NEW HIGGS INFLATION

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Work in collaboration with C. Germani:

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Outline

1. Introduction

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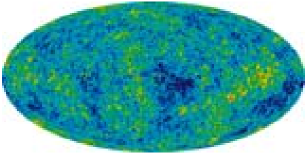
The latest cosmological data



agree impressively well with a Universe which is at large scales,

- homogeneous,
- isotropic
- spatially flat,

(well described by a FRW spatially flat geometry).

A theoretical puzzle:

A flat FRW Universe is *extremely fine tuned* solution in GR.

Many attempts have been put forward to solve this puzzle. However, the most developed and yet simple idea still remains *Inflation*. Inflation solves homogeneity, isotropy and flatness problems in one go just by postulating a rapid expansion of the early time Universe post Big Bang.

A phenomenological implementation of Inflation: “slow rolling” scalar field

the Inflaton

The most economical and yet fundamental candidate for the Inflaton:

Standard Model Higgs boson.

Unfortunately, Standard Model parameters do not allow “slow rolling“ for Higgs, if Higgs minimally coupled to gravity.

A no-go result

The, tree-level, Standard Model Lagrangian for the Higgs boson minimally coupled to gravity is

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2} D_\mu \mathcal{H}^\dagger D^\mu \mathcal{H} - \frac{\lambda}{4} \left(\mathcal{H}^\dagger \mathcal{H} - v^2 \right)^2 \right] ,$$

R is the Ricci scalar, κ the gravitational coupling, $\mathcal{H}$ the Higgs boson doublet, D_μ the covariant derivative with respect to $SU(2) \times U(1)$ and $v = < \mathcal{H} >$ is the vev of the Higgs in the broken phase of the SM.

Assumptions: during Inflation no gauge interactions and the Higgs field is "large" with respect to its vev. Then:

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi - \frac{\lambda}{4}\Phi^4 \right] \,, \qquad (1)$$

where for simplicity $\mathcal{H} \rightarrow \Phi$ (real scalar instead of complex doublet).

To study a FRW solution of this system, we can directly insert into the action the following metric ansatz

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 \delta_{ij} dx^i dx^j \,. \qquad (2)$$

Plugging (2) into (1) we have the following action per unit three-volume

$$\mathcal{S} = \int dta^3 \left[-3\frac{H^2}{\kappa^2 N} + \frac{1}{2}\frac{\dot{\Phi}^2}{N} - N\frac{\lambda^4}{4}\Phi^4 \right] ,$$

where $H \equiv \frac{\dot{a}}{a}$ is the Hubble constant and $\dot{} = \frac{d}{dt}$.

The only independent Einstein equation, with FRW symmetries, is the Hamiltonian constraint (variation of the action with respect to the lapse N) The field equation for Φ corresponds instead to the variation of (1) with respect to Φ.

The Hamiltonian constraint and field equation are then

$$H^2 = \frac{\kappa^2}{6}\left(\dot{\Phi}^2 + \frac{\lambda}{2}\Phi^4\right) \ , \ \ddot{\Phi} + 3H\dot{\Phi} + \lambda\Phi^3 = 0 \ . \tag{3}$$

Slow roll means $\dot{\Phi}^2 \ll \frac{\lambda}{2}\Phi^4$ together with

$$|\ddot{\Phi}| \ll 3H|\dot{\Phi}| \ , \tag{4}$$

so that

$$H^2 \simeq \frac{\kappa^2}{12}\lambda\Phi^4 \tag{5}$$

and

$$\dot{\Phi} \simeq -\frac{\lambda}{3H}\Phi^3 \ . \tag{6}$$

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$$\epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{8}{\kappa^2 \Phi^2} \ll 1 \; . \tag{7}$$

Satisfied for: $\Phi >> M_{Pl}$. and justify our initial assumption to neglect the vev v.

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- *Classical gravity*: During Inflation $R \sim H^2$. A sufficient condition to avoid quantum gravity during slow roll

$$R \simeq 12H^2 \ll \frac{1}{2\kappa^2}.$$

This implies, $\Phi^4 \ll 1/(2\lambda\kappa^4)$.

