QCD-2004

Lesson 3 :Non-perturbative

1) Simple quantities: masses and decay constants
2) Sistematic errors
3) Semileptonic decays
4) Non leptonic decays

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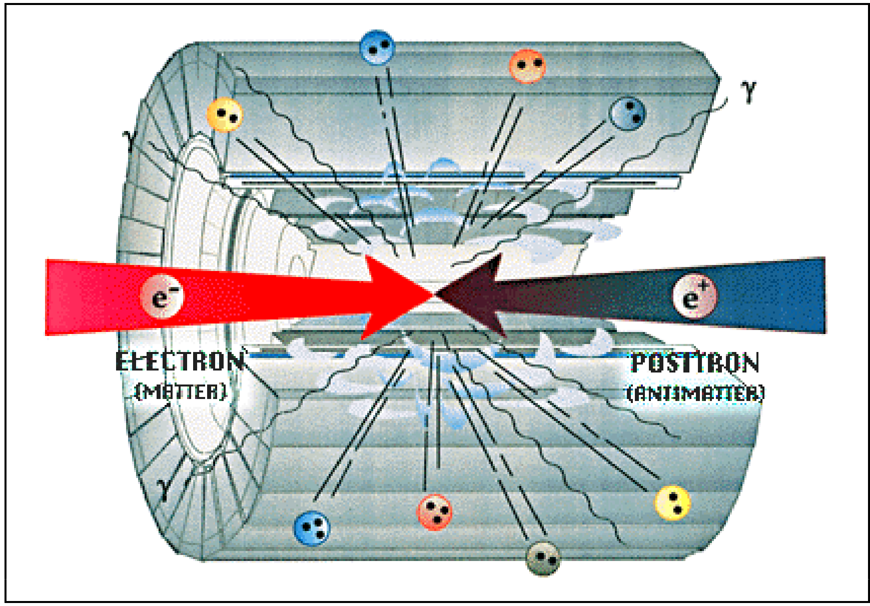
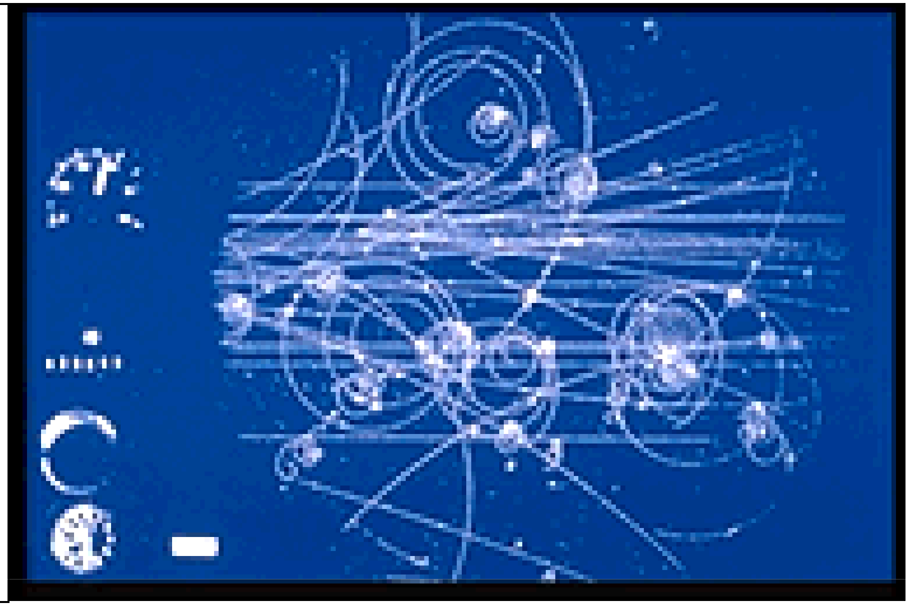
COULD WE COMPUTE THIS PROCESS WITH
SUFFICIENT COMPUTER POWER ?





THE ANSWER IS: NO

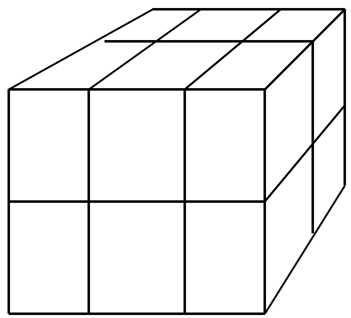
IT IS NOT ONLY A QUESTION OF COMPUTER POWER
BECAUSE THERE ARE COMPLICATED
FIELD THEORETICAL PROBLEMS

LATTICE FIELD THEORY IN FEW SLIDES

$$Z^{-1} \int [d\,\phi]\ \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4)\ e^{i\,S(\phi)}$$

On a finite volume (**L**) and with a finite lattice spacing (**a**) this is now an integral on L^4 real variables which can be performed with

Important sampling techniques



$$Z = \sum_{\{\sigma=\pm 1\}} e^{J_{ij}\, \sigma_i\, \sigma_j} \quad \text{Ising Model}$$

$$2^N = 2^{L^3} \approx 10^{301} \text{ for } L = 10 \ \ !!!$$

Wick Rotation $t \to i\, t_E$

$Z^{-1} \int [d\,\phi]\ \ \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4)\ \ e^{i\,S(\phi)}$

$\to\ Z^{-1} \int [d\,\phi]\ \ \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4)\ \ e^{-S(\phi)}$

This is like a statistical Boltzmann system with

$$b\,H = S$$

Several important sampling methods can be used, for example the Metropolis technique, to extract the fields with weight

$$e^{-S(\phi)}$$

$$< \phi\ \phi\ \phi\ \phi > = Z^{-1} \sum_{\{\phi\,(x)\}_n} \ \phi_n(x_1)\ \ \phi_n(x_2)\ \ \phi_n(x_3)\ \ \phi_n(x_4)$$

$$Z = \sum_{\{\phi\,(x)\}_n} 1 = N$$

$$Z^{-1} \int [d\,\phi]\ \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)\ e^{-S(\phi)}$$

This integral is only a formal definition because of the infrared and ultraviolet divergences. These problems can be cured by introducing an infrared and an ultraviolet cutoff.

1) We introduce an ultraviolet cutoff by defining the fields on a (hypercubic) four dimensional lattice $\phi(x) \to \phi(a\, n)$
where $n=(n_x , n_y , n_z , n_t)$ and a is the lattice spacing

$$\partial_\mu \phi\,(\mathbf{x}) \to \nabla_\mu \phi\,(\mathbf{x}) = (\phi(x+a\, n_\mu) - \phi(x)) / a \quad ;$$

The momentum p is cutoff at the first Brioullin zone, $|p| \leq \pi / a$
The cutoff can be in conflict with important symmetries of the theory, as for example Lorentz invariance or chiral invariance
This problem is common to all regularizations like for example Pauli-Villars, dimensional regularization etc.

$$\mathcal{L} = -\nabla_\mu \phi^\dagger \nabla_\mu \phi + m_0^2 \phi^\dagger \phi + V(\phi^\dagger \phi)$$

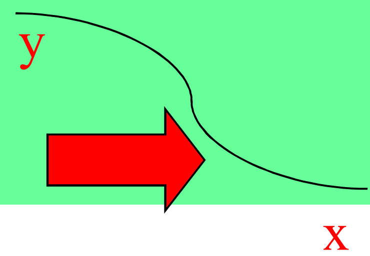
LOCAL GAUGE INVARIANCE

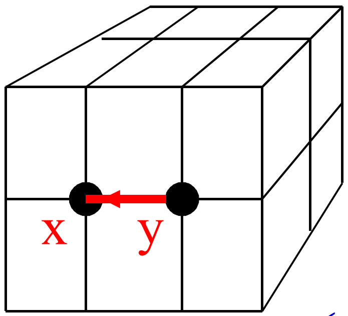
$G^A_\mu(x)\, t^A \;->$

$$V(x)\,[G^A_\mu(x)\, t^A]\, V^\dagger(x) + i/\, g_0\, [\partial_\mu V(x)]\, V^\dagger(x)$$

$q(x) -> V(x)\, q(x)$ $\qquad$ $\bar{q}(x) -> \bar{q}(x)\, V^\dagger(x)$

$$q(y) = \mathrm{P}\,[\,\exp \int_x^y \; i\, g_0\, G^A_\mu(x)\, t^A\, dx_\mu\,]\; q(x)$$





$$\bar{q}(y)\, \mathrm{P}\,[\,\exp \int_x^y \; i\, g_0\, G^A_\mu(x)\, t^A\, dx_\mu\,]\; q(x)$$

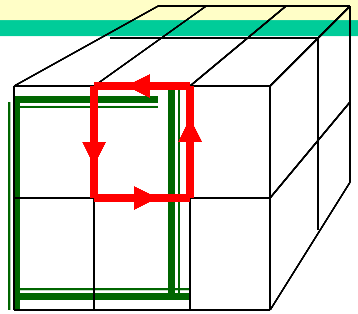
is Gauge invariant

$$\bar{q}(x + a\,\mu\,) \quad \exp\,[i\, g_0\, G^A_\mu(x + a\,\mu\,/2)\, t^A] \quad q(x)$$

LINK $\qquad$ $U^\dagger_\mu(x)$

Plaquette $U_\mu(x) \to V(x)\, U_\mu(x)\, V^\dagger(x + a\,\mu)$

$$W_{\mu\nu}(x) = U_\mu(x)\, U_\nu(x + a\,\mu)\, U^\dagger_\mu(x + a\,\nu)\, U^\dagger_\nu(x)$$
$$\approx 1 + i\, a^2 g_0\, G_{\mu\nu}(x) - a^4 g_0^2/2\, G_{\mu\nu}(x)\, G^{\mu\nu}(x) + \ldots$$



$$\frac{1}{g_0^2} \sum_x \sum_{\mu<\nu} \mathrm{Re}\ \mathrm{Tr}\,[1 - W_{\mu\nu}(x)] \to$$
$$a^4/4 \sum_x \sum_{\mu\nu} G_{\mu\nu}(x)\, G^{\mu\nu}(x) \to$$
$$1/4 \int G_{\mu\nu}(x)\, G^{\mu\nu}(x) + O(a^2)$$

Fermion action(s)

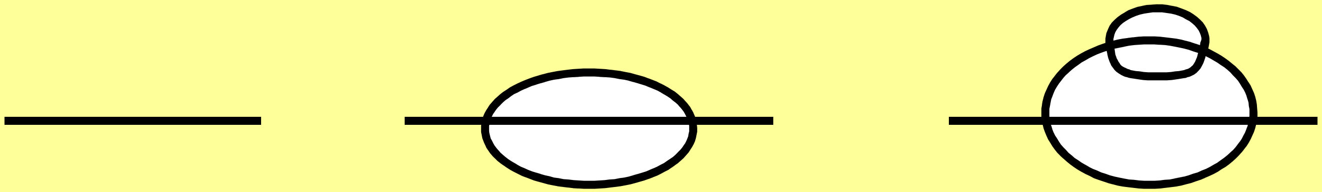
$$S_f = a^4 \sum_x \{ \left(\frac{1}{2a}\right) \bar{q}(x) \sum_\mu \gamma_\mu \left[U_\mu(x) q(x+a\mu) - U_\mu^\dagger(x-a\mu) q(x-a\mu) \right] + m\bar{q}(x)q(x) \} = \sum_{x,y} \bar{q}(x) \Delta_f(U)_{x,y} q(y)$$

We may define many (an infinite number of) lattice actions which all formally converge to the same continuum QCD action:
Naïve, Kogut-Susskind, Wilson, Clover, Domain Wall, Overlap. We postpone the discussion of these formulations and return to the calculation of physical quantities like masses, decay constants etc.

