A new model for confinement

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1. Nucl. Phys. B822 (2009) 1
2. Phys. Lett. B685 (2010) 86

Corfu, 5 Sept. 2010

Background

1. J. M. Drouffe and J. B. Zuber, Phys. Rept. 102 (1983) 1

2. J. Zinn-Justin, Quantum Field Theories and Critical Phenomena

3. I. Montvay and G. Muenster, Quantum Fields on a Lattice

4. M. Creutz, Phys. Rev. Lett. 43 (1979), 553

5. B. Svetitsky and L.G. Yaffe, Nucl. Phys. B210 (1982) 423

6. Y.K. Fu and H.B. Nielsen, Nucl. Phys. B236 (1984), 167

7. S. Ejiri, J. Kubo and M. Murata, Phys. Rev. D62 (2000), 105025

8. M. Luscher and P. Weisz, JHEP 0407:014, 2204

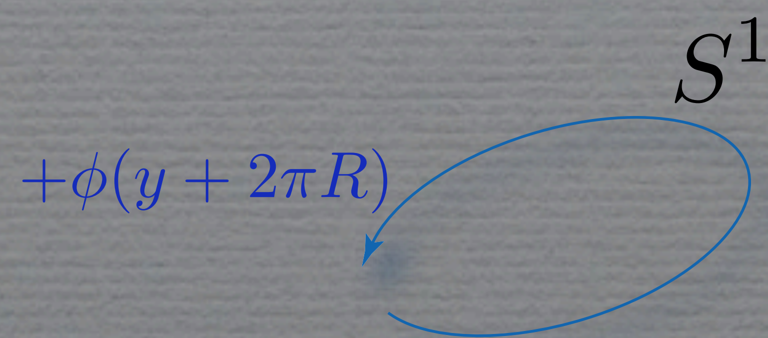
In 5 Dimensions...

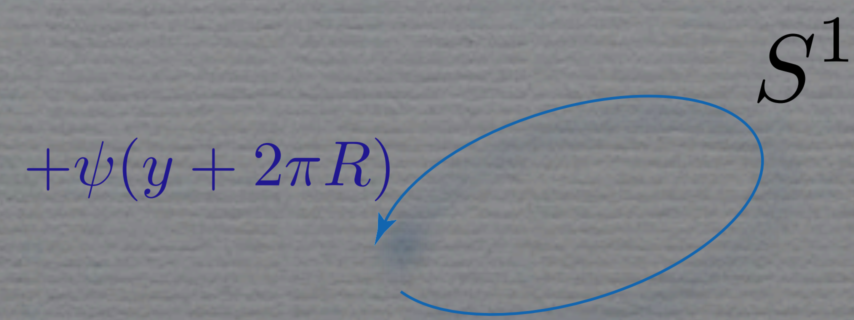
$$A_M \xrightarrow{\mathcal{M}_5 = E_4 \times S^1} \{A_\mu, A_5\}$$

5D gauge field SU(2)

Z: 4D gauge field

h: 4D "Higgs"



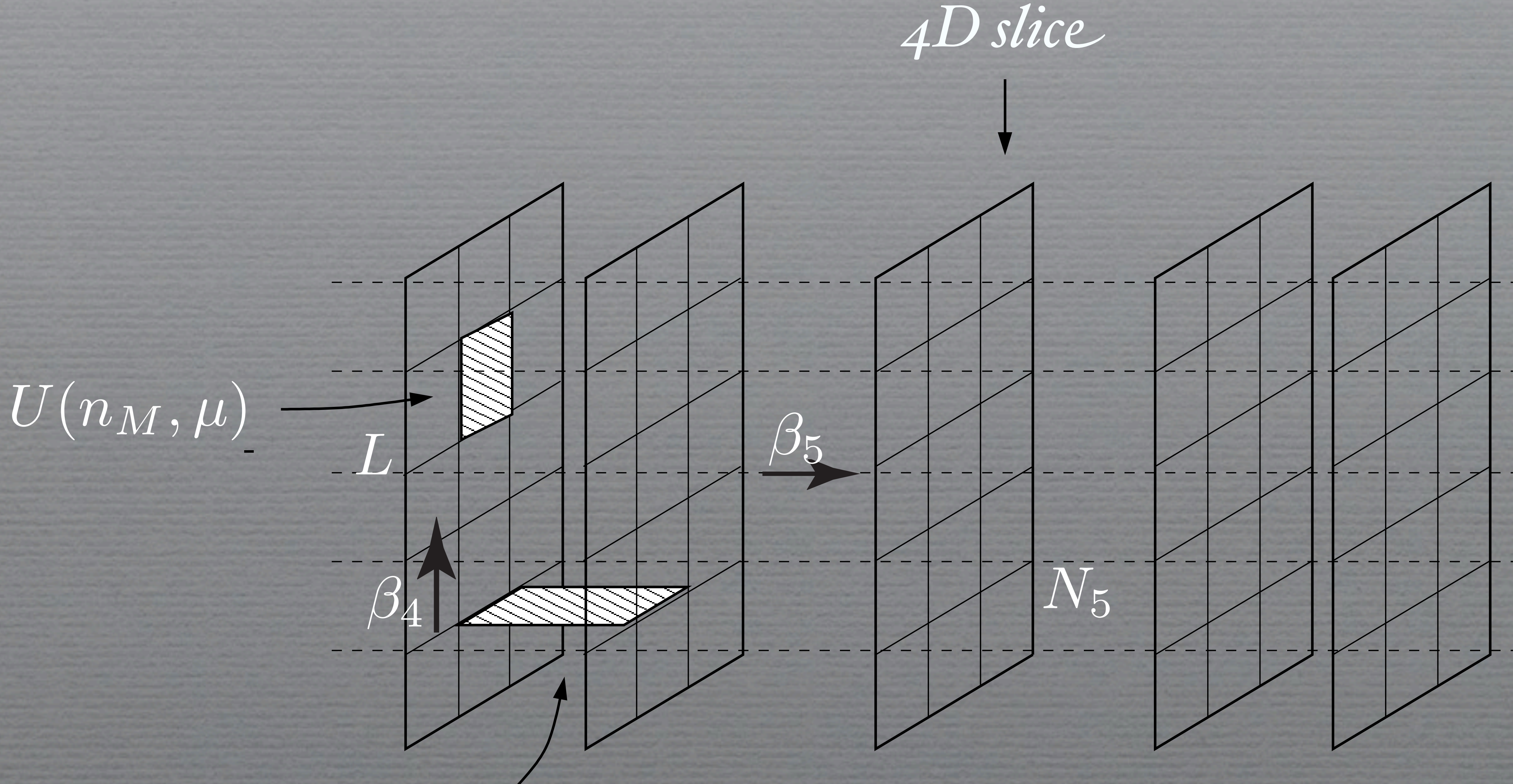


$$\phi(y) = \sum_n \phi_n \, e^{iny/R}$$

$$\psi(y) = \sum_n \psi_x e^{iny/R}$$

4D Georgi-Glashow (G-G) model + KK

The Anisotropic Lattice



$$\beta_4 = \frac{\beta}{\gamma} \quad \beta_5 = \beta\gamma \quad \frac{N_5}{L} = \frac{2\pi R}{l}\gamma$$

$\gamma =$ anisotropy parameter

The mean-field expansion

parameters: $L,\ \beta,\ \gamma$ (we keep $N_5 = L$)

$$Z = \int DU \int DV \int DH e^{(1/N)\mathrm{Re}[\mathrm{tr}H(U-V)]} e^{-S_G[V]}$$

$$Z = \int DV \int DH e^{-Seff[V,H]}, \quad Seff = S_G[V] + u(H) + (1/N)\mathrm{Retr}HV$$

$$e^{-u(H)} = \int DU e^{(1/N)\mathrm{Retr}UH}$$

To 0'th order

The background is determined by

$$\overline{V} = -\frac{\partial u}{\partial H}\Big|_{\overline{H}} \qquad \overline{H} = -\frac{\partial S_G[V]}{\partial V}|_{\overline{V}}$$

The free energy

$$F^{(0)} = -\frac{1}{\mathcal{N}}\ln(Z[\overline{V},\overline{H}]) = \frac{S_{\text{eff}}[\overline{V},\overline{H}]}{\mathcal{N}}$$

To 1st order

$$H = \bar{H} + h \qquad V = \bar{V} + v$$

$$Seff = Seff[\bar{V}, \bar{H}] + \frac{1}{2}\left(\left.\frac{\delta^2 Seff}{\delta H^2}\right|_{\overline{V},\overline{H}} h^2 + 2\left.\frac{\delta^2 Seff}{\delta H \delta V}\right|_{\overline{V},\overline{H}} hv + \left.\frac{\delta^2 Seff}{\delta V^2}\right|_{\overline{V},\overline{H}} v^2 \right)$$

$$\left.\frac{\delta^2 Seff}{\delta H^2}\right|_{\overline{V},\overline{H}} h^2 = h_i K^{(hh)}_{ij} h_j = h^T K^{(hh)} h$$

$$\left.\frac{\delta^2 Seff}{\delta V^2}\right|_{\overline{V},\overline{H}} v^2 = v_i K^{(vv)}_{ij} v_j = v^T K^{(vv)} v$$

$$\left.\frac{\delta^2 Seff}{\delta V \delta H}\right|_{\overline{V},\overline{H}} v^2 = v_i K^{(vh)}_{ij} h_j = v^T K^{(vh)} h$$

The Gaussian fluctuations

$$z = \int Dv \int Dh e^{-S^{(2)}[v,h]} \qquad S^{(2)}[v,h] = \frac{1}{2}\left(h^T K^{(hh)} h + 2v^T K^{(vh)} h + v^T K^{(vv)} v\right)$$

$$z = \frac{(2\pi)^{|h|/2}(2\pi)^{|v|/2}}{\sqrt{\det[(-\mathbf{1} + K^{(hh)}K^{(vv)})]}}$$

$$Z^{(1)} = Z[\overline{V}, \overline{H}] \cdot z = e^{-S_{\text{eff}}[\overline{V}, \overline{H}]} \cdot z$$

The free energy to first order

$$F^{(1)} = F^{(0)} - \frac{1}{\mathcal{N}} \ln(z) = F^{(0)} + \frac{1}{2\mathcal{N}} \ln\Big[\det\left(-\mathbf{1} + K^{(hh)}K^{(vv)}\right) \Delta_{\text{FP}}^{-2}\Big]$$

Observables

$$\mathcal{O}[V] = \mathcal{O}[\overline{V}] + \left.\frac{\delta \mathcal{O}}{\delta V}\right|_{\overline{V}} v + \frac{1}{2} \left.\frac{\delta^2 \mathcal{O}}{\delta V^2}\right|_{\overline{V}} v^2 + \ldots$$

$$= \mathcal{O}[\overline{V}] + \frac{1}{2} \left.\frac{\delta^2 \mathcal{O}}{\delta V^2}\right|_{\overline{V}} \frac{1}{z} \int Dv \int Dh v^2 e^{-S^{(2)}[v,h]}$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int Dv \int Dh \left(\mathcal{O}[\overline{V}] + \frac{1}{2} \left.\frac{\delta^2 \mathcal{O}}{\delta V^2}\right|_{\overline{V}} v^2 \right) e^{-\left(Seff[\overline{V},\overline{H}]+S^{(2)}[v,h]\right)}$$

$$< v_i v_j >= \frac{1}{z} \int Dv \int Dh\, v_i v_j e^{-S^{(2)}[v,h]} = (K^{-1})_{ij}$$

$$K = -K^{(vh)} {K^{(hh)}}^{-1} K^{(vh)} + K^{(vv)}$$

$$\boxed{< \mathcal{O} >= \mathcal{O}[\overline{V}] + \frac{1}{2} \mathrm{tr} \left\{ \left.\frac{\delta^2 \mathcal{O}}{\delta V^2}\right|_{\overline{V}} K^{-1} \right\}}$$

1st order master formula

Correlators

$$C(t) =< \mathcal{O}(t_0+t)\mathcal{O}(t_0) > - < \mathcal{O}(t_0+t) >< \mathcal{O}(t_0) >= C^{(0)}(t) + C^{(1)}(t) + \cdots$$

$$C^{(0)}(t) = 0$$

$$< \mathcal{O}(t_0+t)\mathcal{O}(t_0) >= \mathcal{O}^{(0)}(t_0+t)\mathcal{O}^{(0)}(t_0) + \frac{1}{2}\mathrm{tr}\left\{ \frac{\delta^2(\mathcal{O}(t_0+t)\mathcal{O}(t_0))}{\delta^2 v} K^{-1} \right\} + \cdots$$

$$C^{(1)}(t) = \frac{1}{2}\mathrm{tr}\left\{ \frac{\delta^{(1,1)}(\mathcal{O}(t_0+t)\mathcal{O}(t_0))}{\delta^2 v} K^{-1} \right\} = \frac{1}{2}\mathrm{tr}\left\{ \frac{\tilde{\delta}^{(1,1)}(\mathcal{O}(t_0+t)\mathcal{O}(t_0))}{\delta^2 v} \tilde{K}^{-1} \right\}$$

$$C^{(1)}(t) = \sum_\lambda c_\lambda e^{-E_\lambda t} \qquad E_0 = m_H, \quad E_1 = m_H^*, \cdots$$

To 2nd order

$$S_{\text{eff}} = S_{\text{eff}}[\overline{V},\overline{H}] + \frac{1}{2}\left(\frac{\delta^2 S_{\text{eff}}}{\delta H^2}h^2 + 2\frac{\delta^2 S_{\text{eff}}}{\delta H \delta V}hv + \frac{\delta^2 S_{\text{eff}}}{\delta V^2}v^2\right)$$

$$+\frac{1}{6}\left(\frac{\delta^3 S_{\text{eff}}}{\delta H^3}h^3 + \frac{\delta^3 S_{\text{eff}}}{\delta V^3}v^3\right) + \frac{1}{24}\left(\frac{\delta^4 S_{\text{eff}}}{\delta H^4}h^4 + \frac{\delta^4 S_{\text{eff}}}{\delta V^4}v^4\right) + \cdots$$

$$\mathcal{O}[V] = \mathcal{O}[\overline{V}] + \frac{\delta \mathcal{O}}{\delta V}v + \frac{1}{2}\frac{\delta^2\mathcal{O}}{\delta V^2}v^2 + \frac{1}{6}\frac{\delta^3\mathcal{O}}{\delta V^3}v^3 + + \frac{1}{24}\frac{\delta^4\mathcal{O}}{\delta V^4}v^4 + \cdots$$

