Myers-Perry black holes within asymptotically safe gravity

Kostas Nikolakopoulos

Sussex University

September 18, 2010

Table of contents

1. Classical solutions
2. Quantum gravity effects
3. AS black holes
4. Conclusions

Introduction

- Black holes are objects of extremely strong gravity.
- They could provide information about quantum gravity.
- This information is "hidden" behind an event horizon.
- Spherical symmetric Schwarzschild black hole.

$$ds^2 = -\left(1 - G_N \frac{M}{r}\right) dt^2 + \left(1 - G_N \frac{M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- When we consider rotating black holes the centrifugal repulsion competes the gravitational attractive force and an event horizon exists only up to a maximum value of angular momentum.

Limitations

- Curvature singularities
- Information paradox

Motivations & earlier results

- Curvature singularities?
- TeV scale gravity (Quantum gravity effects at LHC)
- High energy scattering
- Cross section

$$\sigma \sim \pi r_{Sch}^2$$

- Earley & Giddings $\rightarrow$ closed trapped surfaces

$$\sigma \sim \pi r_{Kerr}^2$$

previous results

- Four dimensional Schwarzschild

  [A. Bonanno, M. Reuter 2000]
- Four dimensional Kerr

  [M. Reuter, P. E. Tuiran 2006]
- Higher dimensional, spherical symmetric

  [K. Falls, D. Litim, A. Raghuraman 2010]

Classical solutions

Higher dimensional black holes can rotate in more than one independent planes ($\lfloor \frac{d-1}{2} \rfloor$ angular momentum). For simplicity we look at the case with only one rotation.

[R. Myers, M. Perry 1986]

Myers-Perry metric (one angular momentum)

$$ds^2 = -dt^2 + G_N \frac{M}{r^{d-5}\Sigma}(dt - \alpha \sin^2\theta d\phi)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$
$$+(r^2 + \alpha^2)\sin^2\theta d\phi^2 + r^2\cos^2\theta d\Omega^2_{d-4}$$

with

$$\Sigma = r^2 + \alpha^2\cos^2\theta, \qquad \Delta = r^2 + \alpha^2 - G_N \frac{M}{r^{d-5}},$$

and

$$M = \frac{16\pi M_{phys}}{(d-2)\Omega_{d-2}}, \qquad \alpha = \frac{d-2}{2}\frac{J}{M_{phys}},$$

Classical solutions

Horizons ($g^{rr} = 0 \Rightarrow \Delta = 0$)

- $d = 4 \rightarrow 0$, 1 or 2 horizons depending on the ratio α/M (Kerr B.H.)
- $d = 5 \rightarrow 0$ or 1 horizon depending on the ratio α/M
- $d \geq 6 \rightarrow$ Always 1 horizon (Ultra-spinning black holes)

Properties

- The four laws of black hole thermodynamics still hold.
- Violation of uniqueness theorems. In four dimensions every stationary vacuum black hole belongs to the Kerr-Newman family. In higher dimensions there exist solutions with the same mass and angular momentum but with different horizon topology.
- e.g. Black ring in five dimensions.

Classical solutions

New solutions and instabilities

- There have been found recently new black hole solutions such like black branes.
- It was shown that these solutions suffer from linear instabilities (G-L instabilities).
- For $d \geq 6$ and at the limit of very high rotation it was found that singly spinning rotating black holes approach the black branes solution.
- This led to the heuristic argument that ultra-spinning black holes are unstable.
- This was recently supported numerically for various dimensions.
- Interesting relation between linear and thermodynamical stability.

Temperature

$$T = \frac{1}{4\pi r_+}\left(\frac{2r_+^2}{r_+^2+\alpha^2} + d - 5\right)$$

minimum at $\frac{\alpha}{r_+} = \sqrt{\frac{d-3}{d-5}}$

Quantum gravity effects

- As in any field theory, quantum effects turn the coupling constants into energy dependent quantities through the renormalization process.
- Newton's constant turns into a position dependent coupling:

$$G_N \to G(r)$$

  We assume that the leading quantum corrections of the metric are captured by this replacement.
- The main effect to be discussed is the potential weakening of gravity at high energies (short distances).
- Therefore, we are going to study the effects of the weakening $G_N \to G(r) < G_N$.
- We will study these effects in the context of asymptotic safety for gravity.