Determination of hadron masses and simple matrix elements

An example from the $\lambda\ \phi^4$ theory

$$G(t,\vec{q}) = \int d^3x\, e^{i\vec{q}\cdot\vec{x}} \langle 0|\phi^\dagger(\vec{x},t)\phi(\vec{0},0)|0\rangle$$
$$= \sum_n \langle 0|\phi^\dagger|n\rangle\langle n|\phi|0\rangle \frac{e^{-E_n t}}{2E_n} \quad t>0$$
$$\langle n|m\rangle = (2\pi)^3 2E_n \delta(\vec{q}_n - \vec{q}_m)$$



The field ϕ can excite one-particle, 3-particle etc. states

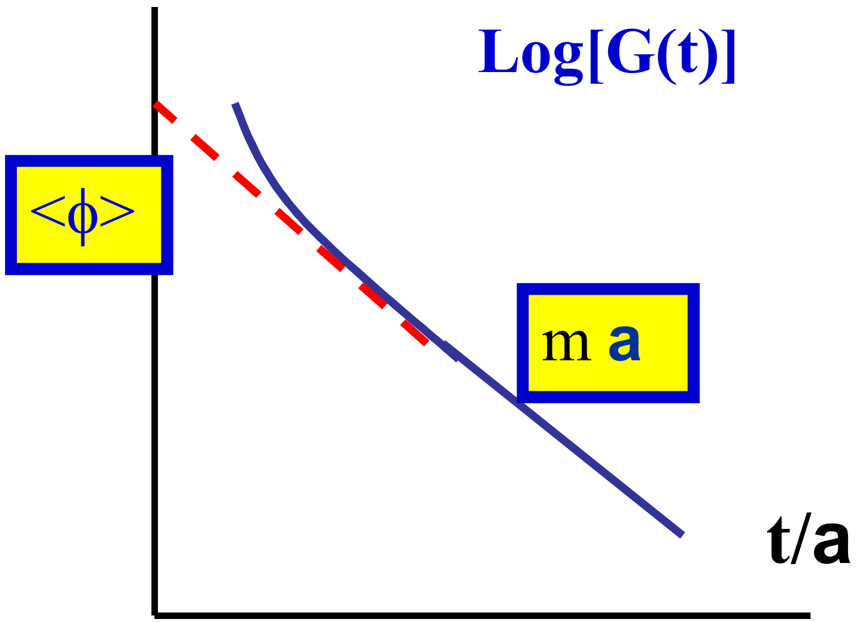
At large time distances the lightest (one particle) states dominate :

$$G(t,\vec{q})=\sum_n \langle 0|\phi^\dagger|n\rangle\langle n|\phi|0\rangle \frac{e^{-E_n t}}{2E_n} \rightarrow \langle 0|\phi^\dagger|\vec{q}\rangle\langle\vec{q}|\phi|0\rangle \frac{e^{-E_q t}}{2E_q}$$

For a particle at rest we have

$$G(t)=|\langle\vec{q}=0|\phi|0\rangle|^2 \frac{e^{-ma(t/a)}}{2m}$$

$\xi = 1/\ m\ a$ is the dimensionless correlation length (and the size of the physical excitations)

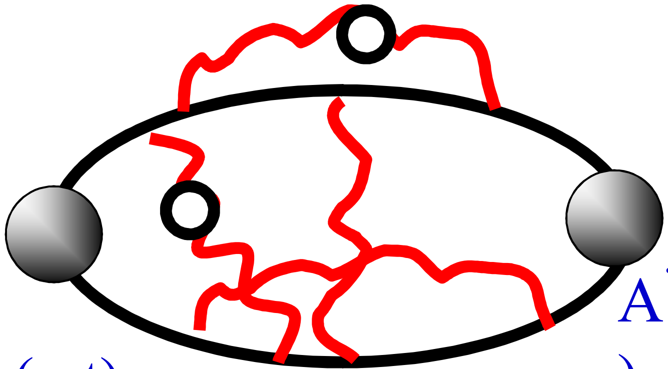


HADRON SPECTRUM AND DECAY CONSTANTS IN QCD

Define a source with the correct quantum numbers :

$$\text{“}\pi\text{”} \equiv A_0(\mathbf{x},t) = u^{\mathbf{a}}_{\alpha}(\mathbf{x},t)\,(\gamma_0\,\gamma_5)^{\alpha\beta}\,d^{\mathbf{a}}_{\beta}(\mathbf{x},t)$$

a=colour
β=spin



$$G(t) = \sum_{\mathbf{x}} \langle A_0(\mathbf{x},t)\, A^{\dagger}_0(\mathbf{x},t) \rangle$$

$$= \sum_n \frac{|\langle 0 | A_0 | n \rangle|^2}{2\,E_n} \exp[-\,E_n\,t]$$

$$\to \frac{|\langle 0 | A_0 | \pi \rangle|^2}{2\,M_\pi} \exp[-\,M_\pi\,t]$$

$$\to \frac{f_\pi^{\,2}\,M_\pi}{2} \exp[-\,M_\pi\,t]$$

$$f_\pi\,M_\pi \sim Z_\pi$$

Mass and decay constant in lattice units $M_\pi = m_\pi\,\mathbf{a}$

$$\boxed{A_\mu^{\eta'} = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s}$$

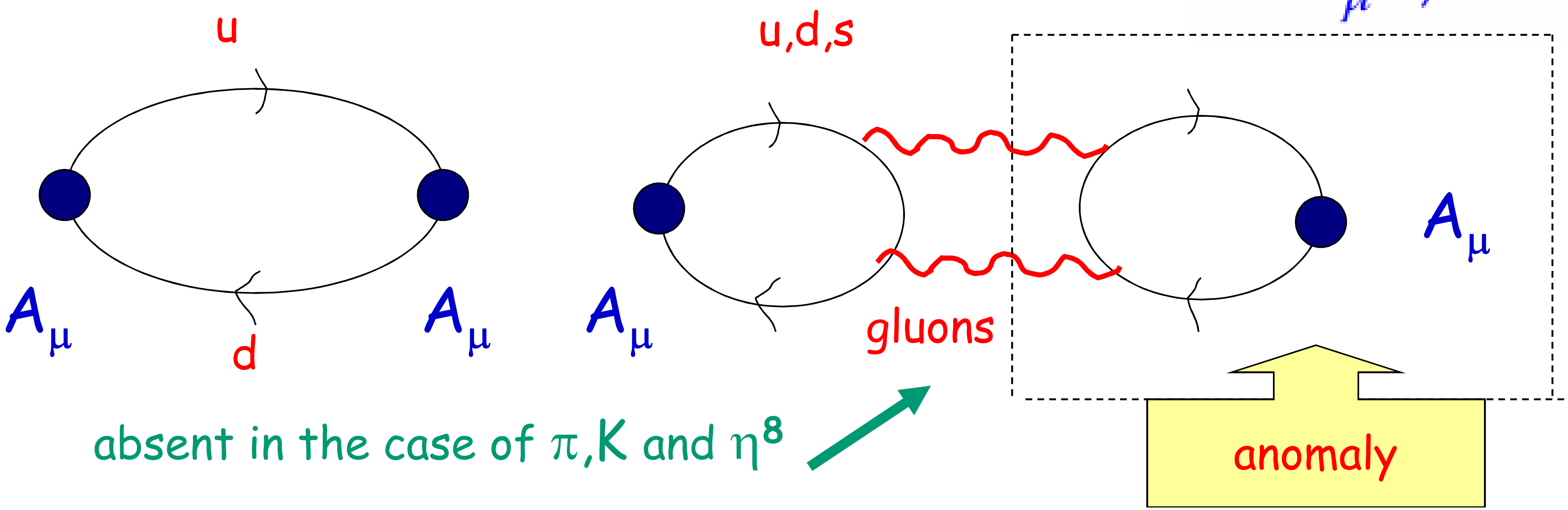
$$\partial^\mu A_\mu^{\eta'} = \frac{g^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu} + 2m_q P^{\eta'}$$

$$P^{\eta'} = \bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s \quad \tilde{G}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}$$

