Towards classification of $SO(10)$ heterotic superstring vacua

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CORFU2010

September 7, 2010

Summary

The Standard Model from Strings

String theory, as a theory of all interactions, should reproduce the Standard Model at low energies.
String theory is formulated in 10D: Heterotic $E_8 \times E_8$,$SO(32)$, Type I, Type IIA,IIB.
However, String Theory in four dimensions contains a huge number of vacua.
Although, various low energy models with (semi)realistic features have been constructed in the past, we have recently started a systematic exploration of string vacua in 4-dimensions.

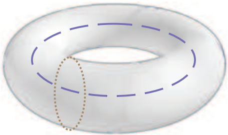
Classification of string vacua

- Type II /orientifolds, see eg,
  T.P.T. Dijkstra1 , L. R. Huiszoon and A.N. Schellekens (2004)
  E. Kiritsis, M. Lennek and B. Schellekens (2008),(2009)...
- Heterotic string orbifolds e.g.
  F. Gmeiner, R. Blumenhagen, G. Honecker, D. Lust and T. Weigand (2006)
  O. Lebedev, H. P. Nilles, S. Ramos-Sanchez, M. Ratz and P. K. S. Vaudrevange (2008)
  F. Gmeiner and G. Honecker (2008)...
- Heterotic Calabi-Yau models
  Maxime Gabella, Yang-Hui He , Andre Lukas, (2008)
  Yang-Hui He , Seung-Joo Lee, Andre Lukas, (2010)...
- Heterotic Gepner models see eg,
  B. Gato-Rivera and A. N. Schellekens, (2010)...
- Heterotic Free Fermionic, e.g.
  A.E. Faraggi , C. Kounnas , S.E.M. Nooij , J. Rizos (2004)
  K. R. Dienes (2006), K. R. Dienes and M. Lennek (2007)
  A. E. Faraggi , C. Kounnas , J. Rizos (2007),(2008)
  B. Assel, K. Christodoulides, A. E. Faraggi , C. Kounnas , J. Rizos, (2009),(2010)

The Free Fermionic Formulation of the heterotic superstring

In the Free Fermionic Formulation of the heterotic superstring we can reduce the critical dimension of the superstring and construct models in $D = 4$ by fermionizing the left movers and introducing non-linear supersymmetry among them.

$$f \to -e^{-i\pi\alpha(f)} f$$



A model is defined by a set of basis vectors $B = \{v_1, v_2, \ldots v_n\}$ and a set of $2^{n(n-1)}$ phases $c\left[{v_i \atop v_j}\right], i > j$.

The basis vectors give rise to a set $\Xi = \{\xi_1 = 0, \xi_2 = 1, \xi_3, \ldots, \xi_M\}$ of string sectors and phases are related to the GSO projections.

The basis vectors and phases are subject to constraints due to modular invariance, string amplitude factorization.

Antoniadis, Bachas, Kounnas (1987)
H. Kawai, D.C. Lewellen, and S.H.-H. Tye (1987)

$Z_2 \times Z_2$ models

The partition function can be written as

$$Z = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3} \frac{1}{\eta^2 \bar{\eta}^2} \sum_{\alpha,\beta\epsilon\Xi} c\begin{bmatrix}\alpha\\\beta\end{bmatrix} \tilde{\zeta}\begin{bmatrix}\alpha\\\beta\end{bmatrix}$$

where

$$\tilde{\zeta}\begin{bmatrix}\alpha\\\beta\end{bmatrix} = \frac{1}{2^n} \prod_{i=1}^{n_L} \left(\frac{\theta\begin{bmatrix}\alpha_i\\\beta_i\end{bmatrix}}{\eta}\right)^{\frac{r_i}{2}} \prod_{i=n_L+1}^{n_R} \left(\frac{\bar{\theta}\begin{bmatrix}\alpha_i\\\beta_i\end{bmatrix}}{\bar{\eta}}\right)^{\frac{r_i}{2}}$$

where $r_i = 1, 2$ if the i fermion is real or complex respectively and n_L/n_R the number of left/right moving fermions.