2nd order master formula

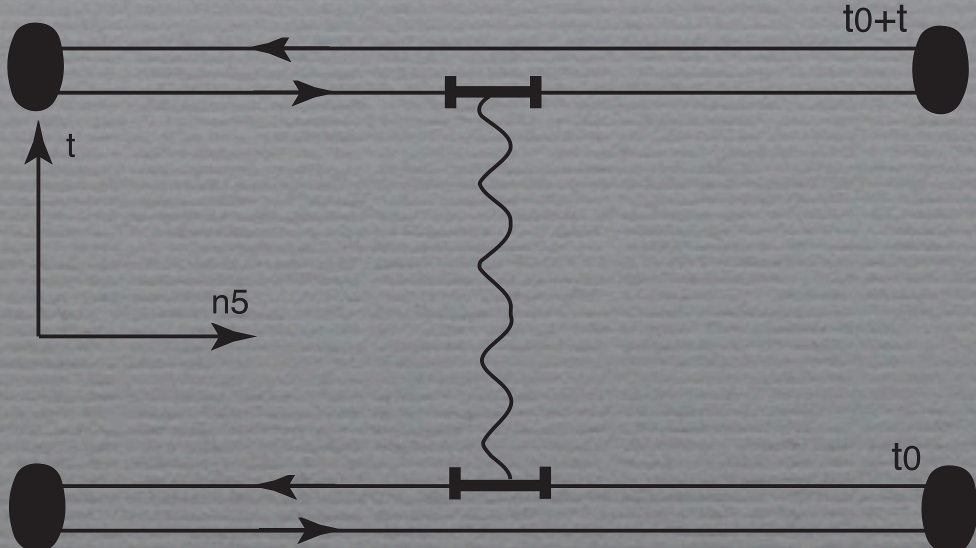
$$< \mathcal{O} >= \mathcal{O}[\overline{V}] + \frac{1}{2}\left(\frac{\delta^2\mathcal{O}}{\delta V^2}\right)_{ij}\left(K^{-1}\right)_{ij}$$

$$+\frac{1}{24}\sum_{i,j,l,m}\left(\frac{\delta^4\mathcal{O}}{\delta V^4}\right)_{ijlm}\left((K^{-1})_{ij}(K^{-1})_{lm} + (K^{-1})_{il}(K^{-1})_{jm} + (K^{-1})_{im}(K^{-1})_{jl}\right)$$

$$C^{(2)}(t) = \frac{1}{24}\sum_{i,j,l,m}\left(\frac{\delta^4\mathcal{O}^c(t)}{\delta v^4}\right)_{ijlm}\left((K^{-1})_{ij}(K^{-1})_{lm} + (K^{-1})_{il}(K^{-1})_{jm} + (K^{-1})_{im}(K^{-1})_{jl}\right)$$

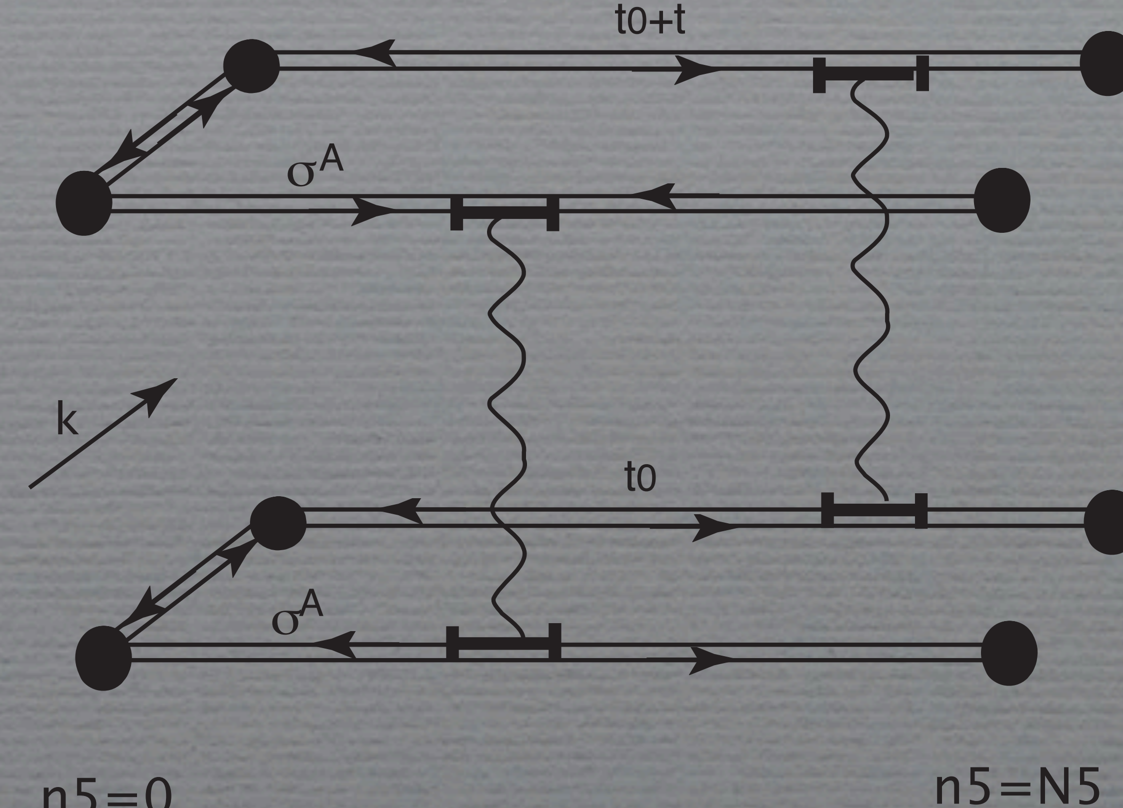
Lattice Observables (Polyakov loops)

The scalar



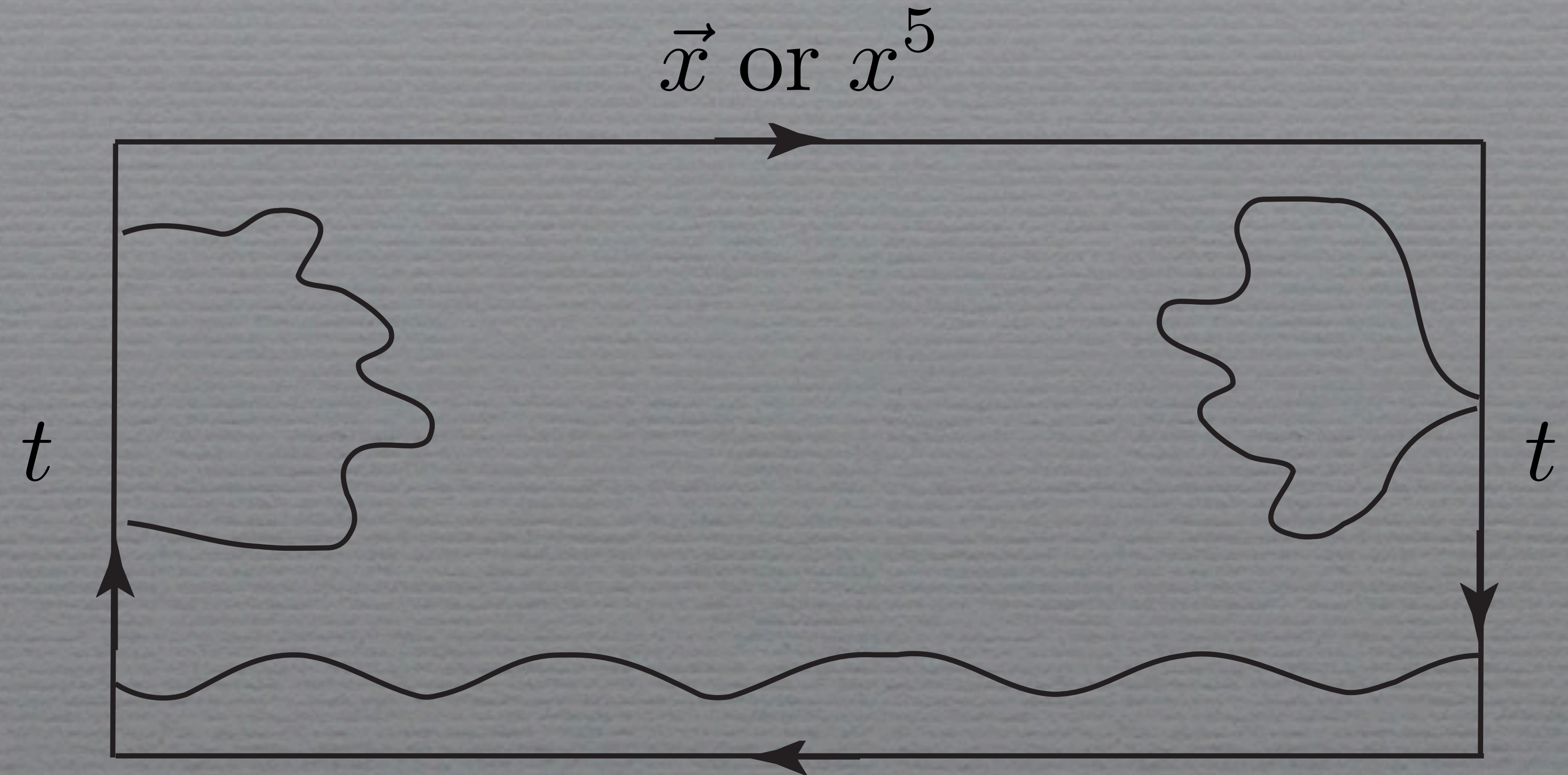
$$m = \lim_{t\to\infty} \ln \frac{C^{(1)}(t)}{C^{(1)}(t-1)}$$

The vector



$$m = \lim_{t\to\infty} \ln \frac{C^{(2)}(t)}{C^{(2)}(t-1)}$$

The Wilson loop



$$t \to \infty : \qquad e^{-Vt} \simeq < \mathcal{O}_W >$$

The static potential

$$V(r) = -2\log(\overline{v}_0) - \frac{1}{2\overline{v}_0^2}\frac{1}{L^3 N_5} \times \left\{ \sum_{p'_{M\neq 0}, p'_0=0} \left[\frac{1}{4}\sum_{N\neq 0}(2\cos(p'_N r)+2)\right] C_{00}^{-1}(p',0) \right]$$

$$+3\sum_{p'_{M\neq 0}, p'_0=0} \left[\frac{1}{4}\sum_{N\neq 0}(2\cos(p'_N r)-2)\right] \frac{1}{C_{00}(p',1)} \right\}$$

The scalar mass

$$C_H^{(1)}(t) = \frac{1}{\mathcal{N}}(P_0^{(0)})^2 \sum_{p'_0} \cos(p'_0 t) \sum_{p'_5} |\tilde{\Delta}^{(\mathcal{N}_5)}(p'_5)|^2 \tilde{K}^{-1}\Big((p'_0, \vec{0}, p'_5), 5, 0; (p'_0, \vec{0}, p'_5), 5, 0\Big)$$

$$\Delta^{(m_5)}(n_5) = \sum_{r=0}^{m_5-1} \frac{\delta_{n_5 r}}{\overline{v}_0(\hat{r})}, \qquad \hat{r} = r + 1/2$$

The vector mass

$$C_V^{(2)}(t) = \frac{2304}{\mathcal{N}^2}(P_0^{(0)})^4(\overline{v}_0(0))^4 \sum_{\vec{p}'}\sum_k \sin^2(p'_k)\left(\overline{\overline{K}}^{-1}(t,\vec{p}',1)\right)^2$$

$$\overline{K}^{-1}((p'_0,\vec{p}'),5,\alpha) = \sum_{p'_5,p''_5}\tilde{\Delta}^{(N_5)}(p'_5)\tilde{\Delta}^{(N_5)}(-p''_5)K^{-1}(p'',5,\alpha;p',5,\alpha)$$

$$\overline{\overline{K}}^{-1}(t,\vec{p}',\alpha) = \sum_{p'_0} e^{ip'_0 t}\overline{K}^{-1}((p'_0,\vec{p}'),5,\alpha)$$

The free energy

$$F^{(1)} = F^{(0)} + \frac{1}{2\mathcal{N}}\sum_p \ln\left[\det\left(-\mathbf{1}+\tilde{K}^{(hh)}_{\alpha'=0}\tilde{K}^{(vv)}_{\alpha'=0}\right)\det\left(-\mathbf{1}+\tilde{K}^{(hh)}_{\alpha'\neq 0}\tilde{K}^{(vv)}_{\alpha'\neq 0}\right)^3\Delta_{\mathrm{FP}}^{-2}\right]$$

A few remarks on the calculation

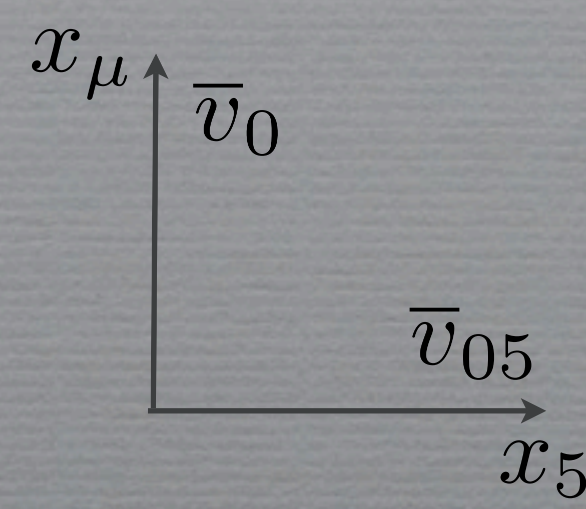
- We choose the Lorentz gauge. No ghosts at this order.
- Torons appear always as a 0/0: “0/0=0 regularization”.
- Formulas generalized to the anisotropic lattice. There are two inequivalent Wilson loops: one along the 4d and one along the 5th dimension.
- Analytical formulas computed numerically.

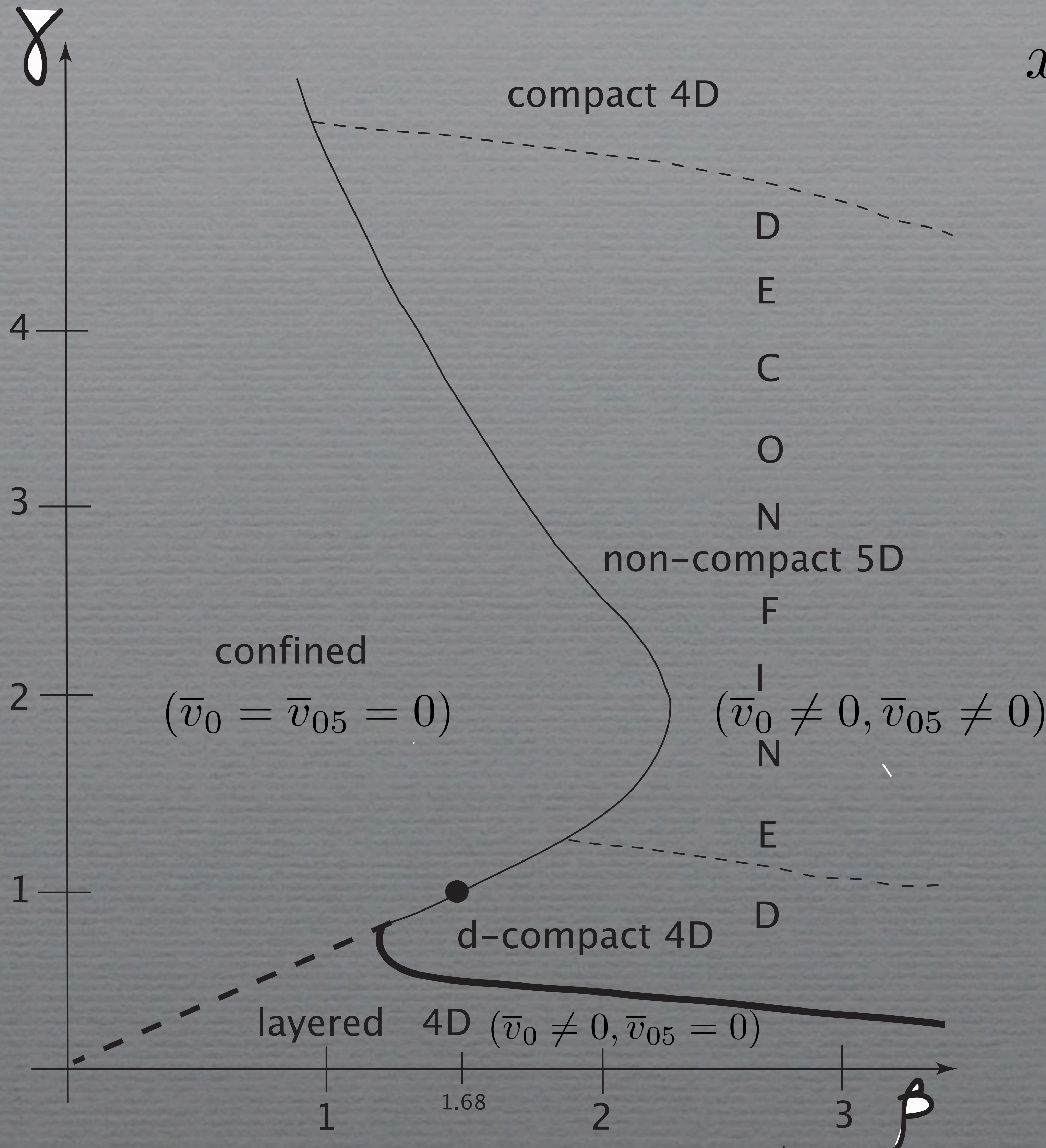
Keep this in mind

- Working assumption: whenever non-trivial, physical.

