Flow equations with momentum dependent vertex functions

ERG 2010

Corfou Summer Institute

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Outline

- Truncation of the eRG flow equations (momentum dependent vertices)

- Application to perturbation theory at finite temperature

- ERG and 2PI relations

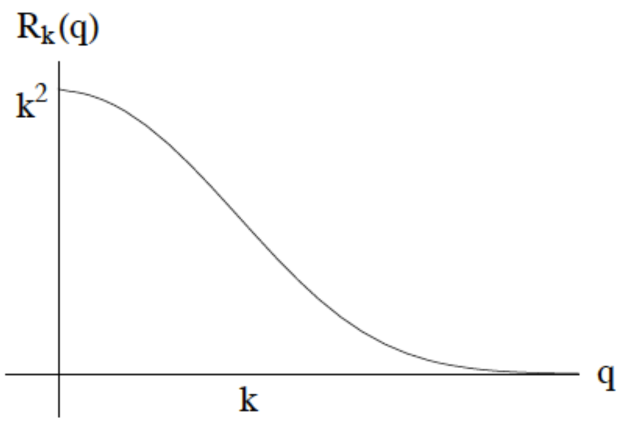
The "exact" Renormalization Group

Basic strategy (ex scalar field theory)

$$S = \int \mathrm{d}^d x \left\{ \frac{1}{2} \left(\partial_\mu \varphi(x) \right)^2 + \frac{m^2}{2} \varphi^2(x) + \frac{u}{4!} \varphi^4(x) \right\}$$

Control infrared with "mass-like" regulator

$$\Delta S_\kappa[\varphi] = \frac{1}{2} \int_q R_\kappa(q^2) \, \varphi(q) \varphi(-q)$$



Exact flow equation

Theory at "scale" κ defined by the regulated action

$$S \longrightarrow S + \Delta S_\kappa \equiv S_\kappa$$

or the "effective action" $\Gamma_\kappa[\phi]$

Exact flow equation (Wetterich, 1993)

$$\partial_\kappa \Gamma_\kappa[\phi] = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\, \partial_\kappa R_\kappa(q)\, G_\kappa(q,-q;\phi) =$$

$$G_\kappa^{-1}[\phi] = \Gamma_\kappa^{(2)}[\phi] + R_\kappa$$

κ runs from 'microscopic scale' Λ to zero (regulator vanishes)

'Initial conditions' $\Gamma_{\kappa=\Lambda}[\phi] \sim S[\phi]$

An infinite hierarchy of equations for n-point functions

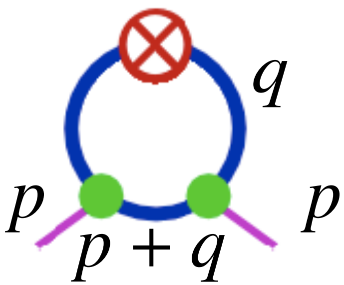
Effective potential

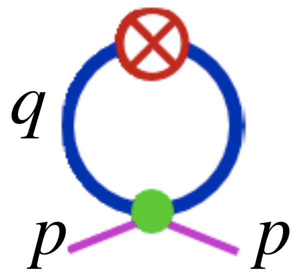
$$\kappa\partial_\kappa V_\kappa(\rho) = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\kappa\,\partial_\kappa R_\kappa(q)\,G_\kappa(q,\rho) =$$

$$\rho \equiv \frac{\phi^2}{2}$$

2-point function

$$\begin{aligned}\partial_\kappa\Gamma^{(2)}_\kappa(p,\rho) \;=\; & \int \frac{d^d q}{(2\pi)^d}\,\partial_\kappa R_\kappa(q)\,G^2_\kappa(q,\rho) \\ & \times\left\{\Gamma^{(3)}_\kappa(p,q,-p-q;\phi)G_\kappa(q+p,\rho)\Gamma^{(3)}_\kappa(-p,p+q,-q;\phi) - \frac{1}{2}\Gamma^{(4)}_\kappa(p,-p,q,-q;\phi)\right\}\end{aligned}$$





Beyond the local potential approximation

J.-P. B, R. Mendez-Galain, N. Wschebor (PLB, 2006)

Two observations

The vertex functions depend weakly on the loop momentum

$$\Gamma_\kappa^{(n)}(p_1, p_2, ..., p_{n-1} + q, p_n - q; \phi) \sim \Gamma_\kappa^{(n)}(p_1, p_2, ..., p_{n-1}, p_n; \phi)$$

The hierarchy can be closed by exploiting the dependence on the field

$$\Gamma_\kappa^{(n+1)}(p_1, p_2, ..., p_n, 0; \phi) = \frac{\partial \Gamma_\kappa^{(n)}(p_1, p_2, ...p_n; \phi)}{\partial \phi}$$

The equation for the 2-point function becomes a closed equation

$$\kappa\partial_\kappa\Gamma_\kappa^{(2)}(p,\rho) = J_3(p)\left(\frac{\partial\Gamma_\kappa^{(2)}(p,\rho)}{\partial\phi}\right)^2 - \frac{1}{2}I_2\,\frac{\partial^2\Gamma_\kappa^{(2)}(p,\rho)}{\partial\phi^2}$$

Structure of truncation

$$\Gamma^{(n)}(0,\cdots,0) = \frac{\partial^n V_\kappa}{\partial\phi^n}$$ (LPA)

$$\Gamma^{(n)}(p,-p,0,\cdots,0) = \frac{\partial^{n-2}\Gamma_\kappa^{(2)}(p;\phi)}{\partial\phi^{n-2}}$$ ('BMW'-LO)

etc.