Some (semi)realistic models

Several $N = 1$ models have been constructed

Flipped $SU(5)$ model $SU(5) \times U(1) \times \text{Hidden}$

Pati-Salam model $SU(4) \times SU(2)_L \times SU(2)_R \times \text{Hidden}$

Standard-like models $SU(3) \times SU(2) \times U(1)^n \times \text{Hidden}$

In simple constructions the gauge group rank r can be reduced by 6 so $r \geq 44/2 - 6 = 16$

Model construction: Gauge group, full massless spectrum, superpotential, flat directions, massless doublets, non-renormalizable interactions, fermions masses, exotic fractional charged particles

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I. Antoniadis, J. Ellis and D.V. Nanopoulos

A. Faraggi

I. Antoniadis, J. Rizos and G. Leontaris

Exotic - fractional charged states

Among the basic features of the string models is the existence of exotic fractional charged states in the string spectrum.
These exotics could be considered as a signature of string theory. Due to their fractional charge they are stable (the lightest cannot decay), however it seems hard to reconcile their existence with

- Stringent experimental limits on fractional charged matter
- In the standard cosmological scenario relics from their production in the early universe could still be present.
- Depending on the model, they could lead to fast hypercharge running and destroy coupling unification (e.g. fractional charge singlets).

X. G. Wen and Witten (1985)
G.G. Athanasiou, J.J. Atick, M. Dine and W. Fishler (1988)
A. N. Schellekens (1989)

Exotic - fractional charged states

The existence of fractional charged states is generic in string theory, as shown by Schellekens, electric charge embedding in string theory, requires to abandon at least one of the following Standard Model features

- Color singlets have integer charge.
- All SM particles are accommodated in singlet or vector representations of $SU(3)$ and $SU(2)$.
- $\sin^2\theta_W = \frac{3}{8}$ at the string scale.

A. N. Schellekens (1989)

Exotic - fractional charged states

Some solutions to the exotic state problem discussed up to now are:

1. Construct models with higher k (higher $SU(3), SU(2)$ reps)
2. Assume the exotics transform under hidden sector (eg. Flipped $SU(5)$ string model , $SU(4)$ hidden, is this enough ?)
3. Search for models where these states are vector-like and find appropriate flat directions to make them massive at the effective field theory level.

Classification strategy

The number of vacua is huge, so we need a classification strategy

- Choose a basis set that contains the realistic models
- Fix basis vectors and vary GSO coefficients
- Choose chiral observable gauge group: $SO(10)$ gauge group
- Identify models by few characteristic properties: # of spinorials, # of antispinorials, # of vectorials
- Derive analytic formulae for the above characteristics
- Use a fast computer program to evaluate formulae

The class of $Z_2 \times Z_2$ $SO(10)$ heterotic models

The free fermions in the light-cone gauge in the traditional notation are:

left: $\psi^\mu, \chi^{1,\dots,6}, \quad y^{1,\dots,6}, \omega^{1,\dots,6}$

right: $\bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \quad \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \quad \bar{\phi}^{1,\dots,8}$

The class of models under consideration is generated by a set of 12 basis vectors $B = \{v_1, v_2, \dots, v_{12}\}$ where

$$
\begin{aligned}
v_1 = 1 \quad &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \\
&\qquad \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\} \\
v_2 = S \quad &= \{\psi^\mu, \chi^{1,\dots,6}\} \\
\textit{shifts}: \quad v_{2+i} = e_i \quad &= \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6 \\
Z_2\ \textit{twist}: v_9 = b_1 \quad &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\} \\
Z_2\ \textit{twist}: v_{10} = b_2 \quad &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\} \\
v_{11} = z_1 \quad &= \{\bar{\phi}^{1,\dots,4}\} \\
v_{12} = z_2 \quad &= \{\bar{\phi}^{5,\dots,8}\}
\end{aligned}
$$

and a set of $2^{12(12-1)/2}$ phases $c[v_i, v_j] = \pm 1, j < i = 1, \dots, 12$

Gauge Group

$1, S, e_i, b_1, b_2, z_1, z_2$

$$\overbrace{\underbrace{SO(10)}\times U(1)^3}^{E_6\times U(1)^2}\overbrace{\times SO(8)^2}^{E_8}$$

$$\Downarrow$$

$\alpha, (\beta)$

$$SU(4)\times SU(2)_L\times SU(2)_R$$
$$SU(5)\times U(1)$$
$$SU(3)\times SU(2)_L\times SU(2)_R\times U(1)$$
$$SU(3)\times SU(2)\times U(1)$$

