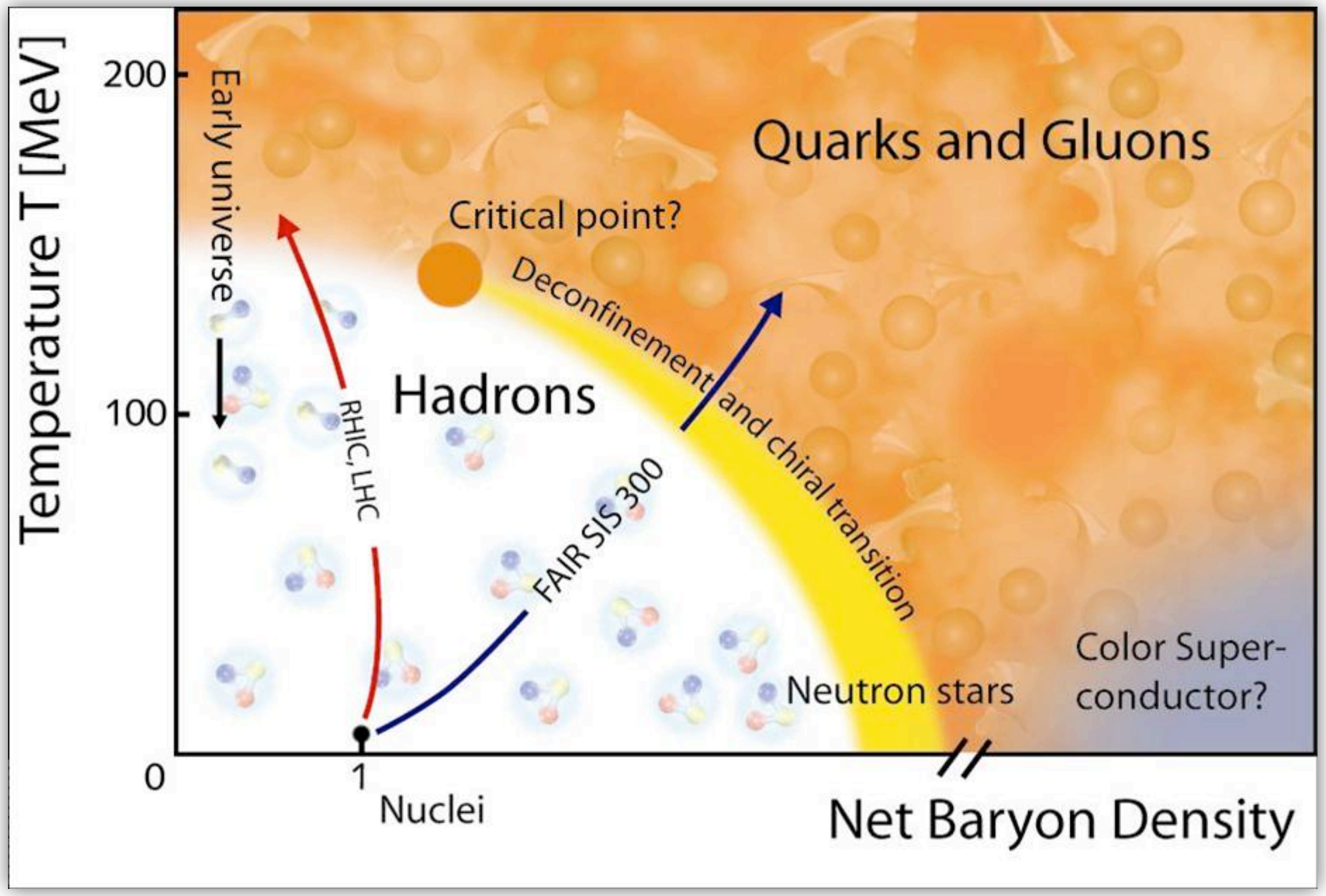
ERG Corfu - September 2010

TECHNISCHE UNIVERSITÄT DARMSTADT

EMMI

Motivation

ultimate goal: computation of physical observables from microscopic dynamics



credits: GSI Darmstadt

experiment:
- thermodynamic potential,
- pressure,
- entropy,
- screening masses, etc.

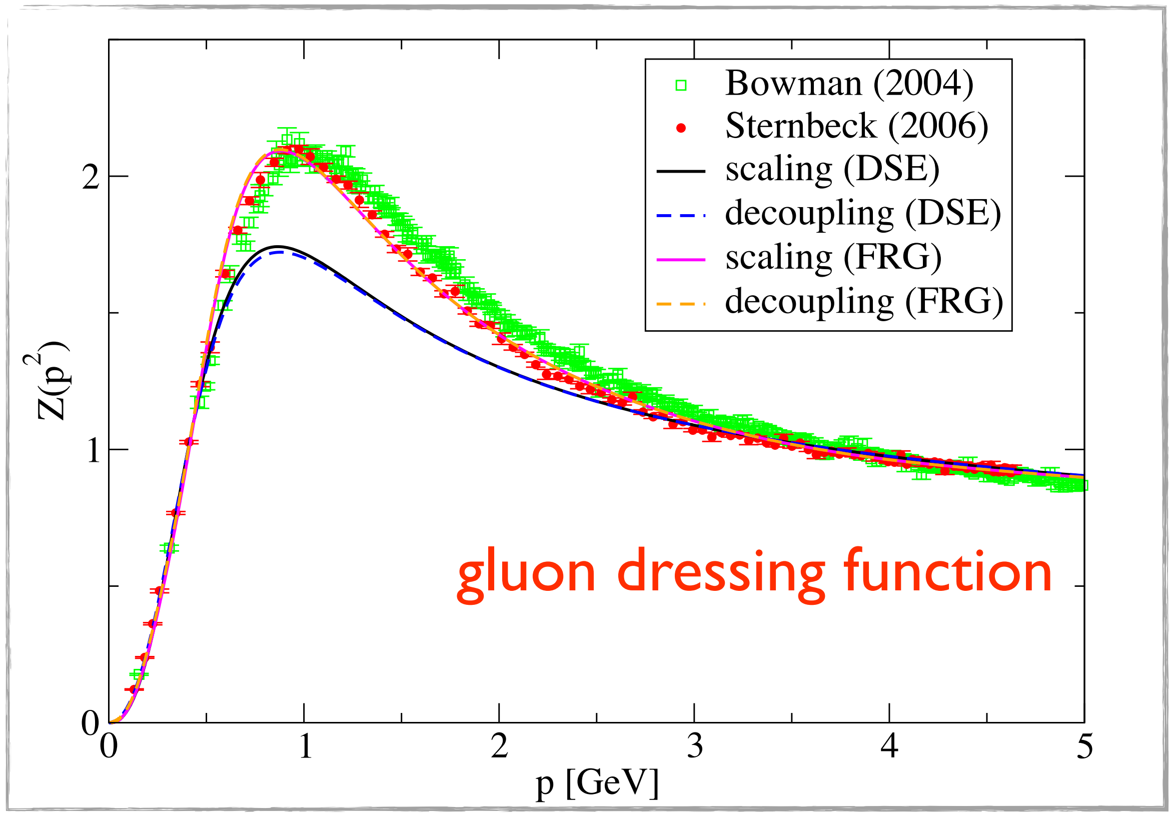
study QCD phase diagram
fully non-perturbatively:

