ERG for a Yukawa Theory in 3 Dimensions

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Abstract

We consider a theory of N Majorana fermions and a real scalar in the 3 dimensional Euclid space. We construct the RG flow of three parameters using ERG. We look for a non-trivial fixed point by loop expansions. For $N = 1$, we compare the critical exponents with those of the Wess-Zumino model at the fixed point. We need to improve the numerical accuracy, perhaps, by going to 2-loop.

Plan of the talk

1. Spinors in $D = 3$
2. A Yukawa model in $\mathbf{R}^3$
3. ERG formalism (a quick review)
4. RG flow for the Yukawa model
5. 1-loop results
6. Comparison with the Wess-Zumino model
7. Summary and conclusions

Spinors in $D = 3$

1. **Minkowski space**

   (a) We can choose the gamma matrices pure imaginary:

$$\gamma^0 = \sigma_y, \quad \gamma^1 = i\sigma_x, \quad \gamma^2 = i\sigma_z \quad \text{satisfy} \quad \{\gamma^\mu \ , \ \gamma^\nu\} = 2\eta^{\mu\nu}\mathbf{1}_2$$

   (b) A Lorentz transformation transforms a two-component spinor ψ as

$$\psi \longrightarrow A\psi \quad \text{where} \quad A \in SL(2, R) \quad \textbf{real!}$$

   Its complex conjugate $\bar{\psi} \equiv \psi^\dagger \gamma^0$ transforms as

$$\bar{\psi} \longrightarrow \bar{\psi} A^{-1}$$

(c) Given $\psi = \frac{1}{\sqrt{2}}(u + iv)$, where u, v are Majorana (real) spinors,

$$\bar{\psi} \underbrace{(i\gamma^\mu \partial_\mu - m)}_{\text{real}} \psi = \frac{1}{2}\Big(\tilde{u}(i\gamma^\mu\partial_\mu - m)u + \tilde{v}(i\gamma^\mu\partial_\mu - m)v\Big)$$

where $\tilde{u} \equiv u^T \sigma_y$ transforms as $\tilde{u} \to \tilde{u} A^{-1}$. $(\sigma_y A^T \sigma_y = A^{-1})$

(d) **Parity**

$$\begin{cases} \psi(x,y,t) & \longrightarrow & -i\gamma^1\psi(-x,y,t) & = & \sigma_x \psi(-x,y,t) \\ \tilde{\psi}(x,y,t) & \longrightarrow & \tilde{\psi}(-x,y,t) i\gamma^1 & = & \tilde{\psi}(-x,y,t)(-)\sigma_x \end{cases}$$

forbids the mass term $\tilde{\psi}\psi$.

2. **Euclid space**

(a) A spatial rotation transforms a two-component spinor ψ as

$$\psi \longrightarrow U\psi \quad \text{where} \quad U \in SU(2) \quad \textbf{complex!}$$

and $\tilde{\psi} \equiv \psi^T \sigma_y$ as

$$\tilde{\psi} \longrightarrow \tilde{\psi} U^{-1}$$

(b) Though a "Majorana" spinor ψ is not real, the action

$$\int d^3x \, \frac{1}{2} \tilde{\psi} \left(\vec{\sigma} \cdot \nabla + m \right) \psi$$

makes sense for the anticommuting ψ.

(c) **Parity**

$$\begin{cases} \psi(x,y,z) \longrightarrow \sigma_y \psi(x,-y,z) \\ \tilde{\psi}(x,y,z) \longrightarrow \tilde{\psi}(x,-y,z)(-)\sigma_y \end{cases}$$

forbids the mass term $\tilde{\psi}\psi$.

(d) **Parity**' — Parity & π-rotation around the y-axis gives

$$\begin{cases} \psi(x,y,z) \longrightarrow i\psi(-x,-y,-z) \\ \tilde{\psi}(x,y,z) \longrightarrow i\tilde{\psi}(-x,-y,-z) \end{cases}$$

A Yukawa model in $\mathbf{R}^3$

1. The classical lagrangian

$$\mathcal{L} = \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}\sum_{I=1}^{N} \tilde{\chi}_I \vec{\sigma}\cdot\nabla\chi_I + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + g\,\phi\frac{1}{2}\sum_{I=1}^{N}\tilde{\chi}_I\chi_I$$

   where ϕ is a real scalar, and $\chi_I\ (I = 1, \cdots, N)$ are "Majorana" spinors.