Massless spectrum

Untwisted sector matter spectrum (universal)
Gauge symmetry (rank 16) $SO(10) \times U(1)^3 \times SO(8)^2$
6 pairs of $SO(10)$ vectorials and a number of $SO(10)$ singlets.
The twisted sectors are generated by $b_1, b_2, b_1 + b_2$ (three $Z_2 \times Z_2$ orbifold planes), labeled by the shifts $(e_i, i = 1, \ldots, 6)$,they contain

Spinorial $SO(10)$ representations $(\mathbf{16} + \bar{\mathbf{16}})$:

$$\begin{aligned} B^{(1)}_{pqrs} &= S + b_1 + p^1\, e_3 + q^1\, e_4 + r^1\, e_5 + s^1\, e_6 \\ B^{(2)}_{pqrs} &= S + b_2 + p^2\, e_1 + q^2\, e_2 + r^2\, e_5 + s^2\, e_6 \\ B^{(3)}_{pqrs} &= S + b_3 + p^3\, e_1 + q^2\, e_2 + r^3\, e_3 + s^3\, e_4 \end{aligned}$$

where $b_3 = b_1 + b_2 + x,\ p^i, q^i, r^i, s^i = \{0, 1\}$.

Vectorial $SO(10)$ representations $(\mathbf{10})$

$$V^{(I)}_{pqrs} = B^{(I)}_{pqrs} + x$$

where $x = 1 + S + \sum_{i=1}^{6} e_i + \sum_{i=1}^{2} z_i$

Analytic formulae for # of spinorials/vectorials

$$\#(S^{(I)}) = \begin{cases} 2^{4-\mathrm{rank}\left(\Delta^{(I)}\right)} & \mathrm{rank}\left(\Delta^{(I)}\right) = \mathrm{rank}\left[\Delta^{(I)}, Y_{16}^{(I)}\right] \\ 0 & \mathrm{rank}\left(\Delta^{(I)}\right) < \mathrm{rank}\left[\Delta^{(I)}, Y_{16}^{(I)}\right] \end{cases}$$

$$\#(V^{(I)}) = \begin{cases} 2^{4-\mathrm{rank}\left(\Delta^{(I)}\right)} & \mathrm{rank}\left(\Delta^{(I)}\right) = \mathrm{rank}\left[\Delta^{(I)}, Y_{10}^{(I)}\right] \\ 0 & \mathrm{rank}\left(\Delta^{(I)}\right) < \mathrm{rank}\left[\Delta^{(I)}, Y_{10}^{(I)}\right] \end{cases}$$

$\Delta^{(I)}$, are 4x4 and $Y^{(I)}$ $I = 1, 2, 3$ are 4x1 GSO coefficient matrices

Analytic formulae for spinorial chiralities

The chirality of the surviving spinorials is given by

$$
\begin{aligned}
X^{(1)}_{pqrs} &= c\begin{bmatrix} b_2 + (1-r)e_5 + (1-s)e_6 \\ B^{(1)}_{pqrs} \end{bmatrix} \\
X^{(2)}_{pqrs} &= c\begin{bmatrix} b_1 + (1-r)e_5 + (1-s)e_6 \\ B^{(2)}_{pqrs} \end{bmatrix} \\
X^{(3)}_{pqrs} &= c\begin{bmatrix} b_1 + (1-r)e_3 + (1-s)e_4 \\ B^{(3)}_{pqrs} \end{bmatrix}
\end{aligned}
$$

where $X^i_{pqrs} = +1$ corresponds to a $\mathbf{16}$ of $SO(10)$($X^i_{pqrs} = -1$ corresponds to a $\overline{\mathbf{16}}$). The net number of families is given by

$$
N_F = \sum_{i=1}^{3} \sum_{p,q,r,s=0}^{1} X^{(i)}_{pqrs} P^{(i)}_{pqrs}
$$

Useful formulae but they cannot be inverted (i.e solve for number of generations)