Applications

- Critical O(N) models (see B. Delamotte's lecture)
- Bose-Einstein condensation
- Finite temperature field theory

Non perturbative renormalization group at finite temperature

(J.-P B, A. Ipp, N. Wschebor, 2010)

Motivation : physics of the quark-gluon plasma

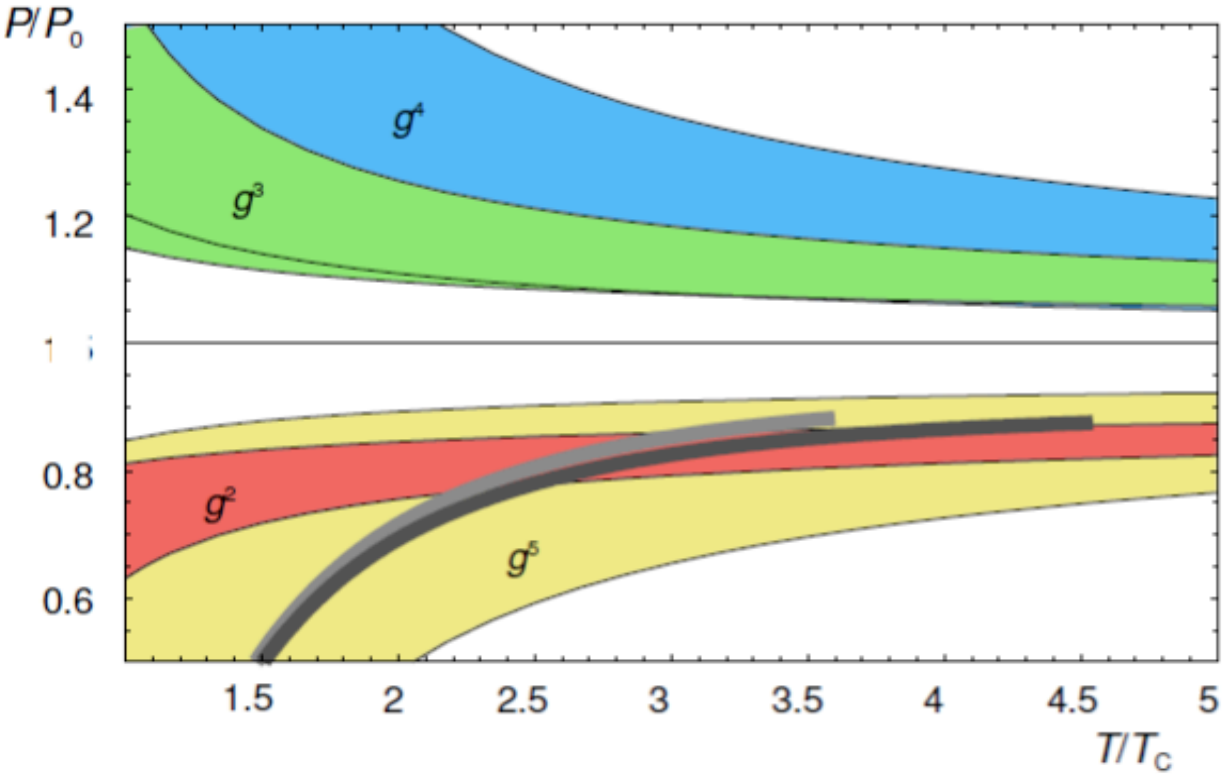
A paradoxical situation :

Naively: QCD asymptotic freedom implies that matter is simple at high temperature (weakly interacting gas of quarks and gluons)

Experiments (heavy ion collisions at RHIC) suggest that the quark-gluon plasma is 'strongly coupled'