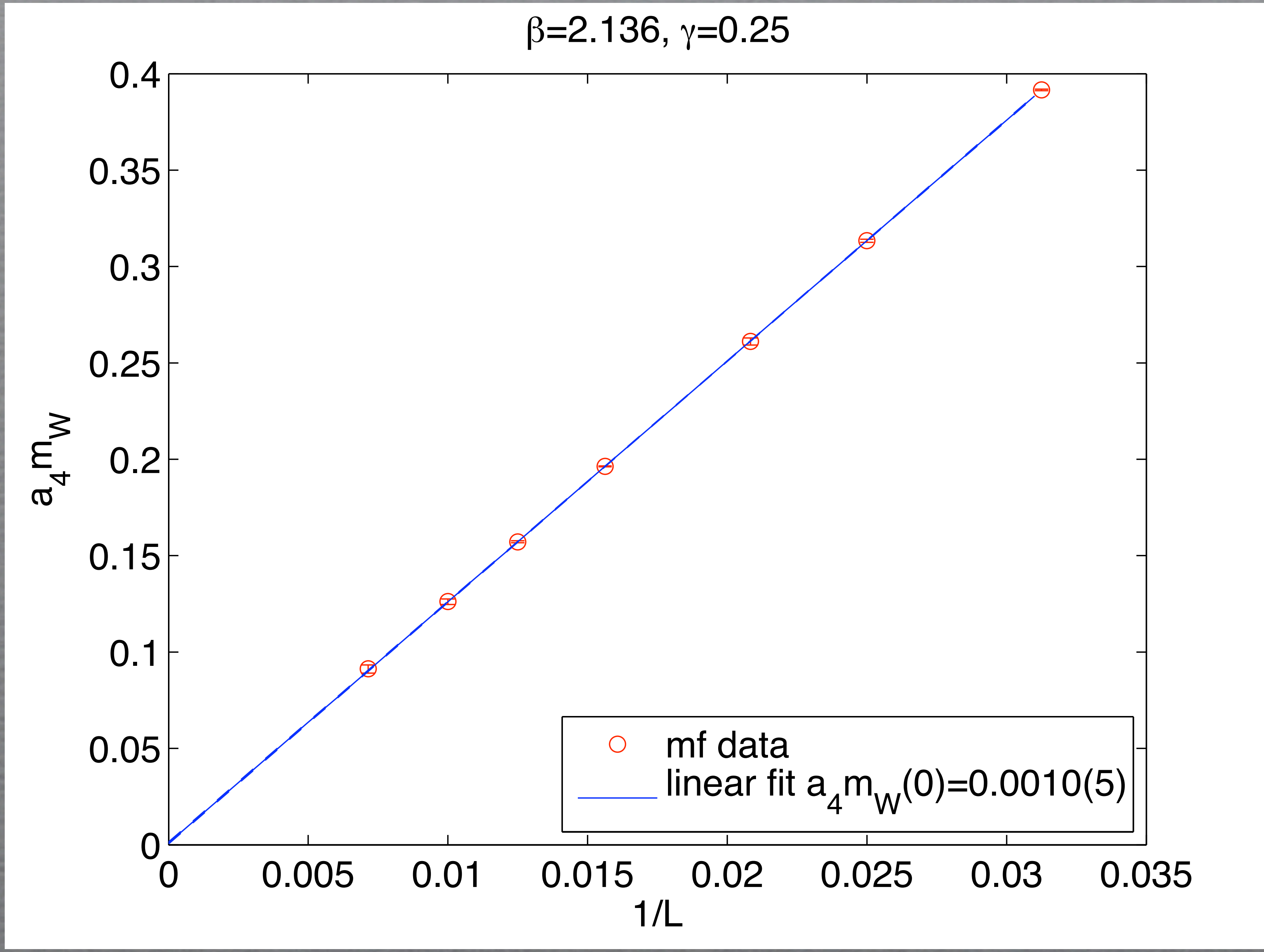
The phase diagram

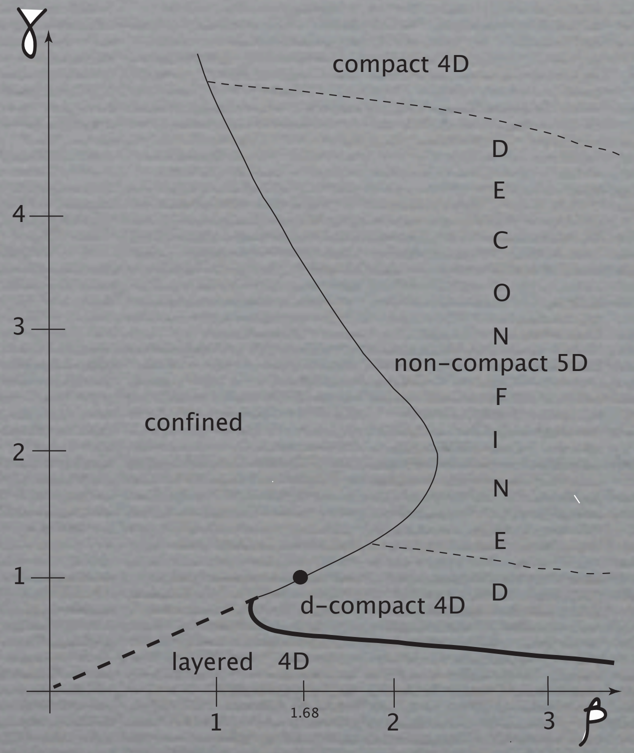
the mean-field background





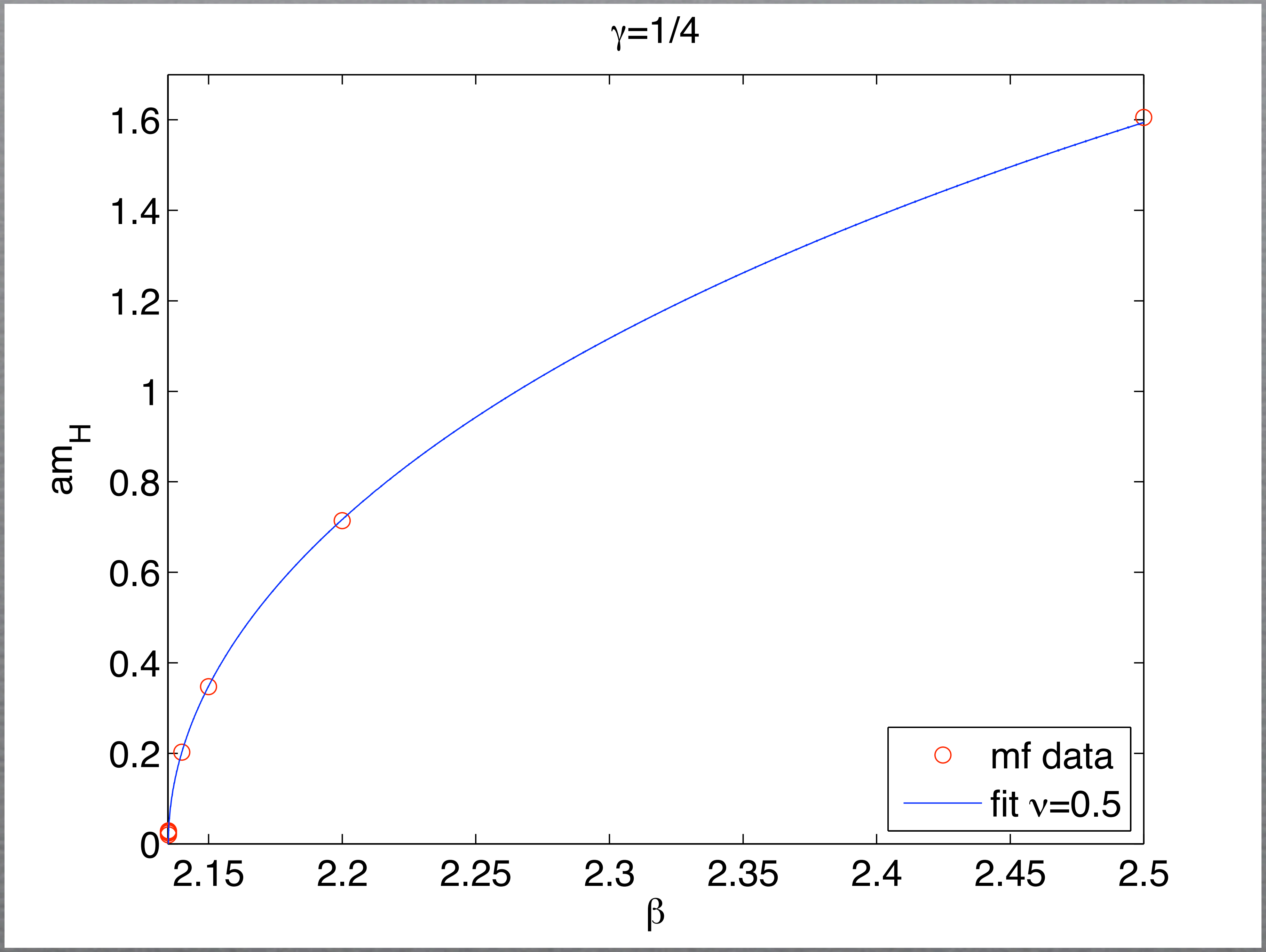
The W is massless or perhaps exponentially light:

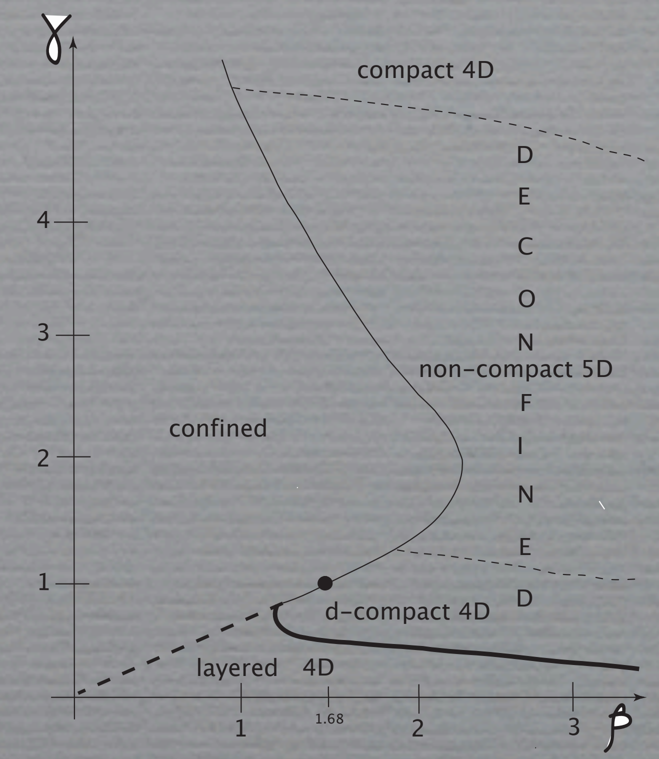




$$a_4 m_W = \frac{c_L}{L}$$

The d-compact phase is separated from the layered phase by a 2nd order phase transition:



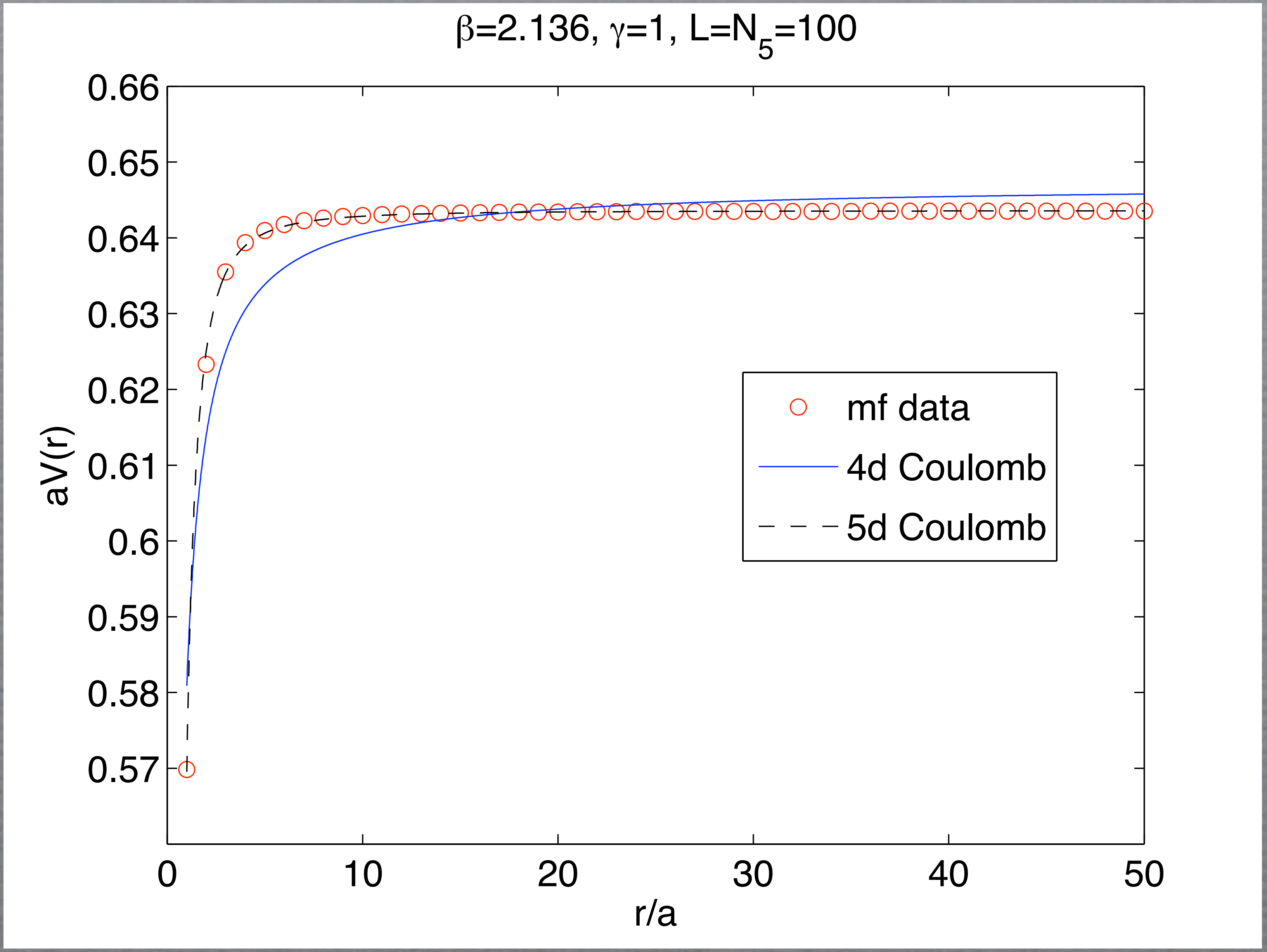


$$\nu = 1/2$$

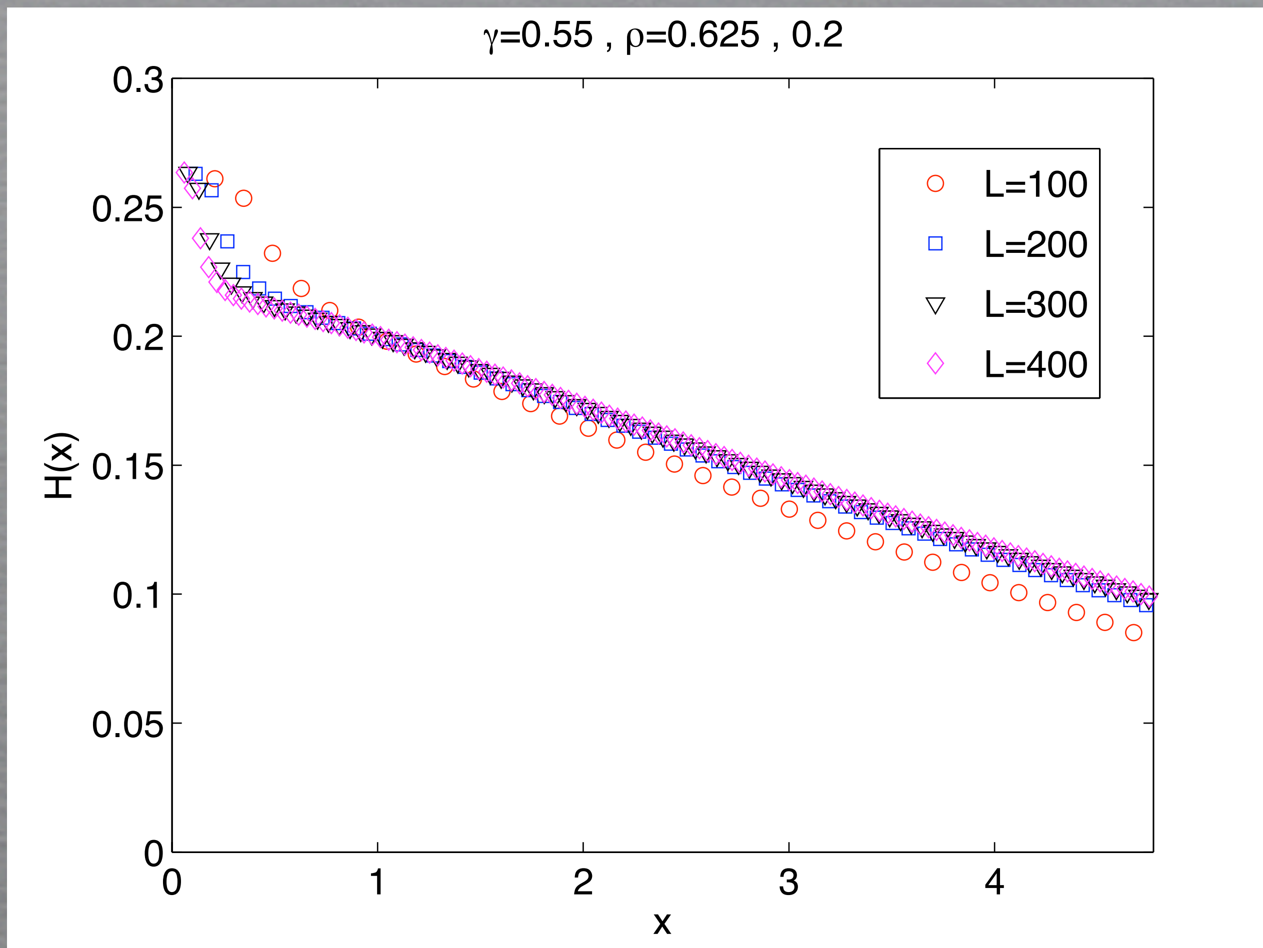
agrees with Svetitski & Yaffe: dim reduction to 4d Ising

The Static Potential

The static potential on the isotropic lattice:

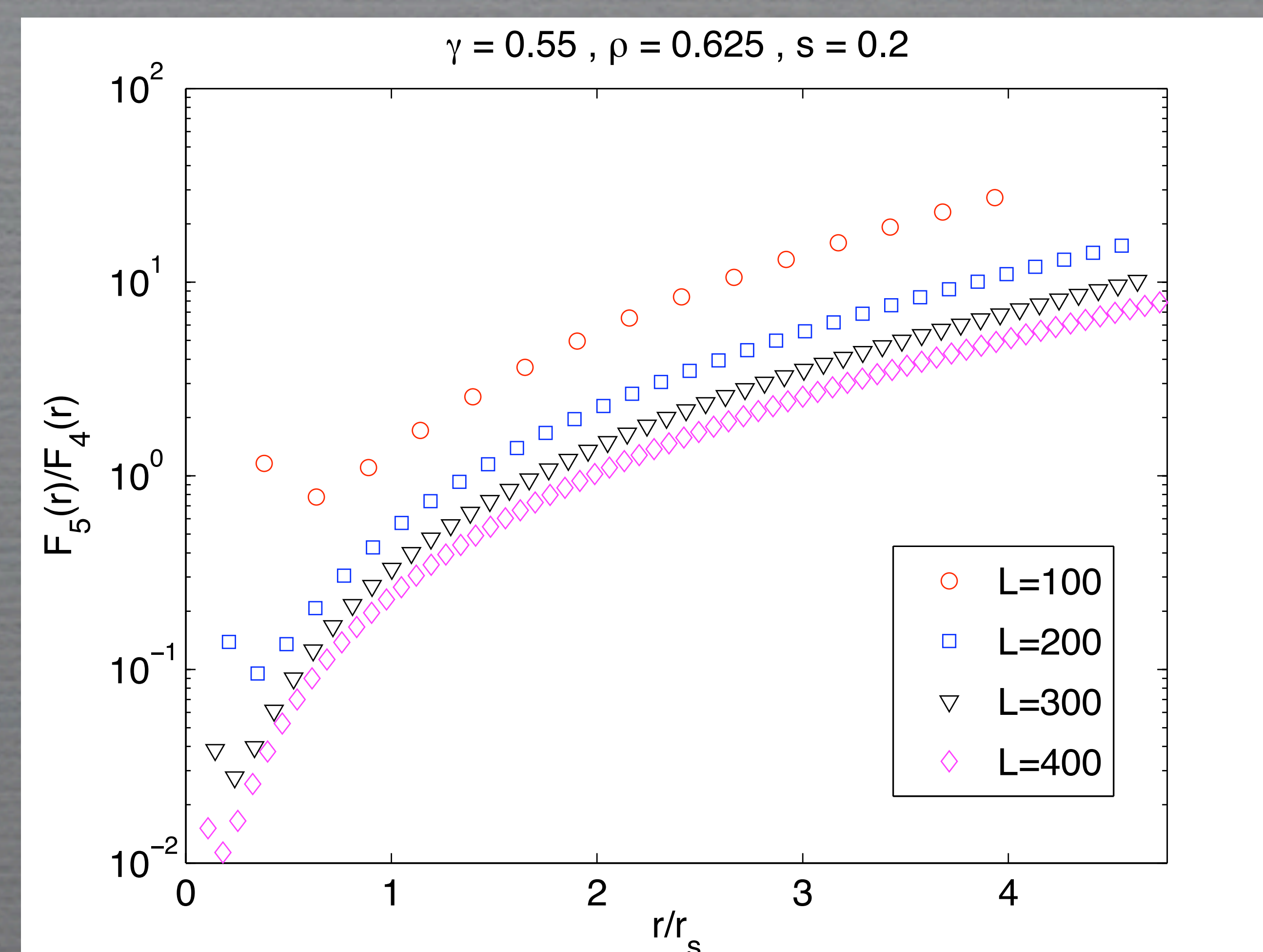


$$H(x) = F_4(r)r^2\big|_{r=xr_s}$$



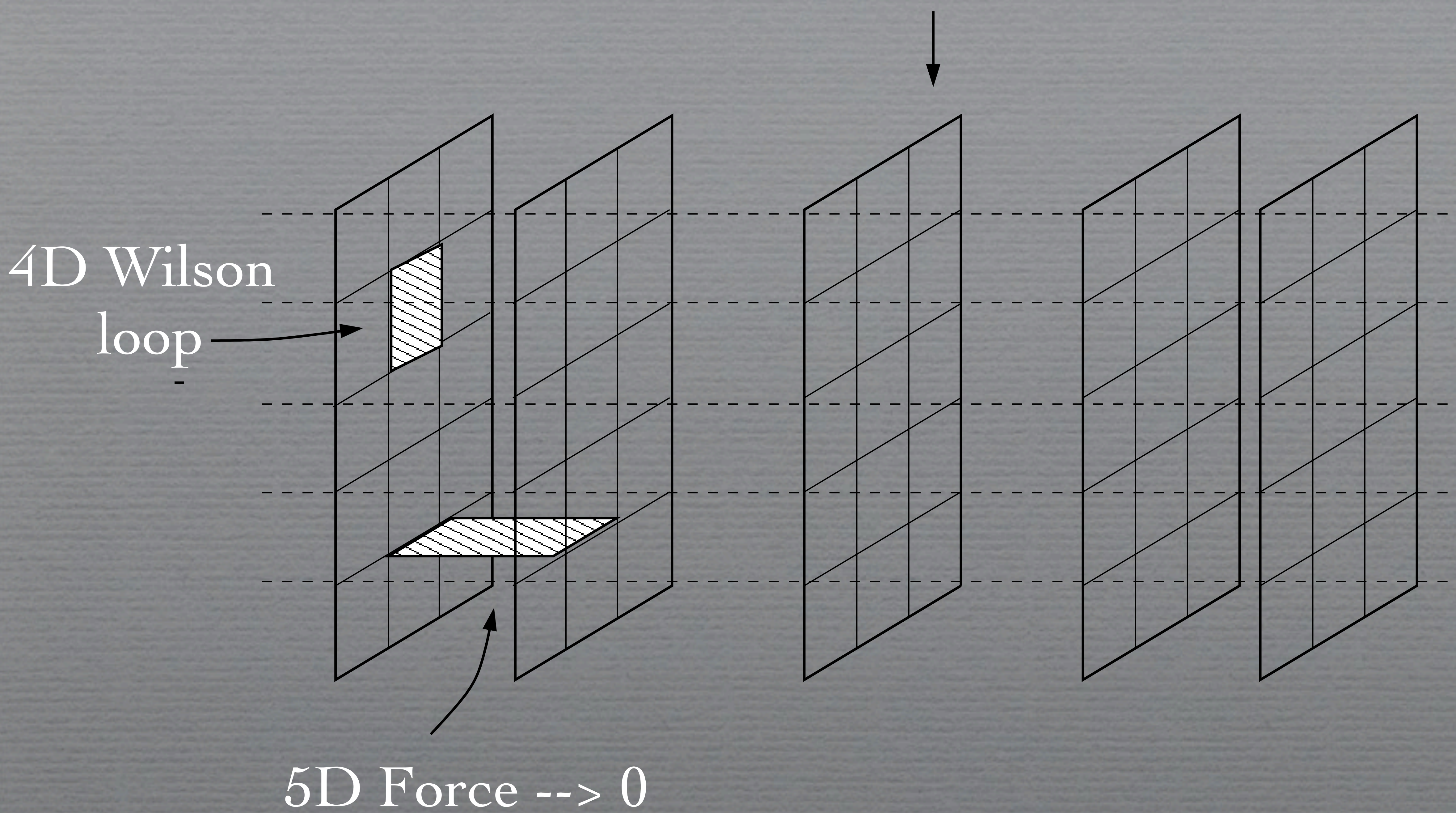
The static force in the d-compact phase along the 4D

$$F_5/F_4$$



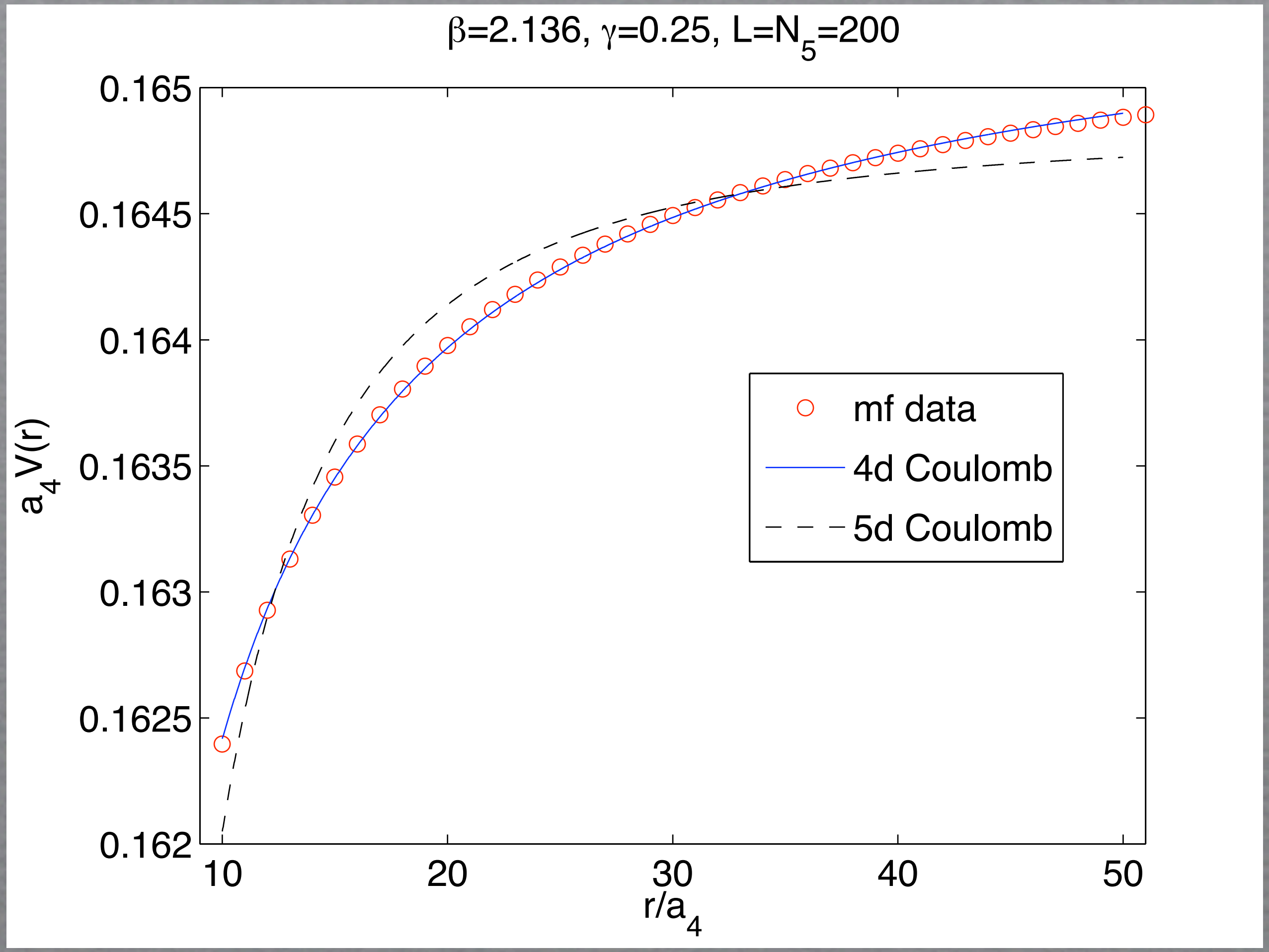
The 5D force vanishes in the infinite volume-continuum limit:

We are looking at an array of non-interacting 4D hyperplanes



The short distance static potential in the d-compact phase along the 4d hyperplanes:

β_4 large and β_5 small



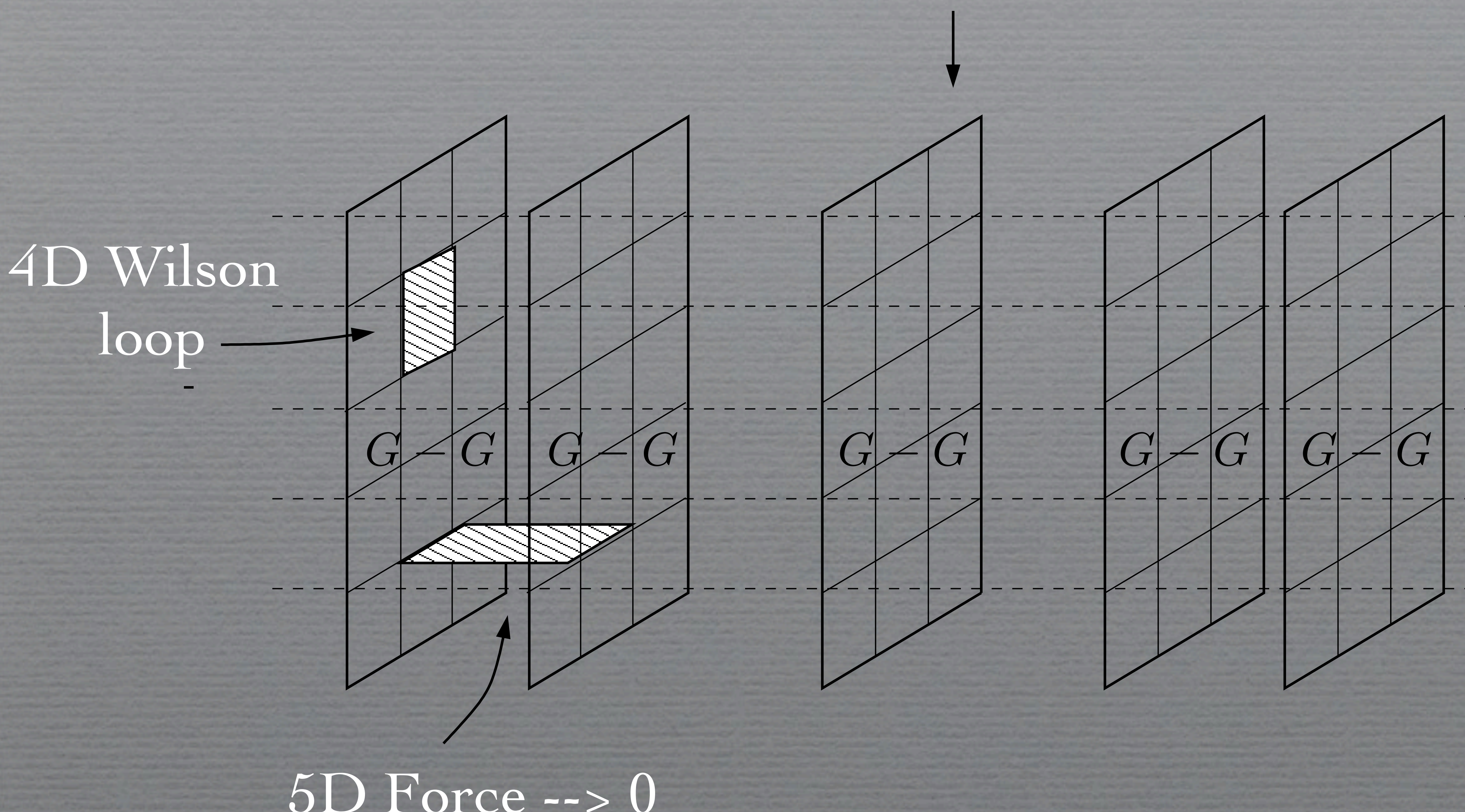
In the stability region r=10...50 the potential is clearly 4d Coulomb, large Yukawa excluded

$\longrightarrow$

dimensional reduction in the d-compact phase

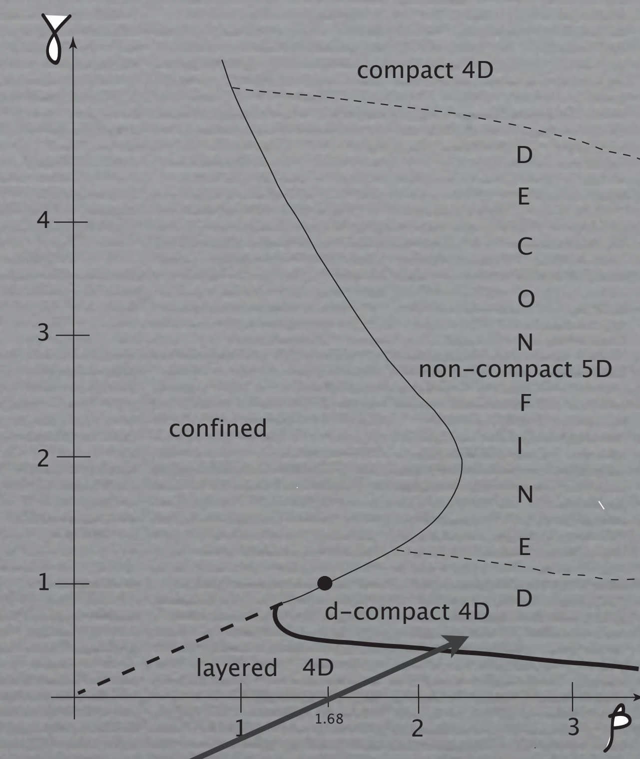
We are looking at an array of non-interacting 4D hyperplanes where gauge interactions are localized.

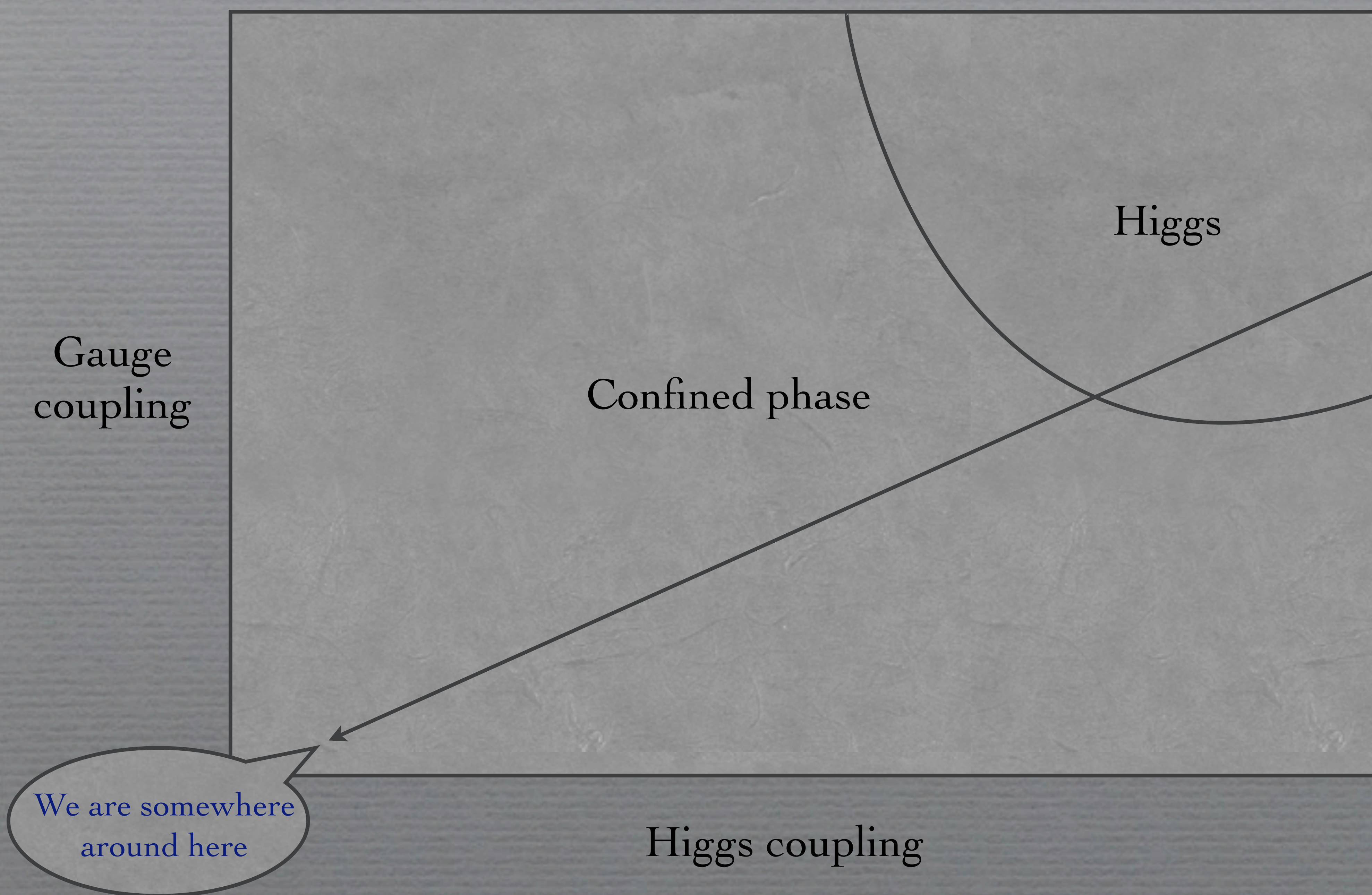
4D Georgi-Glashow on each of the branes



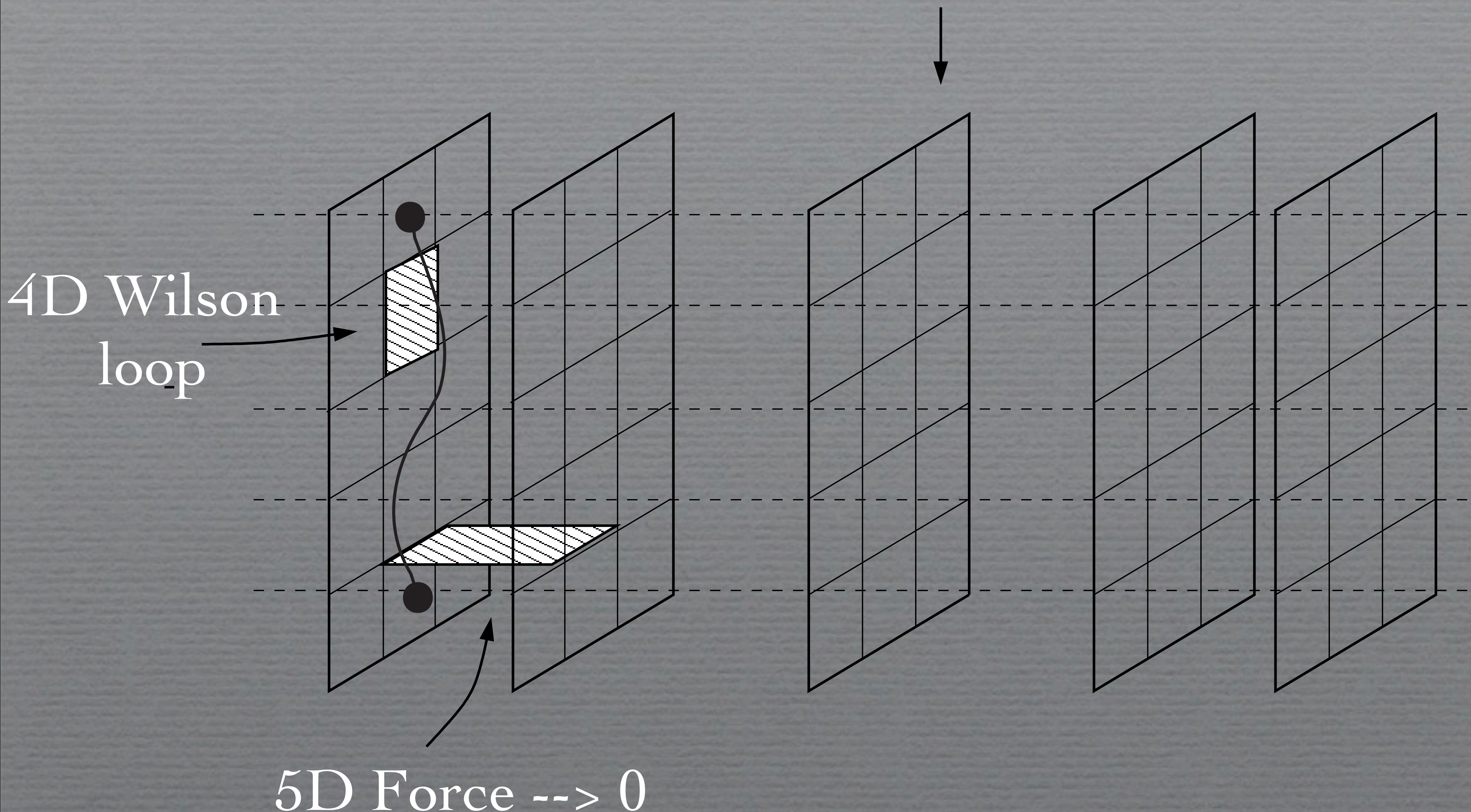
The localization is "perfect" in the infinite L limit