Computer Analysis

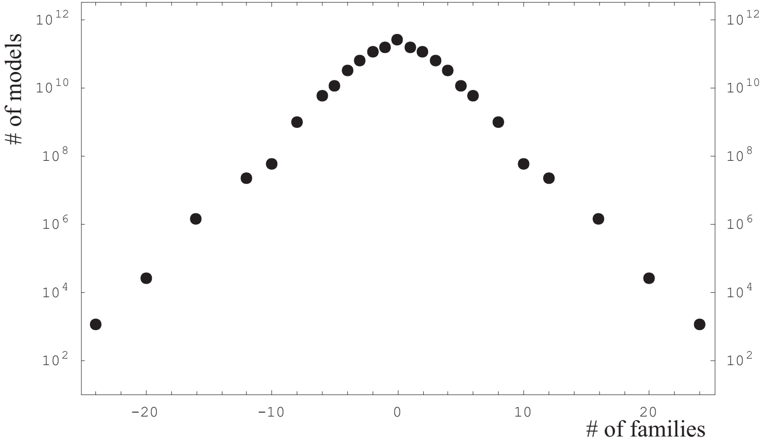
Model characteristics are expressed in term of GSO phase matrices and sums. They can be evaluated for the using a computer program.

1. The program should be fast (at least 10^5 models per second)
2. The program must face the memory and storage problem

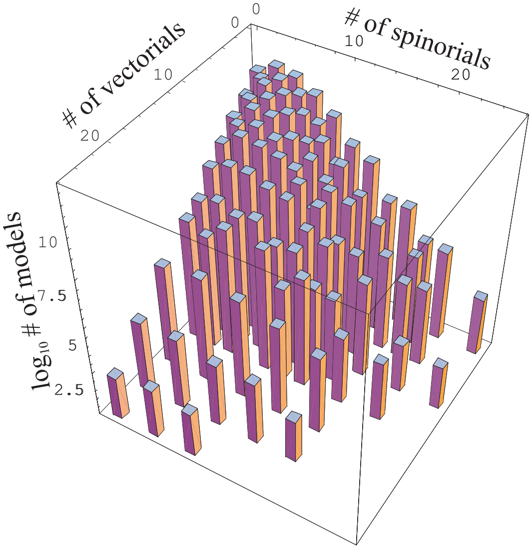
We have constructed such a computer program FORTRAN95. Run on Dual Xeon ⇒ full results in 60 days (200.000 models per second).

Results of computer analysis

Total number of models in this class: $1.016.808.865.792 \sim 10^{12}$



Spinorial-Vectorial analysis



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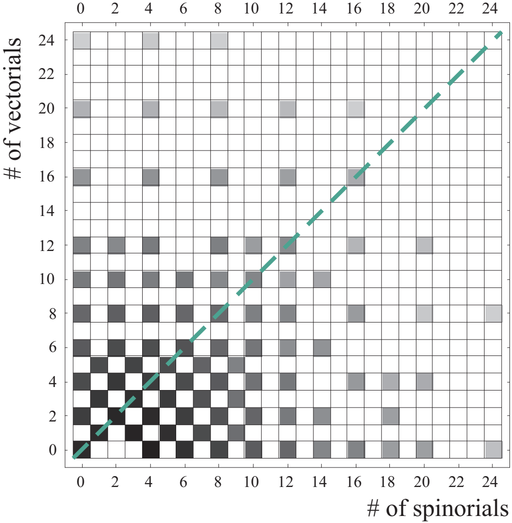

Results of $SO(10)$ model classification

1. Three generation models are quite abundant 15% of the class.
2. Models in this class appear in pairs related with spinor-vector symmetry:

$$(S = \text{spinorinals}, V = \text{vectorials})$$

$$\Updownarrow$$

$$(V = \text{spinorinals}, S = \text{vectorials})$$

The map has been derived analytically and holds to each orbifold plane separately. Self-dual models under this symmetry appear to be anomaly free (no anomalous $U(1)$)

Towards the Standard Model

The next step in our analysis is to break $SO(10)$ and obtain the Standard Model. The simplest way to realize this is through the Pati-Salam GUT model.

$$SO(10) \to SU(4) \times SU(2)_L \times SU(2)_R \to SU(3) \times SU(2) \times U(1)$$

Motivation for Pati-Salam models

1. Technically easier , can be realized with a single additional vector of real spin structures
2. Models constructed up to now contain additional fractional charge matter (exotics)
3. According to recent results, (see e.g. Lust (2009)) this model has very low statistics in Intersecting D-brane models