Technically: perturbation theory breaks down

Perturbation theory is ill behaved at finite temperature

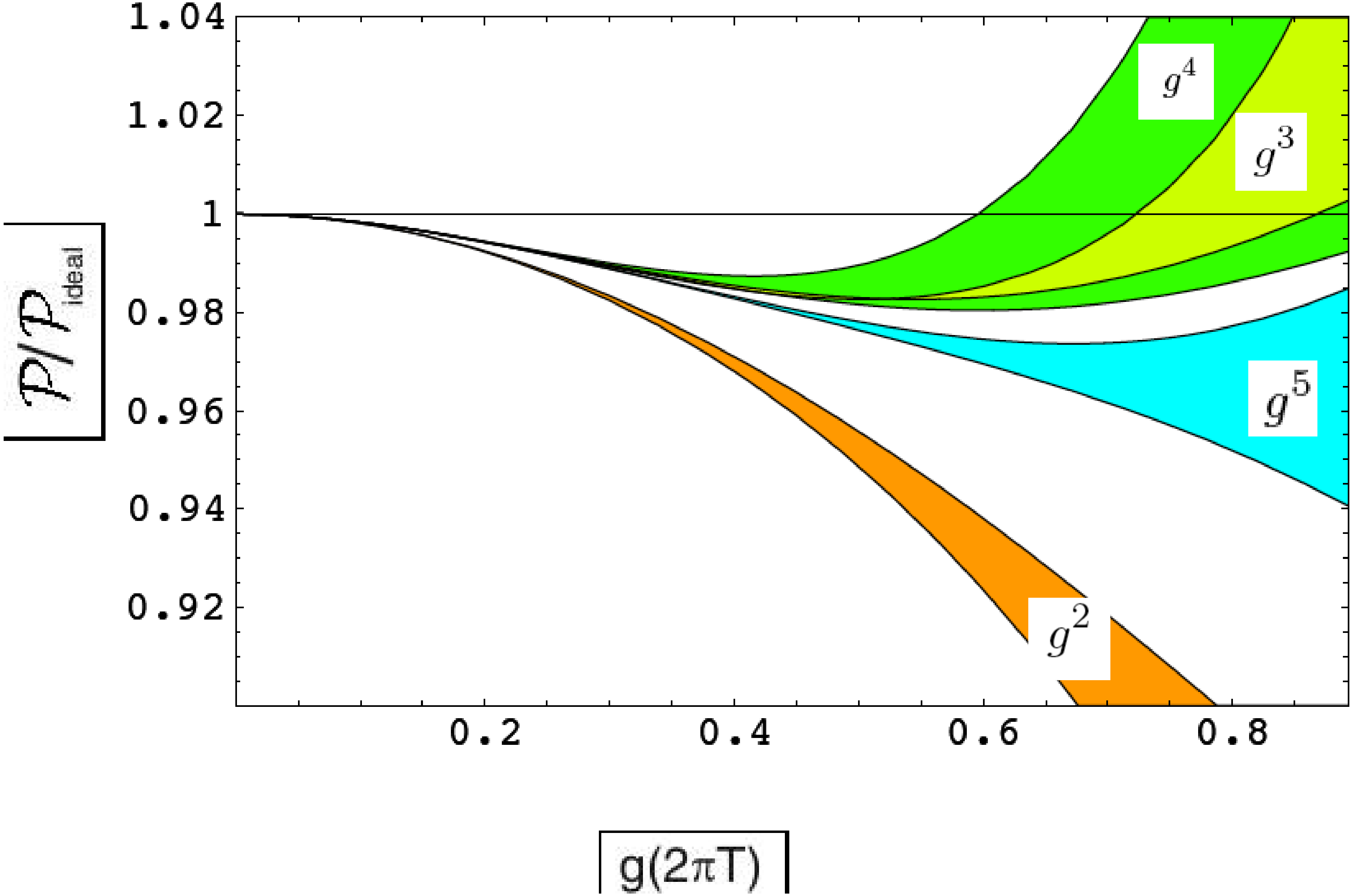


Perturbation theory:

g^2: Shuryak; Chin (1978)
g^3: Kapusta (1979)
g^4 ln g: Toimela (1983)
g^4 : Arnold, Zhai (1994)
g^5 : Zhai, Kastening (1995), Braaten, Nieto (1996)
g^6 ln g: Kajantie, Laine, Rummukainen, Schröder (2002)
g^6 (partly): Di Renzo, Laine, Miccio, Schröder, Torrero (2006)

Lattice data: G. Boyd et al. (1996); M. Okamoto et al. (1999).

Generic feature in (most) field theories,
e.g. in scalar field theory



Weakly AND strongly coupled ...

Degrees of freedom with different wavelengths are differently coupled.

Expansion parameter

$$\gamma_\kappa = \frac{g^2\langle\phi^2\rangle}{\kappa^2} \qquad \langle\phi^2\rangle_\kappa \sim \kappa T \quad (\kappa \lesssim T)$$

Dynamical scales

$$\kappa \sim T \qquad \gamma_\kappa \sim g^2$$

$$\kappa \sim gT \qquad \gamma_\kappa \sim g$$

$$\kappa \sim g^2 T \qquad \gamma_\kappa \sim 1$$

Non perturbative renormalization group at finite temperature

(J.-P B, A. Ipp, N. Wschebor, 2010)

effective action

$$\partial_t \Gamma_\kappa[\phi] = \frac{1}{2} T \sum_n \int \frac{d^3\mathbf{q}}{(2\pi)^3} \partial_t R_\kappa(Q) G_\kappa(Q;\rho) \quad =$$

propagator $G_\kappa^{-1}(Q;\rho) \equiv \Gamma_\kappa^{(2)}(\mathbf{Q},-\mathbf{Q};\rho) + R_\kappa(Q)$

regulator

four-momentum vectors:

$\mathbf{Q} = (\omega_n, \mathbf{q})$

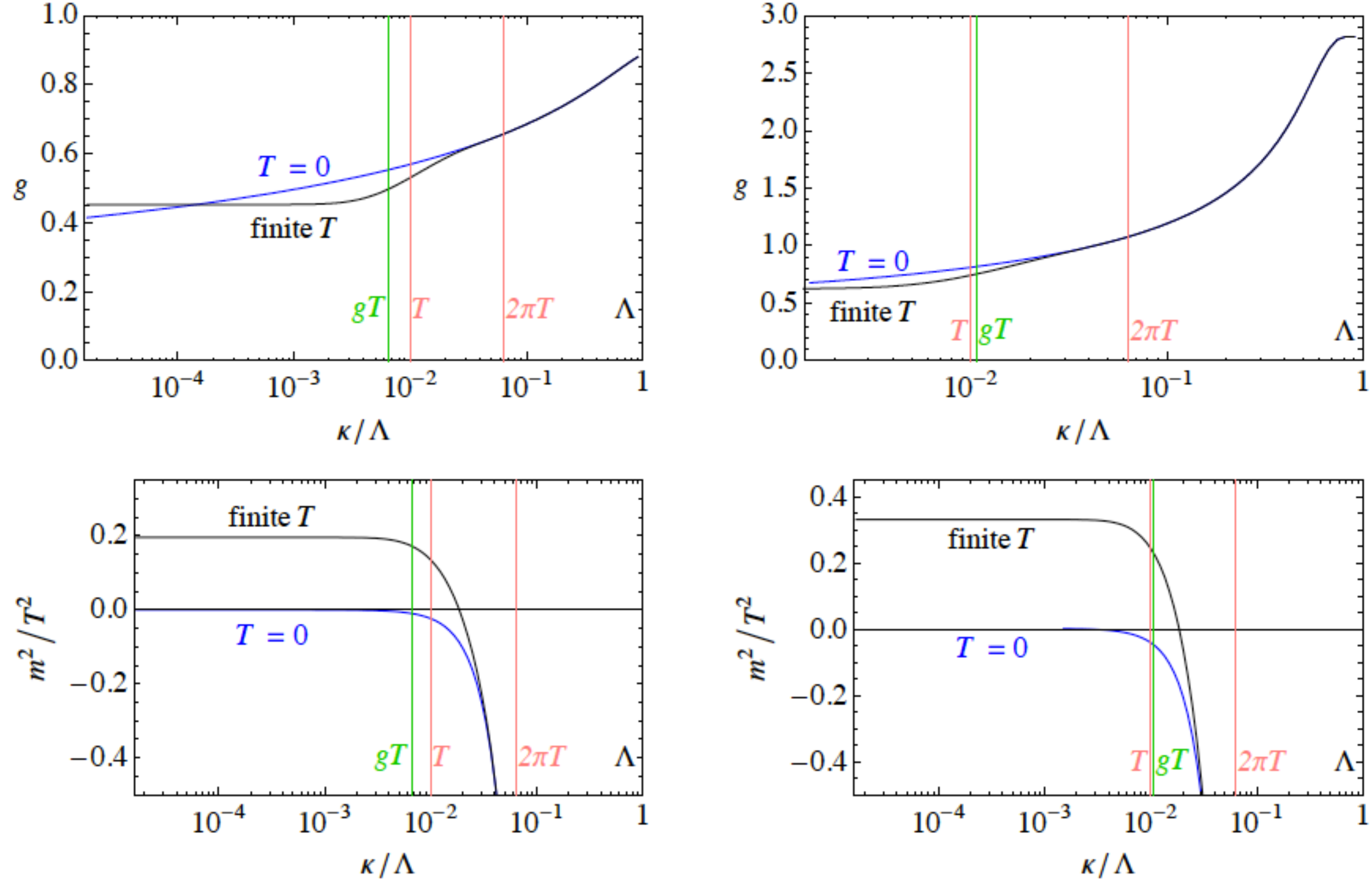
$Q = |\mathbf{Q}|$

Matsubara frequency $\omega_n = 2\pi n T$

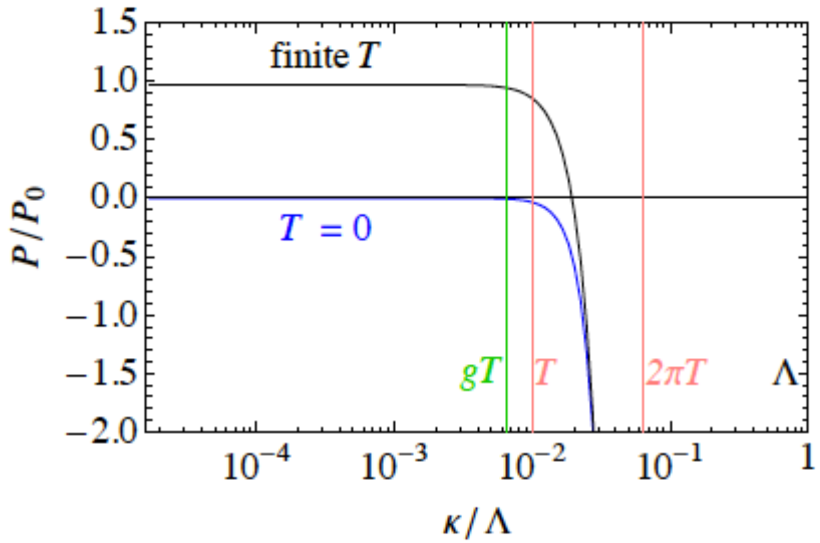
momentum derivative: $\partial_t \equiv \kappa\, \partial_\kappa$