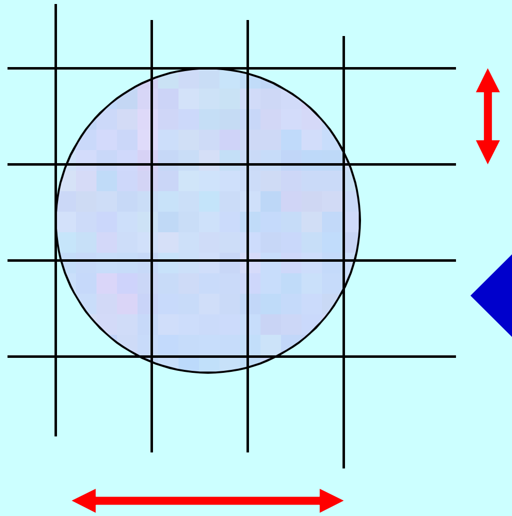
In the chiral limit $m_q \to 0$

$$\partial^\mu A_\mu^{\pi} = 0$$

$$\partial^\mu A_\mu^{\eta'} \neq 0$$



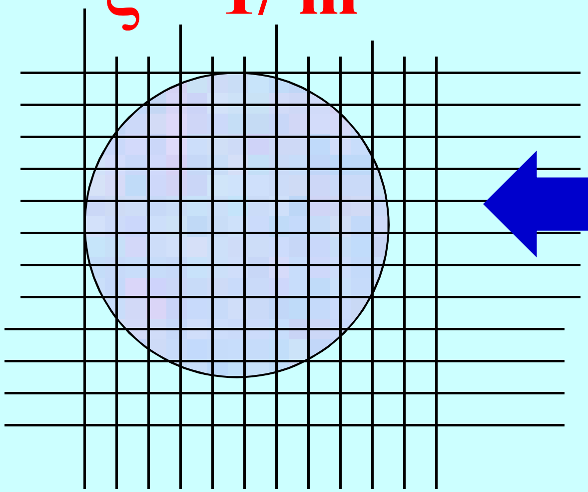
Continuum limit



a Formal $\lim_{a \to 0} S_{Lattice}(\phi) \to S_{Continuum}(\phi)$

a/ ξ = m a ~1 The size of the object is comparable to the lattice spacing

ξ = 1/ m



a/ ξ <<1 i.e. **m a -> 0** The size of the object is much larger than the lattice spacing

Similar to **a** $\sum_n \to \int dx$

Calibration of the lattice spacing **a**

Let us start for simplicity with massless quarks $m_q = 0$

$$M_{proton} = M_{proton}\,(g_0\,,\,\mathbf{a}\,) = m_{proton}\,\mathbf{a}$$

Measured in the numerical simulation

Physical proton mass

$$\boxed{\mathbf{a}\,(g_0\,) = \frac{M_{proton}}{m_{proton}}}$$

Then we predict $m_\Lambda\,,\, m_\Xi\,,\ \ m_\Sigma\,,\ \ f_\pi\ \ ,$

….

we cannot predict m_π since $m^2_\pi \propto m_q$

Calibration of the lattice spacing **a**

$$M_{proton} = M_{proton}(g_0, \mathbf{a}, m_{up} = m_{down}, m_{strange}) = m_{proton}\,\mathbf{a}$$

$$M_{\pi} = M_{\pi}(g_0, \mathbf{a}, m_{up} = m_{down}, m_{strange}) = m_{\pi}\,\mathbf{a}$$

$$M_{K} = M_{K}(g_0, \mathbf{a}, m_{up} = m_{down}, m_{strange}) = m_{K}\,\mathbf{a}$$

....

$$\boxed{\mathbf{a}(g_0, m_{up} = m_{down}, m_{strange})}$$

Then we predict $m_{\Lambda}, m_{\Xi}, m_{\Sigma}, f_{\pi}, \ldots$

.... everything including the quark masses

Continuum limit $\mathbf{a} \longrightarrow \mathbf{0}$

Using asymptotic freedom

$$\mathbf{a}\,\frac{d\,g_0}{d\,\mathbf{a}} = \beta_0 g^{\mathbf{3}}{}_0 + \beta_1 g^{\mathbf{5}}{}_0 + \mathbf{O}\,(g^{\mathbf{7}}{}_0)$$

$$\mathbf{a}\,(g_0) \sim \Lambda_{QCD}{}^{-1}\, e^{-1/(2\,\beta_0 g_0{}^2)}$$

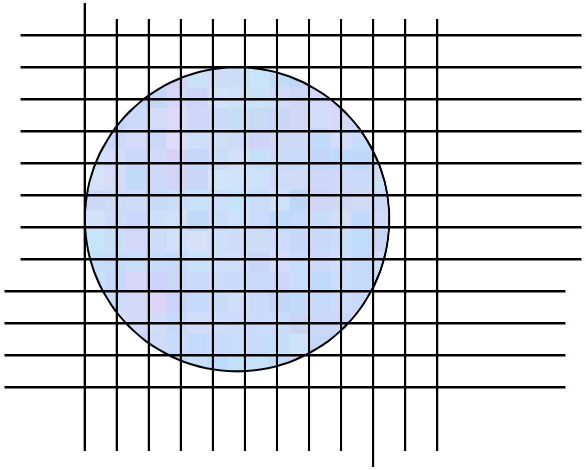
$a \longrightarrow 0$ when $g_0 \longrightarrow 0$

$$M_{proton} = m_{proton}\,\mathbf{a} = m_{proton}\,\Lambda_{QCD}{}^{-1}\, e^{-1/(2\,\beta_0 g_0{}^2)}$$

$$= C_{proton}\; e^{-1/(2\,\beta_0 g_0{}^2)} \longrightarrow 0$$

$a \longrightarrow 0$ when $g_0 \longrightarrow 0$

$$M_{proton} = m_{proton}\, \mathbf{a} = m_{proton}\, \Lambda_{QCD}^{-1}\, e^{-1/(2\beta_0 g_0^2)}$$
$$= C_{proton}\, e^{-1/(2\beta_0 g_0^2)} + O(\mathbf{a}) \longrightarrow 0$$

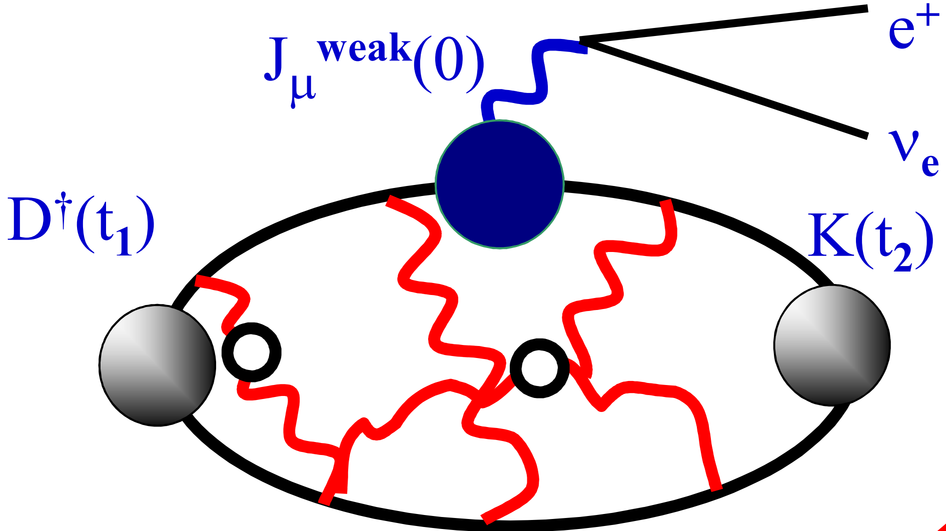


These are discretization errors due to the use of a finite lattice spacing; they vanish exponentially fast in g_0

With inverse lattice spacings of order 2-4 GeV (+improvement/extrapolation) discretization errors range from O(10%) to less than 1%

$$M_{proton} / M_{\pi} \longrightarrow C_{proton} / C_{\pi} = \text{const.}$$

3-point functions



$D^\dagger(t_1) = \sum_x D^\dagger(\mathbf{x}, t_1) \exp[-i\, \mathbf{p_D}\, \mathbf{x}]$
$K(t_2) = \sum_x K(\mathbf{x}, t_2) \exp[+i\, \mathbf{p_K}\, \mathbf{x}]$

from the 2-point functions

$$\langle K(t_2,\vec{p}_K) J_\mu^{weak}(0) D^\dagger(t_1,\vec{p}_D)\rangle \rightarrow \left[\frac{\langle 0|K|K\rangle\langle D|D^\dagger|0\rangle}{2E_D\, 2E_K} e^{-E_D t_1 - E_K t_2}\right] \times \langle K(\vec{p}_K)|J_\mu^{weak}|D(\vec{p}_D)\rangle$$

$< K \,|\, J_\mu^{weak}(0) \,|\, D >$
also electromagnetic form factors, structure functions, dipole moment of the neutron, g_a/g_v, etc.

1) Kl3 namely $K \rightarrow \pi\, l\, \nu_l$

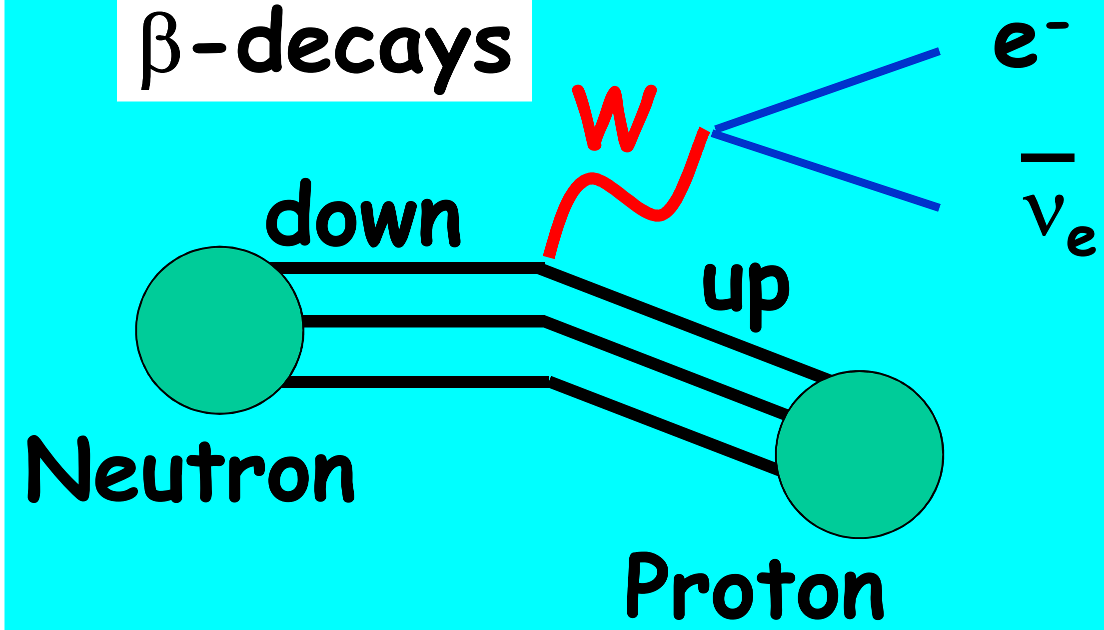
2) $D \rightarrow (K,\pi)\, l\, \nu_l$

3) $B \rightarrow (\pi,\rho)\, l\, \nu_l$

V_{ud}	V_{us}	V_{ub}
V_{cd}	V_{cs}	V_{cb}
V_{tb}	V_{ts}	V_{tb}

Quark masses & Generation Mixing

β-decays



$|V_{ud}|$

$|V_{ud}| = 0.9735(8)$

$|V_{us}| = 0.2196(23)$

$|V_{cd}| = 0.224(16)$

$|V_{cs}| = 0.970(9)(70)$

$|V_{cb}| = 0.0406(8)$

$|V_{ub}| = 0.00363(32)$

$|V_{tb}| = 0.99(29)$

(0.999)