The Pati-Salam model

$$SO(10) \supset SU(4) \times SU(2)_L \times SU(2)_R$$
$$\mathbf{16} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$$
$$\mathbf{10} = (\mathbf{6}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}, \mathbf{2})$$

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$$SU(4) \times SU(2)_L \times SU(2)_R \supset SU(3) \times SU(2) \times U(1)$$

$$F_L(\mathbf{4}, \mathbf{2}, \mathbf{1}) \to q(\mathbf{3}, \mathbf{2}, -\frac{1}{6}) + \ell(\mathbf{1}, \mathbf{2}, \frac{1}{2})$$
$$\bar{F}_R(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \to u^c(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3}) + d^c(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3}) + e^c(\mathbf{1}, \mathbf{1}, -1) + \nu^c(\mathbf{1}, \mathbf{1}, 0)$$
$$D(\mathbf{6}, \mathbf{1}, 1) \to D_3(\mathbf{3}, \mathbf{1}, \frac{1}{3}) + \bar{D}_3(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3})$$
$$h(\mathbf{1}, \mathbf{2}, \mathbf{2}) \to h^d(\mathbf{1}, \mathbf{2}, \frac{1}{2}) + h^u(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$$

J. Pati and A. Salam, *Lepton number as the fourth color* (1974)
I. Antoniadis and G. Leontaris (1988) (SUSY version)
I. Antoniadis, G. Leontaris and J. Rizos (1990) (heterotic superstring version)

The supersymmetric Pati-Salam model

Symmetry breaking The PS symmetry can be broken to the Standard Model by $\langle \nu_H^c \rangle$, $\langle \nu_H \rangle$

$$\bar{H}(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \to u_H^c(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3}) + d_H^c(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3}) + \nu_H^c(\mathbf{1}, \mathbf{1}, 0) + e_H^c(\mathbf{1}, \mathbf{1}, -1)$$
$$H(\mathbf{4}, \mathbf{1}, \mathbf{2}) \to u_H(\mathbf{3}, \mathbf{1}, -\frac{2}{3}) + d_H(\mathbf{3}, \mathbf{1}, \frac{1}{3}) + \nu_H(\mathbf{1}, \mathbf{1}, 0) + e_H(\mathbf{1}, \mathbf{1}, 1)$$

Triplet mass

$$H^2\, D + \bar{H}^2\, D \to d_H\, \bar{D}_3\, \langle \nu_H \rangle + d_H^c\, D_3\, \langle \nu_H^c \rangle$$

We need at least one $(\mathbf{6}, \mathbf{1}, 1)$ to realize this mechanism.

Fermion masses

$$F_L(\mathbf{4}, \mathbf{2}, \mathbf{1}) \bar{F}_R(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})\, \langle h(\mathbf{1}, \mathbf{2}, \mathbf{2}) \rangle \quad (1)$$

Neutrinos mix with additional heavy singlets

$$\mathcal{M}_{\nu, \nu^c, \varphi} = \begin{pmatrix} 0 & v & 0 \\ v & 0 & M_{GUT} \\ 0 & M_{GUT} & M \end{pmatrix} \to \frac{v^2\, M}{M_{GUT}^2} \quad (2)$$

Superstring realization of PS gauge symmetry

Add and extra vector to $SO(10)$ basis

$$v_{13} = \alpha = \{\bar{\psi}^{45}\bar{\phi}^{1,2}\}$$

that introduces 12 new GSO projection phases $c[\alpha, v_j], j = 1, \ldots, 12$.

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Gauge Group: $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^3 \times SU(2)^4 \times SO(8)$

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The α-projection truncates $SO(10)$ multiplets

$$\begin{aligned} SO(10) &\supset SU(4) \times SU(2)_L \times SU(2)_R \\ \mathbf{16} &= (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \\ \overline{\mathbf{16}} &= (\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}, \mathbf{1}, \mathbf{2}) \\ \mathbf{10} &= (\mathbf{6}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}, \mathbf{2}) \end{aligned}$$

So we need $2 \times \mathbf{16}$ for each family and one pair of $\mathbf{16}+\overline{\mathbf{16}}$ for the PS breaking Higgs