2. three dimensionful parameters — $[m^2] = 2$, $[\lambda] = 1$, $[g] = 1/2$

3. A fermion mass term is forbidden by the $\mathbf{Z}_2$ invariance (parity):

$$\begin{cases} \phi(\vec{x}) & \longrightarrow & -\phi(-\vec{x}) \\ \chi_I(\vec{x}) & \longrightarrow & i\chi_I(-\vec{x}) \end{cases}$$

4. Classical phase analysis

$$\begin{cases} m^2 > 0 & \mathbf{Z}_2 \, \text{exact} & \langle \phi \rangle = 0, \, m_\chi = 0 \\ m^2 < 0 & \mathbf{Z}_2 \, \text{broken} & \langle \phi \rangle \neq 0, \, m_\chi \neq 0 \end{cases}$$

5. Related models with the same $\mathbf{Z}_2$ invariance

(a) $N = 1$ — Wess-Zumino model (1 SUSY), classically $\lambda = 3g^2$

$$\mathcal{L}_{WZ} = \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} \tilde{\chi} \vec{\sigma} \cdot \nabla \chi + g \; \phi \frac{1}{2} \tilde{\chi} \chi + \frac{1}{8} g^2 \left(\phi^2 - v^2 \right)^2$$

Classical phase analysis

	$\mathbf{Z}_2$	SUSY	$\langle \phi \rangle$	masses
$v^2 < 0$	exact	broken	0	$m_\chi = 0$, $m_\phi^2 = \frac{g^2}{2}(-v^2)$
$v^2 > 0$	broken	exact	$\pm v$	$m_\chi = m_\phi = g\|v\|$

(b) $N \geq 2$ — Gross-Neveu model with $O(N)$ invariance

$$\mathcal{L}_{GN} = \frac{1}{2} \sum_I \tilde{\chi}_I \vec{\sigma} \cdot \nabla \chi_I - \frac{g_0}{2N} \left(\frac{1}{2} \sum_I \tilde{\chi}_I \chi_I \right)^2$$

Expectation

	$\mathbf{Z}_2$	$\left\langle \sum_I \frac{1}{2} \tilde{\chi}_I \chi_I \right\rangle$	m_χ
$g_0 > g_{0,cr}$	exact	0	$= 0$
$g_0 < g_{0,cr}$	broken	$\neq 0$	$\neq 0$

O(N) unbroken

6. We wish to do two things:
   (a) construct the RG flow of m^2, λ, g for the Yukawa model,
   (b) find a non-trivial fixed point and obtain the critical exponents there.