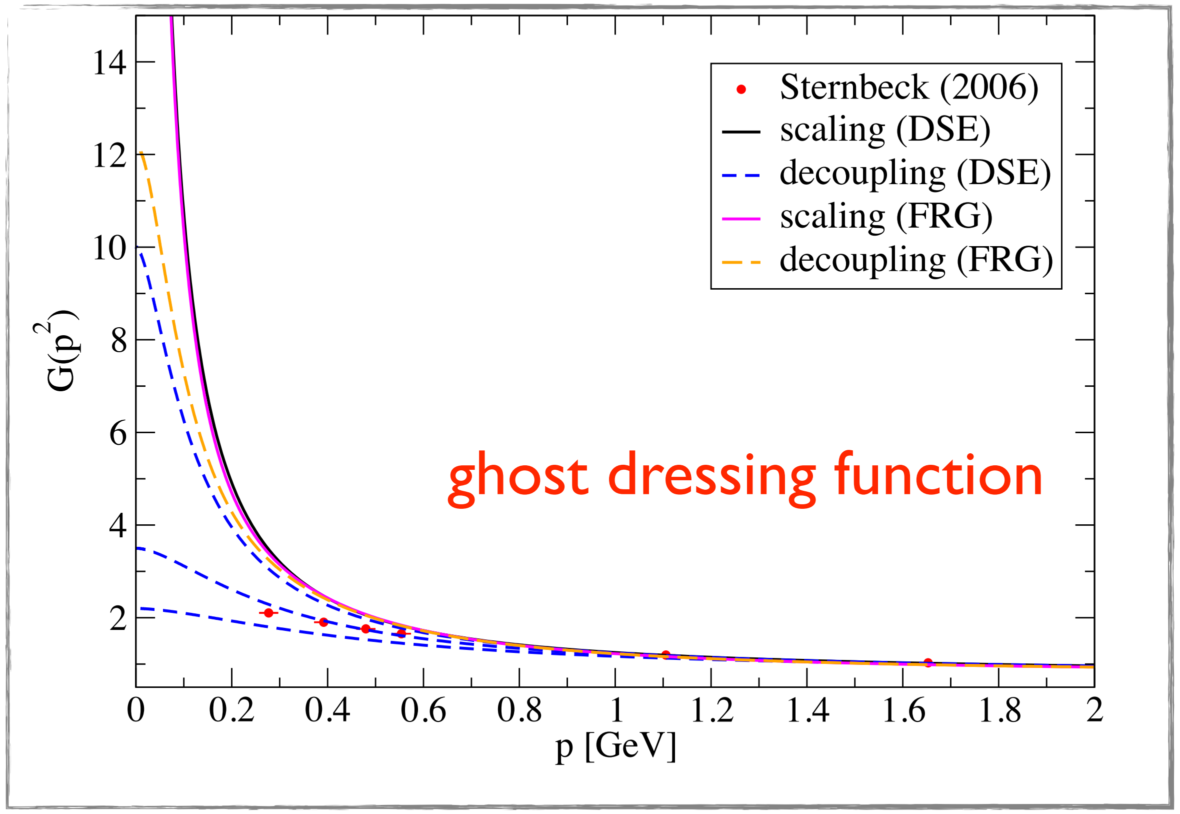
functional renormalisation group

applicable for all temperatures

Outline

- Motivation
- Yang-Mills Flow Equation
- Thermal Flow Equation
- Propagators at Non-Vanishing Temperature
- Summary





taken from: Fischer, Maas, Pawlowski, Annals Phys. 324 (2009).

Yang-Mills Theory - Basics

Yang-Mills action: $$S_{YM} = \int d^4x \left(-\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\xi} (\partial_\mu A^a_\mu)^2 + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right)$$

Landau gauge: $\xi \to 0$

covariant derivative: $$D^a_{\mu\nu} = \delta^{ab} \partial_\mu + g f^{abc} A^c_\mu$$

field-strength tensor: $$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

effective action $\Gamma[A, \bar{c}, c]$

$$Z[J, \eta, \bar{\eta}] \equiv e^{W[J,\eta,\bar{\eta}]} = \int \mathcal{D}A \mathcal{D}\bar{c} \mathcal{D}c \; e^{-S[A,\bar{c},c] + \int (J \cdot A + \bar{\eta} \cdot c - \bar{c} \cdot \eta)}$$

$$\Gamma[A, \bar{c}, c] = \sup_{J,\eta,\bar{\eta}} \left(\int (J \cdot A + \bar{\eta} \cdot c - \bar{c} \cdot \eta) \; - \; W[J, \eta, \bar{\eta}] \right)$$

Flow Equation (for Yang-Mills Theory)

Wetterich, Phys. Lett. B301 (1993) 90-94.

$$\partial_t \Gamma_k[A,\bar{c},c] = \frac{1}{2}\mathrm{Tr}\left\{\frac{1}{\Gamma^{(2)}[A,\bar{c},c]+R_k}\;\;\partial_t R_k\right\} - \partial_t C_k$$

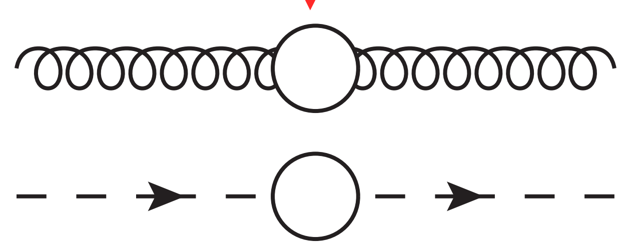
Flow Equation (for Yang-Mills Theory)

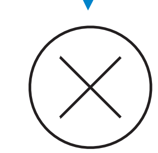
Wetterich, Phys. Lett. B301 (1993) 90-94.

$$\underset{\substack{\downarrow \\ t\,=\,k\ln k}}{\partial_t}\Gamma_k[A,\bar{c},c] = \frac{1}{2}\mathrm{Tr}\left\{\frac{1}{\Gamma^{(2)}[A,\bar{c},c]+R_k}\,\partial_t R_k\right\} - \partial_t C_k$$