Superstring realization of PS gauge symmetry

Hypercharge embedding: $SU(4) \times SU(2)_L \times SU(2)_R$

$$Q_{em} = \frac{1}{\sqrt{6}} T_{15} + \frac{1}{2} I_{3L} + \frac{1}{2} I_{3R}$$

$$\begin{array}{ll} (\mathbf{4}, \mathbf{2}, \mathbf{1}) & q + \ell \\ (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) & d^c + u^c + e^c + \nu^c \\ (\bar{\mathbf{6}}, \mathbf{1}, \mathbf{1}) & D + D^c \end{array}$$

Possible exotics in the PS model

$$(\mathbf{4}, \mathbf{1}, \mathbf{1}) + (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1}) : \pm \frac{1}{6} \text{ exotic colored particles and SM singlets}$$

$$(\mathbf{1}, \mathbf{2}, \mathbf{1}) : \pm \frac{1}{2} \text{ leptons}$$

$$(\mathbf{1}, \mathbf{1}, \mathbf{2}) : \pm \frac{1}{2} \text{ SM singlets}$$

Search strategy

A model is characterized by 9 integers
$(n_g, k_L, k_R, n_6, n_h, n_4, n_{\bar{4}}, n_{2L}, n_{2R})$

$$n_g = n_{4L} - n_{\bar{4}R} = n_{\bar{4}L} - n_{4R} = \text{ net \# of generations}$$

$$k_L = n_{\bar{4}L} = \text{\# of non chiral left pairs (mirrors)}$$

$$k_R = n_{4R} = \text{\# of non chiral right pairs (mirrors)}$$

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n_4 &= \text{\# of } (\mathbf{4}, \mathbf{1}, \mathbf{1}) \text{ (exotic)}\\
n_{\bar{4}} &= \text{\# of } (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1}) \text{ (exotic)}\\
n_{2L} &= \text{\# of } (\mathbf{1}, \mathbf{2}, \mathbf{1}) \text{ (exotic)}\\
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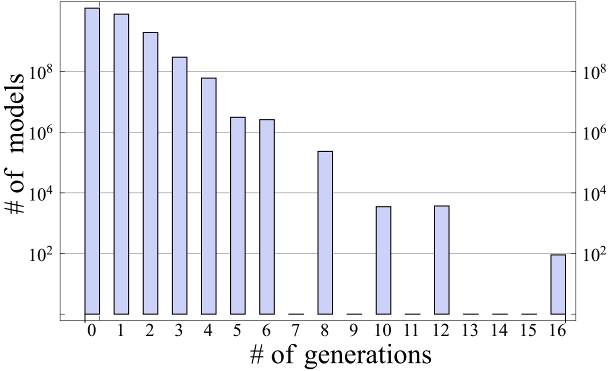
n_{2L} &= \text{\# of } (\mathbf{1},\mathbf{2},\mathbf{1}) \text{ (exotic)} \\
n_{2R} &= \text{\# of } (\mathbf{1},\mathbf{1},\mathbf{2}) \text{ (exotic)}
\end{aligned}
$$

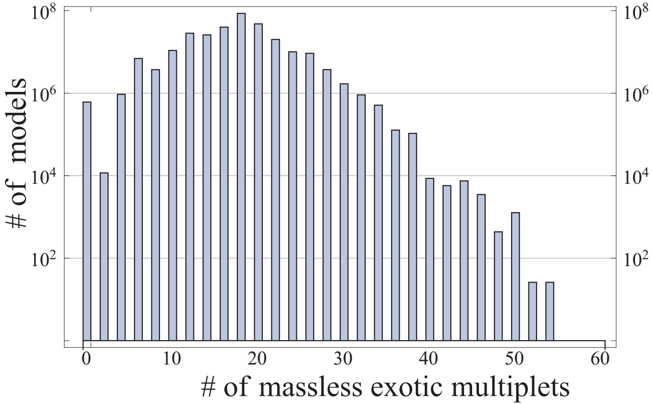
We have derived analytic formulae for all these quantities, similar to the $SO(10)$ case ↰.
It turns out that they depend on 51 GSO phases , that lead to a class of $2^{51} \sim 2 \times 10^{15}$ models.

