Emergent Geometry and Gravity from Matrix Models

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Motivation

- expect quantum structure of space-time at Planck scale

  due to Gravity $\leftrightarrow$ Quantum Mechanics

- $\left\{ \begin{array}{l} \text{fine-tuning problems} \\ \text{quantum gravity} \end{array} \right\}$ $\rightarrow$ not just deformation of GR ?!

$\Rightarrow$ maybe a different approach to gravity is needed:

pre-geometric theory of gravity:

Matrix Models $\rightarrow$ noncommutative space-time & gravity

Outline:

- geometry from matrix models:

  NC branes, effective geometry
  dynamics

- quantization
- examples, curvature, etc. $\rightarrow$ talk by D. Blaschke

review: H.S., arXiv:1003.4134
D. Blaschke, H. S. arXiv:1003.4132, arXiv:1005.0499

Matrix Models

candidate for quantum theory of fundamental interactions

$$S = -Tr\left([X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'} \; + \bar{\Psi}\Gamma^a[X_a, \Psi]\right)$$

$$X^a \in Mat(\infty, \mathbb{C}), \qquad a = 1, ..., D(= 10)$$

(IKKT Model 1996)

- no geometrical pre-requisites, extremely simple
- $\left\{ \begin{array}{l} \text{NC space-time} \\ \text{metric } (\rightarrow \text{gravity}) \end{array} \right\}$ emerge
- $\left\{ \begin{array}{l} \text{nonabelian gauge fields} \\ \text{gravitons} \end{array} \right\}$ ... fluctuations of NC space
- promising new approach for quantization of gravity
  interesting physical perspectives

Ishibashi, Kawai, Kitazawa and Tsuchiya 1996, ff

Rivelles 2002, Yang 2006, H.S. 2007 ff, ...

Space-time & geometry from matrix models:

e.o.m.: $\delta S = 0 \;\Rightarrow\; [X^a, [X^{a'}, X^{b'}]]\eta_{aa'} = 0$

solutions:

- 1) prototype:

  $[X^a, X^b] = i\theta^{ab}\,\mathbb{1}$, rank $\theta^{ab} = 2n$

  separate $X^a = (X^\mu, \Phi^i)$, $\mu = 1, ..., 2n$

$$\begin{aligned} [X^\mu, X^\nu] &= i\theta^{\mu\nu}\,\mathbb{1} \\ \Phi^i &= 0 \end{aligned}$$

"(Moyal-Weyl) quantum plane" $\mathbb{R}^{2n}_\theta \subset \mathbb{R}^D$

("single-brane configurations": X^a generates $Mat(\infty, \mathbb{C})$)

the Moyal-Weyl quantum plane $\mathbb{R}^4_\theta$:

$$[X^\mu, X^\nu] = i\theta^{\mu\nu}\, \mathbb{1}, \qquad \mu,\nu = 1,...,4$$
$$\phi^i = 0$$

... Heisenberg algebra, interpreted as space of functions on $\mathbb{R}^4_\theta$

uncertainty relations $\Delta x^\mu \Delta x^\nu \geq |\theta^{\mu\nu}|$

relation with classical $\mathbb{R}^4$:

$$\phi(x) = \int d^4k\, e^{ik_\mu x^\mu} \hat\phi(k) \;\leftrightarrow\; \int d^4k\, e^{ik_\mu X^\mu} \hat\phi(k) =: \Phi(X) \;\in Mat(\infty, \mathbb{C})$$

note:

$X^\mu \in Mat(\infty, \mathbb{C})$ ... quantized coordinate functions on $\mathbb{R}^4_\theta$

$\Phi(X^\mu) \in Mat(\infty, \mathbb{C})$ ... general function on $\mathbb{R}^4_\theta$

$[X^\mu, \Phi] =: i\theta^{\mu\nu}\partial_\nu\Phi \;\rightarrow\;$ NC field theory