7. The non-trivial fixed point is accessible from the gaussian fixed point.

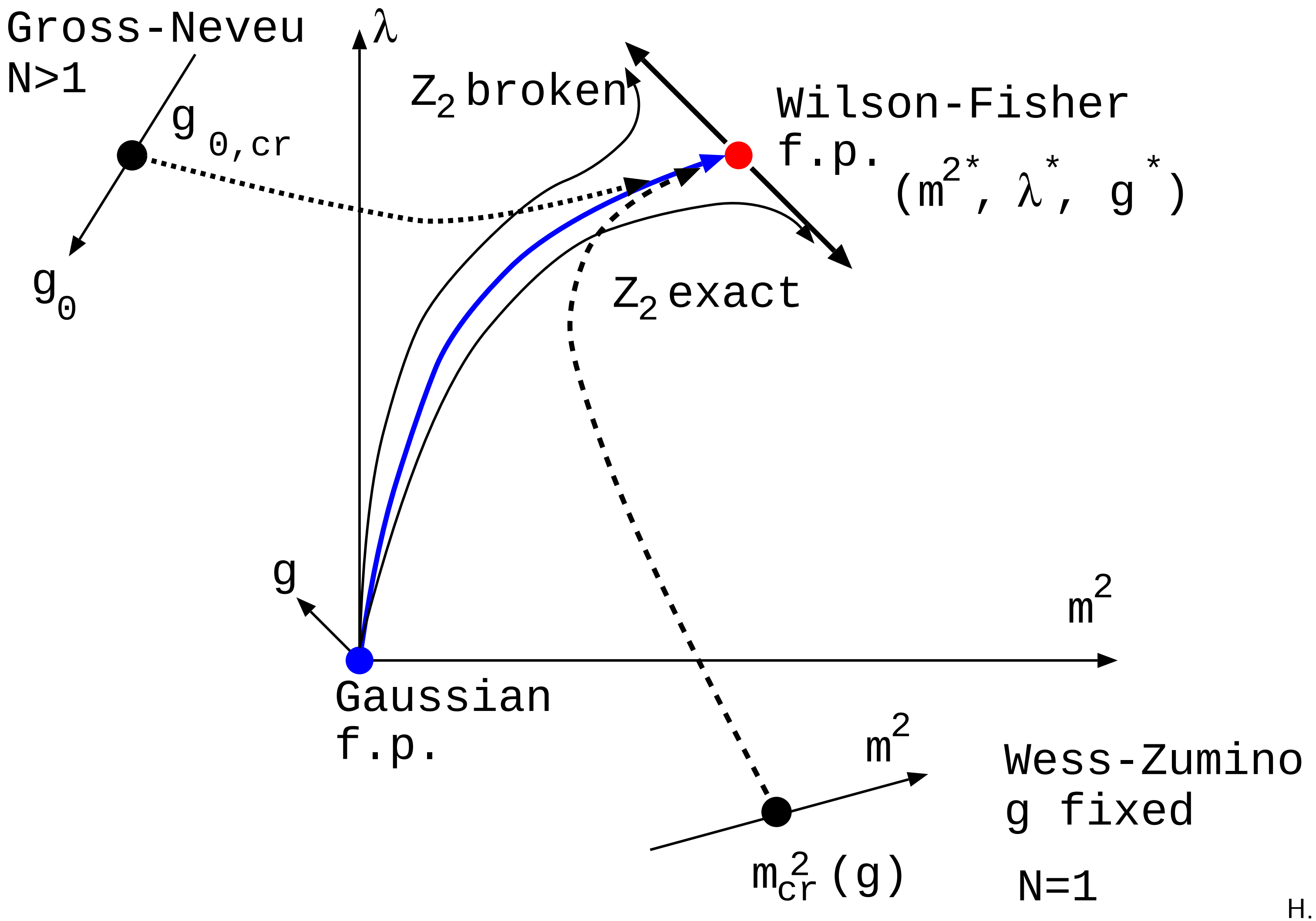
$$\Downarrow$$

Perturbation theory is applicable.

8. Motivated by works by Wipf, Synatschke-Czerwonka, Braun, ... on SUSY models in 1 & 2 dimensions.

9. Talks on related subjects by Wipf, Synatschke-Czerwonka, Scherer.

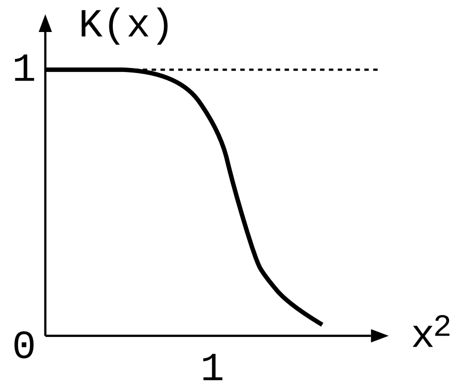
RG flows for the Yukawa theory with N Majorna fermions



ERG formalism (a quick review)

1. **Wilson action** with a momentum cutoff Λ

$$S_\Lambda = -\frac{1}{2}\int_p \frac{p^2}{K\left(\frac{p}{\Lambda}\right)}\phi(p)\phi(-p) + S_{I,\Lambda}$$



2. **the ERG differential equation** by J. Polchinski (1983)

$$-\Lambda\frac{\partial}{\partial\Lambda}S_{I,\Lambda} = \frac{1}{2}\int_p \frac{\Delta(p/\Lambda)}{p^2}\left\{\frac{\delta S_{I,\Lambda}}{\delta\phi(-p)}\frac{\delta S_{I,\Lambda}}{\delta\phi(p)} + \frac{\delta^2 S_{I,\Lambda}}{\delta\phi(-p)\delta\phi(p)}\right\}$$

where $\Delta(p/\Lambda) \equiv \Lambda \frac{\partial K\left(\frac{p}{\Lambda}\right)}{\partial \Lambda}$.

3. Λ independence of the correlation functions

$$\begin{cases} \langle\, \phi(p)\phi(-p)\,\rangle & \equiv & \frac{1}{K\left(\frac{p}{\Lambda}\right)^2} \langle\, \phi(p)\phi(-p)\,\rangle_{S_\Lambda} + \frac{1-1/K\left(\frac{p}{\Lambda}\right)}{p^2} \\ \langle\, \phi(p_1)\cdots\phi(p_n)\,\rangle & \equiv & \prod_{i=1}^{N} \frac{1}{K\left(\frac{p_i}{\Lambda}\right)} \cdot \langle\, \phi(p_1)\cdots\phi(p_N)\,\rangle_{S_\Lambda} \end{cases}$$

4. Vertices defined by

$$\begin{aligned} S_\Lambda &= \sum_{n=1}^{\infty} \frac{1}{(2n)!} \int_{p_1,\cdots,p_{2n}} \delta(p_1 + \cdots p_{2n}) \\ &\qquad \times u_{2n}(\Lambda; p_1, \cdots, p_{2n}) \phi(p_1) \cdots \phi(p_{2n}) \end{aligned}$$

5. S_Λ determined uniquely by the ERG diff eq & two sets of conditions: one at $\Lambda = \mu$, another at $\Lambda \to \infty$.

   (a) conditions at $\Lambda = \mu$ introduce m^2 and λ:

$$\left\{ \begin{array}{ccc} u_2(\mu; 0, 0) & = & -m^2 \\ \frac{\partial}{\partial p^2} u_2(\mu; p, -p)\Big|_{p^2=0} & = & -1 \\ u_4(\mu; 0, \cdots, 0) & = & -\lambda \end{array} \right.$$

   (b) asymptotic conditions for $\Lambda \to \infty$ for renormalizability

$$\left\{ \begin{array}{rcl} u_2(\Lambda; p, -p) & \stackrel{\Lambda\to\infty}{\longrightarrow} & \text{linear in } p^2 \\ u_4(\Lambda; p_1, \cdots, p_4) & \stackrel{\Lambda\to\infty}{\longrightarrow} & \text{independent of } p_i \\ u_{2n\geq 6}(\Lambda; p_1, \cdots, p_{2n}) & \stackrel{\Lambda\to\infty}{\longrightarrow} & 0 \end{array} \right.$$

   This guarantees that the flow starts from the gaussian fixed point.