Combining the above, SM higgs can drive inflation for

$$\lambda << 10^{-2} \tag{8}$$

Current experimental bounds coming from direct Higgs boson searches (also from global fit to electroweak precision data), favors a value of λ $0.11 < \lambda \lesssim 0.27$

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Thus:

SM higgs minimally coupled to gravity cannot be the slow-roll Inflaton.

Non-minimally coupled Higgs

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What kind of couplings we should look for?

Couplings that should not change the degrees of freedom.

The number of degrees of freedom of gravity and the higgs is 3 (2 from gravity and 1 from the higgs). If we allow non-minimal couplings of the higgs to gravity, this number may change. We don't want this. (In this case, is more elegant and even economical to add a second scalar, the Inflaton back to the game). So the question is:

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What are the allowed couplings of a scalar to gravity such that no extra propagating degrees of freedom are introduced

There are three solutions:

There are three solutions:

- One you know

There are three solutions:

- One you know
- One you may guess

There are three solutions:

- One you know
- One you may guess
- And one I will describe.

I. The one you know

Standard non-minimal coupling (Brans-Dicke)

The theory described by the action

$$S = \int d^4\sqrt{-g}\left[(1+\kappa^2\xi\Phi^2)\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{\lambda}{4}\Phi^4\right] , \qquad (9)$$

propagates 3 degrees of freedom as the minimal coupled one.

Φ^2 can be replaced by any $U(\Phi)$ and Φ^4 by any potential $V(\Phi)$.

II. The one you may guess

The theory described by the action

$$S = \int d^4\sqrt{-g}\left[\frac{R}{2\kappa^2} + \alpha\Phi^2 R^2_{GB} - \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{\lambda}{4}\Phi^4\right] \ , \qquad (10)$$

where

$$R^2_{GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \qquad (11)$$

is the Gauss-Bonnet tensor also propagates 3 degrees of freedom

The one I will describe

The action

$$S=\int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2}-\frac{1}{2}\left(g^{\mu\nu}-\frac{w^2}{2}G^{\mu\nu}\right)\partial_\mu\Phi\partial_\nu\Phi-\frac{\lambda}{4}\Phi^4\right], \quad (12)$$

where

$$G^{\mu\nu}=R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R$$

is the Einstein tensor, also propagate 3 degrees of freedom.

Standard non-minimal coupling

Consider the following non-minimally coupled action

$$S = \int d^4\sqrt{-g}\left[(1+\kappa^2\xi\Phi^2)\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{\lambda}{4}\Phi^4\right] , \qquad (13)$$

where ξ is a parameter.

Conformal transformation to Einstein frame

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = (1+\kappa^2\xi\Phi^2)^{-1}.$$

During slow roll (*i.e.* neglecting terms in $\dot{\Phi}$ and $\dot{H}$), if $\kappa^2\xi\Phi_0^2 \gg 1$ (where Φ_0 is the scalar field value during Inflation), the action in the Einstein frame is approximately

$$S \simeq \int d^4\sqrt{-g}\left[\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2}\partial_\mu\tilde{\Phi}\partial^\mu\tilde{\Phi} - \frac{\tilde{\lambda}}{4}\tilde{\Phi}^4\right] .$$

Here $\tilde{\Phi} = \Phi/(\kappa\sqrt{\xi}\Phi_0)$ the canonically normalized field and $\tilde{R}\kappa^2 \simeq \lambda\xi^{-2} \ll 1$. Then, for

$$\xi \sim 10^4$$

it seems that Inflation might be obtained within the SM value for λ without reaching the quantum gravity regime $H^2 > M_{Pl}^2$.