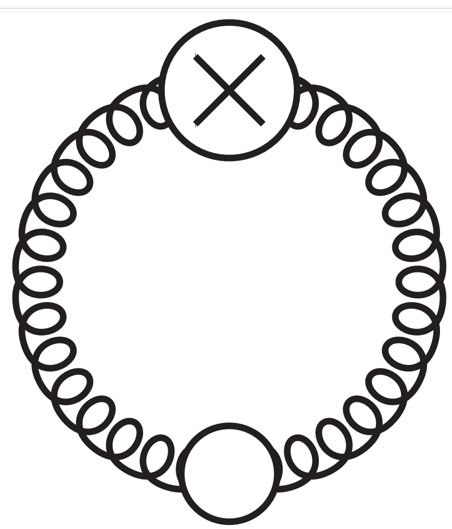
full propagator

regulator

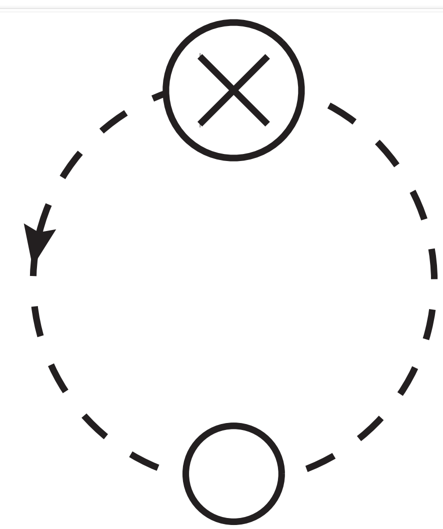




$$\partial_t\Gamma[A,\bar{c},c] \;=\; \frac{1}{2}$$

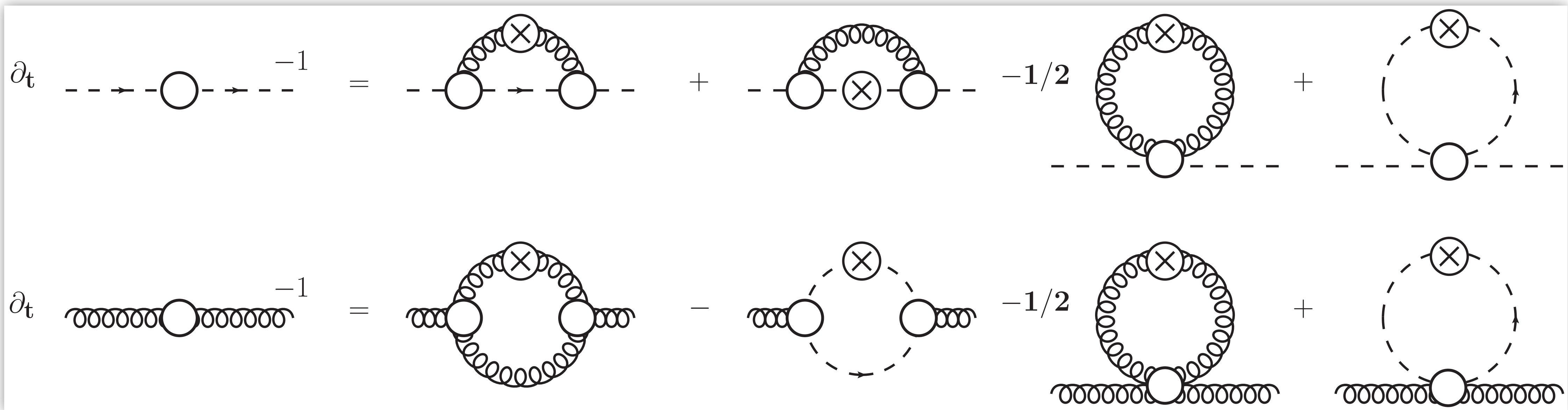


$-$



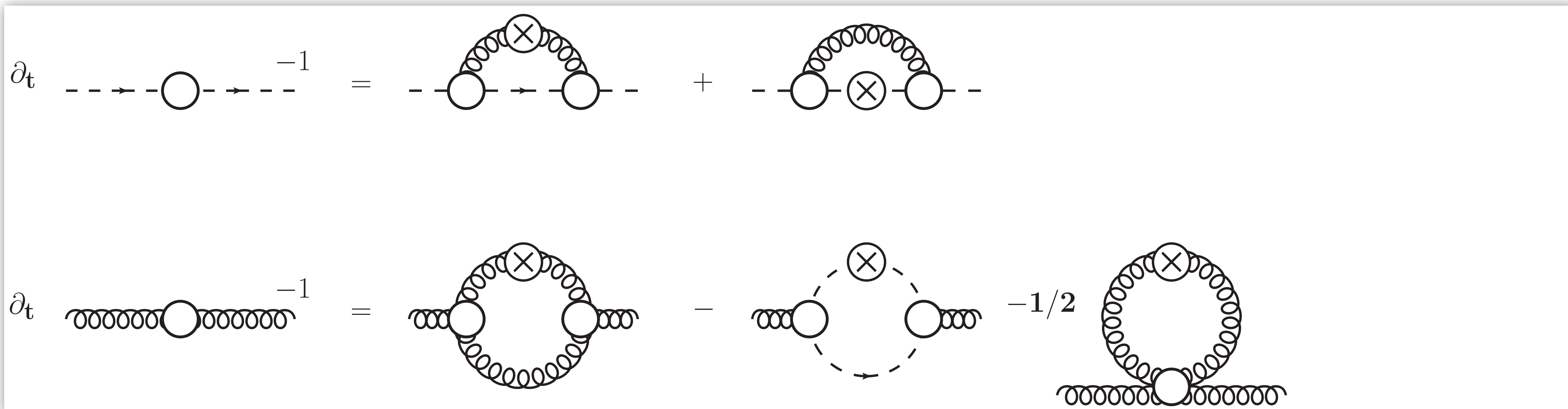
Yang-Mills Propagators

obtained from generating flow equation via functional derivation wrt the in-/out-going fields



Yang-Mills Propagators (truncated)

obtained from generating flow equation via functional derivation wrt the in-/out-going fields



Yang-Mills Propagators - Parametrisation

zero temperature:

ghost propagator $D^{ab}_{\mathrm{gh}}(p) = -\frac{G(p)}{p^2}\delta^{ab}$

gluon propagator $D^{ab}_{\mathrm{gl},\mu\nu}(p^2) \; = \Pi_{\mu\nu}\frac{Z(p^2)}{p^2}\delta^{ab}$, $\Pi_{\mu\nu}\ldots$ transversal 4d-projector

Yang-Mills Propagators - Parametrisation

zero temperature:

ghost propagator $D^{ab}_{\mathrm{gh}}(p) = -\frac{G(p)}{p^2}\delta^{ab}$

gluon propagator $D^{ab}_{\mathrm{gl},\mu\nu}(p^2) = \Pi_{\mu\nu}\frac{Z(p^2)}{p^2}\delta^{ab}$, $\Pi_{\mu\nu}\ldots$ transversal 4d-projector

finite temperature (Matsubara formalism): $p_0 = 2\pi T n_p$, $n_p \ldots$ Matsubara modes

ghost propagator $D^{ab}_{\mathrm{gh}}(p_0^2, \vec{p}^{\,2}) = -\delta^{ab}\frac{G(p_0^2, \vec{p}^{\,2})}{p_0^2 + \vec{p}^{\,2}}$

gluon propagator $D^{ab}_{\mathrm{gl},\mu\nu}(p_0^2, \vec{p}^{\,2}) = \delta^{ab} P^T_{\mu\nu}\frac{Z_T(p_0^2, \vec{p}^{\,2})}{p_0^2 + \vec{p}^{\,2}} + \delta^{ab} P^L_{\mu\nu}\frac{Z_L(p_0^2, \vec{p}^{\,2})}{p_0^2 + \vec{p}^{\,2}}$

transversal- longitudinal-

3d projector

magnetic electric

Flow Equation for Thermal Fluctuations

at non-vanishing temperature: quantum and thermal fluctuations

idea:

(1) calculate quantum fluctuations at zero temperature
(2) project onto thermal fluctuations and add to (1)

thermal flow:

$$\bar{\Gamma}_{k,T}[A,\bar{c},c] = \Gamma_{k,T} - \Gamma_{k,T=0}$$

Litim, Pawlowski, arXiv: hep-th/9901063.
Litim, Pawlowski, JHEP 11 (2006) 026.

advantages:

- $T \to 0$ limit trivially satisfied
- truncations for (1) and (2) may differ

Flow Equation for Thermal Fluctuations

technique make a guess for the finite temperature result and iterate around it:

- does not change the final result
- numerical stability in iteration procedure as one starts „closer to physics“

$$\Gamma^{(2)}_{k,T} = \Gamma^{(2)}_{k,0} + \underbrace{\left(\Gamma^{(2)}_{k,T} - \Gamma^{(2)}_{k,0}\right)}_{thermal\ \ contribution} + \underbrace{\left(\Gamma^{(2)}_{k,T,trial} - \Gamma^{(2)}_{k,T,trial}\right)}_{0}$$

Flow Equation for Thermal Fluctuations

technique make a guess for the finite temperature result and iterate around it:

- does not change the final result
- numerical stability in iteration procedure as one starts „closer to physics“

$$\Gamma^{(2)}_{k,T} = \Gamma^{(2)}_{k,0} + \underbrace{\left(\Gamma^{(2)}_{k,T} - \Gamma^{(2)}_{k,0}\right)}_{thermal\ \ contribution} + \underbrace{\left(\Gamma^{(2)}_{k,T,\color{red}{trial}} - \Gamma^{(2)}_{k,T,\color{red}{trial}}\right)}_{0}$$

$\longrightarrow$ *„re-ordering“*

$$\Gamma^{(2)}_{k,T} = \Gamma^{(2)}_{k,0} + \left(\Gamma^{(2)}_{k,T,\color{red}{trial}} - \Gamma^{(2)}_{k,0}\right) + \left(\Gamma^{(2)}_{k,T} - \Gamma^{(2)}_{k,T,\color{red}{trial}}\right)$$

$$\Gamma^{(2)}_{k,T,\color{red}{trial}} = \int_k T \sum_n \int_{\vec{p}} \dot{\Gamma}^{(2)}_{k,T=0}$$

Flow Equation for Thermal Fluctuations

technique make a guess for the finite temperature result and iterate around it:

- does not change the final result
- numerical stability in iteration procedure as one starts „closer to physics“

$$\Gamma^{(2)}_{k,T} = \Gamma^{(2)}_{k,0} + \underbrace{\left(\Gamma^{(2)}_{k,T} - \Gamma^{(2)}_{k,0}\right)}_{thermal\ \ contribution} + \underbrace{\left(\Gamma^{(2)}_{k,T,trial} - \Gamma^{(2)}_{k,T,trial}\right)}_{0}$$

$\longrightarrow$ *„re-ordering“*

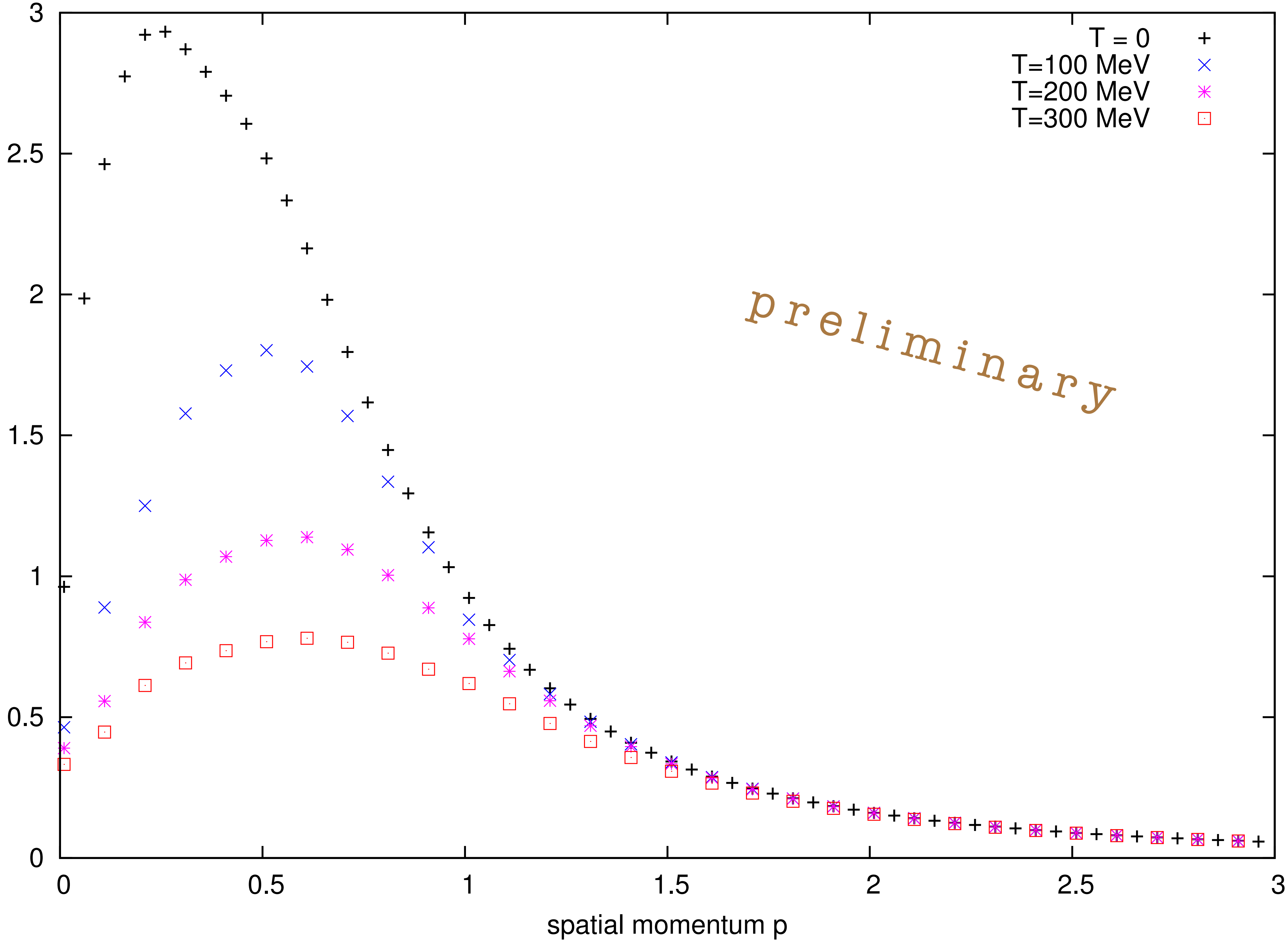
$$\boxed{\Gamma^{(2)}_{k,T} = \Gamma^{(2)}_{k,0} + \left(\Gamma^{(2)}_{k,T,trial} - \Gamma^{(2)}_{k,0}\right)} + \left(\Gamma^{(2)}_{k,T} - \Gamma^{(2)}_{k,T,trial}\right)$$