Generation structure of PS models



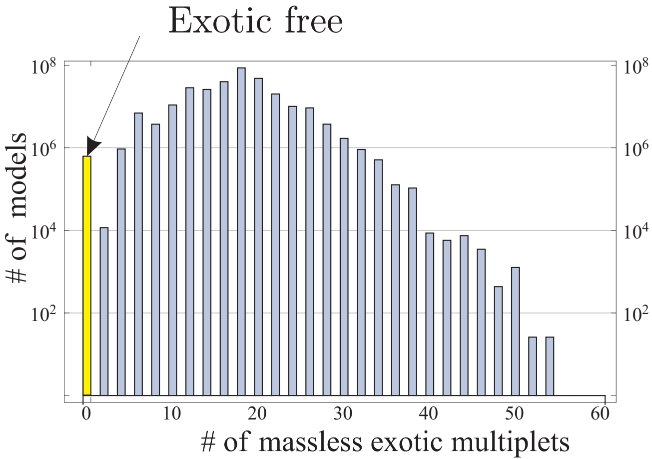
Number of models versus number of generations (n_g) in a random sample of 10^{11} vacua.

Exotic particle structure of PS models



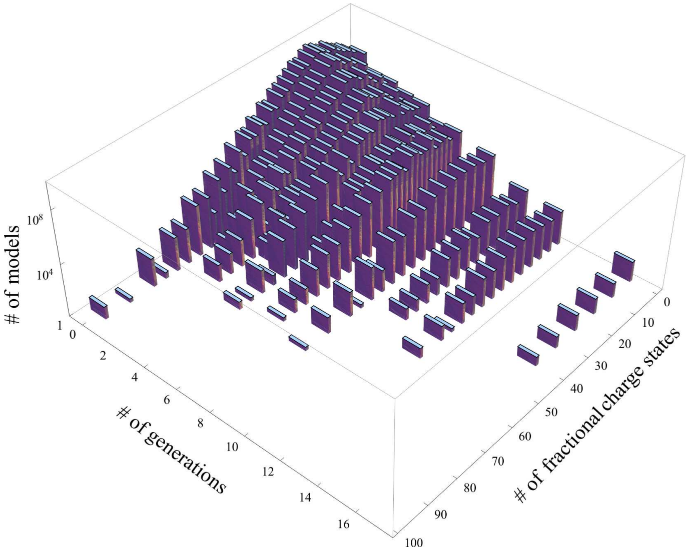
Number of 3 generation models versus total number of exotic multiplets in a random sample of 10^{11} vacua.

Exotic particle structure of PS models



Number of 3 generation models versus total number of exotic multiplets in a random sample of 10^{11} *vacua.*

Generation structure of exotic free models



Summary of the Pati-Salam model landscape

constraint	# of models in sample	probability	estimated # of models in class
None	100000000000	1	2.25×10^{15}
+ No group enhancements.	78977078333	7.90×10^{-1}	1.78×10^{15}
+ Complete families	22497003372	2.25×10^{-1}	5.07×10^{14}
+ 3 generations	298140621	2.98×10^{-3}	6.71×10^{12}
+ PS breaking Higgs	23694017	2.37×10^{-4}	5.34×10^{11}
+ SM breaking Higgs	19191088	1.92×10^{-4}	4.32×10^{11}
+ No massless exotics	121669	1.22×10^{-6}	2.74×10^{9}

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Model degeneracy. How many phenomenologically interesting models ?

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Model degeneracy. How many phenomenologically interesting models ? Impose additional phenomenological constraints, eg top mass.

Conclusions

- We have developed tools that allow the exploration of $2^{51} \sim 10^{15}$ Pati-Salam heterotic $Z_2 \times Z_2$ $N = 1$ vacua.
- The heterotic PS vacua seem to be very rich, realistic models (3 generations, PS breaking Higgs, SM breaking Higgs) correspond to 3×10^{-4} of this class
- We have identified an interesting subclass of realistic models, (1×10^{-6} of the vacua) where the massless string spectrum is free of fractionally charged states.

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- We have identified an interesting subclass of realistic models, (1×10^{-6} of the vacua) where the massless string spectrum is free of fractionally charged states.
- Explore more details of the models (Fermion masses, hidden sector) and study their phenomenology.
- Try to explore other models including the SM in this framework.