However, although quantum gravity is not reached, the unitarity bound of the theory seems to be violated. The operator descrining the non-minimal couplings is dimenision 5 when written in terms of canonical normalized fields and suppressed by the cutoff of the theory (above of which tree-level unitarity is lost). Expanding

$$\Phi = \Phi_0 + \varphi\,, \quad g_{\mu\nu} = \gamma_{\mu\nu} + \frac{1}{\sqrt{\xi}\,\Phi_0} h_{\mu\nu},$$

(with $h_{\mu\nu}$ and φ canonically normalized) we get, among others, the non-renormalizable operator

$$\frac{\sqrt{\xi}}{2\Phi_0}\varphi^2\gamma^{\mu\nu}\partial^2 h_{\mu\nu}.$$

By considering the $2\varphi \to 2\varphi$ scattering amplitude, we find the unitarity violation scale of the theory is

$$\Lambda = \frac{\Phi_0}{\sqrt{\xi}}$$

With the Standard model value of λ,

$$H \sim \Lambda \tag{14}$$

the energy scale of the inflationary background is so close to the unitarity bound to challenge the robustness of the Inflationary background against quantum corrections.

New Higgs Inflation

Higher curvature or curvature derivative coupling automatically introduce new degrees of freedom. We will therefore only study the following tree-level action

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}g^{\mu\nu}(1+\zeta R) + \frac{w^2}{2}G^{\mu\nu}\right]\partial_\mu\Phi\partial_\nu\Phi\,, \quad (15)$$

where

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$$

is the Einstein tensor and w and ζ are some inverse mass scales.

Appropriate framework to analyse this theory: *ADM formalism*.

Metric decomposition in ADM :

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \ . \tag{16}$$

N: the lapse function,

N_i : the shift vector.

All geometry is described by defining:

$$D_i, \quad {}^{(3)}R \, , \qquad K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - D_i N_j - D_j N_i\right) \tag{17}$$

the spatial covariant derivative, a three-dimensional curvature

(both constructed by h^{ij}) and the extrinsic curvature, respectively.

Time evolution is then only related to the extrinsic curvature. In General Relativity minimally coupled to a scalar field, in the gauge in which Φ propagates, N is not propagating.

The term that containing higher than two time derivatives is

$$S_{hd} \sim \zeta \int d^3x dt \sqrt{h}\, \frac{\dot{K}}{N^2} \dot{\phi}^2 .$$

Clearly, this term increases the number of degrees of freedom of the theory, with respect to General Relativity, by making N a propagating degree of freedom. To cancel this term one should then take $\zeta = 0$.

As a result, unique non-minimally derivative coupled Higgs theory to gravity, propagating no more degrees of freedom than General Relativity minimally coupled to a scalar field is

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}\left(g^{\mu\nu} - w^2 G^{\mu\nu}\right)\partial_\mu \Phi \partial_\nu \Phi - \frac{\lambda}{4}\Phi^4\right]. \qquad (18)$$

This is the tree-level theory we will discuss in the following.

En passant, we note that the non-minimal coupling (18) appears in heterotic String Theory for the universal Dilaton.

It is easy to see that the non-minimal coupling in (18) may lower the effective self coupling of the Higgs boson. In a FRW background we have that $G^{tt} \sim H^2$. During slow roll (*i.e.* $H \simeq const.$) we can rewrite the action (18) as

$$S \simeq \int d^4\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\bar{\Phi}\partial^\mu\bar{\Phi} - \frac{\bar{\lambda}}{4}\bar{\Phi}^4\right] ,$$

$\bar{\Phi} \sim \frac{1}{wH}\Phi$ canonically normalized. Then effective self coupling constant is

$$\bar{\lambda} \sim \frac{1}{w^4H^4}\lambda \ll \lambda.$$

Thus, for big enough $wH >> 1$, $\bar{\lambda}$ can be lowered to $<< 10^{-2}$ as required for SM value of λ.

