UV Behavior of the Einstein–Yang-Mills system

[J.-E. Daum, U. Harst and M. Reuter, JHEP 01 (2010) 084.]

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Introduction - Recent Results

Are there gravity corrections to the beta function of the Yang-Mills (YM) coupling constant?

- perturbative calculations by several authors:

  [Robinson and Wilczek (2006), Pietrykowski (2007), Toms (2007), Ebert, Plefka, Rodigast (2008)]

  - only the former find a non-trivial result
  - problematic either due to dimensional or cutoff regularization
- Tang and Wu (“loop regularization”) find a non-zero correction [Tang and Wu (2008)]

Introduction - The ERG Approach

We analyzed the running of the gauge coupling constant in the Asymptotic Safety scenario for quantum gravity.

Main tools:

- FRGE for the effective average action
- the background field method

Advantages:

- sensitive to quadratic divergences
- background field method preserves gauge invariance

Nevertheless:

- resulting beta-function may still be gauge fixing dependent

The Truncation

$$\Gamma_k = \Gamma_k^{\mathrm{EH}} + \Gamma_k^{\mathrm{YM}} + \Gamma_k^{\mathrm{gf}} + S_{\mathrm{gh}} = \breve{\Gamma}_k + S_{\mathrm{gh}}$$

with

$$\Gamma_k^{\mathrm{EH}}[g] = \frac{Z_N(k)}{16\pi \hat{G}} \int \mathrm{d}^4x \, \sqrt{g} \, \left(-R(g) + 2\bar{\lambda}(k)\right)$$

$$\Gamma_k^{\mathrm{YM}}[g, A] = \frac{Z_F(k)}{4 \, \hat{g}_{\mathrm{YM}}^2} \int \mathrm{d}^4x \, \sqrt{g} \, g^{\mu\rho} g^{\nu\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}$$

$$\Gamma_k^{\mathrm{gf}}[\bar{h}, \bar{a}; \bar{g}, \bar{A}] = \int \mathrm{d}^4x \, \sqrt{\bar{g}} \, \left(\frac{Z_N(k)}{2\alpha_{\mathrm{D}}} \bar{g}^{\mu\nu} \mathrm{F}_\mu \mathrm{F}_\nu + \frac{Z_F(k)}{2\alpha_{\mathrm{YM}}} \mathrm{G}^a \mathrm{G}^a \right)$$

Motivation:

- Γ_k^{EH} contains the all essential features of gravity close to the non-Gaussian fixed point (NGFP)
- in pure YM theory Γ_k^{YM} approximates the perturbative 2-loop result within a few percent

Gauge Fixing and Ghost Action

Gauge conditions:

$$\mathrm{F}_{\mu}(\bar{h};\bar{g}) = \frac{1}{\sqrt{16\pi\hat{G}}}\left(\delta_{\mu}^{\beta}\bar{g}^{\alpha\gamma}\bar{D}_{\gamma} - \frac{1}{2}\bar{g}^{\alpha\beta}\bar{D}_{\mu}\right)\bar{h}_{\alpha\beta}$$

$$\mathrm{G}^{a}(\bar{a};\bar{g},\bar{A}) = \hat{g}_{\mathrm{YM}}^{-1}\,\bar{g}^{\mu\nu}\bar{\mathcal{D}}_{\mu}\bar{a}_{\nu}^{a}$$

S_{gh} is then obtained by the Faddeev-Popov method

Further simplifications:

- neglect renormalization effects in the ghost sector
  $\longrightarrow$ fluctuations in S_{gh} are set to zero
- gauge parameters chosen to $\alpha_{\mathrm{D}} = 1 = \alpha_{\mathrm{YM}}$

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Functional Renormalization Group Equation

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \Big[\Big(\Gamma_k^{(2)} + \mathcal{R}_k(\Delta) \Big)^{-1} \partial_t \mathcal{R}_k(\Delta) \Big]$$

Inserting the truncation we have

$$\partial_t \Gamma_k = \frac{1}{2} \,\mathrm{Tr} \left[\frac{\partial_t \mathcal{R}_k(\breve{\Delta})}{\breve{\Gamma}_k^{(2)} + \mathcal{R}_k(\breve{\Delta})} \right] - \mathrm{Tr} \left[\frac{\partial_t \mathcal{R}_k^{\mathrm{gh}}(\Delta_{\mathrm{gh}})}{S_{\mathrm{gh}}^{(2)} + \mathcal{R}_k^{\mathrm{gh}}(\Delta_{\mathrm{gh}})} \right]$$

where

$$\mathcal{R}_k(x) = \mathcal{Z}_k k^2 R^{(0)}(x/k^2), \qquad \mathcal{R}_k^{\mathrm{gh}}(x) = \mathcal{Z}_k^{\mathrm{gh}} k^2 R^{(0)}(x/k^2)$$

- shape function $R^{(0)}(y)$ with $R^{(0)}(0) = 1$ and $\lim_{y\to\infty} R^{(0)}(y) = 0$
- $\mathcal{Z}_k$ and $\mathcal{Z}_k^{\mathrm{gh}}$ constant matrices in field space, chosen such that modes of $\Gamma_k^{(2)} \to \Gamma_k^{(2)} + \mathcal{R}_k$ get shifted $\zeta_k p^2 \to \zeta_k (p^2 + k^2 R^{(0)})$

Spectrally Adjusted Cutoff Operator

As cutoff operator we choose

$$\breve{\Delta} = \mathcal{Z}_k^{-1} \breve{\Gamma}_k^{(2)} \qquad \Delta_{\text{gh}} = \mathcal{Z}_{\text{gh}}^{-1} S_{\text{gh}}^{(2)}$$

This has been called "spectrally adjusted" or "type III" cutoff in the literature.