Remember: 4D Georgi-Glashow model
+ ~~KK~~





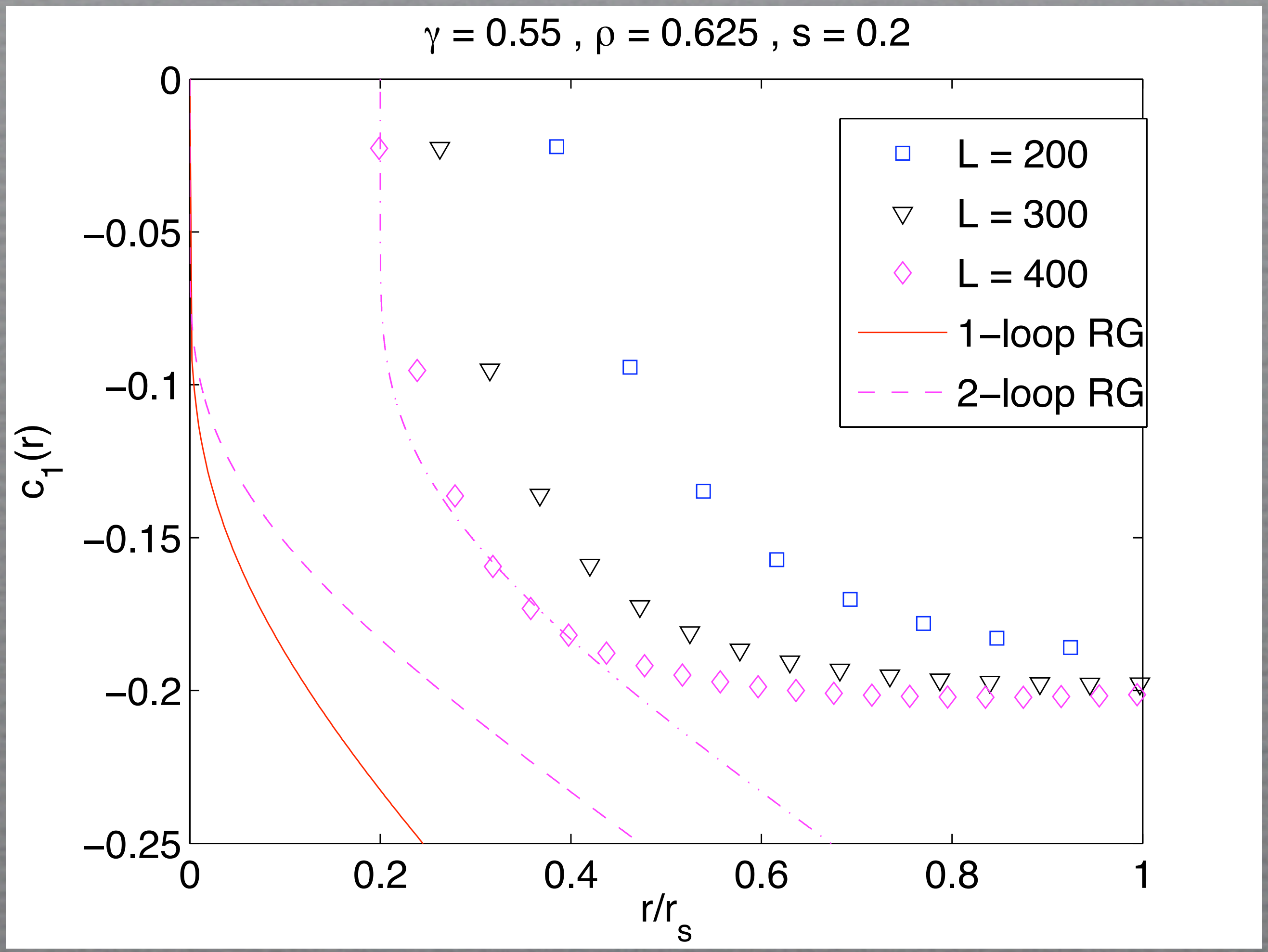
We are looking at an array of non-interacting 4D hyperplanes where gauge interactions are localized.
4D G-G on each of the branes.
The 4D Wilson loop at small r should run like an asymptotically free coupling and at large r must describe a string

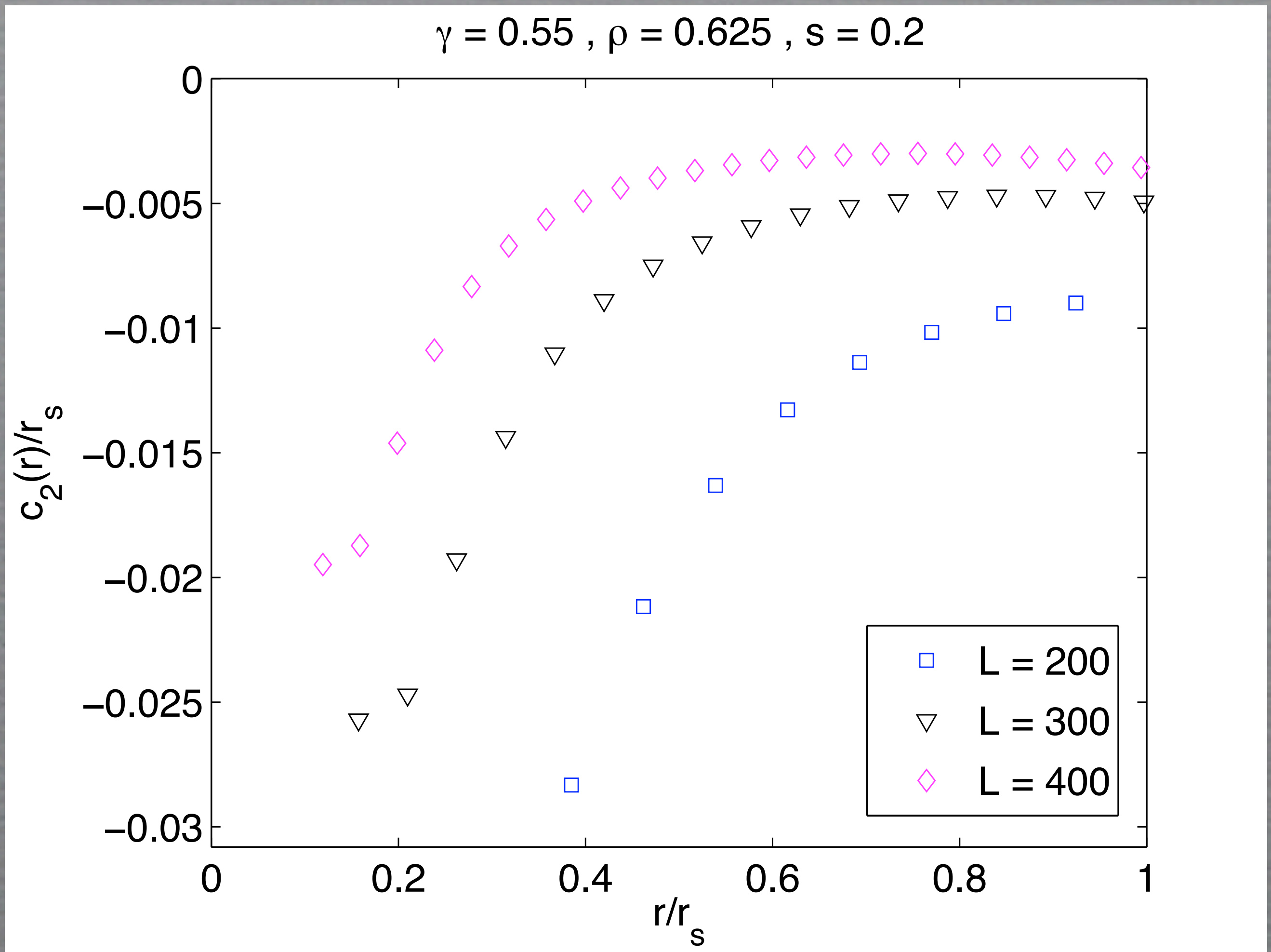


Short distance

ansatz: $$V(r) = \mu + \frac{c_1}{r} + \frac{c_2}{r^2}$$

$$\Lambda_{q\bar{q}}\, r_s = 0.277$$



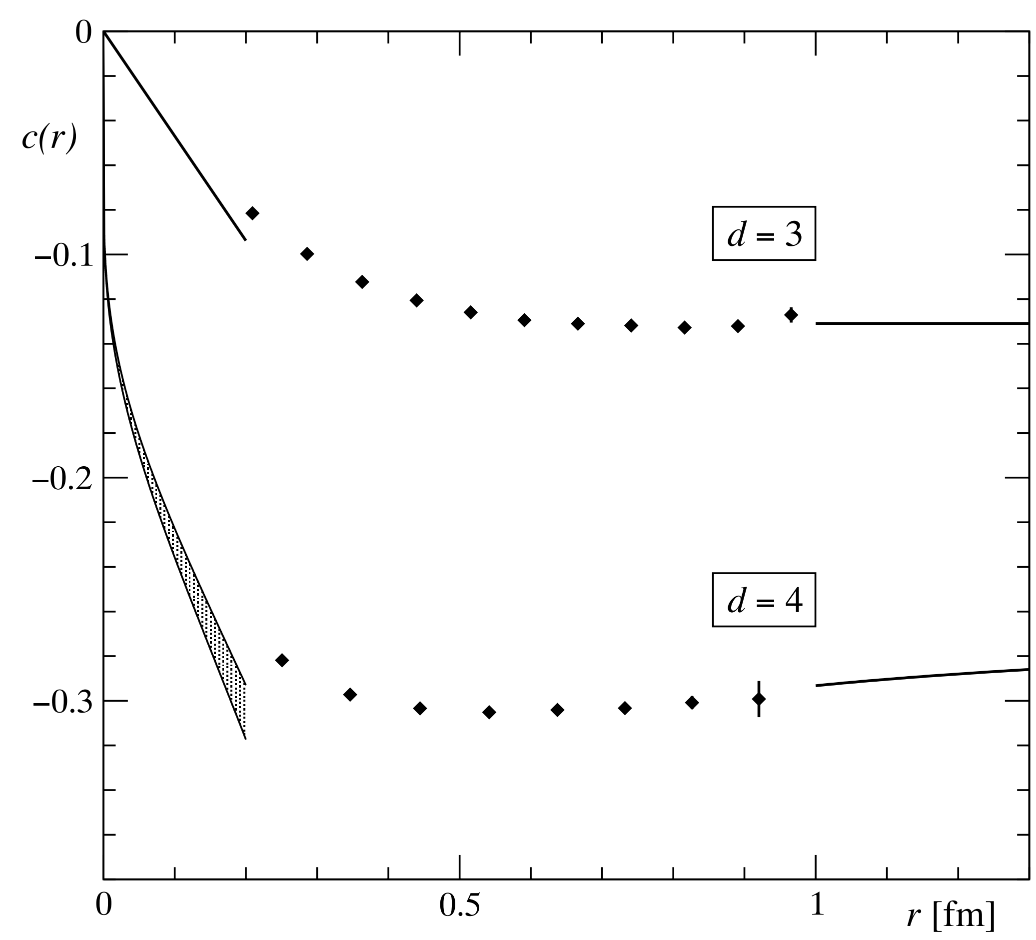


The short distance is 4D asymptotically free

The 5D behavior gradually disapperas

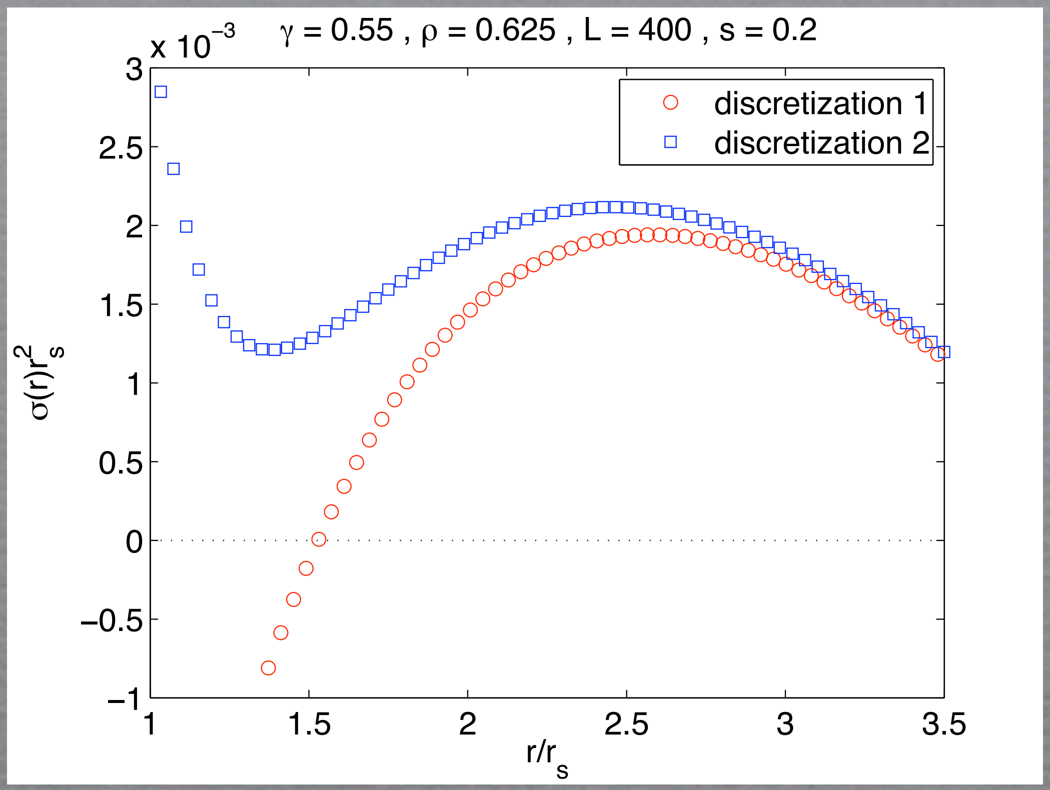
Long distance

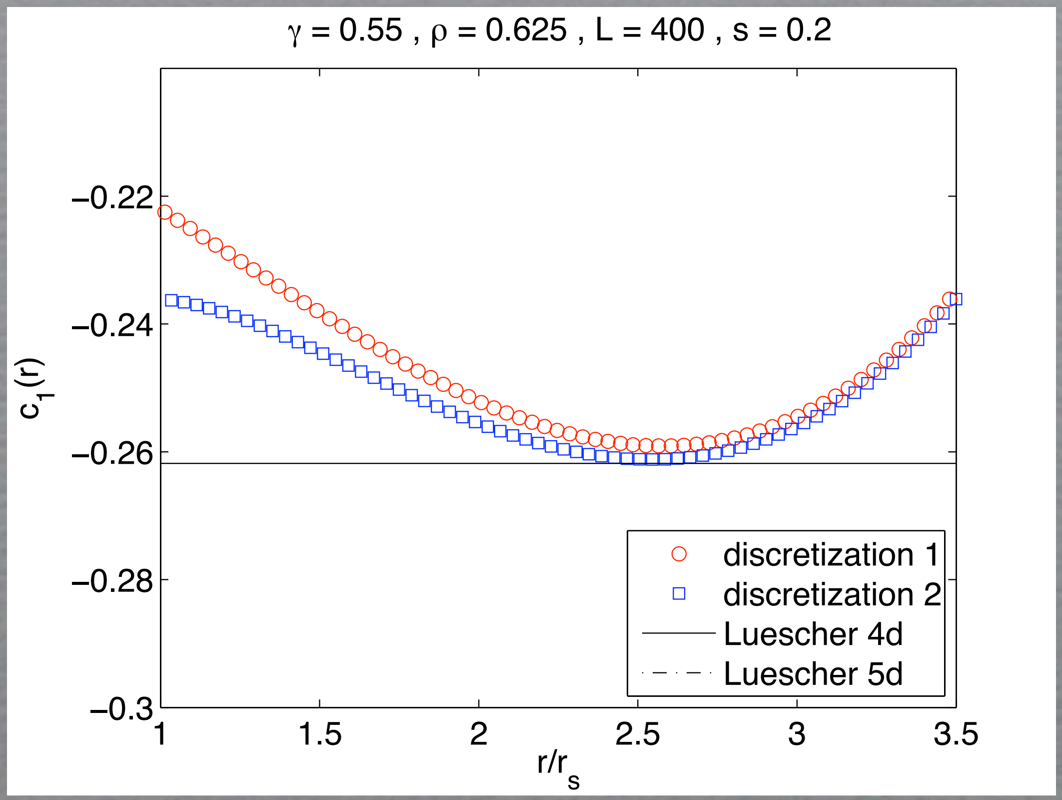
Luescher & Weisz:

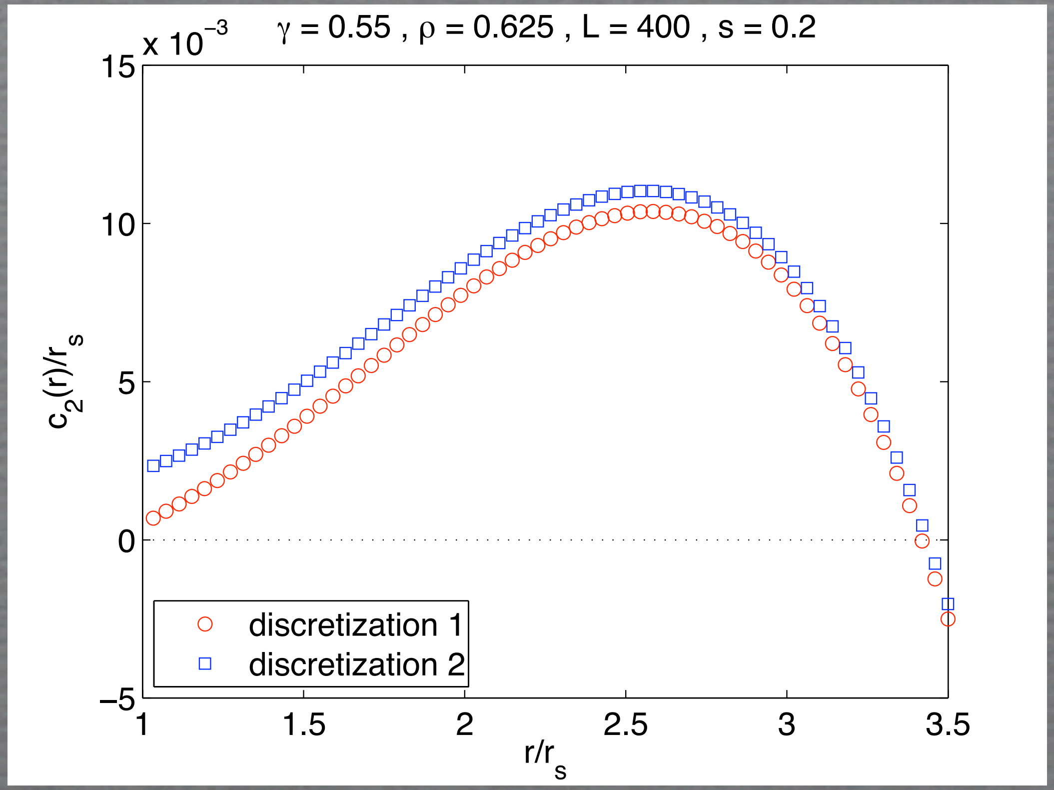


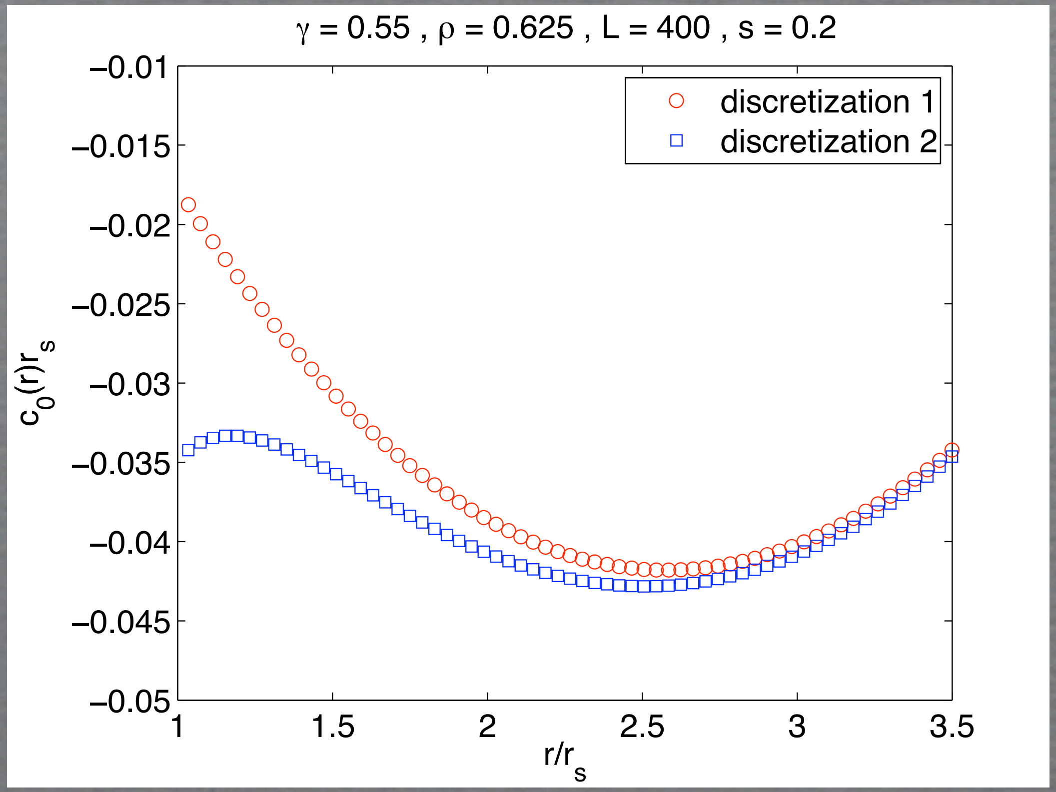
simultaneous plateaus for $r/r_s \in [2.15\,,\,2.80]$

ansatz: $$V(r) = \mu + \sigma r + c_0 \log(r) + \frac{c_1}{r} + \frac{c_2}{r^2}$$

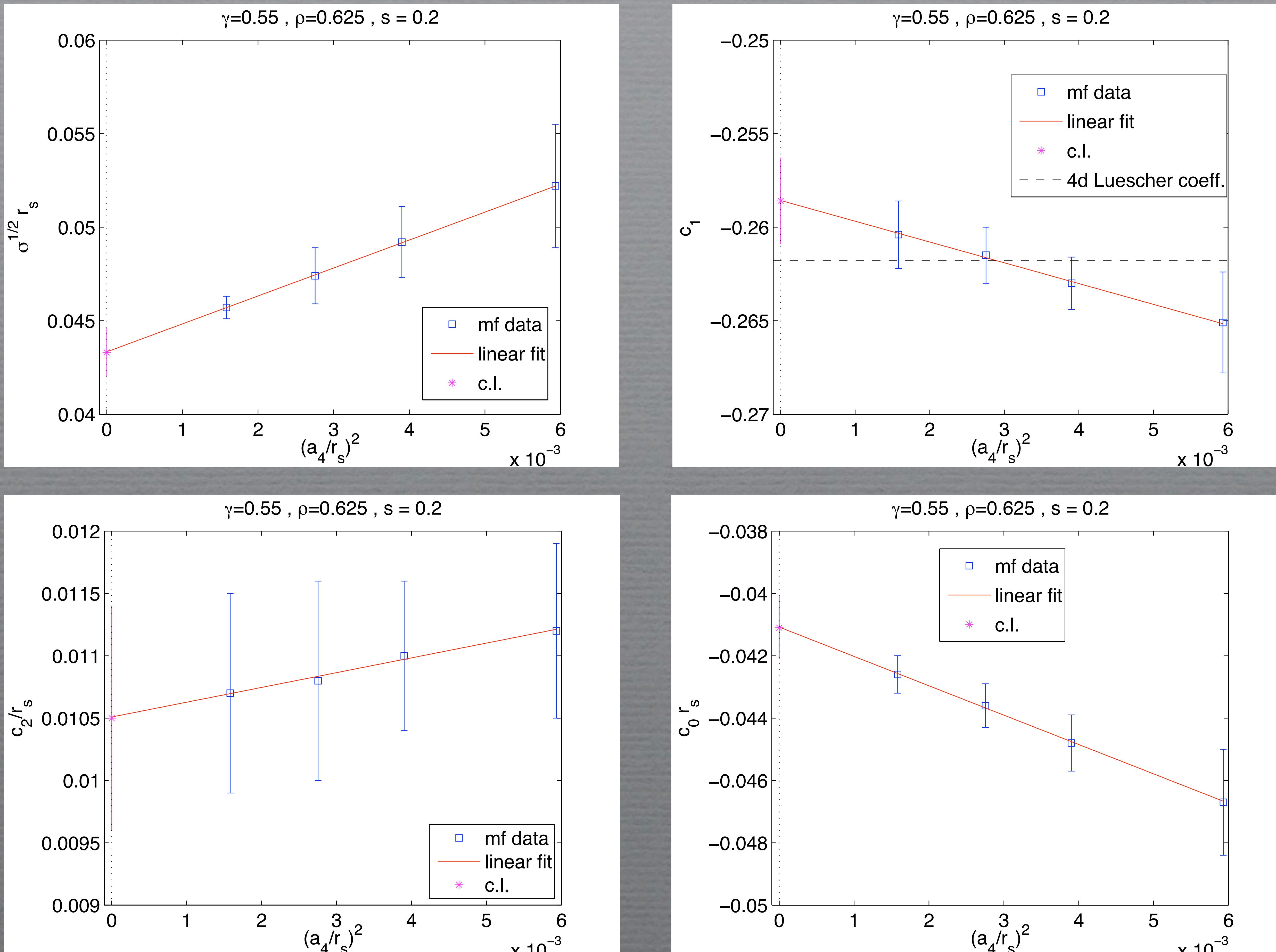


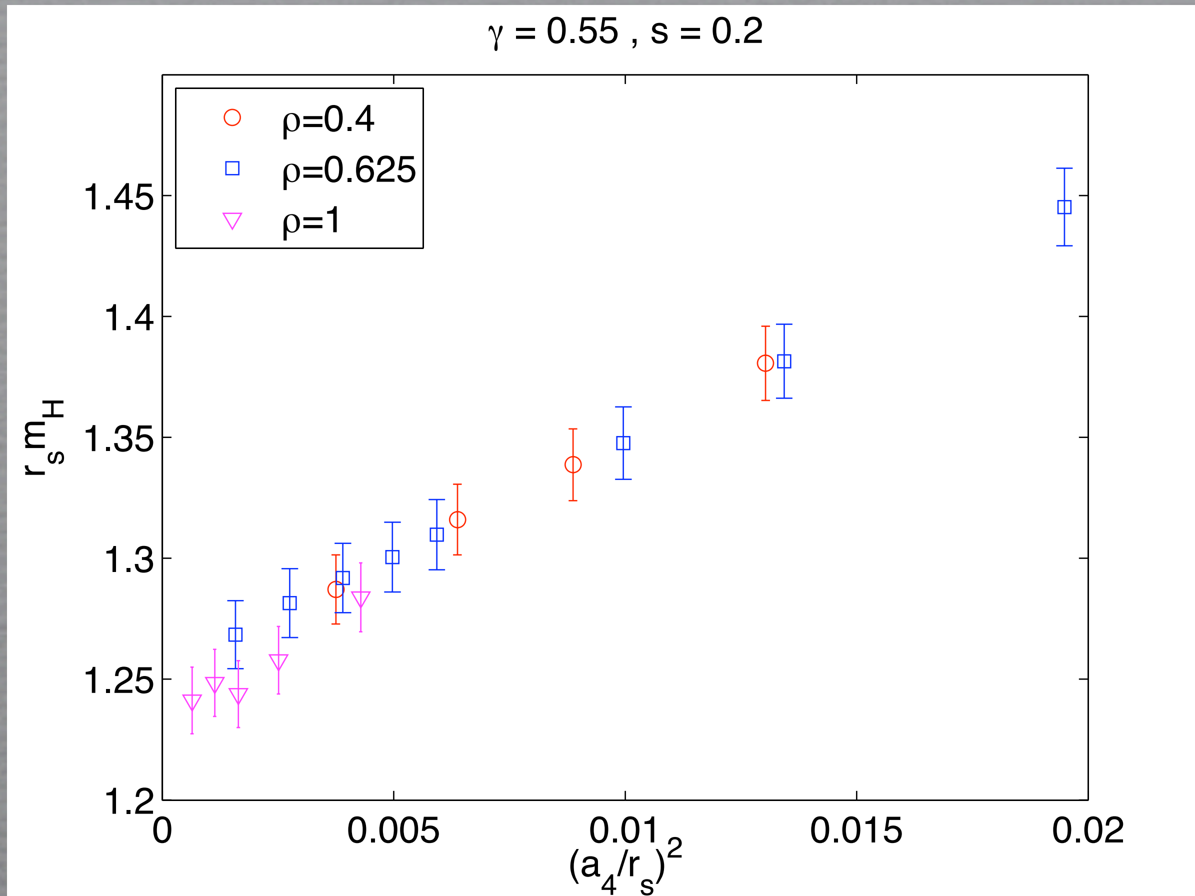


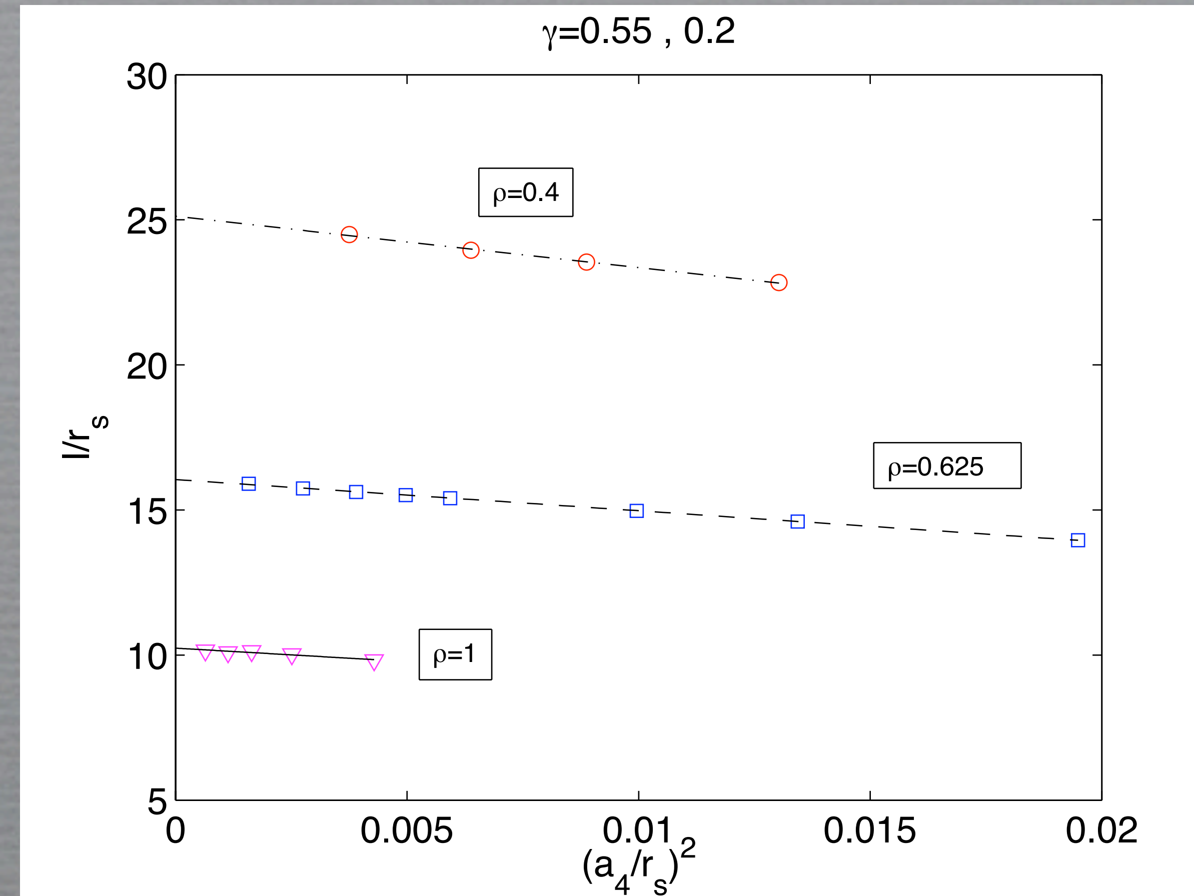




continuum limit: $(L \longrightarrow \infty, \ \beta \longrightarrow \beta_c)|_{\gamma,\rho=\text{const}} \iff$ continuum limit