IN THE ELICITY BASIS:

1^-

$$< K(p_K) \mid J_\mu^{weak}(0) \mid D(p_D) > =$$
$$[(p_D + p_K)_\mu - q_\mu (M^2_D - M^2_K)/q^2] \times f^+(q^2) +$$
$$q_\mu (M^2_D - M^2_K)/q^2 \times f^0(q^2)$$

0^+

$$< K^*(p_{K^*},\eta) \mid J_\mu^{weak}(0) \mid D(p_D) > = \eta^{*\beta} T_{\mu\beta}$$

Vector meson polarization

0^- 1^- 1^+ = t-channel quantum numbers

1^-

$$T_{\mu\beta} = 2 V(q^2) / (M_D + M_{K^*}) \times (p_D)^\gamma (p_{K^*})^\delta \varepsilon_{\mu\gamma\delta\beta} +$$

1^+

$$- i (M_D + M_{K^*}) A_1(q^2) \times g_{\mu\beta}$$
$$+ i A_2(q^2) / (M_D + M_{K^*}) \times (p_D + p_{K^*})_\mu q_\beta +$$
$$- i A(q^2) 2 M_{K^*} / q^2 \times (p_D + p_{K^*})_\beta q_\mu$$

0^-

$$\mathbf{A(q^2) = A_0(q^2) - A_3(q^2)}$$

Radiative Decays: $B \to K^* \gamma$ $(e^+ e^-)$

$$< K^*(p_K, \eta) \mid \bar{s}\, \sigma_{\mu\nu} q^\nu b \mid B(p_B) > = \sum_{i=1,3} C_\mu^{\,i}\, T_i(q^2)$$

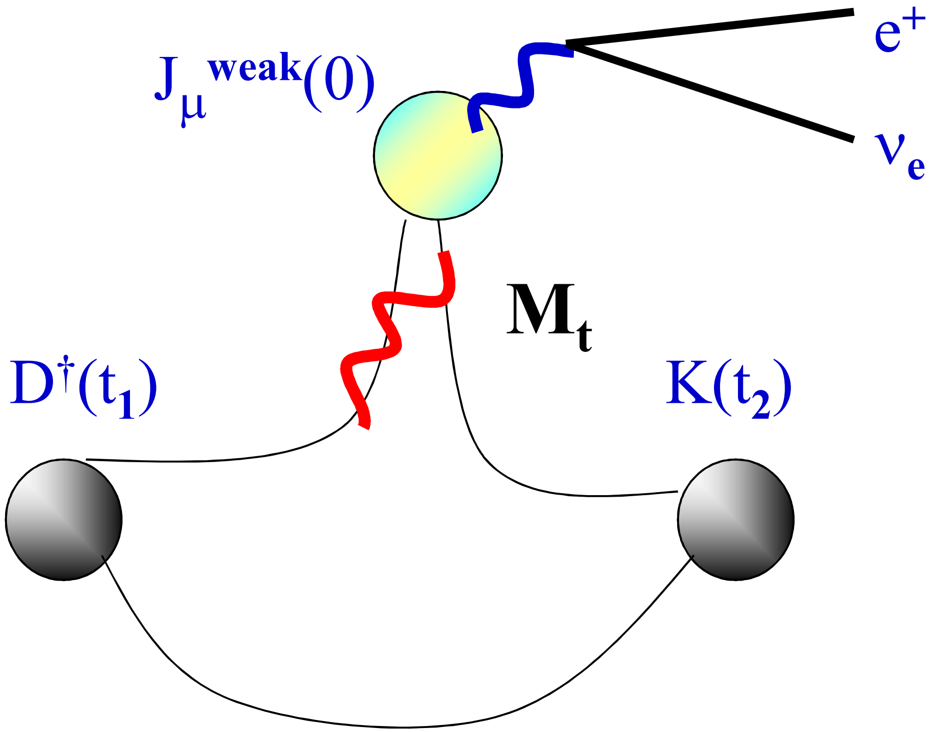
Vector meson polarization

$$C_\mu^{\,1} = 2\, \eta^\gamma (p_B)^\delta (p_{K^*})^\beta \varepsilon_{\mu\gamma\delta\beta}$$
$$C_\mu^{\,2} = \eta_\mu (M^2_B - M^2_{K^*}) - (\eta.q)\,(p_B + p_{K^*})_\mu$$
$$C_\mu^{\,3} = (\eta.q)\,[q_\mu - q^2 / (M^2_B - M^2_{K^*})\,(p_B + p_{K^*})_\mu\,]$$

AT $q^2 = 0$ $\quad T_1(q^2) = i\, T_2(q^2)$

$T_3(q^2)$ does not contribute

Pole Dominance



$$f^+(q^2) = \frac{f^+(0)}{1 - q^2/\mathbf{M_t}^2}$$

works well for the pion electromagnetic form factor ,

dipole in the case of the proton

$$\frac{f\ (0)}{(1 - q^2/\mathbf{M_t}^2)^{\ 2}}$$

Scaling behavior for the Form Factors
at $q^2 \approx (q^2)^{max}$

Form Factor	t-channel	m_Q dependence
	B -> π	
f^+	1^-	$m_Q^{1/2}$
f^0	0^+	$m_Q^{-1/2}$
	B -> ρ	
V	1^-	$m_Q^{1/2}$
A_1	1^+	$m_Q^{-1/2}$
A_2	1^+	$m_Q^{1/2}$
A_3	1^+	$m_Q^{3/2}$
A_0	0^-	$m_Q^{1/2}$

Kinematical constraints & scaling at $q^2 \approx 0$

$f^+(0) = f^0(0)$ $\qquad$ $f^+(0) \approx m_Q^{-3/2}$ from Light cone behaviour

A POPULAR PARAMETRIZATION WHICH TAKES INTO ACCOUNT
THE SCALING AT LARGE AND SMALL MOMENTUM TRANSFER
THE POLE CONTRIBUTION, THE KINEMATICAL CONSTRAINT AND THE
ANALITICITY PROPERTY OF THE FORM FACTORS
IS THE BK PARAMETRIZATION

$$f^+(q^2) = \frac{C^+(1-\alpha^+)}{(1-q^2/\mathbf{M_t}^2)(1-\alpha^+ q^2/\mathbf{M_t}^2)}$$

$$f^0(q^2) = \frac{C^+(1-\alpha^+)}{(1-q^2/(\beta^+\mathbf{M_t}^2))}$$

$$\boxed{\begin{array}{l} C^+ \approx m_Q^{-1/2} \\ (1-\alpha^+) \approx m_Q^{-1} \\ (1-\beta^+) \approx m_Q^{-1} \end{array}}$$

General consideration on non-perturbative methods/approaches/models

Models a) bag-model b) quark model

not based on the fundamental theory; at most QCD "inspired"; cannot be systematically improved

Effective theories c) chiral lagrangians d) Wilson Operator Product Expansion (OPE) e) Heavy quark effective theory (HQET)

based on the fundamental theory; limited range of applicability; problems with power corrections (higher twists), power divergences & renormalons; need non perturbative inputs (f_π, $< x >$, λ_1, Λ)

Methods of effective theories used also by QCD sum rules and Lattice QCD

f) QCD sum rules

based on the fundamental theory + **"condensates"** (non-perturbative matrix elements of higher twist operators, which must be determined phenomenologically; very difficult to improve; share with other approaches the problem of renormalons etc.

LATTICE QCD

Started by Kenneth Wilson in 1974

★ Based on the fundamental theory [Minimum number of free parameters, namely Λ_{QCD} and m_q]

★ Systematically improvable [errors can me measured and corrected, see below]

★ Lattice QCD is not at all numerical simulations and computer programmes only. A real understanding of the underline Field Theory, Symmetries, Ward identities, Renormalization properties is needed.

LATTICE QCD IS REALLY EXPERIMENTAL FIELD THEORY

Major fields of investigation

QCD

- QCD thermodynamics
- Hadron spectrum
- Hadronic matrix elements

(K -> ππ , structure functions, etc. see below)

EW

- Strong interacting Higgs Models
- Strong interacting chiral models

- Surface dynamics
- Quantum gravity

LATTICE QCD

α_S and the Quark Masses

Leptonic decay constants : f_π , f_K , f_D , f_{D_S}, f_B , f_{B_S}, f_ρ .

Electromagnetic form factors : $F_\pi(Q^2)$, $G_M(Q^2)$, ...