Asymptotic safety

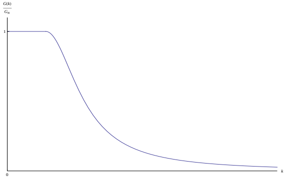
- Using the exact renormalization group the higher dimensional Callan-Symanzik equation is

  [D. Litim, P. Fischer 2006]

$$\frac{dg(k)}{d\ln k} = (d-2+\eta)g(k) \qquad g(k) = k^{d-2}G_k$$

- Non-Gaussian fixed point when $\eta_* = 2-d$
- In the vicinity of the Planck scale we can approximate the momentum dependence of the gravitational coupling by

$$\frac{1}{G(k)} = \frac{1}{G_N} + \omega k^{d-2}, \qquad \omega = 1/g_*$$



Quantum gravity effects

Matching the momentum and position scales

- We have the running gravitational coupling as a function of momentum and we want to convert it into a function of position.
- We have to make an identification between the momentum scale and the coordinate variable r.
- We are going to use the IR matching with $k = 1/r^{\gamma}$
- The gravitational coupling takes the form

$$G(r) = G_N \frac{r^{\gamma(d-2)}}{r^{\gamma(d-2)} + \omega G_N}$$

- In what follows we are going to use the value $\gamma = 1$
- The qualitative results remain the same for every $\gamma > \frac{d-3}{d-2}$.

Asymptotically safe black holes

We make the substitution $G_N \to G(r) = G_N \frac{r^{d-2}}{r^{d-2}+\omega G_N}$ and we are first interested for the horizon structure of the quantum black holes

Horizons

- The horizons are found from the relation $\Delta = 0$ or

$$r^2 + \alpha^2 - MG_N \frac{r^3}{r^{d-2} + \omega G_N} = 0$$

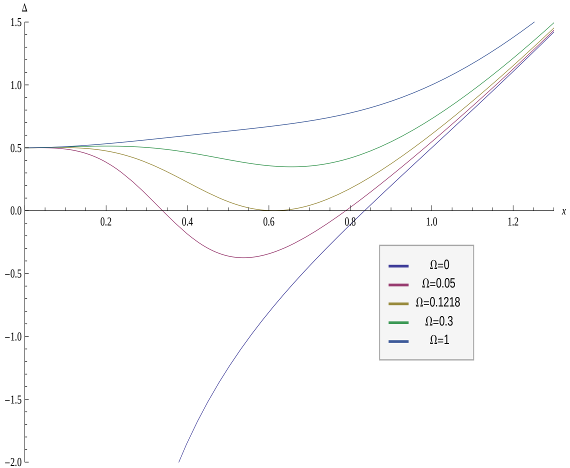
- In contrast to the classical case, quantum corrected black holes have *always* 0, 1, or 2 horizons.
- Now the condition for the existence of horizons is a relation between all the parameters of our spacetime $F(M, \alpha, \omega) = 0$.

Dimensionless variables:

$$x = \frac{r}{r_{cl}}, \qquad \Omega = \frac{\omega G_N}{r_{cl}^{d-2}}, \qquad A = \frac{\alpha^2}{r_{cl}^2}$$

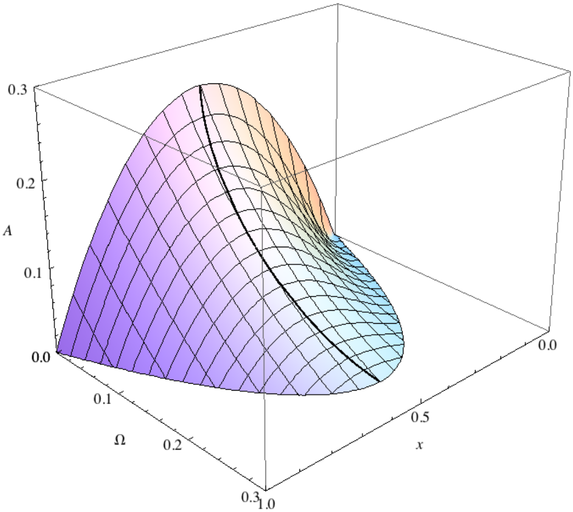
with $r_{cl} = (G_N M)^{\frac{1}{d-3}}$

The lapse function in 6 dimensions for various values of Ω.