... 0-th iteration

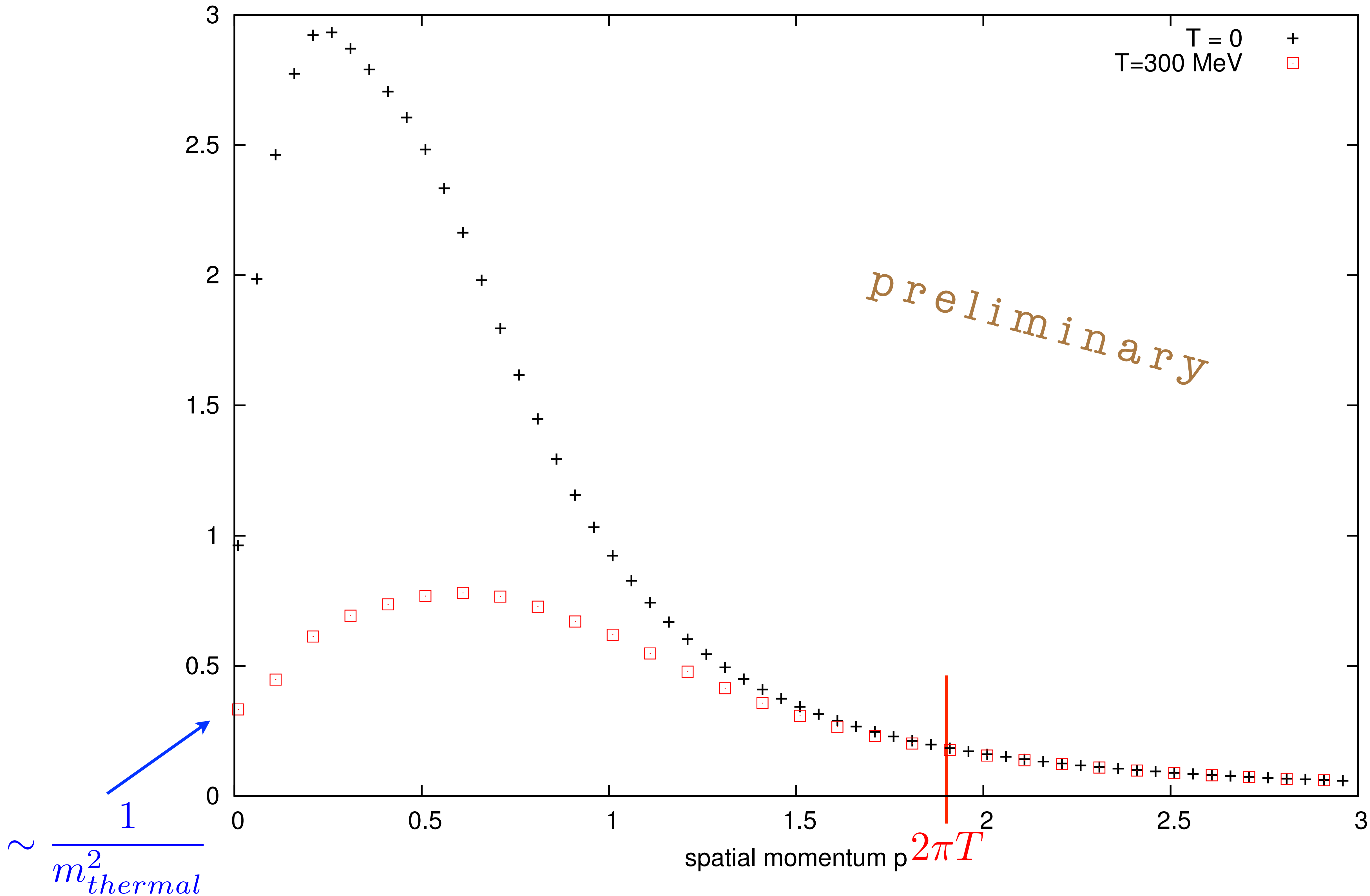
preliminary

Results

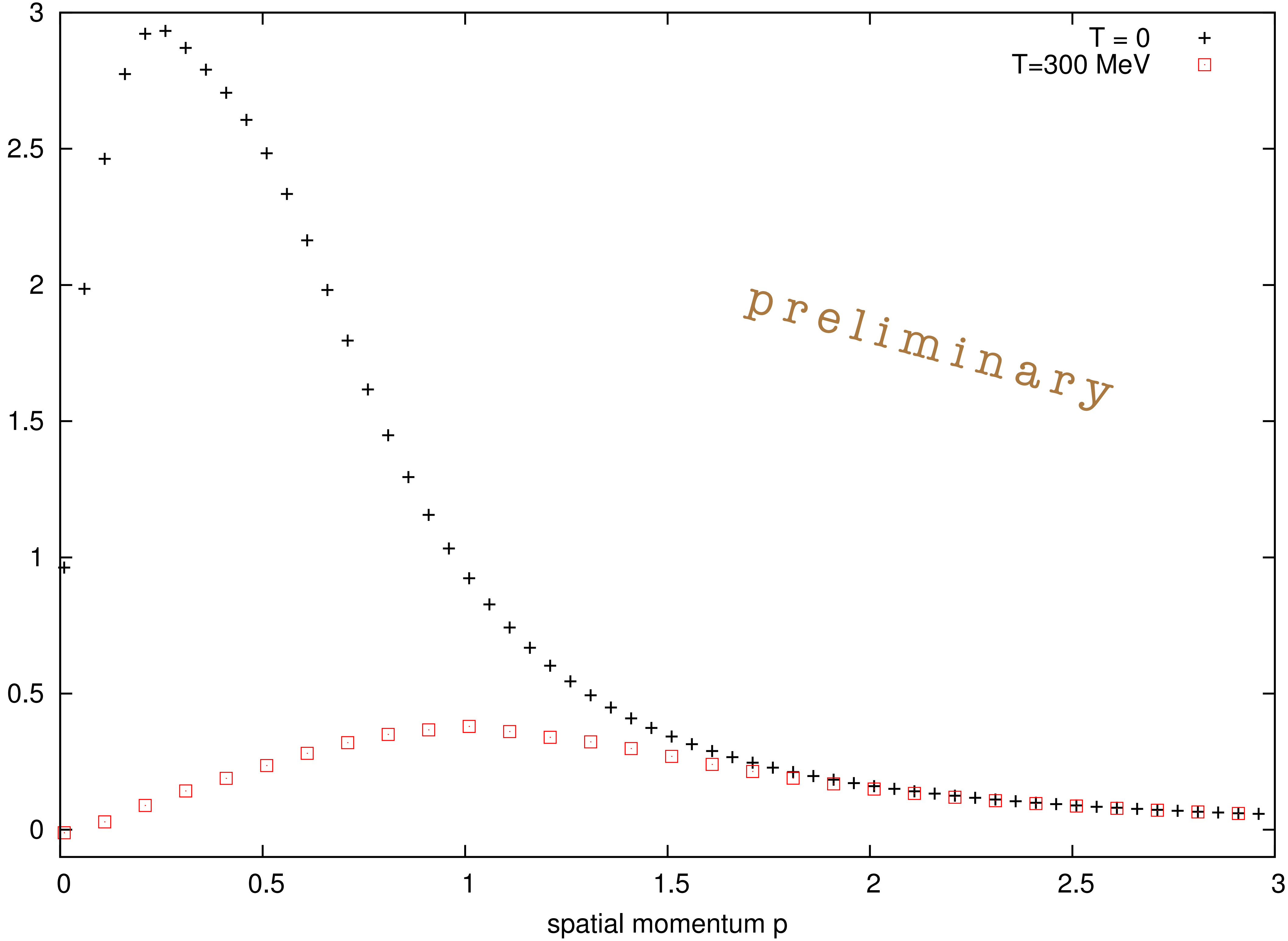
Electric Gluon-Propagator (after 0th iteration)