Semileptonic form factors : $f^{+,0}(Q^2)$, $A_{0,..3}(Q^2)$, $V(Q^2)$

K -> π, D -> K, K*, π, ρ, B -> D, D*, π, ρ B -> K* γ

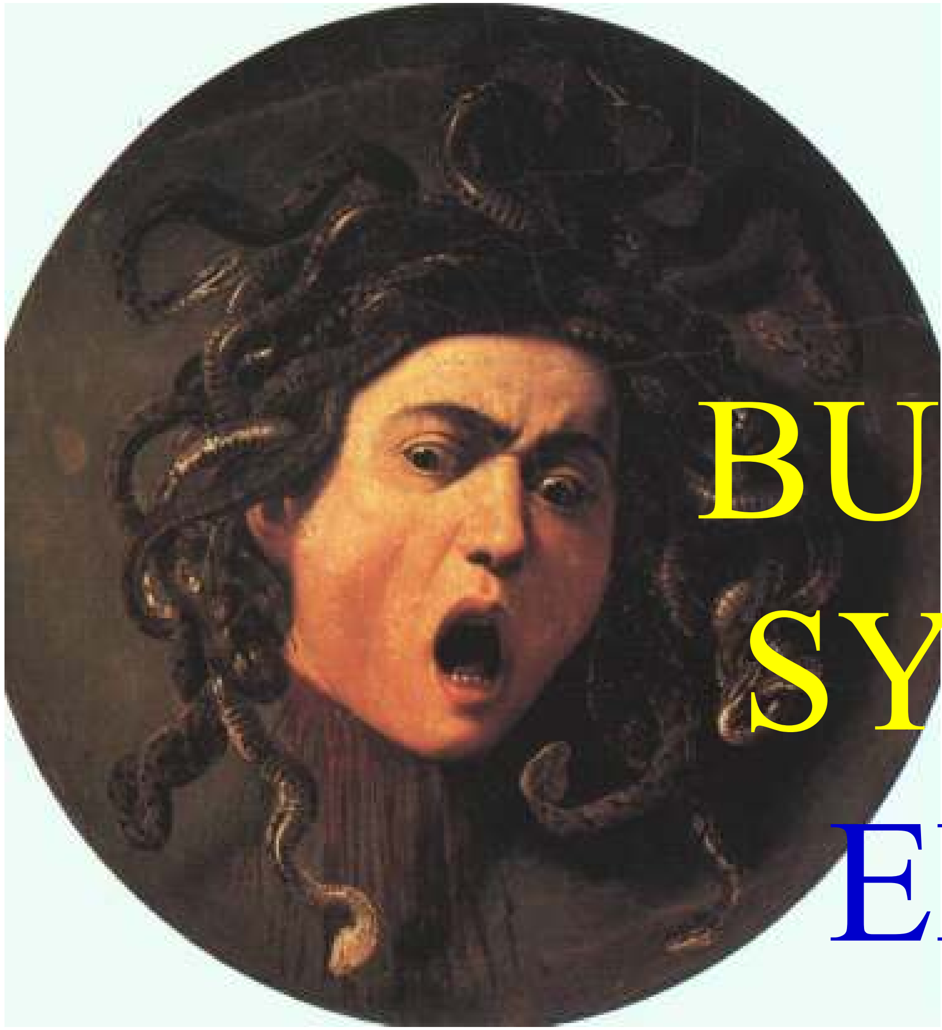
The Isgur-Wise function

B-parameters : $\langle \overline{K^0} \,|\, Q^{\Delta S=2} \,|\, K^0 \rangle$ and $\langle \overline{B^0} \,|\, Q^{\Delta B=2} \,|\, B^0 \rangle$

Weak decays : $\langle \pi \,|\, Q^{\Delta S=1} \,|\, K \rangle$ and $\langle \pi\, \pi \,|\, Q^{\Delta S=1} \,|\, K \rangle$

Matrix elements of leading twist operators :

Lattice QCD is really a powerful approach



BUT … FOR SYSTEMATIC ERRORS

Lattice QCD is really a powerful approach

SYSTEMATIC ERRORS

Quenching errors

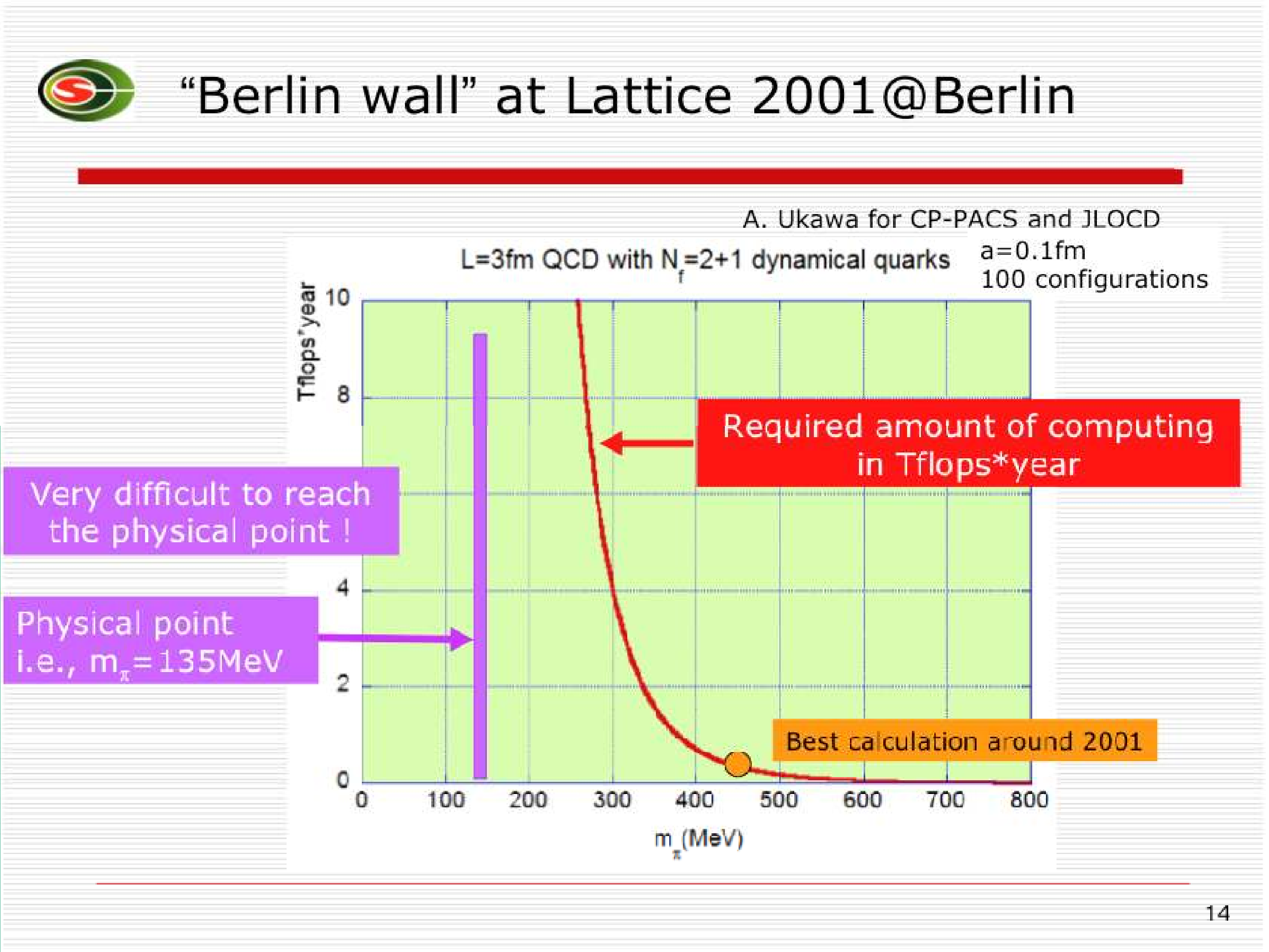
ALL MODERN LATTICE CALCULATIONS ARE UNQUENCHED:

Nf=2, 2+1 or 2+1+1

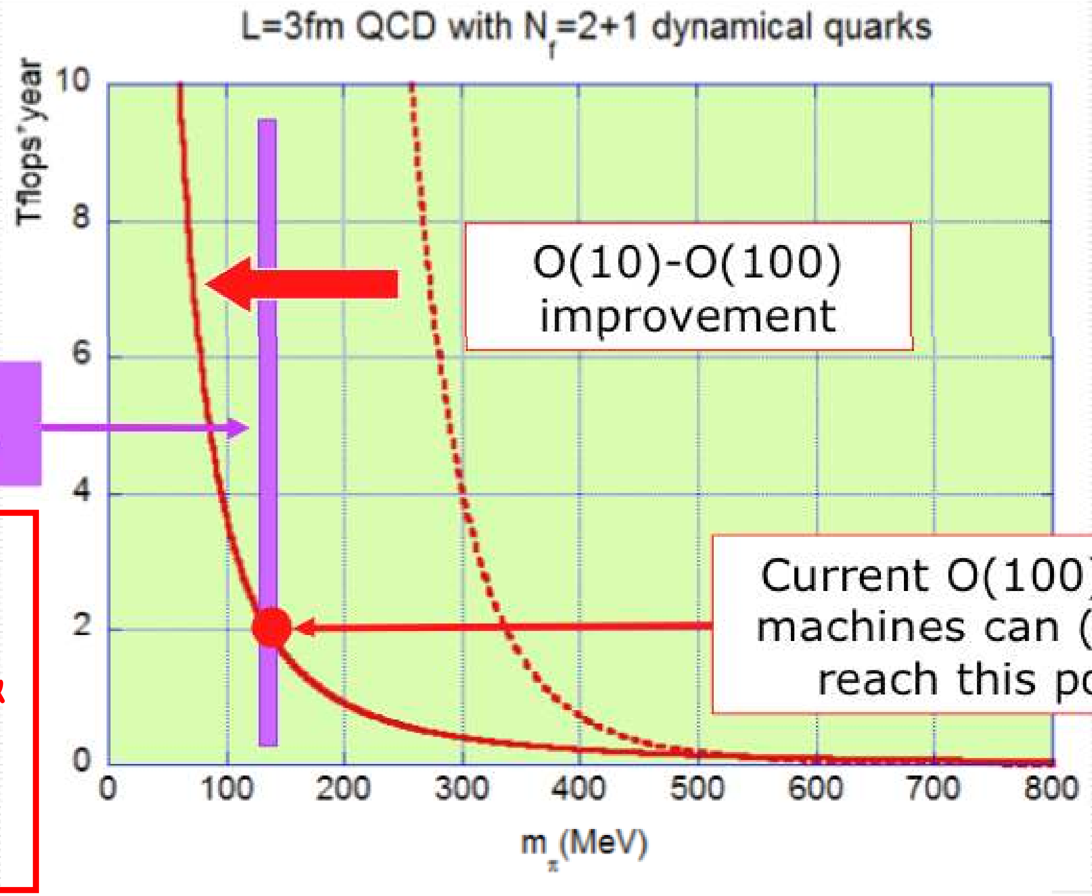
- nucleon σ-term and polarized structure functions wrong
- problems with chiral logarithms
- problems with unitarity for two-body decays

Almost all groups are now moving to unquenched calculations

Many slides from he Workshop Future Directions in lattice gauge theory LGT10 July 19th- August 13th CERN