6. μ **dependence**

$$-\mu \frac{\partial S_\Lambda}{\partial \mu} = \beta \mathcal{O}_\lambda + \beta_m \mathcal{O}_m + \gamma \mathcal{N}$$

where

$$\begin{cases} \mathcal{O}_m & \equiv & -\partial_{m^2} S_\Lambda \\ \mathcal{O}_\lambda & \equiv & -\partial_\lambda S_\Lambda \\ \mathcal{N} & \equiv & -\int_p \left[\phi(p) \frac{\delta S_\Lambda}{\delta \phi(p)} + \frac{K\left(\frac{p}{\Lambda}\right)\left(1-K\left(\frac{p}{\Lambda}\right)\right)}{p^2} \left\{ \frac{\delta S_\Lambda}{\delta \phi(p)} \frac{\delta S_\Lambda}{\delta \phi(-p)} + \frac{\delta^2 S_\Lambda}{\delta \phi(p) \delta \phi(-p)} \right\} \right] \end{cases}$$

$\mathcal{N}$ counts ϕ:

$$\langle\, \mathcal{N}\, \phi(p_1) \cdots \phi(p_n) \,\rangle = n \,\langle\, \phi(p_1) \cdots \phi(p_n) \,\rangle$$

RG equation

$$\left(-\mu \frac{\partial}{\partial \mu} + \beta \partial_\lambda + \beta_m \partial_{m^2} \right) \langle\, \phi(p_1) \cdots \phi(p_n) \,\rangle = n\gamma \,\langle\, \phi(p_1) \cdots \phi(p_n) \,\rangle$$

7. **dimensional analysis**

$$\left(\Lambda\frac{\partial}{\partial\Lambda} + \mu\frac{\partial}{\partial\mu} + 2m^2\partial_{m^2} + \lambda\partial_\lambda\right) S_\Lambda = \int_p \left(p_i\frac{\partial\phi(p)}{\partial p_i} + \frac{5}{2}\phi(p)\right)\frac{\delta S_\Lambda}{\delta\phi(p)}$$

8. Combining the ERG diff eq, μ dependence, and dimensional analysis, we obtain

$$\begin{aligned}&\left\{(\lambda+\beta)\partial_\lambda + (2m^2+\beta_m)\partial_{m^2}\right\} S(m^2,\lambda)\\ &= \int_p \Bigg[\left\{p_i\frac{\partial\phi(p)}{\partial p_i} + \left(\frac{5}{2} - \gamma + \frac{\Delta(p/\mu)}{K(p/\mu)}\right)\phi(p)\right\}\frac{\delta S}{\delta\phi(p)}\\ &\quad + \frac{\Delta(p/\mu) - 2\gamma K(1-K)}{p^2}\frac{1}{2}\left\{\frac{\delta S}{\delta\phi(p)}\frac{\delta S}{\delta\phi(-p)} + \frac{\delta^2 S}{\delta\phi(p)\delta\phi(-p)}\right\}\Bigg]\end{aligned}$$

where we have set $\Lambda = \mu$ (fixed).

9. RG flow in the parameter space

$$\begin{cases} \frac{d}{dt}m^2 &= 2m^2 + \beta_m(m^2, \lambda) \\ \frac{d}{dt}\lambda &= \lambda + \beta(m^2, \lambda) \end{cases}$$

10. **Scaling formula**

$$\begin{aligned} &\left\langle\, \phi(p_1 \mathrm{e}^{\Delta t}) \cdots \phi(p_n \mathrm{e}^{\Delta t}) \,\right\rangle_{m^2 \mathrm{e}^{2\Delta t}(1+\Delta t \cdot \beta_m)\ ,\ \lambda \mathrm{e}^{\Delta t}(1+\Delta t \cdot \beta)} \\ &= \mathrm{e}^{\Delta t \left\{3 + n\left(-\frac{5}{2}+\gamma\right)\right\}} \cdot \left\langle\, \phi(p_1) \cdots \phi(p_n) \,\right\rangle_{m^2, \lambda} \end{aligned}$$

11. **Universality** — The dependence on the choice of K is absorbed into the parameters m^2, λ. Hence, the critical exponents are universal.