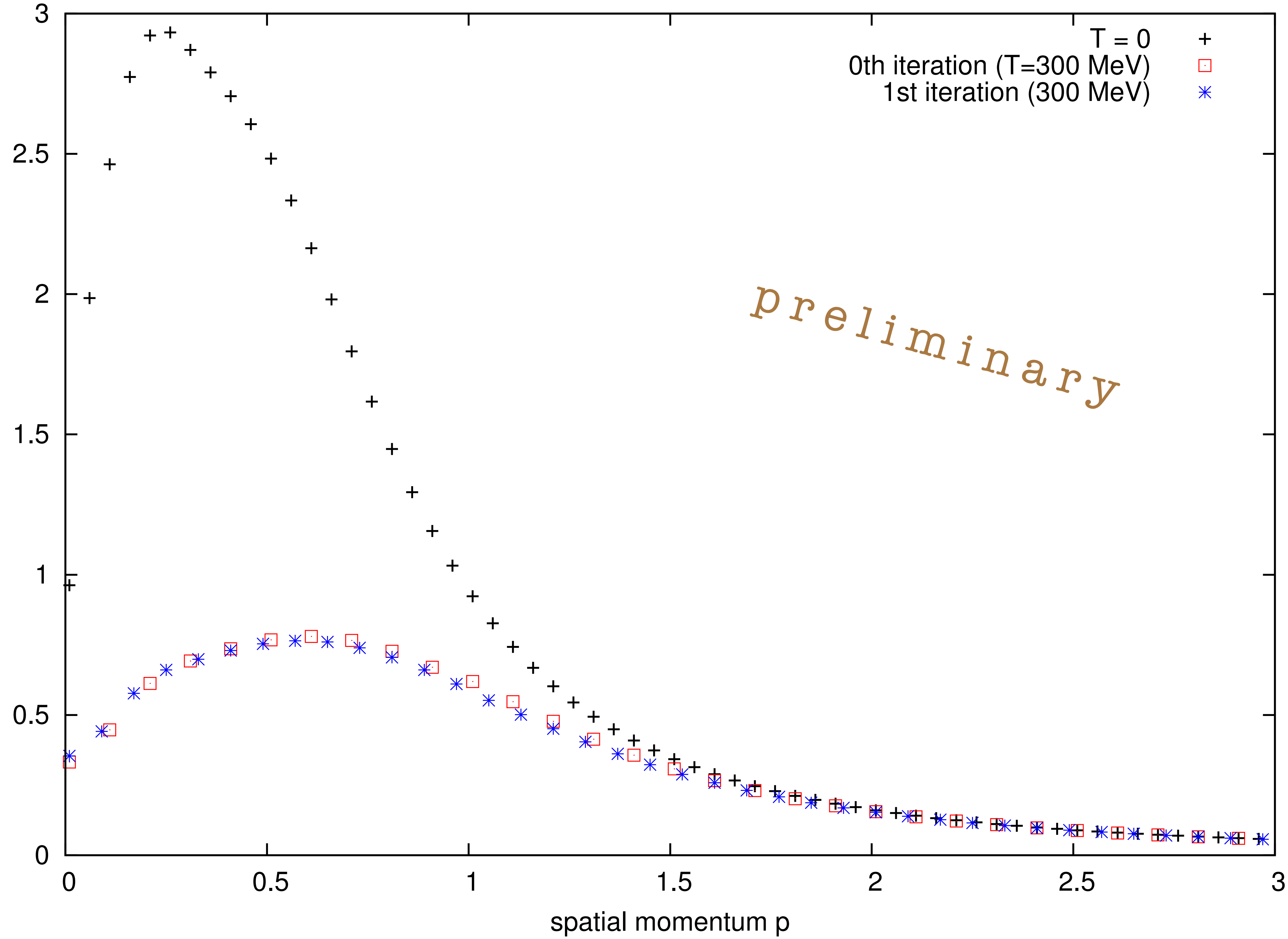
Electric Gluon-Propagator (after 0th iteration)



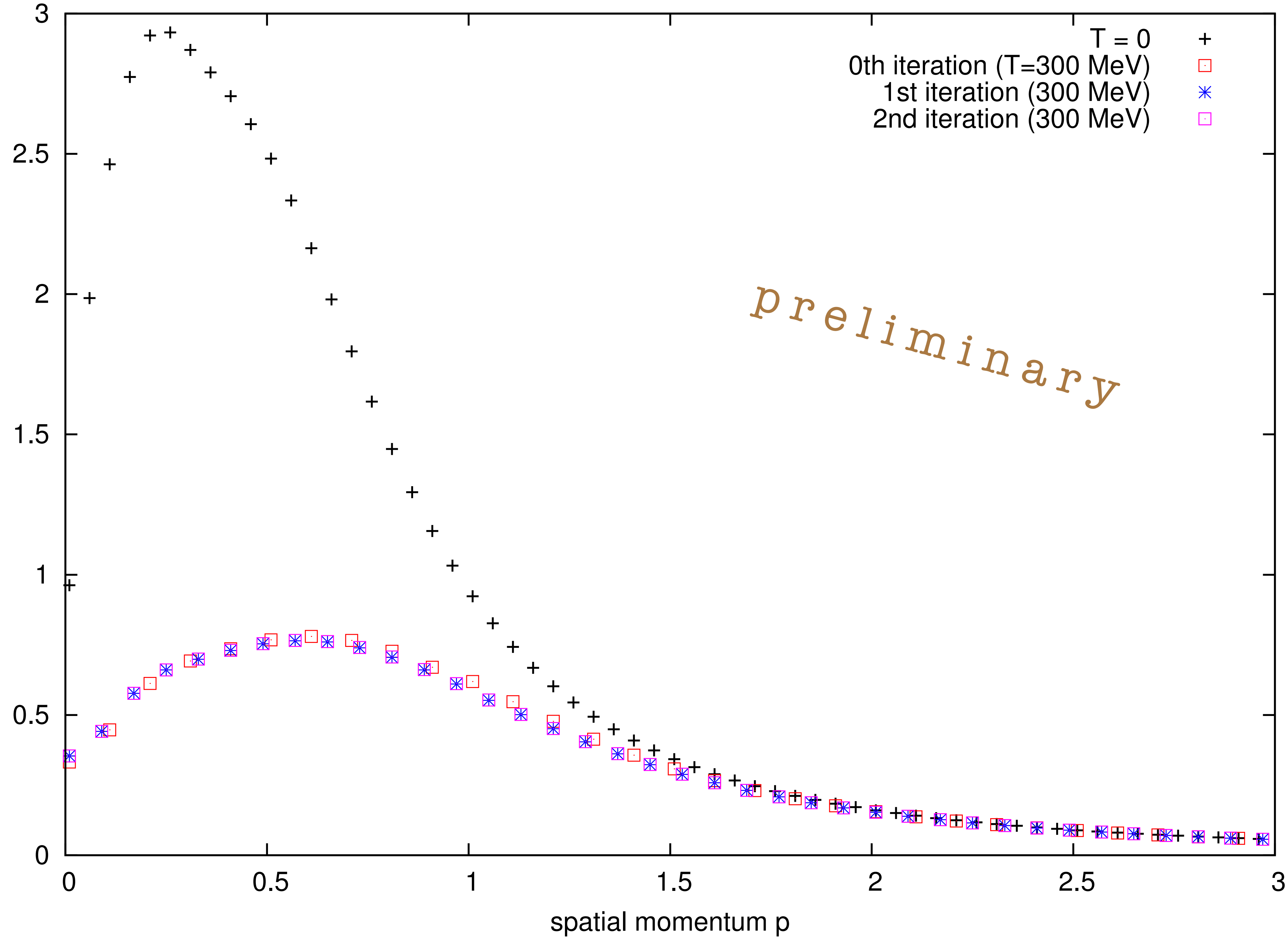
Magnetic Gluon-Propagator (after 0th iteration)