Revolutionary progress since 2005 ; beating the critical slowing down



Physical point i.e., m_π=135MeV

Still Extrapolation to continuum &

Finite volume effects

Physical Point Simulation has become reality

Key observation

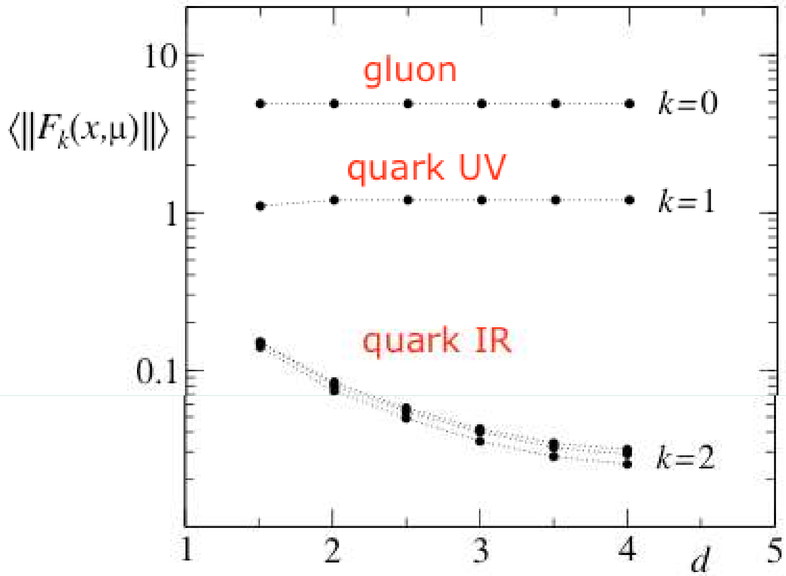
M. Luescher, C

- Separate UV and IR modes of quark fluctuations
- gluon:UV:IR forces are order of magnitude different!

$$F_{gluon} >> F_{quark,UV} >> F_{quark,I}$$

- This invites a multi-time step integration:

$$\delta\tau_{gluon} << \delta\tau_{quark,UV} << \delta\tau_{quark,}$$



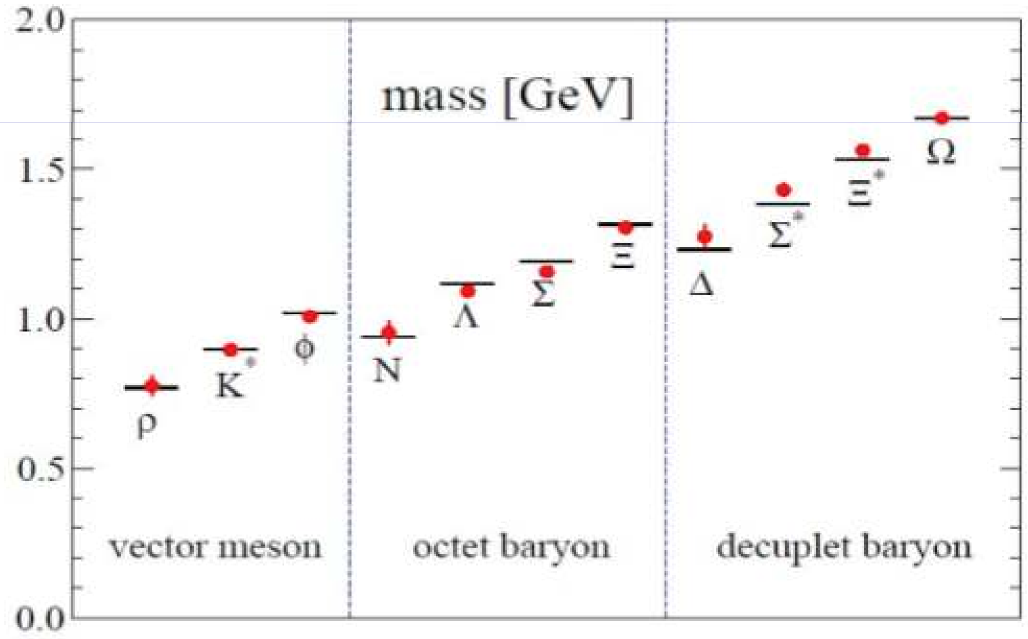
i.e., one can enlarge the time step for the most compute intensive IR quark force, *leading to a large reduction in the computing requirement.*

This is physics!

Our conscious effort toward physical pion mass (II)

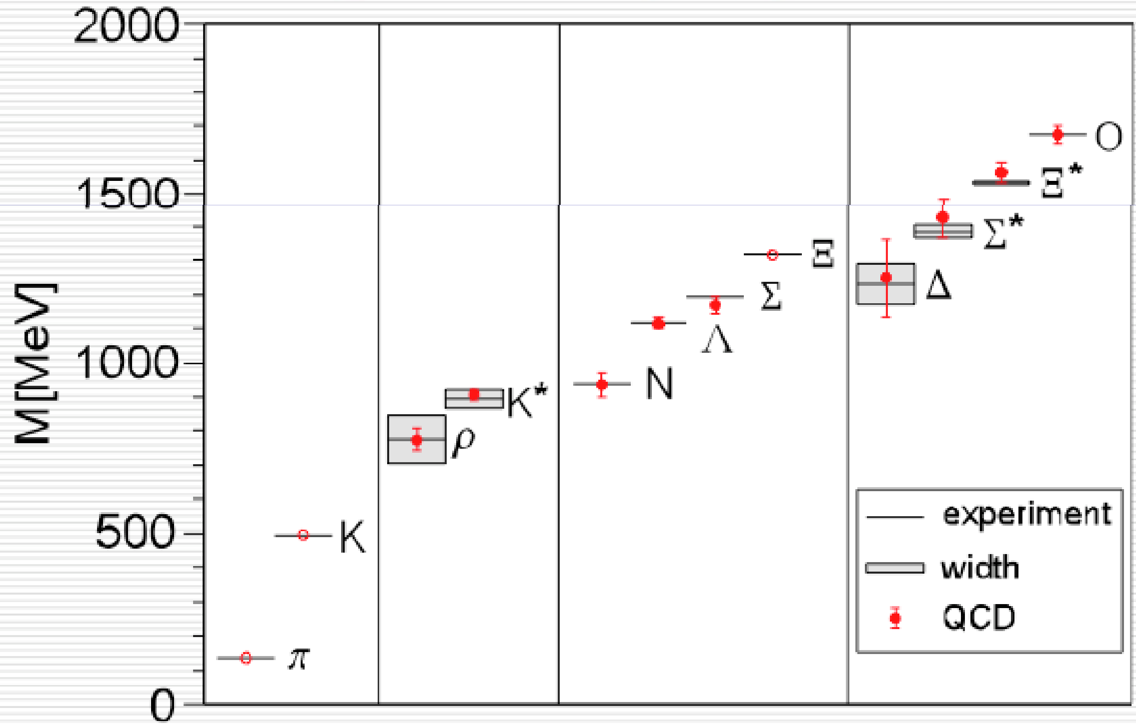
PACS-CS Collaboration Phys. Rev. D79 034504 (2008)

pion mass down to $m_\pi \approx 156 MeV$ $\quad 32^3 \times 64, \quad a = 0.907(13)\, fm$



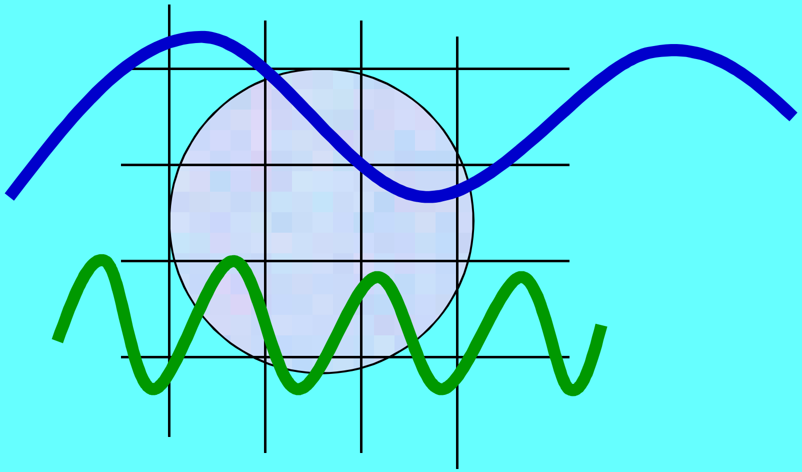
In the mean time,
came along the BMW Collaboration

BMW Collaboration (Butapest-Marseille-Wuppertal)
Science 322(2008) 1224
m_π>200MeV but large lattices ($m_\pi L$>4) and continuum extrapolated!



Extrapolation in the heavy quark mass

DISCRETIZATION ERRORS



THE ULTRAVIOLET PROBLEM

$$1/M_H \gg \mathbf{a}$$

O(a) errors $\left\{ \begin{array}{l} m_q\, \mathbf{a} \ll 1 \\ p\, \mathbf{a} \ll 1 \end{array} \right.$

For a good approximation of the continuum

Typically $\mathbf{a}^{-1} \sim 2 \div 5$ GeV

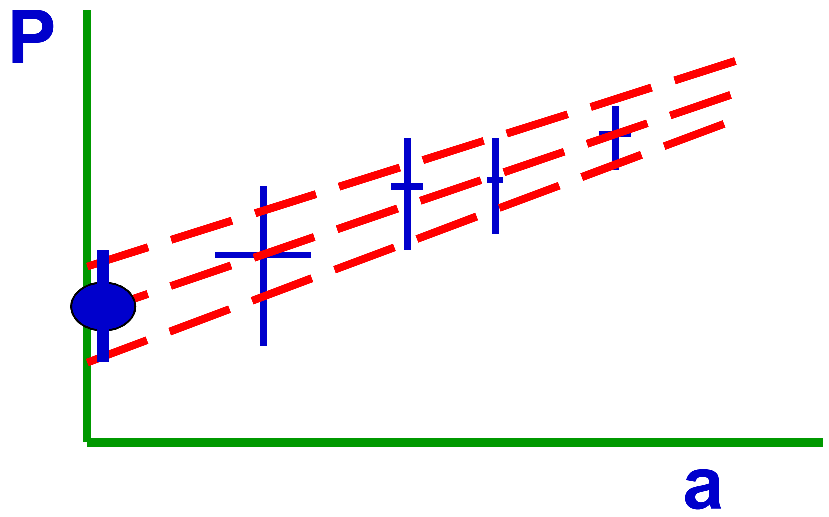
$\mathbf{m_{charm} \sim 1.3}$ **GeV** $\quad \mathbf{m_{charm}\, a \sim 0.3}$