Noncommutative spaces and Poisson structure

$(\mathcal{M}, \theta^{\mu\nu}(x))$ ... $2n$-dimensional manifold with Poisson structure

Its quantization $\mathcal{M}_\theta$ is NC algebra such that

$$\begin{aligned} \mathcal{I}: \ \mathcal{C}(\mathcal{M}) &\to \mathcal{A} \cong Mat(\infty, \mathbb{C}) \\ f(x) &\mapsto F = F(X) \\ x^i &\mapsto X^i, \qquad e^{ikx} \mapsto e^{ikX} \end{aligned}$$

such that $\qquad [F, G] = \mathcal{I}(i\{f, g\}) + O(\theta^2)$

$\Phi \in Mat(\infty, \mathbb{C}) \quad \leftrightarrow \quad$ quantized function on $\mathcal{M}$
semi-class. limit: replace $F \to f, \quad [F, G] \to i\{f, g\}$. notation: $\sim$

furthermore:

$$\begin{aligned} (2\pi)^2 \, Tr(F) &\sim \int d^4x \, \rho(x) \, f(x) \\ \rho(x) &= \mathrm{Pfaff}\,(\theta^{-1}_{\mu\nu}) \ \ldots \ \text{symplectic volume} \end{aligned}$$

(cf. Bohr-Sommerfeld quantization)

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(cf. Bohr-Sommerfeld quantization)

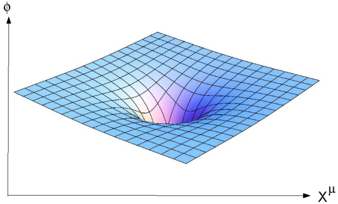
- 2) generic NC brane:

$[X^a, X^b] = i\theta^{ab}(X)$ such that e.g.

$$\begin{aligned} X^a &= (X^\mu, \Phi^i), \quad \mu = 1, ..., 2n \\ \Phi^i &= \Phi^i(X^\mu) \sim \phi^i(x^\mu) \end{aligned}$$

i.e. $X^a \sim x^a : \mathcal{M}^{2n} \to \mathbb{R}^D$

... (quantized) 4-dim. brane



in particular: $X^\mu \sim x^\mu$ ...quantized coord. functions

$$[X^\mu, X^\nu] = i\theta^{\mu\nu}(X^\mu) \sim i\theta^{\mu\nu}(x^\mu)$$

"generic quantum space" $\mathcal{M}_\theta^{2n} \subset \mathbb{R}^D$

Effective geometry of NC brane:

consider scalar field coupled to Matrix Model (“test particle”)

use $[X^\mu, \varphi] \sim i\theta^{\mu\nu}(x)\partial_\nu\varphi \qquad \Rightarrow \qquad \theta^{\mu\nu} = \{x^\mu, x^\nu\}$

$$
\begin{aligned}
S[\varphi] &= Tr\,[X^a,\varphi][X^b,\varphi]\,\eta_{ab} \qquad (U(\mathcal{H}) \quad \text{gauge inv.!}) \\
&\sim \int d^4x\,\sqrt{|G_{\mu\nu}|}\,G^{\mu\nu}(x)\,\partial_\mu\varphi\partial_\nu\varphi
\end{aligned}
$$

$$
\begin{aligned}
G^{\mu\nu}(x) &= e^{-\sigma}\theta^{\mu\mu'}(x)\theta^{\nu\nu'}(x)\,g_{\mu'\nu'}(x) \qquad \text{effective metric} \\
g_{\mu\nu}(x) &= \partial_\mu x^a\partial_\nu x^b\eta_{ab} = \eta_{\mu\nu} + \partial_\mu\phi^i\partial_\nu\phi^j\delta_{ij} \qquad \text{induced metric on } \mathcal{M}^4_\theta
\end{aligned}
$$

$$
e^{-2\sigma} = \frac{|\theta^{-1}_{\mu\nu}|}{|g_{\mu\nu}|}, \qquad |G_{\mu\nu}| = |g_{\mu\nu}| \qquad \text{for } \dim(\mathcal{M}) = 4
$$