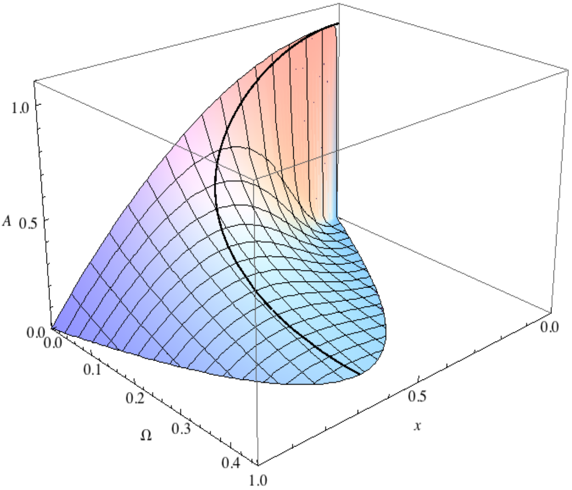


$A = 0.5$

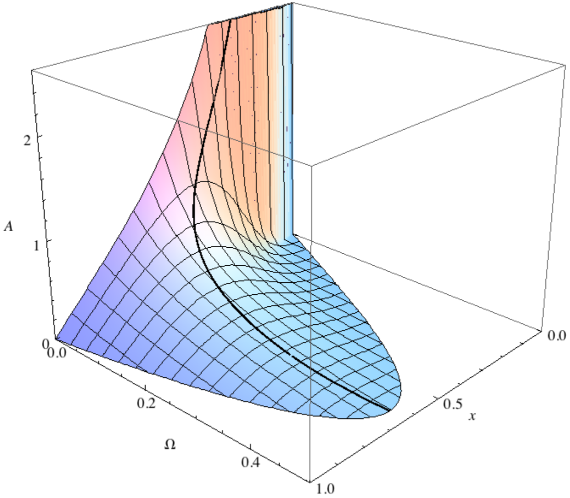
Four dimensions ($d = 4$)



Five dimensions ($d = 5$)



Six dimensions ($d = 6$)



Asymptotically safe black holes

Review of black hole properties

- The radius of the horizon gets smaller as the mass of the black hole approaches M_{Pl}, until it reaches the critical radius.
- The radius of the horizon gets smaller as the angular momentum grows until it reaches the critical radius.
- For every mass there is a maximum value of angular momentum for which horizons exist.
- There is a minimum mass M_{cr} below which there are no horizons, which corresponds to the non-rotating limit.
- Since there is a maximum value of angular momentum, ultra-spinning black holes cease to exist.

Asymptotically safe black holes

Temperature

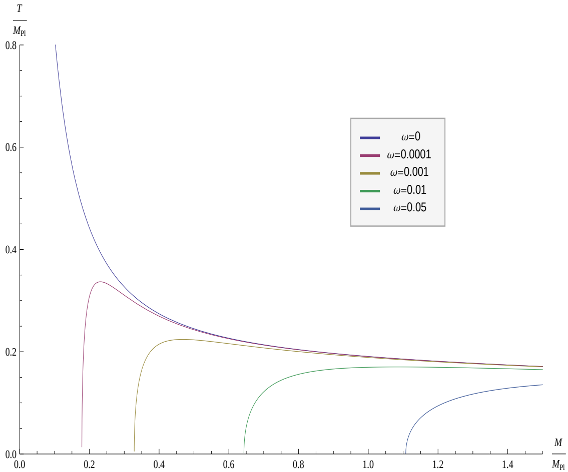
- The temperature is found to be

$$T = \frac{\kappa}{2\pi} = \frac{\partial_r \Delta|_{r=r_+}}{4\pi(r_+^2 + \alpha^2)} = \frac{1}{4\pi r_+}\left[\frac{2r_+^2}{r_+^2 + \alpha^2} + (d-5) - \eta(r_+)\right].$$

where $\eta(r_+) = r_+ \frac{G'(r_+)}{G(r_+)}$ is the anomalous dimension of gravity.

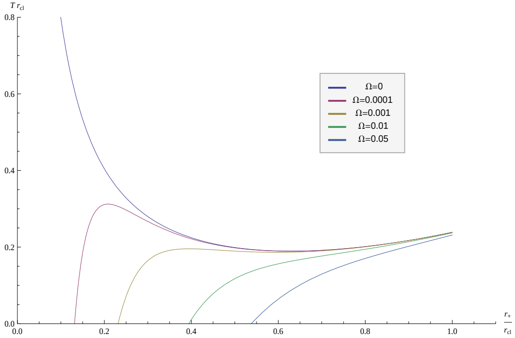
- Temperature is always positive and vanishes for extremal black holes.
- Classically, for $d \geq 6$ there are not extreme black holes and the temperature diverges for small masses
- When quantum corrections are considered there is a change of behavior and temperature vanishes for M_{cr}.
- Interesting applications for the thermodynamical stability of these black holes.

The temperature in 6 dimensions



Asymptotically safe black holes

Ultra-spinning instabilities (6 dimensions)



- Classically, temperature displays a minimum and for large angular momentum diverges.
- Depending on the value of Ω the divergence is "softened" or it is absent.

Asymptotically safe black holes

Area & angular velocity

- Area

$$\mathcal{A}_H = r_+^{d-4}(r_+^2 + \alpha^2)\Omega_{d-2}.$$

- Angular velocity of the horizon

$$\Omega_H = \left.\frac{d\phi}{dt}\right|_{r=r_+} = \frac{\alpha}{r_+^2 + \alpha^2}$$

Conclusions & current work

Conclusions

- The horizon structure is different for $d \geq 5$.
- The horizon radius gets smaller when quantum corrections are "turned on".
- Ultra-spinning black holes cease to exist.
- The temperature is always smaller than the classical and it vanishes at extremality.
- The ultra-spinning instability is "weakened"

Current work

- Entropy and specific heat.
- Laws of black hole thermodynamics.
- Fate of classical instabilities.
- Black hole production at the LHC.

Thank you for your attention!