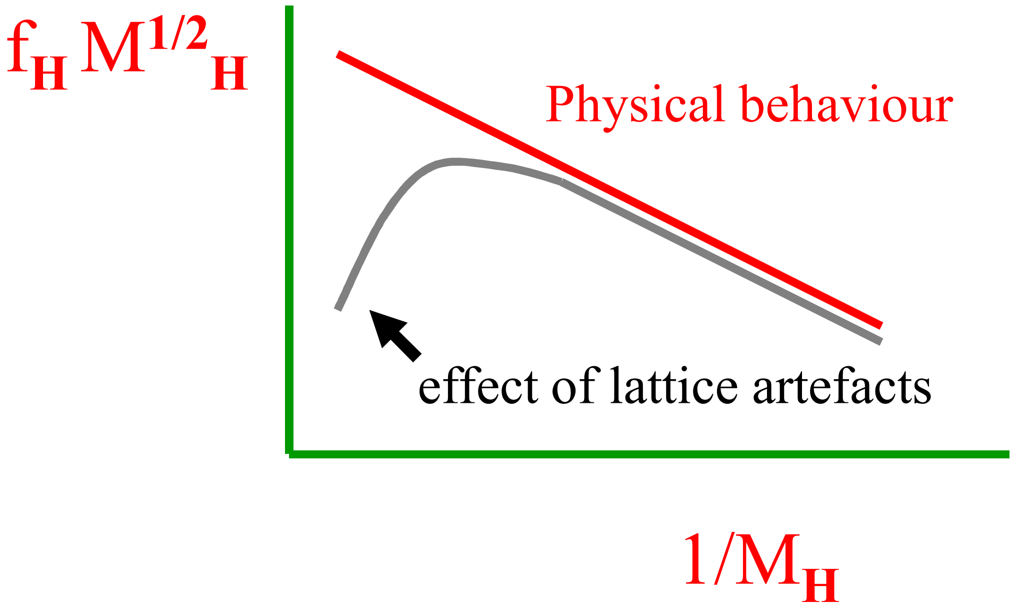
$\mathbf{m_{bottom} \sim 4.5}$ **GeV** $\quad \mathbf{m_{bottom}\, a \sim 1}$

SYSTEMATIC ERRORS

DISCRETIZATION ERRORS

Naïve solution: extrapolate measures performed at different values of the lattice spacing. Price: the error increases



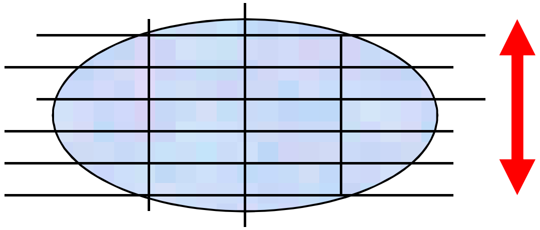


IMPROVEMENT

SYSTEMATIC ERRORS

FINITE VOLUME EFFECTS

THE INFRARED PROBLEM



$$L >> \xi = 1/M_H >> \mathbf{a}$$

To avoid finite size effects

For a good approximation of the continuum

Finite size effects were not really a problem for quenched calculations; potentially more problematic for the unquenched case

$O(\exp[- \xi /L])$

Is $L \geq 4 \div 5 \; \xi$ sufficient ?

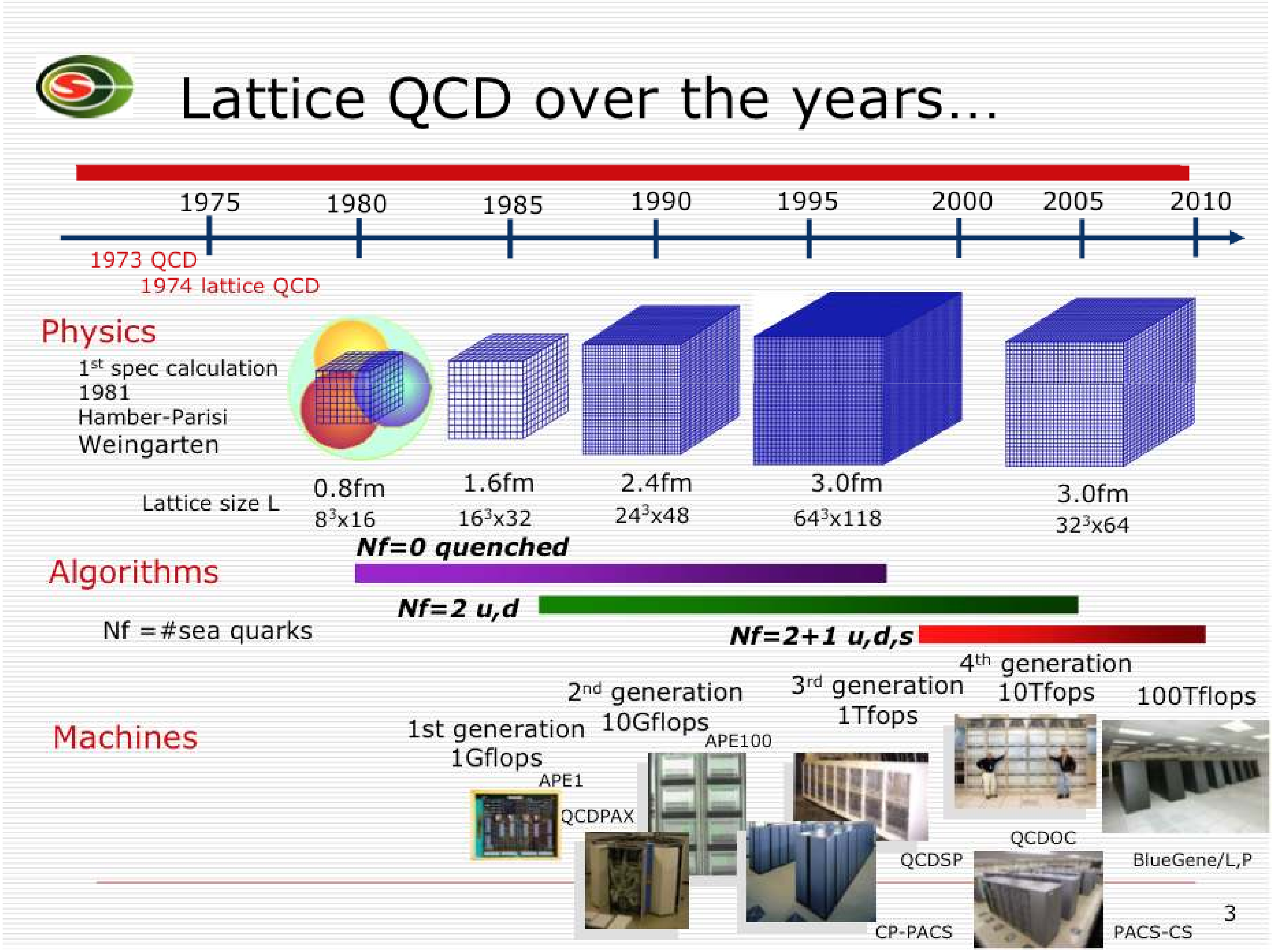
an extrapolation in m_{light} to the physical point is in many cases still necessary

Test if the quark mass dependence is described by Chiral perturbation Theory (χPT),
Then the extrapolation with the functional form suggested by χPT is justified

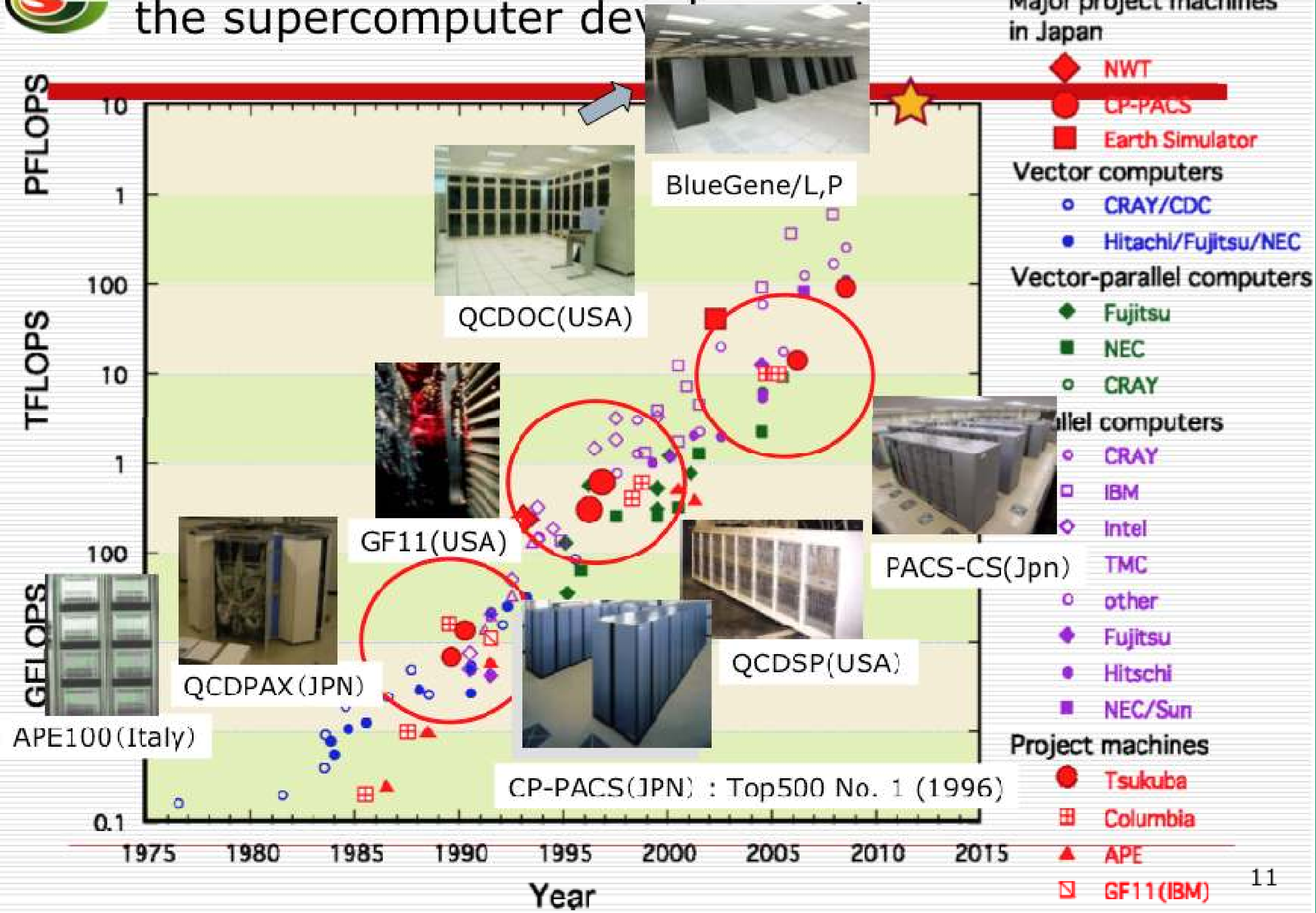
For heavy quark the extrapolation is suggested by the Heavy quark effective theory (HQET)