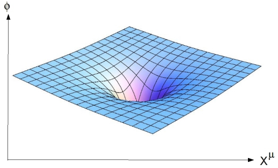
φ couples to metric $G^{\mu\nu}(x)$, determined by $\theta^{\mu\nu}(x)$ & embedding $\phi^i(x)$

same for gauge fields, fermions

... quantized Poisson manifold with metric $(\mathcal{M}, \theta^{\mu\nu}(x), G_{\mu\nu}(x))$

Hence:

$X^a = (X^\mu, \phi^i(X^\mu)) : \quad \mathcal{M}^4 \hookrightarrow \mathbb{R}^D$ ...(quantized) embedding function



all matter couples to dynamical metric $G_{\mu\nu}$ on $\mathcal{M}^4 \Rightarrow$ effective gravity

however: metric is not fundamental d.o.f.
rather: matrices X^a resp. $(\phi^i, \theta^{\mu\nu})$ resp. $(\phi^i, F_{\mu\nu})$

different from GR, might be close enough to observation (?)

note: $D = 10$ just enough to describe most general $g_{\mu\nu}(x)$ (locally)

$su(n)$ gauge fields: same model, new vacuum ("n branes")

$$Y^a = \begin{pmatrix} Y^\mu \\ Y^i \end{pmatrix} = \begin{pmatrix} X^\mu \otimes \mathbf{1}_n \\ \phi^i \otimes \mathbf{1}_n \end{pmatrix}$$

include fluctuations:

$$Y^a = (1 + \mathcal{A}^\rho \partial_\rho) \begin{pmatrix} X^\mu \otimes \mathbf{1}_n \\ \phi^i \otimes \mathbf{1}_n + \Phi^i \end{pmatrix}$$

where

$$\begin{aligned} \mathcal{A}^\mu &= -\theta^{\mu\nu} A_{\nu,\alpha} \otimes \lambda^\alpha, \qquad \lambda^\alpha \in \mathfrak{su}(n) \\ \Phi^i &= \Phi^i_\alpha \otimes \lambda^\alpha \end{aligned}$$

$\Rightarrow$ effective action:

$$S_{YM} \sim \int d^4x \sqrt{G}\, e^\sigma\, G^{\mu\mu'} G^{\nu\nu'}\, tr\, F_{\mu\nu} F_{\mu'\nu'} + 2 \int \eta(x)\, tr\, F \wedge F$$

(H.S., JHEP 0712:049 (2007), JHEP 0902:044,(2009))

... $\mathfrak{su}(n)$ Yang-Mills coupled to metric $G^{\mu\nu}(x)$

"previous point of view" :

$$X^\mu = \bar{X}^\mu + A^\mu, \qquad [\bar{X}^\mu, \bar{X}^\nu] = \bar{\theta}^{\mu\nu}$$

$\rightarrow$ $U(1)$ rep. $U(n)$ field theory on $\mathbb{R}^4_\theta$; $U(1)$ is strange

here: absorb $U(1)$ field strength $F_{\mu\nu}$ in $\theta^{\mu\nu}(x)$

$\rightarrow$ geometrical understanding, gravity

effective (“Poisson-Riemann”) geometry:

<u>result:</u>

$(\mathcal{M}, \omega)$ symplectic manifold, $\omega = \frac{1}{2}\theta^{-1}_{\mu\nu} dx^\mu \wedge dx^\nu$
$x^a : \mathcal{M} \hookrightarrow \mathbb{R}^D$ ... embedding in $\mathbb{R}^D$
induced metric $g_{\mu\nu}$ and $G^{\mu\nu}$ as above. Then:

$$\begin{aligned} \{x^a, \{x^b, \varphi\}\}\eta_{ab} &= e^\sigma \Box_G \varphi \\ \nabla^\mu_G (e^\sigma \theta^{-1}_{\mu\nu}) &= G_{\nu\rho}\, \theta^{\rho\mu} \left(e^{-\sigma} \partial_\mu \eta + \partial_\mu x^a \Box_G x^b \eta_{ab}\right) \end{aligned}$$

for $\varphi \in \mathcal{C}^\infty(\mathcal{M})$, ∇_G ... Levi-Civita, $\Box_G$ ... Laplace- Op. w.r.t. $G_{\mu\nu}$, and

$$\eta(x) := \frac{1}{4} e^\sigma\, G^{\mu\nu} g_{\mu\nu}.$$

(H.S., 2008)

in particular:

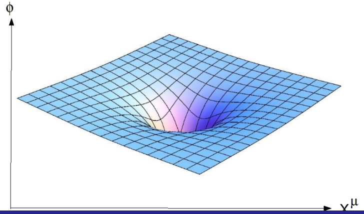
matrix e.o.m: $[X^a,[X^b,X^{a'}]]\eta_{aa'} = 0 \quad \Longleftrightarrow$

$$\begin{aligned} \Delta_G \Phi^i &= 0, \quad \Delta_G x^\mu = 0 \\ \nabla^\mu(e^\sigma \theta^{-1}_{\mu\nu}) &= e^{-\sigma} G_{\rho\nu}\theta^{\rho\mu}\partial_\mu \eta \\ \eta &= \tfrac{1}{4} e^\sigma G^{\mu\nu} g_{\mu\nu} \end{aligned}$$

... covariant formulation in semi-classical limit

in particular:

$\mathcal{M}^4 \hookrightarrow \mathbb{R}^D$ is harmonic embedding (w.r.t. $G_{\mu\nu}$)
minimal surface



dynamics of NC structure $\theta^{\mu\nu}$:

$$S_{YM} = -Tr[X^a,X^b][X^a,X^b] \sim \int d^4x\,\sqrt{g}(G^{\mu\nu}g_{\mu\nu})$$

Euclidean case: at $p \in \mathcal{M}$, diagonalize $g_{\mu\nu} = (1,1,1,1)$
using $SO(4)$ $\rightarrow$ standard form

$$\theta^{\mu\nu} = \theta \begin{pmatrix} 0 & -\alpha & 0 & 0 \\ \alpha & 0 & 0 & 0 \\ 0 & & 0 & \pm\alpha^{-1} \\ 0 & 0 & \mp\alpha^{-1} & 0 \end{pmatrix}.$$

effective metric $G^{\mu\nu} = (\alpha^2,\alpha^2,\alpha^{-2},\alpha^{-2})$.
Note

$$\begin{aligned} \tfrac{1}{4}(G^{\mu\nu}g_{\mu\nu}) &= \tfrac{1}{2}(\alpha^2+\alpha^{-2}) \geq 1 \\ \star\theta^{-1}_{\mu\nu} &= \pm\theta^{-1}_{\mu\nu} \Leftrightarrow \tfrac{1}{4}(Gg) = 1 \Leftrightarrow G_{\mu\nu} = g_{\mu\nu} \Leftrightarrow S_{YM} \text{ minimal} \end{aligned}$$

minimum of S_{YM} $\Leftrightarrow$ $\theta^{-1}_{\mu\nu}$ (A)SD $\Leftrightarrow$ $G_{\mu\nu} = g_{\mu\nu}$.

$\Rightarrow$ special class of solutions:

$$
\begin{aligned}
g_{\mu\nu} &= G_{\mu\nu}, \\
\Delta_G \phi^i &= 0 \\
\nabla^\mu \theta^{-1}_{\mu\nu} &= 0
\end{aligned}
$$

holds for (anti)self-dual symplectic structure $\theta^{-1}_{\mu\nu}$,

$$
\begin{aligned}
\star(\theta^{-1}) &= \pm\theta^{-1} && \text{Euclidean} \\
\star(\theta^{-1}) &= \pm i\theta^{-1} && \text{Minkowski (Wick rotation } X^0 \to it \text{)}
\end{aligned}
$$

then

$$
S_{MM} \sim Tr[X^a, X^b][X^{a'}, X^{\beta'}] = \int d^4x \sqrt{|g_{\mu\nu}|}
$$

... same structure as vacuum energy, “brane tension”.