Unitarity bounds

Expanding around inflationary background,

$$\Phi = \Phi_0 + \frac{1}{\sqrt{3}\, w\, H}\varphi\,, \quad g_{\mu\nu} = \gamma_{\mu\nu} + \kappa\, h_{\mu\nu},$$

with $\varphi, h_{\mu\nu}$ canonically normalized, the first non-renormalizable operator appearing in the expansion of the theory is

$$I = \frac{\kappa}{2H^2}\partial^2 h^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\ . \tag{19}$$

Again, a $2\varphi \to 2\varphi$ scattering indicates a cutoff

$$\Lambda = (2H^2/\kappa)^{1/3}.$$

Requiring that the inflationary scale is much lower than the unitarity bound (*i.e.* $R \ll \Lambda^2$), we get

$$H \ll 1/\kappa$$

i.e., this minimal coupling is free of unitarity problems during Inflation. Consequently, quantum corrections are expected to be subdominant thanks to their suppression by the unitarity bound scale.

Cosmology

Using
$$ds^2 = -N(t)^2 dt^2 + a(t)^2 \delta_{ij} dx^i dx^j \ . \tag{20}$$

the following action per unit three-volume is obtained:

$$\mathcal{S} = \int dt\, a^3 \left[-3\frac{H^2}{\kappa^2 N} + \frac{1}{2}\frac{\dot{\Phi}^2}{N} + \frac{3}{2}\frac{H^2 w^2}{N^3}\dot{\Phi}^2 - N\frac{\lambda}{4}\Phi^4 \right] \ .$$

The Hamiltonian constraint and field equation are then

$$H^2 = \frac{\kappa^2}{6}\left[\dot{\Phi}^2\left(1 + 9H^2 w^2\right) + \frac{\lambda}{2}\Phi^4\right] \ ,$$
$$\partial_t \left[a^3 \dot{\Phi}\left(1 + 3H^2 w^2\right)\right] = -a^3 \lambda \Phi^3 \ .$$

We will constraint the solution to obey the following inequalities

$$H \gg \frac{1}{3w} \ , \quad 9H^2 w^2 \dot{\Phi}^2 \ll \frac{\lambda}{2}\Phi^4 \ , \quad -\frac{\dot{H}}{H^2} \ll 1 \ , \tag{21}$$

where the last two are the usual slow roll conditions. With (21) we find

$$H^2 \simeq \frac{\kappa^2}{12}\lambda\Phi^4 \ , \tag{22}$$

and

$$\ddot{\Phi} + 3H\dot{\Phi} = -\frac{4}{w^2\kappa^2\Phi}.$$

By considering the extra slow roll condition

$$|\ddot{\Phi}| \ll 3H|\dot{\Phi}| \ , \tag{23}$$

we finally get

$$\dot{\Phi} \simeq -\frac{4}{3Hw^2\kappa^2\Phi} \ . \tag{24}$$

Constraints:

- Classical gravity constraint:

  The constraint $R \simeq 12H^2 \ll 1/(2\kappa^2)$ implies

$$\Phi^4 \ll \Phi_M^4 \equiv \frac{1}{2\kappa^4 \lambda} \ . \tag{25}$$

- Perturbative higgs potential :

  $\lambda < 1$ gives

$$w \gg 10 \times \kappa \, \lambda^{1/6}$$

which for $\lambda \geq .11$ gives,

$$w \gg 7\kappa.$$

Cosmological perturbations

Fluctuations around the slow-roll inflationary solution in the ADM formalism where the metric is written as

$$ds^2 = -N^2 dt^2 + h_{ij} \left(dx^i - N^i dt\right) \left(dx^j - N^j dt\right) \ , \tag{26}$$

Scalar field in a FRW background is rotational invariant, scalar and tensor perturbations are decoupled. The theory is invariant under the diffeomorphisms group, as a result of reparametrization invariance.

A convinient gauge is

$$\delta\Phi = 0\,, \quad h_{ij} = a(t)^2\Big[(1+2\zeta)\delta_{ij} + \gamma_{ij}\Big]\,, \qquad \partial^i\gamma_{ij} = \gamma^i{}_i = 0 \quad (27)$$

- ζ parametrizes the scalar perturbations
- γ_{ij} the gravitational waves.