RG flow for the Yukawa model

1. RG flow

$$\begin{cases} \frac{d}{dt}m^2 &= 2m^2 + \beta_m(m^2, \lambda, g) \\ \frac{d}{dt}\lambda &= \lambda + \beta_\lambda(\lambda, g) \\ \frac{d}{dt}g &= \frac{1}{2}g + \beta_g(\lambda, g) \end{cases}$$

2. A fixed point (m^{2*}, λ^*, g^*) is determined by

$$\begin{cases} 2m^{2*} + \beta_m(m^{2*}, \lambda^*, g^*) &= 0 \\ \lambda^* + \beta_\lambda(\lambda^*, g^*) &= 0 \\ \frac{1}{2}g^* + \beta_g(\lambda^*, g^*) &= 0 \end{cases}$$

3. $y_E = \frac{1}{\nu}$ determined by the linearized RG equation at the fixed point:

$$\frac{d}{dt}\left(m^2 - m^{2*}\right) = y_E\left(m^2 - m^{2*}\right)$$

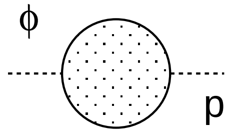
Hence,

$$y_E = 2 + \frac{\partial}{\partial m^2}\beta_m\Big|_{m^{2*},\lambda^*,g^*}$$

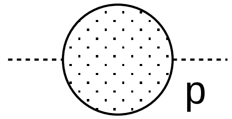
4. The anomalous dimensions γ_ϕ^*, γ_χ^* determined by

$$\gamma_\phi^* = \gamma_\phi(\lambda^*, g^*), \quad \gamma_\chi^* = \gamma_\chi(\lambda^*, g^*)$$

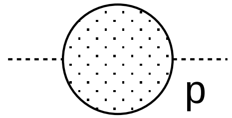
5. parametrization and normalization determined through the low momentum expansions of the vertices



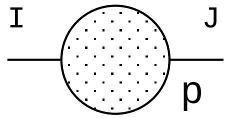
$$\Big|_{p^2=m^2=0} = 0$$

$\dfrac{\partial}{\partial m^2}$ 

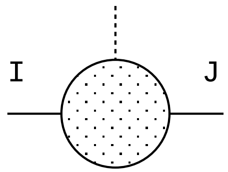
$$\Big|_{p^2=m^2=0} = -1$$

$\dfrac{\partial}{\partial p^2}$ 

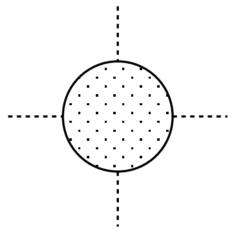
$$\Big|_{p^2=m^2=0} = -1$$



$$\Big|_{m^2=0} = -i\vec{p}\cdot\vec{\sigma}\delta_{IJ} + \mathrm{O}(p^2)$$



$$\Big|_{p_i=m^2=0} = -\frac{g}{\sqrt{N}}\delta_{IJ}$$



$$\Big|_{p_i=m^2=0} = -\lambda$$

6. The beta functions are computed from the RG equation

$$-\mu\frac{\partial}{\partial\mu}S_\Lambda = \beta_\lambda\left(-\partial_\lambda S_\Lambda\right) + \beta_g\left(-\partial_g S_\Lambda\right) + \beta_m \mathcal{O}_m + \gamma_\phi \mathcal{N}_\phi + \gamma_\chi \mathcal{N}_\chi$$

where $\mathcal{O}_m \simeq -\partial_{m^2} S_\Lambda$ since we have incorporated m^2 in the scalar propagator.

$$\begin{aligned}\mathcal{O}_m &= -\partial_{m^2} S_\Lambda \\ &\quad - \int_q \frac{K(1-K)}{(q^2+m^2)^2}\frac{1}{2}\left\{\frac{\delta S_\Lambda}{\delta\phi(q)}\frac{\delta S_\Lambda}{\delta\phi(-q)} + \frac{\delta^2 S_\Lambda}{\delta\phi(q)\delta\phi(-q)}\right\}\end{aligned}$$

1-loop results

1. 1-loop calculations give

$$\beta_m = \frac{1}{2}\lambda\mu \int_q \frac{\Delta}{q^2} - 2\frac{g^2}{N}\mu \int_q \frac{\Delta(1-K)}{q^2}$$
$$-m^2 \left[\frac{1}{2}\frac{\lambda}{\mu} \int_q \frac{\Delta}{q^4} + 2\frac{g^2}{N\mu} \int_q \frac{\Delta}{q^4} \left\{ \frac{1}{3}(1-K) + \frac{1}{6}(\Delta + \tilde{\Delta}) \right\} \right]$$

$$\beta_\lambda = -3\frac{\lambda^2}{\mu} \int_q \frac{\Delta(1-K)}{q^4} + 24\frac{g^4}{N\mu} \int_q \frac{\Delta(1-K)^3}{q^4}$$
$$-4\lambda\frac{g^2}{N\mu} \int_q \frac{\Delta}{q^4} \left\{ \frac{1}{3}(1-k) + \frac{1}{6}(\Delta + \tilde{\Delta}) \right\}$$

$$\beta_g = -g\frac{g^2}{N\mu}\left[3\int_q \frac{\Delta(1-K)^2}{q^4} + \int_q \frac{\Delta(1-K)}{q^4} \right.$$

$$+ \int_q \frac{\Delta}{q^4} \left\{ \frac{1}{3}(1-K) + \frac{1}{6}(\Delta + \tilde{\Delta}) \right\} \Big]$$