Examples of solutions (“bare” M.M.):

... (NC) minimal surfaces $\mathcal{M} \subset \mathbb{R}^D$, deformed by matter

- prototype: flat space $\mathbb{R}^4_\theta \subset \mathbb{R}^{10}$
  insensitive to vacuum energy (minimal surface) !
- interesting near-realistic cosmological solution
  (FRW, big bounce) D. Klammer, H.S. arXiv:0903.0986, PRL 102
  compatible with type Ia supernovae without fine-tuning
- matter $\rightarrow$ Newtonian gravity H.S:, arXiv:0909.4621
  (but post-Newtonian corrections probably (?) not acceptable)

Quantization

$$Z = \int dX^a d\Psi \, e^{-S[X]-S[\Psi]} \;=\; e^{-S_{eff}}$$

...suitable for non-perturbative approach

- semi-classical geometry:

  matter coupled to non-trivial metric $G_{\mu\nu}(x)$
  $\to$ gravity, induced E-H. action

$$S_{eff} \sim \int d^4x \sqrt{|G|} \, (\Lambda^4 + c\Lambda_4^2 \, R[G] + ...)$$

  e.g. 1-loop resp. Seeley-de Witt coeff. (fermions)

- fully NC action:

  expect effective M.M. action $S_{eff}[X] \sim \sum (P_{a_1 \ldots a_n} X^{a_1} \ldots X^{a_n})$
  cf. higher-order terms $\leftrightarrow$ curvature etc. ... talk by D. Blaschke

$\exists$ alternative interpretation of M.M. in terms of NC gauge theory on $\mathbb{R}^4_\theta$

- suitable for perturbative quantization !
- explanation for UV/IR mixing & $U(1)$ entanglement
  $U(1)$ absorbed in $\theta^{\mu\nu}(x)$
- $\Lambda^2 R[G]$ due to UV/IR mixing

  H. Grosse, M. Wohlgenannt, H.S., *JHEP* 0804:023,2008.

$\Rightarrow$ need model with finite effective UV cutoff Λ

IKKT model $\Leftrightarrow$ $\mathcal{N} = 4$ SUSY Yang-Mills,
(supposed to be) finite, free of UV/IR mixing
6 scalar fields $\Rightarrow$ $D = 10$

assume (soft, spontaneous ?) SUSY breaking at Λ

$\Rightarrow$ IKKT model $\supset$ finite quantum theory of gravity ?!?

e.o.m. for emergent NC gravity II

effective action

$$S = \int d^4x \sqrt{|g|}\,(-2\Lambda^4 + \Lambda_4^2 R) + S_{\text{matter}}$$

leads to

$$\begin{aligned} \delta S &= \int d^4x \sqrt{|g|}\,\delta g_{\mu\nu}(-\Lambda^4 g^{\mu\nu} + 8\pi T^{\mu\nu} - \Lambda_4^2 \mathcal{G}^{\mu\nu}) \\ &= -2\int \delta\phi^i \partial_\mu(\sqrt{|g|}\,(-\Lambda^4 g^{\mu\nu} + 8\pi T^{\mu\nu} - \Lambda_4^2 \mathcal{G}^{\mu\nu}))\partial_\nu \phi^i \end{aligned}$$

since $g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \phi^i \partial_\nu \phi^i$

1. "Einstein branch"

$$\Lambda^4 g^{\mu\nu} + \Lambda_4^2 \mathcal{G}^{\mu\nu} = 8\pi T^{\mu\nu}$$

2. "harmonic branch"

$$\Lambda^4 \Box_g \phi = (8\pi T^{\mu\nu} - \Lambda_4^2 \mathcal{G}^{\mu\nu})\nabla_\mu \partial_\nu \phi$$

prototype: flat space $\mathbb{R}^4_\theta \subset \mathbb{R}^{10}$, even for $\Lambda \gg 0$!