The expanded second order action coming from (18) is then

$$S = \frac{1}{2\kappa^2}\int d^3x dt\sqrt{h}N\left({}^{(3)}R(1+\frac{w^2\kappa^2\dot{\Phi}^2}{2N^2})+ (K_{ij}K^{ij}-K^2)(1-\frac{w^2\kappa^2\dot{\Phi}^2}{2N^2})+\frac{\kappa^2\dot{\Phi}^2}{N^2}-2\kappa^2 V\right) \quad (28)$$

The Hamiltonian (variation with respect to N) and momentum (variations with respect to N^i) constraints are found to be

$$D_i\left[(1-\frac{w^2\kappa^2\dot{\Phi}^2}{2N^2})(K^{ij}-h^{ij}K)\right]=0\,, \tag{29}$$

$$(1-\frac{w^2\kappa^2\dot{\Phi}^2}{2N^2})^{(3)}R-(1-\frac{3w^2\kappa^2\dot{\Phi}^2}{2N^2})(K_{ij}K^{ij}-K^2)-\frac{\kappa^2\dot{\Phi}^2}{N^2}-2\kappa^2V=0\ . \tag{30}$$

Scalar Perturbations

Considering a perturbative solution of the form

$$N = 1 + N_1, \quad N^i = \partial_i \psi + N_T^i$$

we find

$$N_1 = \frac{\dot{\zeta}}{H}\,,$$
$$\psi = -\frac{2 - w^2\kappa^2\dot{\Phi}^2}{2 - 3w^2\kappa^2\dot{\Phi}^2}\frac{\dot{\zeta}}{a^2 H} + \chi\,,$$
$$\partial^2\chi = -\kappa^2\frac{1 + 9w^2H^2}{2 - 3w^2\kappa^2\dot{\Phi}^2}\frac{\dot{\Phi}^2}{H^2}\dot{\zeta}\,, \quad \nabla_i N_T^i = 0\,. \qquad (31)$$

The action to second order in ζ is ($a(t) = e^{\rho(t)}$ and $H = \dot{\rho}$)

$$
\begin{aligned}
S_\zeta &= \frac{1}{2\kappa^2}\int d^3x dt \left\{ e^{\rho+\zeta}(1+\frac{\dot{\zeta}}{H})\left[\left(1+\frac{w^2\kappa^2\dot{\Phi}^2}{2(1+\frac{\dot{\zeta}}{H})^2}\right)[-4\partial^2\zeta - \right.\right. \\
& 2(\partial\zeta)^2] - 2\kappa^2 V e^{2(\rho+\zeta)}] + \\
&+ e^{3(\rho+\zeta)}\frac{1}{1+\frac{\dot{\zeta}}{H}}\left[-6(H+\dot{\zeta})^2\left(1-\frac{w^2\kappa^2\dot{\Phi}^2}{2(1+\frac{\dot{\zeta}}{H})^2}\right)+\kappa^2\dot{\Phi}^2\right] - \\
&- 2w^2\kappa^2(\partial\zeta)^2\,\partial_t\left(\frac{\dot{\Phi}^2}{H}\frac{2-w^2\kappa^2\dot{\Phi}^2}{2-3w^2\kappa^2\dot{\Phi}^2}e^{\rho+\zeta}\right) + \\
& \left. 4w^2\kappa^4 e^{3(\rho+\zeta)}\frac{1+9w^2H^2}{2-3w^2\kappa^2\dot{\Phi}^2}\frac{\dot{\Phi}^4}{H^2}\dot{\zeta}^2\right\} \,, \qquad (32)
\end{aligned}
$$

Using the background equations, we get

$$S_\zeta = \frac{1}{2}\int d^3x dt \left\{ 3w^2\kappa^2\dot{\Phi}^2 \left[e^{3\rho}(1+6w^2\kappa^2\dot{\Phi}^2)\dot{\zeta}^2 - e^{\rho}(1-\frac{13}{3}w^2\kappa^2\dot{\Phi}^2)(\partial\zeta)^2 \right] \right\} . \qquad (33)$$

In order to quantize the system, we need to canonically normalize the kinetic term.