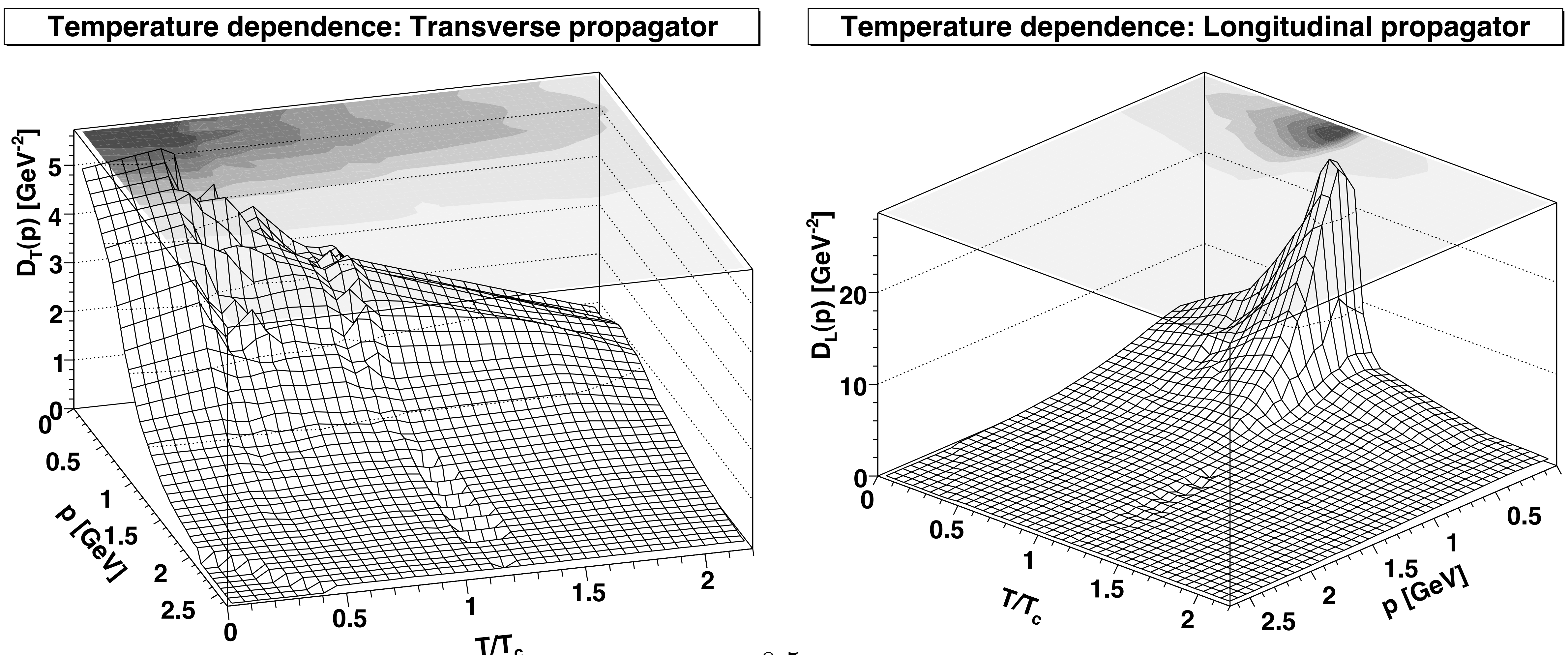
Electric Gluon-Propagator (higher iterations)



Electric Gluon-Propagator (higher iterations)