Einstein branch

- not yet fully understood, complicated
  (due to e.g. dilaton-like terms, presence of $\theta^{\mu\nu}$ etc.
- deviation from GR expected
- need (at least) full 1-loop effective action

illustration: Schwarzschild geometry $\mathcal{M} \subset \mathbb{R}^7$
(Blaschke, H.S. arXiv:1005:0499)

... talk by D. Blaschke

Next steps;

- need: (quantum) effective M.M. action
  e.g. higher-order terms, $\rightarrow$ talk by D.Blaschke
  more structure for vacuum energy sector
- fuzzy extra dimensions $M^4_\theta \times S^2_N$ etc. natural in M.M.
  allow to obtain non-trivial gauge groups, towards particle physics
  Chatzistavrakidis, H.S., Zoupanos arXiv:1002.2606
  Grosse, Lizzi, H.S. arXiv:1001.2703

Summary and Conclusion

- matrix-model $Tr[X^a, X^b][X^{a'}, X^{b'}]\,\eta_{aa'}\eta_{bb'}$

  dynamical NC spaces $\leftrightarrow$ emergent gravity & gauge thy
- *not* same as G.R., long-distance corrections (extrinsic geometry)
- intriguing cosmological solutions,
  physics of vacuum energy different from GR
- suitable for quantizing gravity !
  (IKKT model, $N = 4$ SUSY in $D = 4$)
- ... more work is needed !

cf. UV/IR mixing:
”effective cutoff“ for non-planar diagrams

$$\Lambda_{eff}^2(p) = \frac{1}{1/\Lambda^2 + \frac{1}{4}p^2/\Lambda_{NC}^4} = \Lambda^2(1 - \frac{1}{4}p^2\Lambda^2/\Lambda_{NC}^4 + ...)$$

semi-classical regime: $\frac{p^2\Lambda^2}{\Lambda_{NC}^4} < 1$

at one loop $\to$ induced gravity action

$$\Gamma_\Phi[G] \cong \int d^4y \sqrt{G}\left(\Lambda^4 + c\Lambda^2 R[G]\right)$$

fermions

natural (only?) action

$$S[\Psi] = \mathrm{Tr}\, \overline{\Psi}\gamma_a[X^a, \Psi]$$
$$\sim \int d^4x\, \rho(x)\, \overline{\Psi} i\gamma_\mu \theta^{\mu\nu}(x)\partial_\nu \Psi, \qquad \{\gamma_\mu, \gamma_\nu\} = 2G_{\mu\nu}$$

note:

- naturally SUSY $\to$ IKKT model
- couple to $G_{\mu\nu}$, but non-standard spin connection (submanifold!)
- quantization induces E-H action plus additional terms

$$\Gamma_\Psi = \frac{1}{4\pi^2}\int d^4x\sqrt{|g|}\Big(2\Lambda^4 + \Lambda^2\Big(-\frac{1}{3}R[g] + \frac{1}{4}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{8}e^{-\sigma}R[g]_{\mu\nu\rho\sigma}\theta^{\mu\nu}\theta^{\rho\sigma} + \frac{1}{4}(\Box_g x^a)(\Box_g x^b)\eta_{ab}\Big) + \mathcal{O}(\log\Lambda)\Big).$$

- precise matching with UV/IR mixing (checked in $D = 4$)