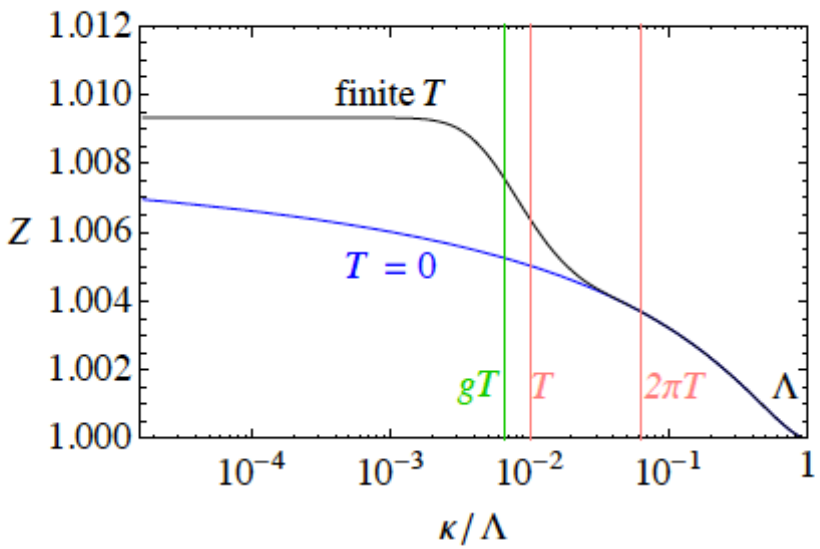
scalar field: $\rho \equiv \frac{1}{2}\phi^2$