- conceptually not as clear as the covariant Laplacian $\Box$ $\leftrightarrow$ additional terms on RHS due ∂_t acting on $\breve{\Delta}$
- admits a simple spectral representation of the RHS $\rightarrow$ simplifies evaluation of the traces (no operator inversion necessary)

Calculation and RG Improvement

Calculational steps:

- compute the Hessian $\Gamma_k^{(2)}$ and $S_{\mathrm{gh}}^{(2)}$
- equate background and full classical fields ($\bar{g}_{\mu\nu} = g_{\mu\nu},\ \bar{A}_\mu^a = A_\mu^a$)
- expand RHS in A_μ^a to second order to extract the $(F_{\mu\nu}^a)^2$-term (we may set $g_{\mu\nu} = \delta_{\mu\nu}$)
- all terms combine to an F^2-contribution due to the built-in gauge invariance

Different degrees of RG improvement:

- ∂_t acts only on explicit k-dependence $\leftrightarrow$ 1-loop calculation
- ∂_t acts in addition on $\mathcal{Z}_k$-factors
- ∂_t acts also on $\breve{\Gamma}_k^{(2)}$ in $\breve{\Delta}$

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1-Loop Result

Switching to dimensionless couplings

$$g_{\mathrm{YM}}^2(k) \equiv \frac{\hat{g}_{\mathrm{YM}}^2}{Z_F(k)}, \quad g(k) \equiv k^2 \frac{\hat{G}}{Z_N(k)}, \quad \lambda(k) \equiv k^{-2} \bar{\lambda}(k)$$

we obtain the 1-loop result

$$\boxed{\partial_t g_{\mathrm{YM}}^2 = -\frac{6\,\Phi_1^1(0)}{\pi}\; g\, g_{\mathrm{YM}}^2 - \frac{11\,N}{24\,\pi^2}\; g_{\mathrm{YM}}^4}$$

[Daum, U.H., Reuter (2009)]

where

$$\Phi_1^1(w) = \int_0^\infty \mathrm{d}z\, \frac{R^{(0)}(z) - zR^{(0)\prime}(z)}{z + R^{(0)}(z) + w}$$

Classical Regime

Newton's constant

$$G(k) \approx G_0 = \text{const}, \quad g(k) = G_0 k^2$$

For an Abelian field ($N = 0$) we obtain

$$\partial_t g_{\text{YM}}^2 = -\frac{6\,\Phi_1^1(0)}{\pi}\, G_0\, k^2\, g_{\text{YM}}^2$$

with solution

$$g_{\text{YM}}^2(k) = g_{\text{YM}}^2(0) \cdot \exp\left(-\omega_{\text{YM}}(k/m_{\text{Pl}})^2\right)$$
$$= g_{\text{YM}}^2(0) \cdot \left[1 - \omega_{\text{YM}}(k/m_{\text{Pl}})^2 + O(k^4/m_{\text{Pl}}^4)\right]$$

where $m_{\text{Pl}} = G_0^{-1/2}$ and $\omega_{\text{YM}} = 3\Phi_1^1(0)/\pi$.

Asymptotic Safety

Free Maxwell field does not destroy NGFP of the EH truncation. Therefore in the UV

$$g(k) \to g^* \quad \Longrightarrow \quad G(k) = g^*/k^2 \to 0 \quad \text{as} \quad k \to \infty$$

which implies

$$\partial_t g_{\rm YM}^2 = -\frac{6\,\Phi_1^1(0)}{\pi}\, g^*\, g_{\rm YM}^2$$

with solution

$$g_{\rm YM}^2(k) \propto k^{-\Theta_{\rm YM}}, \quad \Theta_{\rm YM} = \frac{6\,\Phi_1^1(0)}{\pi}\, g^*$$

- total system has a NGFP with $(g_{\rm YM}^* = 0, g^* > 0, \lambda^* > 0)$
- gravity speeds up approach of asymptotic freedom in the YM sector (power law instead of logarithmic)

Summary

- we obtain a gravitational correction to the Yang-Mills beta function in a setting which both preserves gauge invariance and retains quadratic divergences
- the coefficient of the correction is scheme and gauge fixing dependent, but $g_{\mathrm{YM}}(k)$ is not an observable quantity
- the form of the correction corresponds to the result of [Tang and Wu (2008)]
- the result is consistent with the Asymptotic Safety scenario for QEG, with vanishing gauge coupling at the NGFP
- RG improvements do not change the picture substantially