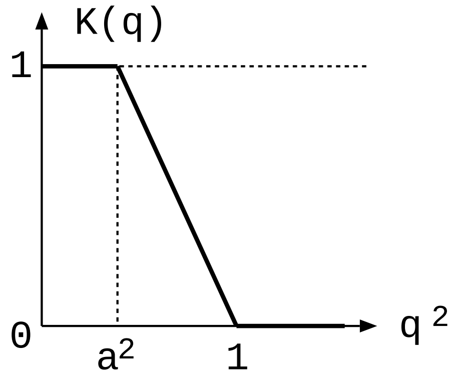
where $\tilde{\Delta}(q) \equiv -2q^2 \frac{d}{dq^2} \Delta(q) = \left(-2q^2 \frac{d}{dq^2} \right)^2 K(q)$.

The anomalous dimensions are given by

$$\begin{aligned} \gamma_\phi &= \frac{g^2}{\mu} \int_q \frac{1}{q^4} (1-K) \left(\frac{5}{6}\Delta + \frac{1}{6}\tilde{\Delta} \right) \\ \gamma_\chi &= \frac{g^2}{N\mu} \frac{1}{2} \int_q \frac{1}{q^4} \Delta (1-K) \end{aligned}$$

2. Choice of K with one parameter a for optimization

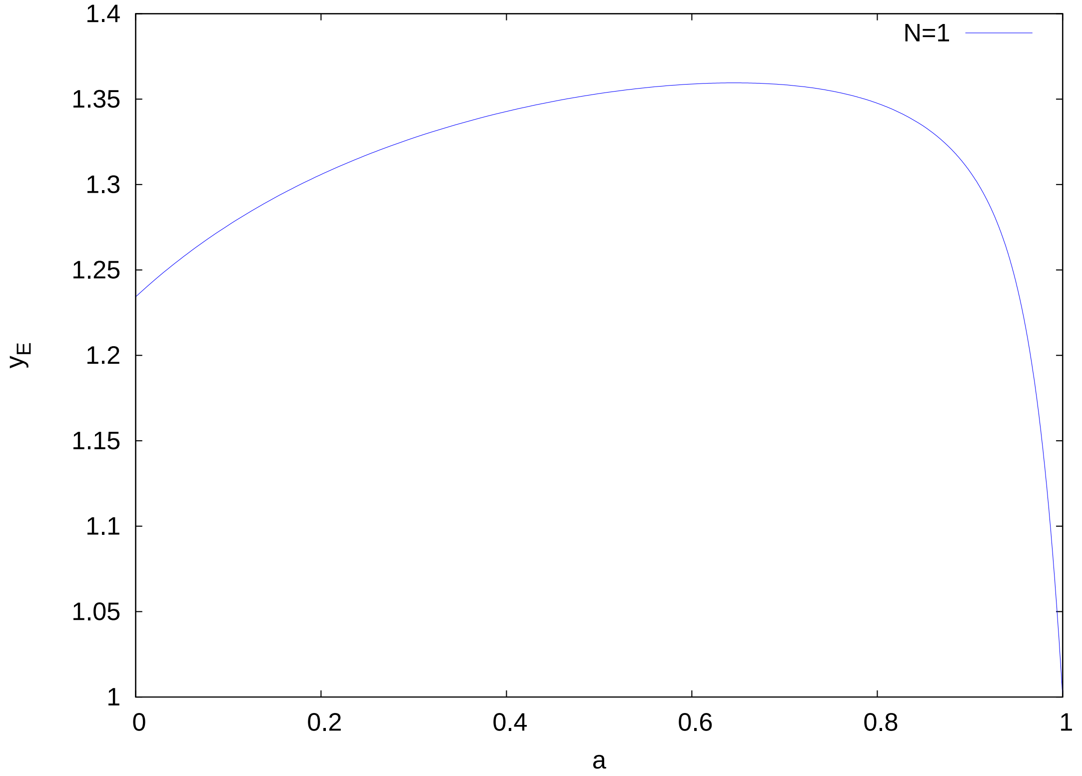
$$K(q) = \begin{cases} 1 & \text{if } 0 < q^2 < a^2 \\ \frac{1-q^2}{1-a^2} & \text{if } a^2 < q^2 < 1 \\ 0 & \text{if } 1 < q^2 \end{cases}$$



Two limits:

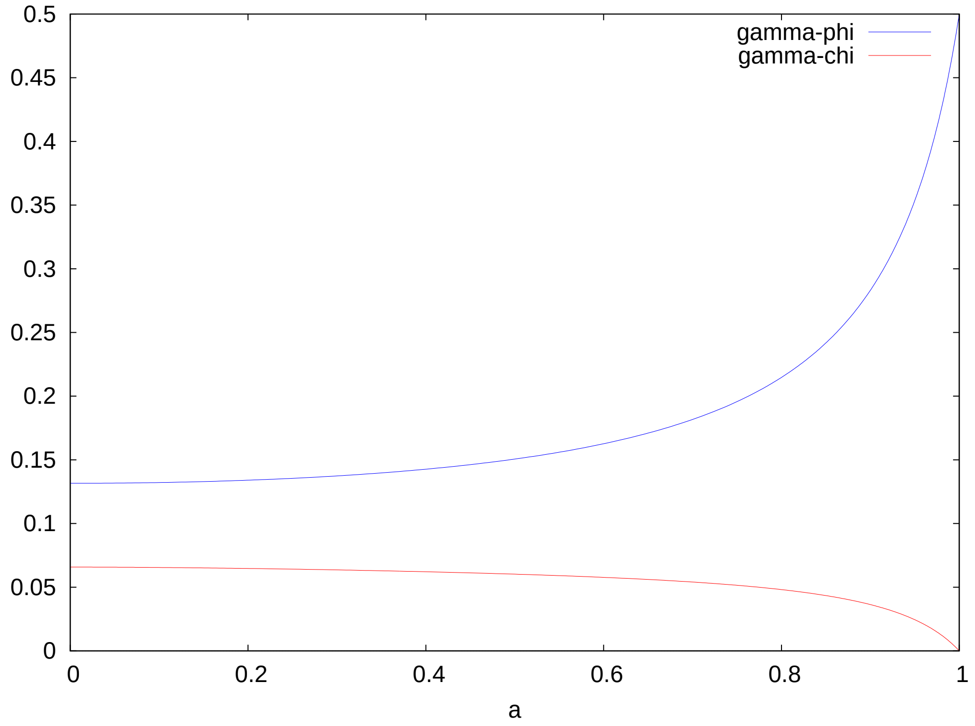
$$\begin{cases} \text{Litim} & a^2 = 0 \\ \text{Wegner-Houghton} & a^2 = 1 \end{cases}$$

3. a dependence of y_E for $N = 1$



Optimized at $a \simeq 0.646$, giving $y_E \simeq 1.36$.

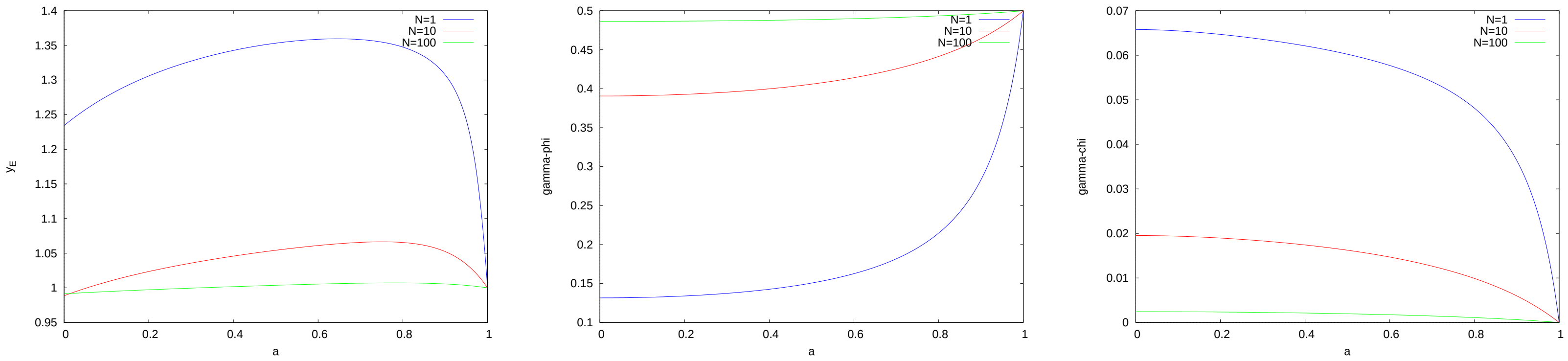
4. $\gamma_\phi^* = \gamma_\chi^*$ is expected from SUSY, but



$$\gamma_\phi^* \simeq 0.170, \quad \gamma_\chi^* \simeq 0.056 \quad \text{at } a = 0.646$$

5. N dependence

$$y_E \xrightarrow{N\to\infty} 1 \qquad \gamma_\phi^* \xrightarrow{N\to\infty} \frac{1}{2} \qquad \gamma_\chi^* \xrightarrow{N\to\infty} 0$$



The large N limits are independent of a.

The above results agree with the large N limit of the Gross-Neveu model.

Comparison with the Wess-Zumino model

1. Classical action with an auxiliary field

$$\mathcal{L} = \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}\tilde{\chi}\vec{\sigma}\cdot\nabla\chi + \frac{1}{2}F^2 + g\left\{ iF\frac{1}{2}\left(\phi^2 - v^2\right) + \phi\frac{1}{2}\tilde{\chi}\chi \right\}$$

where $v^2 = -\frac{2}{g^2}m^2$.