weak coupling   strong coupling

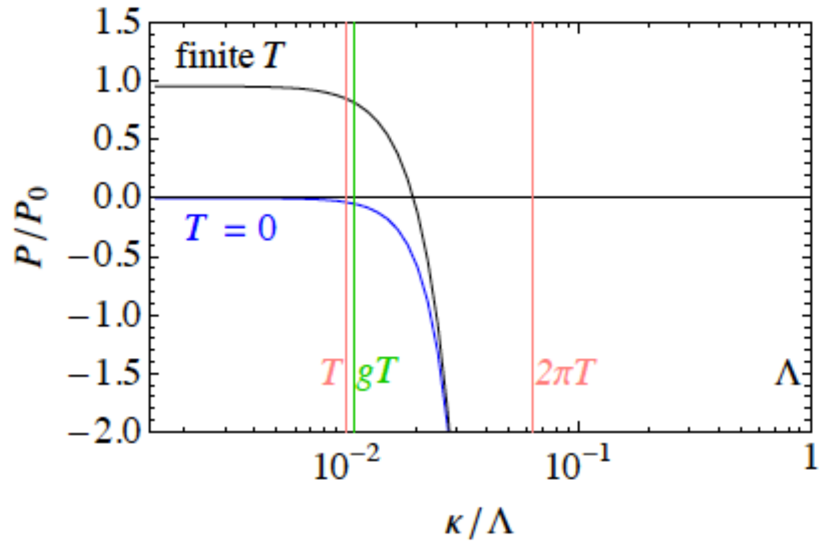


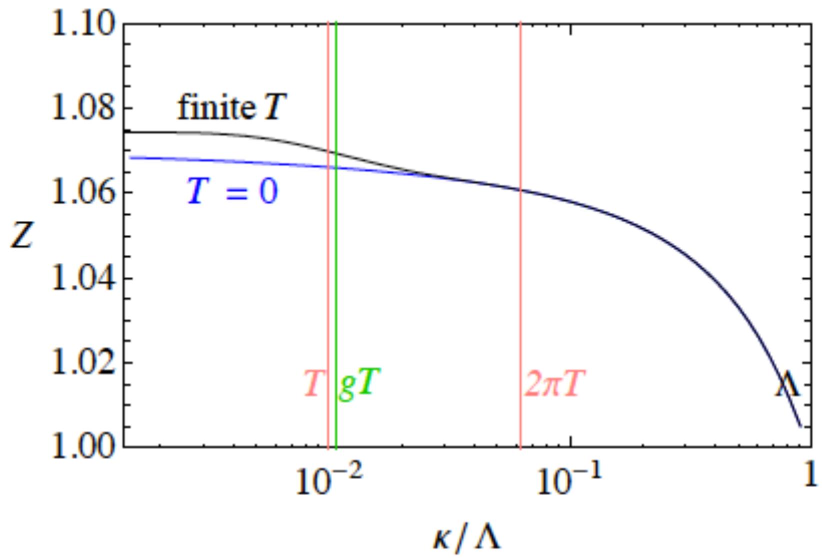
weak coupling





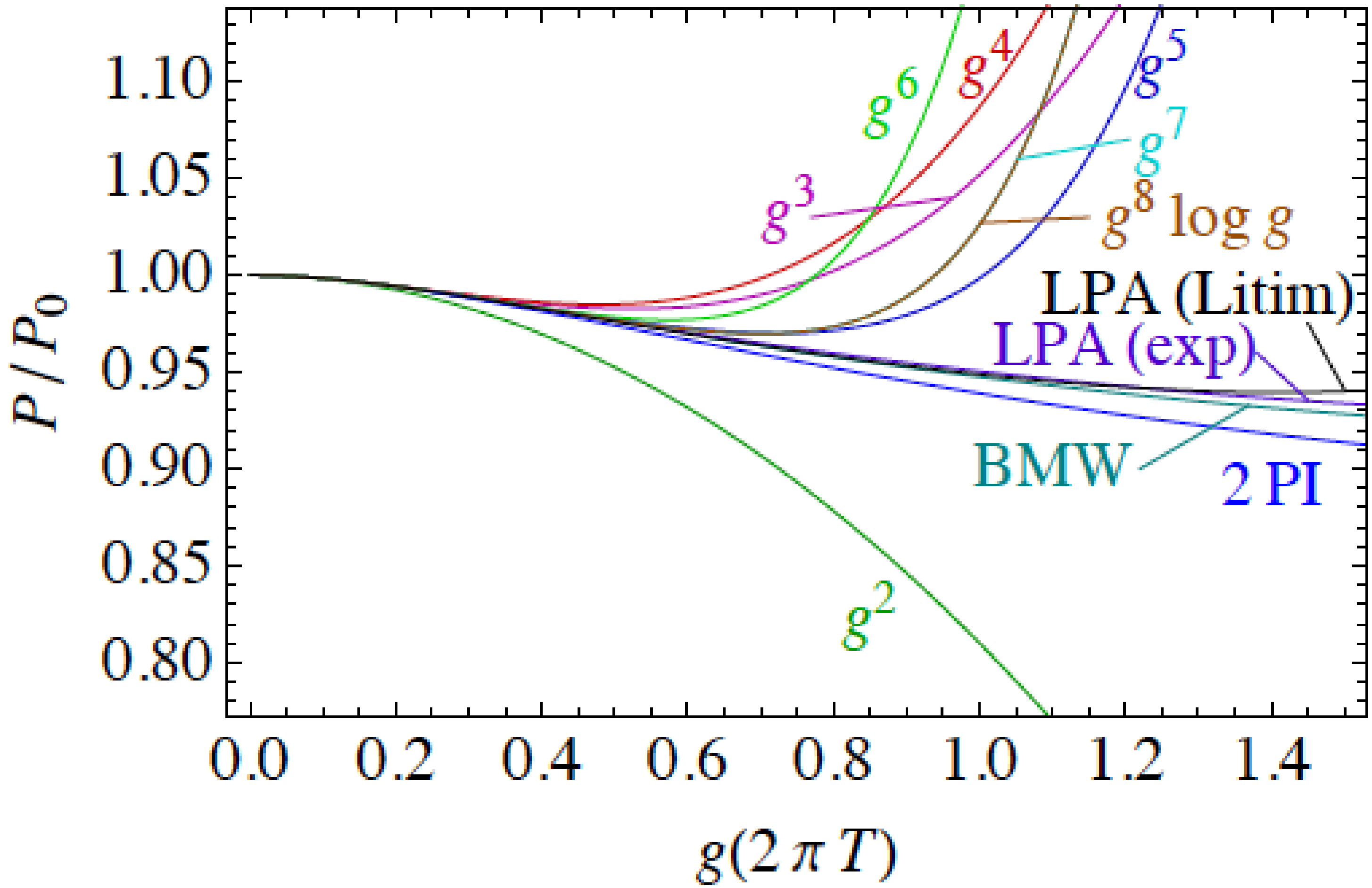