Define the canonically normalized variable

$$\tilde{\zeta} = \sqrt{3} w \kappa \dot{\Phi} \sqrt{1 + 6 w^2 \kappa^2 \dot{\Phi}^2} \; \zeta \; . \tag{34}$$

Then, the quadratic action is written as

$$S_\zeta = \frac{1}{2} \int d^3x dt e^{3\rho} \left[\dot{\tilde{\zeta}}^2 - e^{-2\rho} c_s^2 (\partial \tilde{\zeta})^2 \right] \; , \tag{35}$$

where the sound speed of the scalar perturbations is

$$0 < c_s^2 = \frac{3 - 13 w^2 \kappa^2 \dot{\Phi}^2}{3 + 18 w^2 \kappa^2 \dot{\Phi}^2} < 1 \; . \tag{36}$$

The power spectrum $\mathcal{P}_S$ of scalar perturbations is defined as the two point correlation function of ζ at the horizon crossing $c_s k = aH$,

$$\langle \zeta_k \zeta_{k'} \rangle \Big|_{c_s k = aH} = \frac{2\pi^2}{k^3} \mathcal{P}_S \, \delta^{(3)}(\mathbf{k} + \mathbf{k}') \ , \tag{37}$$

where

$$\zeta = \int \frac{d^3 k}{(2\pi)^{3/2}} \zeta_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}} \ . \tag{38}$$

The result is

$$\mathcal{P}_S = \frac{\kappa^2 H^2}{4\pi^2} \frac{1}{12 w^2 \kappa^2 \dot{\Phi}^2} (1 + \frac{19}{2} w^2 \kappa^2 \dot{\Phi}^2) \Big|_{aH = c_s k} \ . \tag{39}$$

Defining the slow-roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{3}{2}\kappa^2 w^2 \dot{\Phi}^2 \,, \quad \eta_H = -\frac{\ddot{\Phi}}{H\dot{\Phi}} = -\frac{9}{4}\kappa^2 w^2 \dot{\Phi}^2 = -\frac{3}{2}\,\epsilon \,, \tag{40}$$

where (22,24) has been used, we finally find that

$$\mathcal{P}_S = \frac{\kappa^2 H^2}{4\pi^2}\frac{1}{8\epsilon} \,, \tag{41}$$

to leading order in ϵ.

The spectral index of scalar perturbation is given by [?]

$$n_S - 1 = \frac{d \ln \mathcal{P}_S}{d \ln k} = 2\frac{d \ln H}{d \ln k} - \frac{d \ln \epsilon}{d \ln k} \ . \tag{42}$$

By using (41) we find

$$n_S = 1 - 2\epsilon + 2\eta_H = 1 - 5\epsilon \ . \tag{43}$$

The running of the spectral index can also be specified to be

$$\alpha = \frac{dn_S}{d \ln k} = -15\epsilon^2 \ , \tag{44}$$

Taking for n_s the maximum likelihood value as specified by the 7-years WMAP observations of the Cosmic Microwave Background Radiation (CMBR) $n_S = 0.969$, we get

$$\epsilon = 0.0062 \; . \tag{45}$$

Gravitational Waves

Let us now consider tensor perturbations for the action (18). In the ADM formalism, a convenient gauge to analyze tensor perturbations is

$$h_{ij} = a(t)^2(\delta_{ij} + \kappa\gamma_{ij})\,, \quad \partial^i\gamma_{ij} = \gamma^i{}_i = 0\,. \tag{46}$$

After inserting this into the action (18) and keeping only quadratic terms in γ_{ij}, we get the quadratic action for gravitational waves

$$S_g = \int d^3x dt \frac{1}{8}\left[(1 - \frac{1}{2}w^2\kappa^2\dot{\Phi}^2)\, a^3\, \dot{\gamma}_{ij}\dot{\gamma}_{ij} - (1 + \frac{1}{2}w^2\kappa^2\dot{\Phi}^2)\, a\,(\partial_k\gamma_{ij})^2\right]\,.$$