$r_s m_H$ has a ρ independent limit

ρ controls the size of the box

There are limits on the physical size (for a given L):

- The box cannot be arbitrarily large: $\rho < \rho_{\rm inst}$
- The box cannot be arbitrarily small: $\rho > \rho_{\rm dr}$

Observations

$$\sqrt{\sigma}\, r_s = 0.0433(13) \quad \longrightarrow \quad \Lambda_{q\bar{q}}/\sqrt{\sigma} \simeq 6.4$$

compare to: $\Lambda_{q\bar{q}}^{(\mathrm{YM})}/\sqrt{\sigma} \simeq 0.68(10)$ why so small?

from the figures: $\sqrt{\sigma}\, l = 0.69$ the box is barely large enough

but it is large enough: $c_1 = -0.2586(23)$

the 4d Luscher term! $-\dfrac{\pi}{12} \simeq -0.261$

to increase the box size: $$\beta = \beta_c \frac{1}{1 - k\left(\frac{c_L}{\rho L}\right)^2}$$

Conclusions

1. The non-perturbative regime of 4d gauge theories can be probed analytically by the mean-field expansion in 5d

2. The phase diagram has a 2nd order phase transition where the system reduces dimensionally to an array of non-interacting “3-branes” and where the continuum limit can be taken

3. In a dimensionally reduced phase there are effects of asymtptic freedom, confinement and of the associated string

Outlook

1. On the interval (i.e. the orbifold) the Higgs Mechanism can be attacked by such methods (work in progress with F. Knechtli and K. Yoneyama).

2. Monte Carlo simulations can be performed to compare with the mean-field results (work in progress by F. Knechtli and A. Rago).

3. If the general principle is correct one should be able to apply this method to describe any (high enough dimensional) non-renormalizable system, say gravity.
Perhaps a different approach to gravity, ala Horava-Lifshitz...

Appendix

The definitions of the various couplings:

$$\alpha = \frac{g^2}{4\pi} \qquad \alpha_{q\bar{q}}(1/r) = -\frac{4}{3}c_1(r) \qquad g^2 = -(16\pi/3)\, c_1$$

The 2-loop RG evolution for the G-G theory:

$$\Lambda_{q\bar{q}}\, r = \left(b_0 g^2\right)^{-b_1/(2b_0^2)} e^{-1/(2b_0 g^2)}$$

$$b_0 = 7/(4\pi)^2 \qquad b_1 = (115/3)/(4\pi)^4$$

The Sommer scale:

$$r_{\rm s}^2 F_4(r_{\rm s}) = s = 0.2$$

Discretization formulae:

$$\overline{V}'(x) = \{\overline{V}(x+1) - \overline{V}(x-1)\}/2$$

$$\overline{V}''(x) = \overline{V}(x+1) + \overline{V}(x-1) - 2\overline{V}(x)$$

$$\overline{V}'''(x) = \overline{V}(x+2) - \overline{V}(x-2) - 2[\overline{V}(x+1) - \overline{V}(x-1)]/2$$

$$\overline{V}''''(x) = \overline{V}(x+2) + \overline{V}(x-2) - 4[\overline{V}(x+1) + \overline{V}(x-1)] + 6\overline{V}(x)$$

The short distance coefficients: $V(r) = \mu + \frac{c_1}{r} + \frac{c_2}{r^2}$

$$c_1(x) = 2x^3\overline{V}''(x) + 1/2x^4\overline{V}'''(x)$$

$$\overline{c_2}(x) = -1/2x^4\overline{V}''(x) - 1/6x^5\overline{V}'''(x)$$

The long distance coefficients: $V(r) = \mu + \sigma r + \frac{c}{r} + \frac{d}{r^2} + l\log(r)$

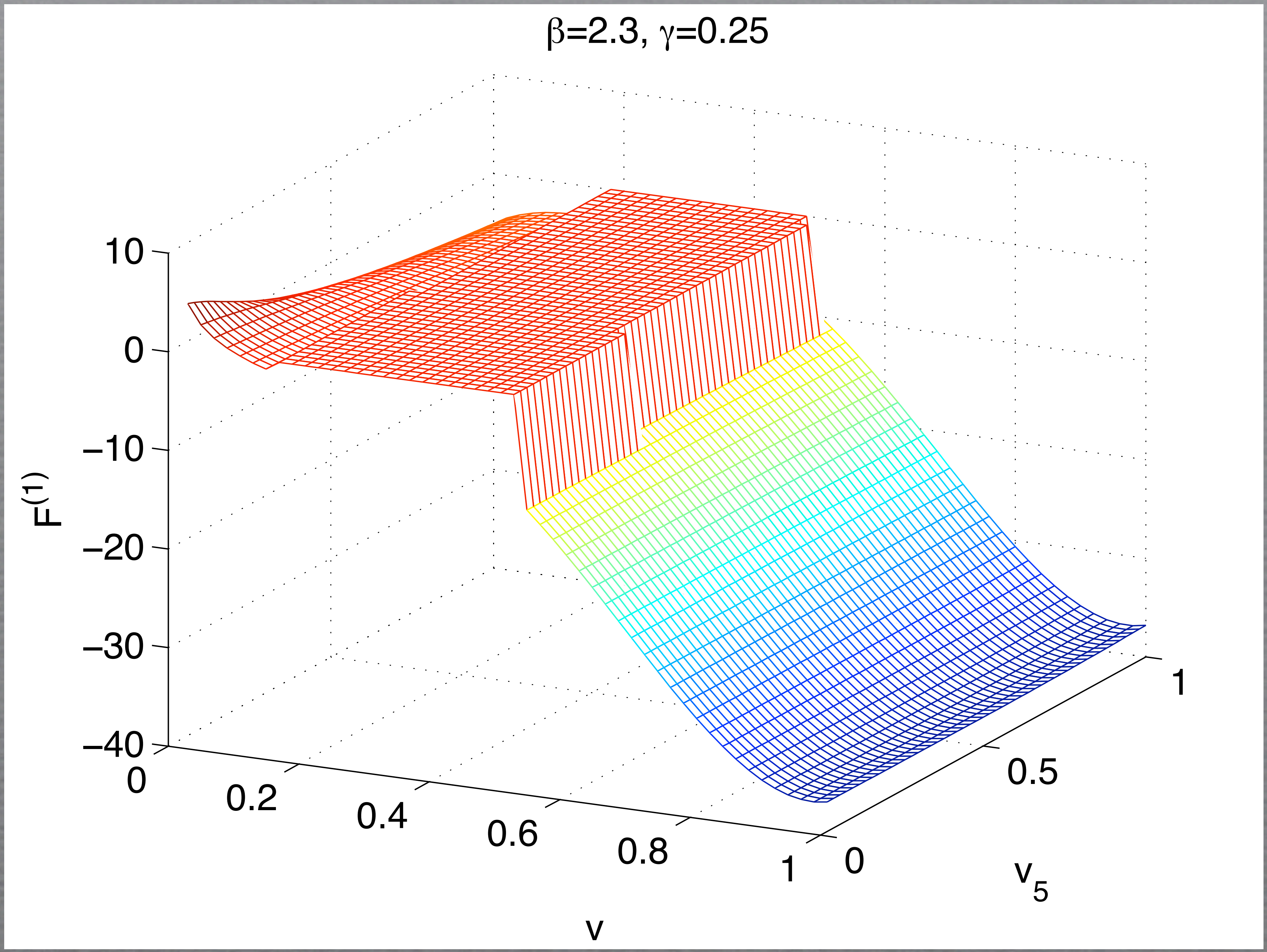
$$\overline{\sigma}(x) = \overline{V}'(x) + 3x\overline{V}''(x) + 3/2x^2\overline{V}'''(x) + 1/6x^3\overline{V}''''(x)$$

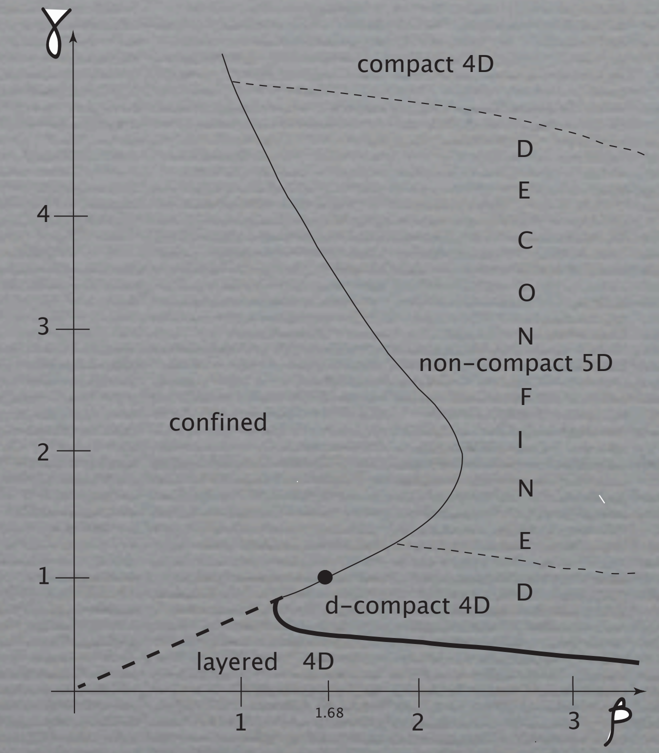
$$\overline{c_0}(x) = -6x^2\overline{V}''(x) - 4x^3\overline{V}'''(x) - 1/2x^4\overline{V}''''(x)$$

$$c_1(x) = -4x^3\overline{V}''(x) - 7/2x^4\overline{V}'''(x) - 1/2x^5\overline{V}''''(x)$$

$$c_2(x) = 1/2x^4\overline{V}''(x) - 1/2x^5\overline{V}'''(x) + 1/12x^5\overline{V}''''(x)$$

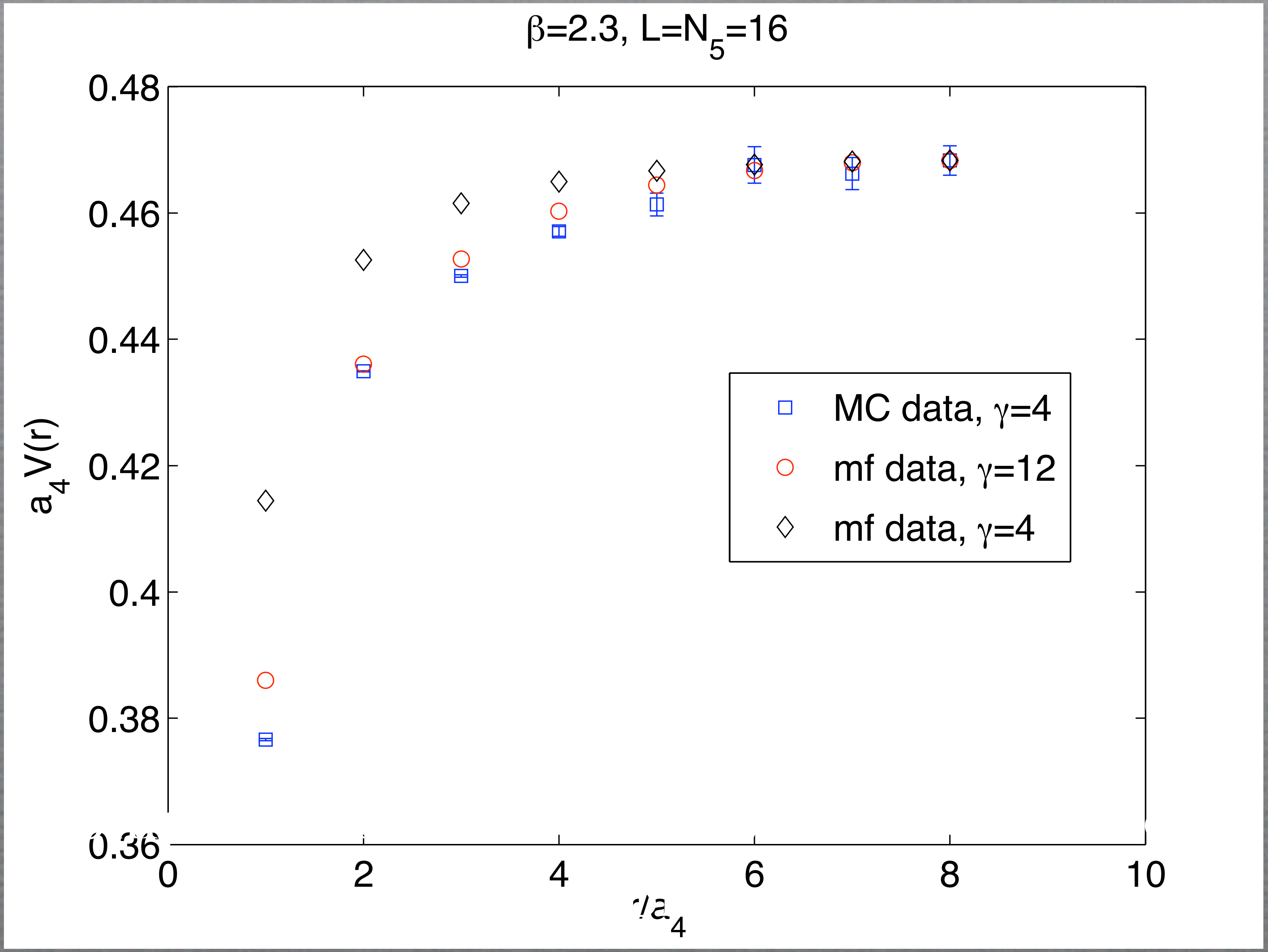
The d-compact phase is stable:





The compact phase (for small β) is unstable to this order

Comparison Mean-Field vs Monte Carlo: (sample)



MC data generated by M. Luz