(D. Klammer, H.S., arXiv:0901.2322, arXiv:0909.5298)

alternative interpretation of M.M: NC gauge theory

parametrize matrices as fluctuations around $\mathbb{R}^4_\theta$:

$$X^\mu = \bar{X}^\mu + \bar{\theta}^{\mu\nu} A_\nu, \qquad \bar{X}^\mu \text{...Moyal-Weyl}$$

$$\begin{aligned} [X^\mu, X^\nu] &= i\bar{\theta}^{\mu\mu'}\bar{\theta}^{\nu\nu'}(\partial_{\mu'}A_{\nu'} - \partial_{\nu'}A_{\mu'} + [A_{\mu'}, A_{\nu'}]) + i\bar{\theta}^{\mu\nu} \\ &= i\bar{\theta}^{\mu\mu'}\bar{\theta}^{\nu\nu'} F_{\mu'\nu'} + i\bar{\theta}^{\mu\nu} \end{aligned}$$

$F_{\mu\nu}(x)$ ... $\mathfrak{u}(1)$ field strength

action:

$$S_{YM} \sim \int d^4x (F_{\mu\nu} + i\bar{\theta}^{-1}_{\mu\nu})(F_{\mu'\nu'} + i\bar{\theta}^{-1}_{\mu'\nu'})\bar{G}^{\mu\mu'}\bar{G}^{\nu\nu'}$$

... NC $U(1)$ gauge theory on $\mathbb{R}^4_\theta$

<u>however:</u>

- $U(1)$ sector does not decouple from $SU(n)$ sector, ...
- one-loop: UV/IR mixing, except in $\mathcal{N} = 4$ SUSY case: finite (!?)

.... understood in interpretation in terms of emergent gravity.

Deformations of Moyal-Weyl plane: gravitons

dynamical $X^\mu \quad \Rightarrow \quad$ dynamical $\quad (\theta^{\mu\nu}(x),\, G^{\mu\nu}(x))$

parametrize fluctuations

$$X^\mu = \bar{X}^\mu + \bar{\theta}^{\mu\nu} A_\nu$$

$$\begin{aligned} i\theta^{\mu\nu}(x) &\sim [X^\mu, X^\nu] \\ &= i\bar{\theta}^{\mu\mu'}\bar{\theta}^{\nu\nu'}(\partial_{\mu'}A_{\nu'} - \partial_{\nu'}A_{\mu'} + [A_{\mu'}, A_{\nu'}]) + i\bar{\theta}^{\mu\nu} \\ &= i\bar{\theta}^{\mu\mu'}\bar{\theta}^{\nu\nu'} F_{\mu'\nu'} + i\bar{\theta}^{\mu\nu} \\ G^{\mu\nu}(x) &= \overline{\eta}^{\mu\nu} - h^{\mu\nu} \qquad (+O(F^2)) \end{aligned}$$

$F_{\mu\nu}(x)$ ... $\mathfrak{u}(1)$ field strength

therefore

$$\boxed{h_{\mu\nu} = \bar{\eta}_{\nu\nu'}\bar{\theta}^{\nu'\rho} F_{\rho\mu} + \bar{\eta}_{\mu\mu'}\bar{\theta}^{\mu'\eta} F_{\eta\nu} - \tfrac{1}{2}\bar{\eta}_{\mu\nu}\,(\bar{\theta}^{\rho\eta} F_{\rho\eta})}$$

... linearized metric fluctuation (cf. Rivelles [hep-th/0212262]) e.o.m for fluctuations of Moyal-Weyl plane (linearized):

$$\begin{aligned} [X^\mu,[X^\nu,X^{\mu'}]]\eta_{\mu\mu'} &= 0 \\ \Rightarrow\ \partial^\mu F_{\mu\nu} &= 0 \\ \Rightarrow\ R_{\mu\nu}[G] &= 0 \quad (\partial^\mu h_{\mu\nu} = 0 \ldots \text{harm. gauge}) \end{aligned}$$

cf. Rivelles [hep-th/0212262]

while $R_{\mu\nu\rho\eta} \neq 0$ ... nonvanishing curvature

$\Rightarrow$ on-shell d.o.f. of gravitational waves on Minkowski space

i.e.: trace-$U(1)$ photons on $\mathbb{R}^4_\theta$ are actually gravitons

NC $U(1)$ does not decouple, couples like graviton