We can now canonically normalize the graviton as

$$\tilde{\gamma}_{ij} = \sqrt{1 - \frac{1}{2} w^2 \kappa^2 \dot{\Phi}^2 / 2} \gamma_{ij} \ , \tag{48}$$

as no ghosts are propagated thanks to $\frac{1}{2} w^2 \kappa^2 \dot{\Phi}^2 \ll 1$. Then

$$S_g = \int d^3x dt \ a^3 \frac{1}{2} \left[\dot{\tilde{\gamma}}_{ij} \dot{\tilde{\gamma}}_{ij} - c_g^2 \, a^{-2} \left(\partial_k \tilde{\gamma}_{ij} \right)^2 \right] \ , \tag{49}$$

where we defined the sound speed

$$c_g^2 = \frac{2 + w^2 \kappa^2 \dot{\Phi}^2}{2 - w^2 \kappa^2 \dot{\Phi}^2} \simeq 1 + w^2 \kappa^2 \dot{\Phi}^2 > 1 \ . \tag{50}$$

Gravitational waves have superluminal velocity during Inflation.

Tensor perturbation may be expanded in Fourier modes

$$\gamma_{ij} = \int \frac{d^3k}{(2\pi)^{3/2}} \gamma_{\mathbf{k}}(t) e_{ij}(\mathbf{k}) \mathbf{e}^{\mathbf{ik\cdot x}} \tag{51}$$

where $e_{ij}(\mathbf{k})$ is the polarization tensor. The power spectrum of gravitational waves is then defined as

$$\langle \gamma_{\mathbf{k}} \gamma_{\mathbf{k}'} \rangle \Big|_{aH = c_g k} = \frac{2\pi^2}{k^3} \mathcal{P}_T \, \delta^{(3)}(\mathbf{k} + \mathbf{k}') \ , \tag{52}$$

where the correlator is easily found by canonically quantizing the action (49).

Explicitly we have

$$\mathcal{P}_T = \frac{\kappa^2 H^2}{4\pi^2 \Omega^2 c_g^3}\Big|_{aH=c_g k} \tag{53}$$

where

$$\Omega^2 = 1 - \frac{1}{2} w^2 \kappa^2 \dot{\Phi}^2 \ . \tag{54}$$

To leading order in $w^2\kappa^2\dot{\Phi}^2$, we have $\Omega \approx 1\,,\ c_s \approx 1.$ Therefore, the spectrum of gravitational waves for the New Higgs Inflation is approximately the same as in chaotic inflation.

Tensor-to-scalar ratio

To leading order in slow roll, the tensor-to-scalar ratio $r = \mathcal{P}_T / \mathcal{P}_S$ is

$$r = 12 w^2 \kappa^2 \dot{\Phi}^2 \Big|_{aH = c_g k} = 8\epsilon \,. \tag{55}$$

The measured Power spectrum has an amplitude

$$P_S = 2.38 \times 10^{-9} \,, \tag{56}$$

at the pivot scale $k = 0.002\ \mathrm{Mpc}^{-1}$. With this data and the value of ϵ found before, we may fix the parameters appearing in New Higgs Inflation.

All constraints are satisfied for

$$2.1 \times 10^{-2}\ M_p < \Phi_0 < 2.7 \times 10^{-2}\ M_p\,,$$
$$7 \times 10^{-8}\ M_p < w^{-1} < 8.8 \times 10^{-8}\ M_p\ \ , \qquad (57)$$

and we may conclude that

$$\Phi_0 \sim 10^{16}\mathrm{GeV}$$

and

$$w^{-1} \sim 10^9 - 10^{10}\mathrm{GeV}.$$

E-foldings: The number of e-foldings N during Inflation is [?]

$$N = \int_{t_i}^{t_f} dtH = \int_{\Phi(t_f)}^{\Phi(t_f)} \frac{H}{\dot{\Phi}} d\Phi \ . \quad (58)$$

Considering that $\Phi(t_f) \ll \Phi(t_i)$ we get

$$N \simeq 54 \ , \quad (59)$$

which is enough to solve the cosmological problems.

Conclusions

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SM Higgs may drive inflation if non-minimally coupled to gravity