Lattice Results



0.5

A. Maas, arXiv: 0911.0348 [hep-lat].

Summary / Outlook

motivation: Yang-Mills propagators at non-vanishing temperature

idea: introduce thermal flow equation and add to zero temperature part

$$\Gamma_{k,T} = \Gamma_{k,0} + \bar{\Gamma}_{k,T}$$

flow at finite temperature

(purely) thermal flow

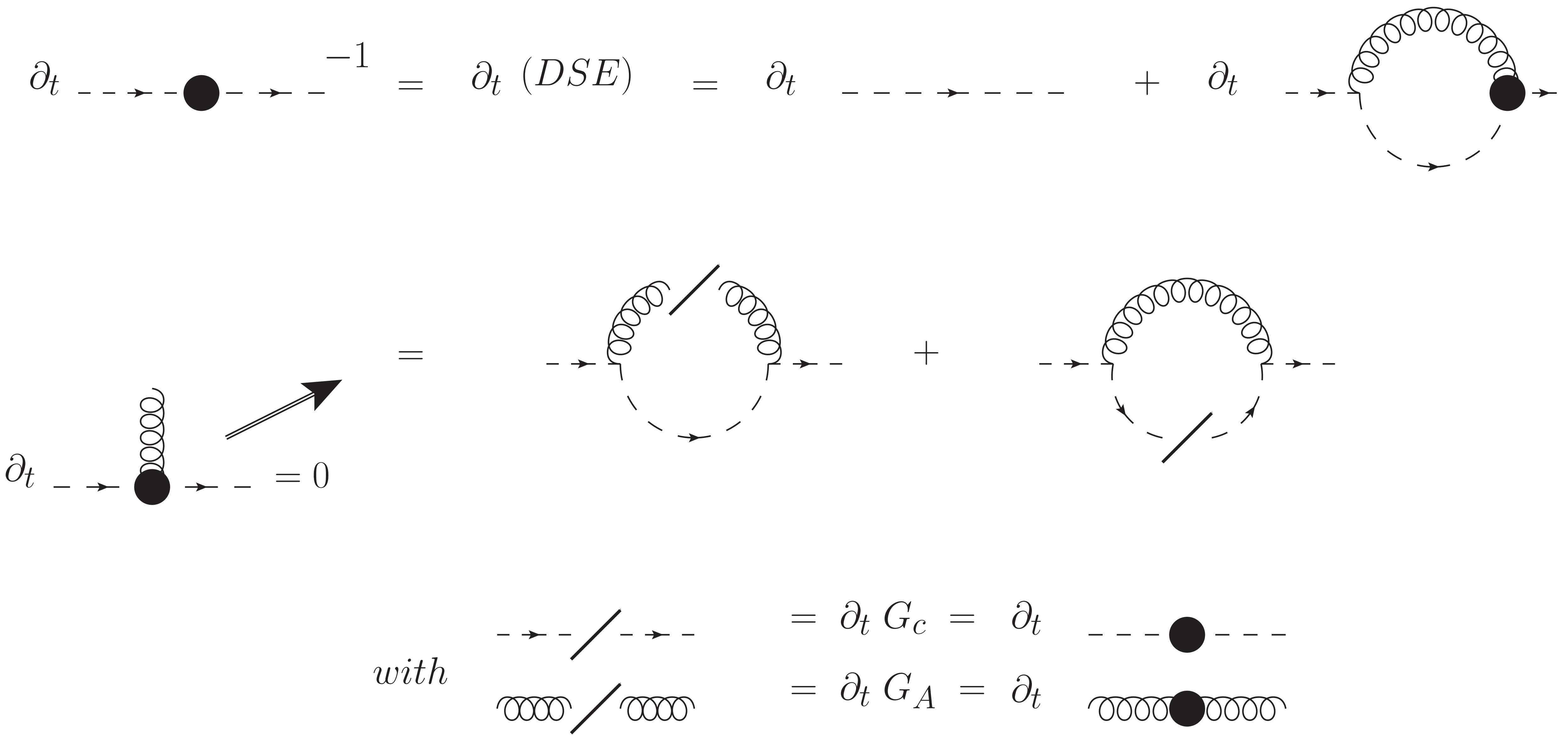
zero temperature result („input“)

have seen: first results for non-perturbative propagators (preliminary)

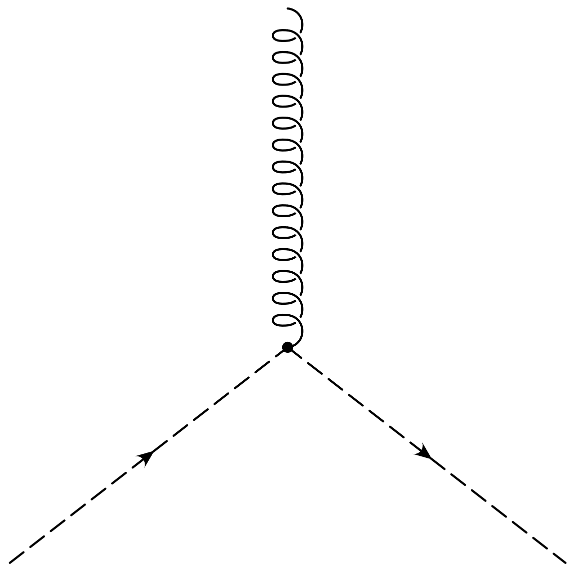
outlook: couple to full QCD calculation

talk J. M. Pawlowski
talk L. M. Haas
Braun, Gies, Pawlowski, Phys. Lett. B684 (2010) 262-267.
Braun, Haas, Marhauser, Pawlowski, arXiv: 0908.0008 [hep-ph].

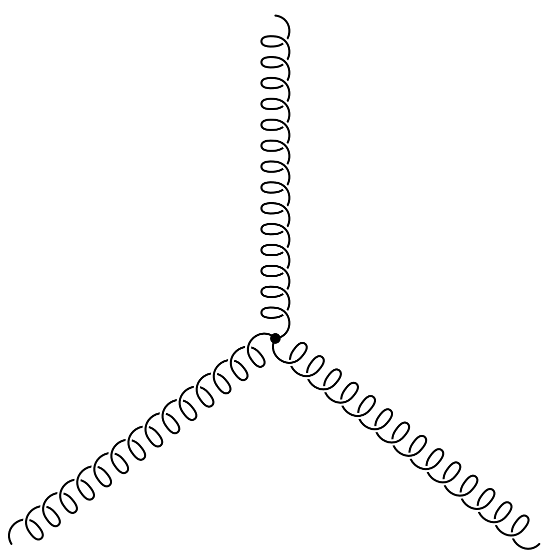
Dyson-Schwinger Approx. for the Ghost-Eq.



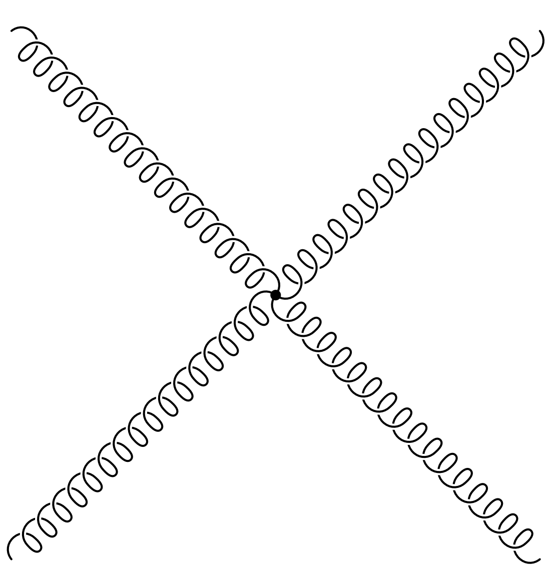
Vertices



$$\sim \; S^{(3)}_{\bar{c}Ac}$$



$$\sim \; g \; Z^{3/2}_{A,k}(k) \; \frac{S^{(3)}_{AAA}}{g_0}$$



$$\sim \; g^2 \; Z^{2}_{A,k}(k) \; \frac{S^{(4)}_{AAAA}}{g_0^2}$$

Full Method

$$\Gamma^{(2)}_{k,T} = \Gamma^{(2)}_{k,T=0} + \underbrace{\left[\Gamma^{(2)}_{\Lambda,T} - \Gamma^{(2)}_{\Lambda,0}\right]}_{\stackrel{\frac{T}{\Lambda}\to 0}{\approx} 0} + \int \frac{dk}{k} \hat{\Delta}\dot{\Gamma}^{(2)}_{k,T}\left[\Gamma^{(2)}_{k,T=0}\right] + \int \frac{dk}{k} \Delta\dot{\Gamma}^{(2)}_{k,T}\left[\Gamma^{(2)}_{k,T}, \Gamma^{(2)}_{k,T=0}\right]$$

$$\hat{\Delta}\dot{\Gamma}^{(2)}_{k,T} = T\sum_n \int \frac{d^3p}{(2\pi)^3}\dot{\Gamma}^{(2)}_{k,0} - \int \frac{d^4p}{(2\pi)^4}\dot{\Gamma}^{(2)}_{k,0}$$

$$\Delta\dot{\Gamma}^{(2)}_{k,T} = T\sum_n \int \frac{d^3p}{(2\pi)^3}\left(\dot{\Gamma}^{(2)}_{k,T} - \dot{\Gamma}^{(2)}_{k,T=0}\right)$$