strong coupling



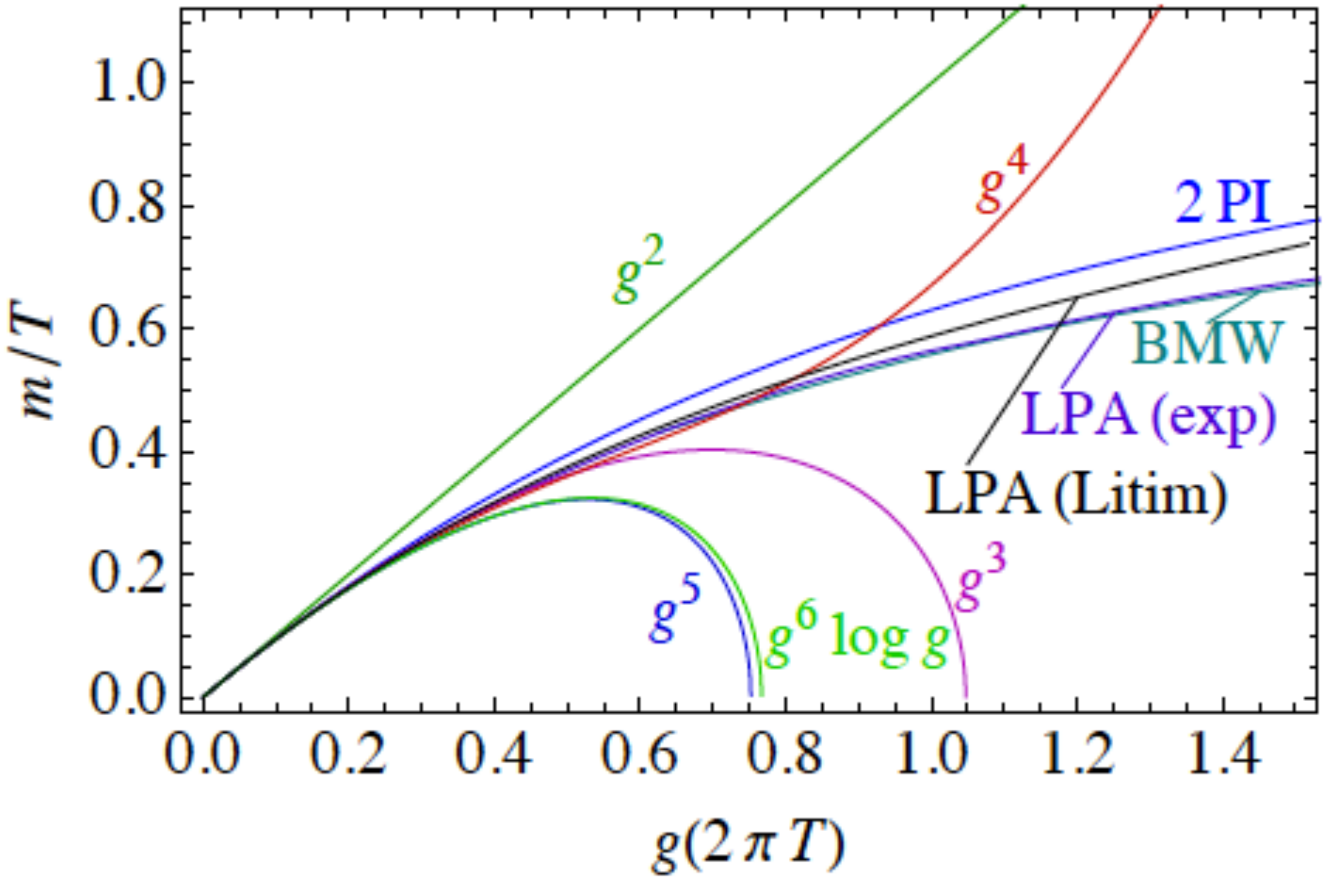


RG techniques yield smooth extrapolation to strong coupling (scalar field theory)

(JPB, A. Ipp, N. Wschebor, 2010)