2. Linear SUSY transformation (arbitrary constant spinor ξ)

$$\begin{cases} \delta\phi &= \tilde{\chi}\xi \\ \delta\chi &= \left(\vec{\sigma}\cdot\nabla\phi - iF\right)\xi \\ \delta iF &= \nabla\tilde{\chi}\cdot\vec{\sigma}\xi \end{cases}$$

3. The supersymmetric $\phi i F + \frac{1}{2}\tilde{\chi}\chi$ is forbidden by the $\mathbf{Z}_2$

$$\left\{ \begin{array}{lcl} \phi(\vec{x}) & \longrightarrow & -\phi(-\vec{x}) \\ \chi(\vec{x}) & \longrightarrow & i\chi(-\vec{x}) \\ F(\vec{x}) & \longrightarrow & F(-\vec{x}) \end{array} \right.$$

4. R-invariance

$$g \to -g, \quad \phi \to -\phi, \quad F \to -F$$

5. ERG is consistent with the linear SUSY.

6. RG equation

$$-\mu \frac{\partial}{\partial \mu} S_\Lambda = \beta_g \left(-\partial_g S_\Lambda \right) + \beta_{v^2} \left(-\partial_{v^2} S_\Lambda \right) + \gamma \mathcal{N}$$

7. RG flow

$$\begin{cases} \frac{d}{dt} g &= \frac{1}{2} g + \beta_g \\ \frac{d}{dt} v^2 &= v^2 + \beta_{v^2} \end{cases}$$

8. 1-loop results

$$\begin{cases} \gamma &= \frac{1}{2} \frac{g^2}{\mu} \int_q \frac{\Delta(1-K)}{q^4} \\ \beta_g &= -\frac{g^3}{\mu} \left[3 \int_q \frac{\Delta(1-K)^2}{q^4} + \frac{1}{2} \int_q \frac{\Delta(1-K)}{q^4} \right] \\ \beta_{v^2} &= -\mu \int_q \frac{\Delta}{q^2} + v^2 \frac{g^2}{\mu} \left[3 \int_q \frac{\Delta(1-K)^2}{q^4} + \int_q \frac{\Delta(1-K)}{q^4} \right] \end{cases}$$

9. $y_E = \frac{1}{\nu}$ determined by

$$y_E = 1 + \frac{\partial}{\partial v^2} \beta_{v^2} \Big|_{v^{2*}, g^*}$$

10. At 1-loop, we have reproduced the relation found by Wipf, ...

$$\gamma^* + y_E = \frac{3}{2}$$

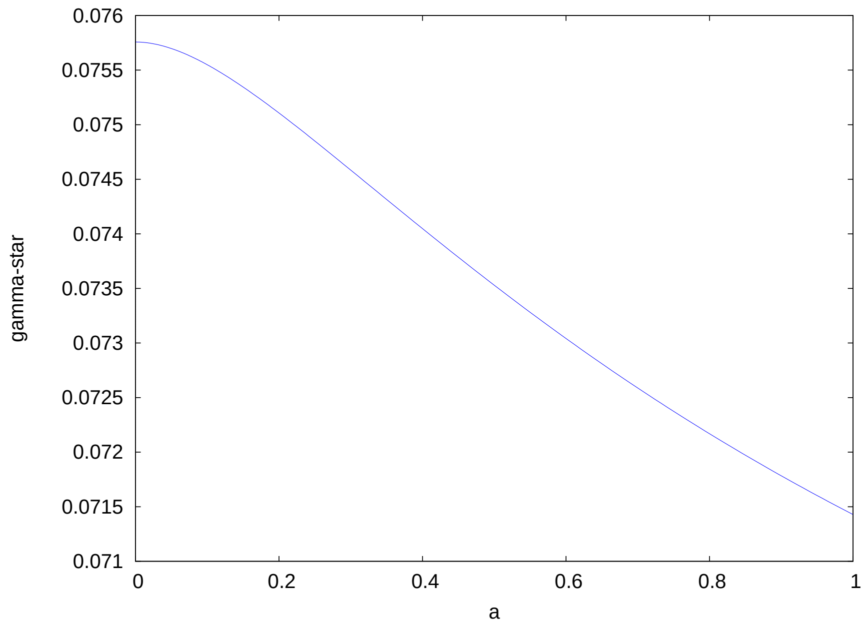
11. No optimization available, but γ^* depends little on a.

$$\gamma^* = \frac{5(2a^2 + 3a + 1)}{6(26a^2 + 33a + 11)}$$

Comparable with

$$\gamma^*_\chi \simeq 0.056$$

for the Yukawa model.



Summary and conclusions

	WZ	Y ($N=1$)	Y (large N)	GN (large N)
y_E	1.429-1.424	1.36	1	1
γ_ϕ^*	0.071-0.076	0.17	0.5	0.5
γ_χ^*	$=\gamma_\phi^*$	0.056	0	0

1. ERG gives an exact formulation of the RG flows for the Yukawa model in $D=3$.

2. Perturbative approximations give reasonable numerical results.

3. Hopefully two-loop calculations (in progress) will improve the numerical accuracy.