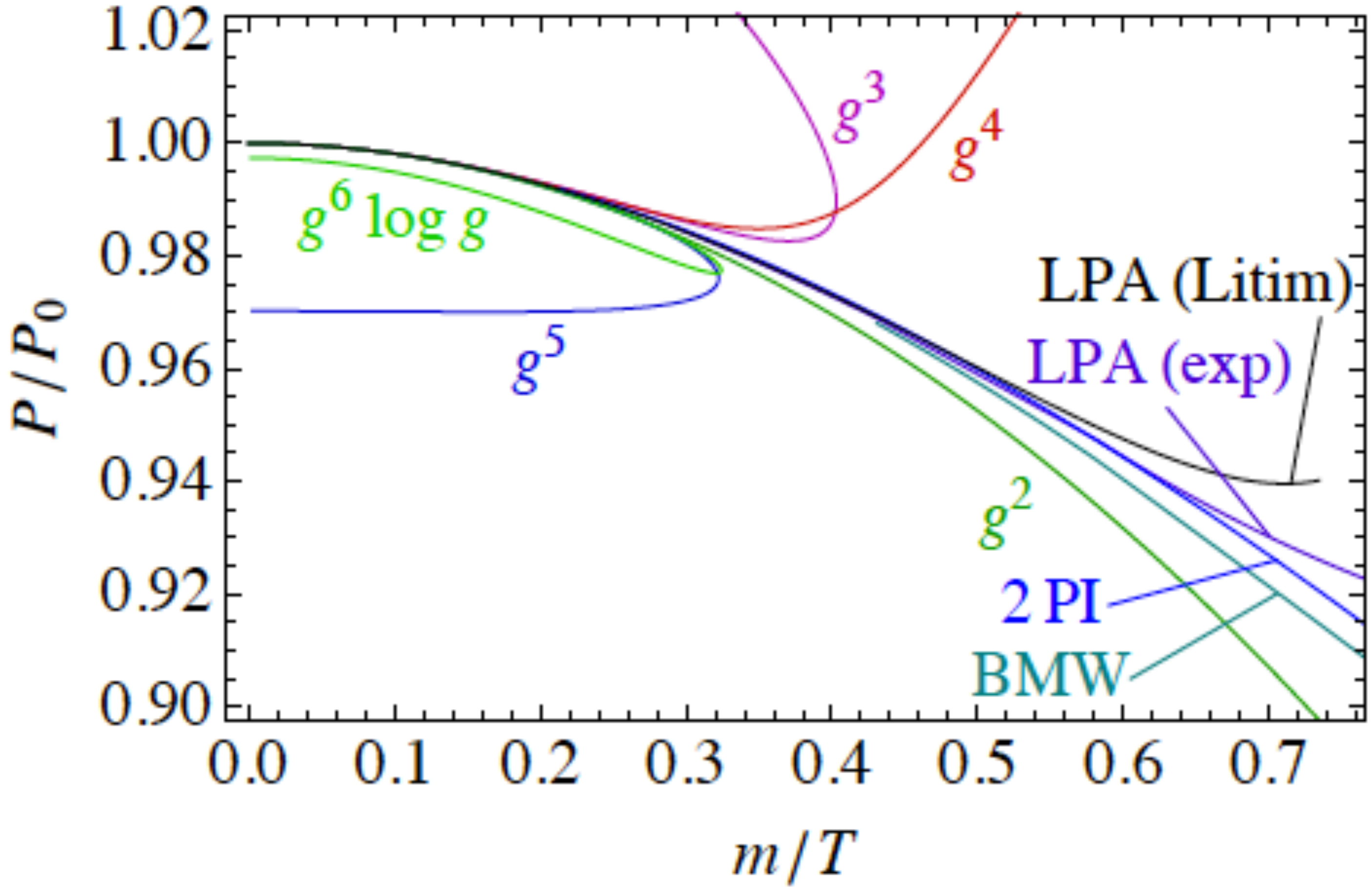


(high orders from J. O. Andersen et al, arXiv 0903.4596)



m/T
g(2πT)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
g^2
g^4
2 PI
BMW
LPA (exp)
LPA (Litim)
g^3
g^5
g^6 log g

Removing scheme dependence



eRG and 2PI techniques

(J.-P B, J. Pawlowski and U. Reinosa - see talk by Reinosa)

Luttinger-Ward expression for the thermodynamic potential

$$\frac{1}{T}\Omega[G] = \frac{1}{2}\mathrm{Tr}\ln G^{-1} - \frac{1}{2}\mathrm{Tr}\,(\Sigma G) + \Phi[G]$$

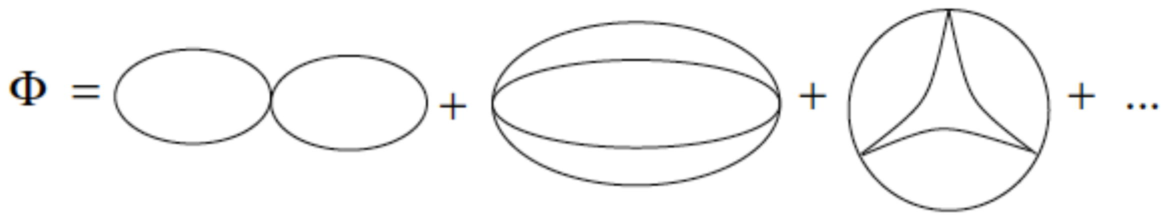
Gap equation

$$\frac{\delta\Omega[G]}{\delta G} = 0 \qquad \frac{\delta\Phi}{\delta G} = \frac{1}{2}\Sigma \qquad G^{-1} = G_0^{-1} + \Sigma[G]$$

Equation for the four-point function

$$\frac{\delta\Sigma}{\delta G} = \frac{1}{2}\mathcal{I} \qquad \Gamma^{(4)}(q,p) = \mathcal{I}(q,p) - \frac{1}{2}\int_k \Gamma^{(4)}(q,k)\,G^2(k)\,\mathcal{I}(k,p)$$

Phi-derivable approximation : choose a set of skeleton diagrams and solve the corresponding gap equation



eRG and 2PI truncation

Eq. for the 2-point function

$$\partial_\kappa \Gamma_\kappa^{(2)}(p) = -\frac{1}{2}\int_q \partial_\kappa R_\kappa(q)\, G_\kappa^2(q)\, \Gamma_\kappa^{(4)}(q,p) \qquad G_\kappa^{-1}[\phi] = \Gamma_\kappa^{(2)}[\phi] + R_\kappa$$

Truncate with 2PI relation

$$\Gamma_\kappa^{(4)}(q,p) = \mathcal{I}_\kappa(q,p) - \frac{1}{2}\int_l \Gamma_\kappa^{(4)}(q,l)\, G_\kappa^2(l)\, \mathcal{I}_\kappa(l,p)$$

The resulting flow is an exact derivative

$$\partial_\kappa \Gamma_\kappa^{(2)}(p) = \partial_\kappa \Sigma_\kappa(p) \qquad \Sigma_\kappa = \Sigma[G_\kappa]$$

Consequences : flow eq. 'solves' the gap equation

no residual dependence on 'regulator'

Conclusions

eRG is a nice tool

why does it work so well ?