Precision Lattice QCD: from simulations to calculations

1) Better theoretical understanding
2) Better Algorithms
3) More powerful machines



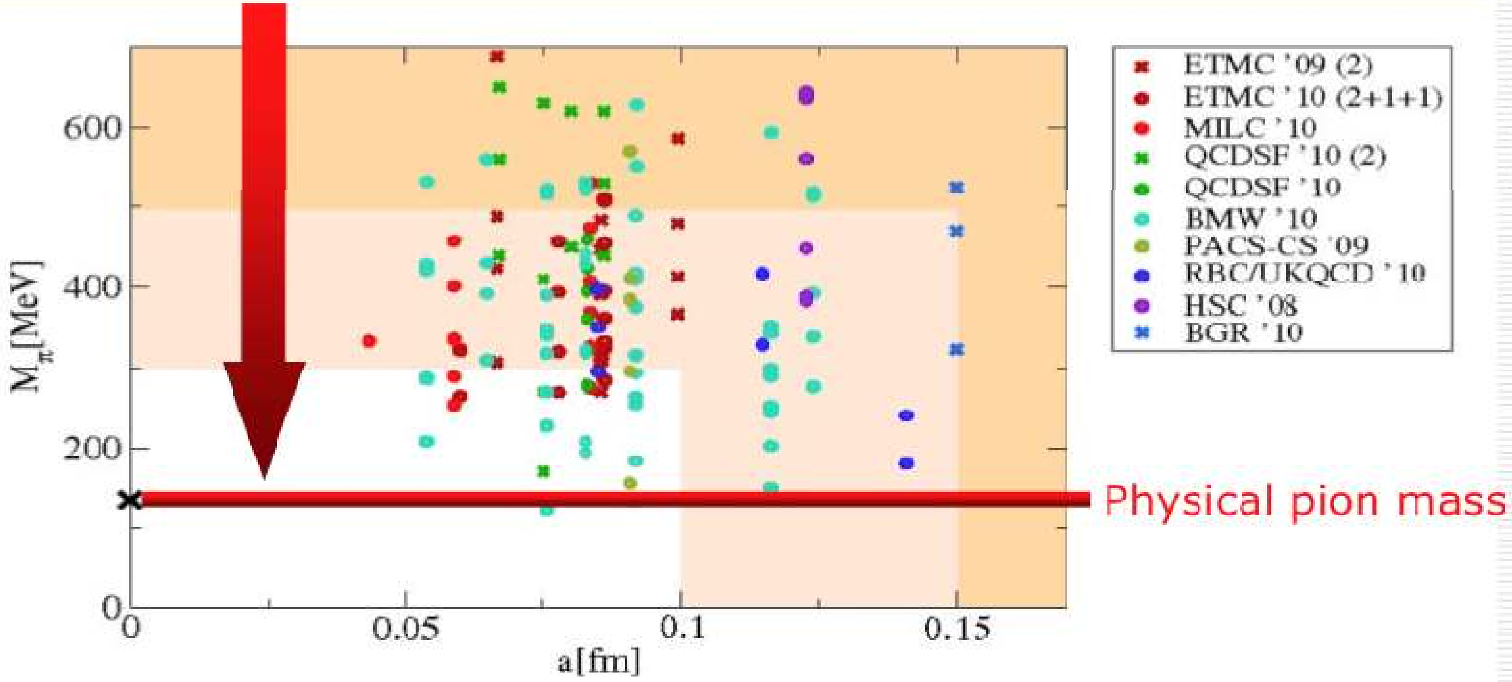
Impact of lattice QCD machines on the supercomputer development



Status this year:
pion mass vs lattice spacing

Continuum Landscape

From C. Hoelbling's review at Lattice 2010



Quality Criteria

FLAG: Flavianet Lattice Averaging Group

A. Vladikas

Quality Criteria

- chiral extrapolation:
  - ★ $M_{\pi,\min} < 250$ MeV
  - ● 250 MeV $\leq M_{\pi,\min} \leq$ 400 MeV
  - ■ 400 MeV $\leq M_{\pi,\min}$

  NB: at least 3 points requested (otherwise there is a "special mention")
- continuum extrapolation:
  - ★ at least 3 lattice spacings, at least two below 0.1 fm
  - ● 2 or more lattice spacings, at least one below 0.1 fm
  - ■ otherwise
- finite volume effects:
  - ★ $[M_\pi \; L\,]_{\min} > 4$ or at least 3 volumes
  - ● $[\,M_\pi L\,]_{\min} > 3$ and at least 2 volumes
  - ■ otherwise, and in any case if $L < 2$ fm

  NB: p-regime
- renormalization (where applicable):
  - ★ non perturbative
  - ● 2-loop perturbation theory
  - ■ otherwise
- renormalization group running (where applicable):
  - ★ non perturbative
  - ● otherwise

Only:
Decay constants
KL3 Form Factors
BK for Neutral Kaon Mixing

Form factor, decay constants and unitarity

- unitarity: $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$
- experiment: $$|V_{ub}| = 3.93\ (36) \cdot 10^{-3}$$
- Kaon decays: $$|V_{us}|\ f_+(0) = 0.21661\ (47)$$

form factor @ zero momentum transfer $K^0 \to \pi^- \nu l^+$

$$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.27599\ (59)$$

Form factor, decay constants and unitarity

- unitarity: $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$
- experiment: $$|V_{ub}| = 3.93\,(36) \cdot 10^{-3}$$
- Kaon decays: $$|V_{us}|\ f_+(0) = 0.21661\,(47)$$
$$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.27599\,(59)$$
- 3 expressions, 4 unknowns; need one more input

Form factor, decay constants and unitarity

- unitarity: $$|V_{ud}|^2 \; + \; |V_{us}|^2 \; + \; |V_{ub}|^2 \; = \; 1$$
- experiment: $$|V_{ub}| \; = \; 3.93\;(36)\;\cdot 10^{-3}$$
- Kaon decays: $$|V_{us}|\; f_+(0) \; = \; 0.21661\;(47)$$
$$\left|\frac{V_{us}\, f_K}{V_{ud}\, f_\pi}\right| \; = \; 0.27599\;(59)$$
- 3 expressions, 4 unknowns; need one more input

Form factor, decay constants and unitarity

$$f_+(0) = 0.964\ (3)\ (4) \qquad (N_f = 2+1)$$
$$f_+(0) = 0.956\ (6)\ (6) \qquad (N_f = 2)$$

Precision at the per mille level !!

most systematics OK

Collaboration	N_f	publication status	chiral extrapolation	finite volume errors	continuum extrapolation	$f_+(0)$
RBC/UKQCD 07	2+1	A	●	★	■	0.9644(33)(34)(14)
ETM 09A	2	A	●	●	●	0.9560(57)(62)
QCDSF 07	2	C	■	★	■	$0.9647(15)_{stat}$
RBC 06	2	A	■	★	■	0.968(9)(6)
JLQCD 05	2	C	■	★	■	0.967(6)

Table 1: Colour code for the data on $f_+(0)$.

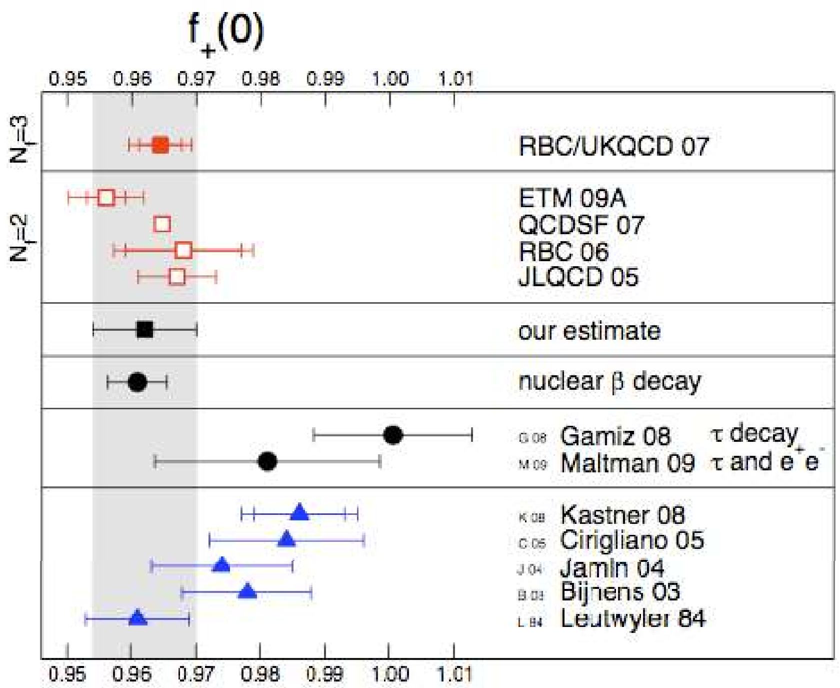
$$f_K/f_\pi = 1.190\ (2)\ (10) \qquad (N_f = 2+1)$$
$$f_K/f_\pi = 1.210\ (6)\ (17) \qquad (N_f = 2)$$

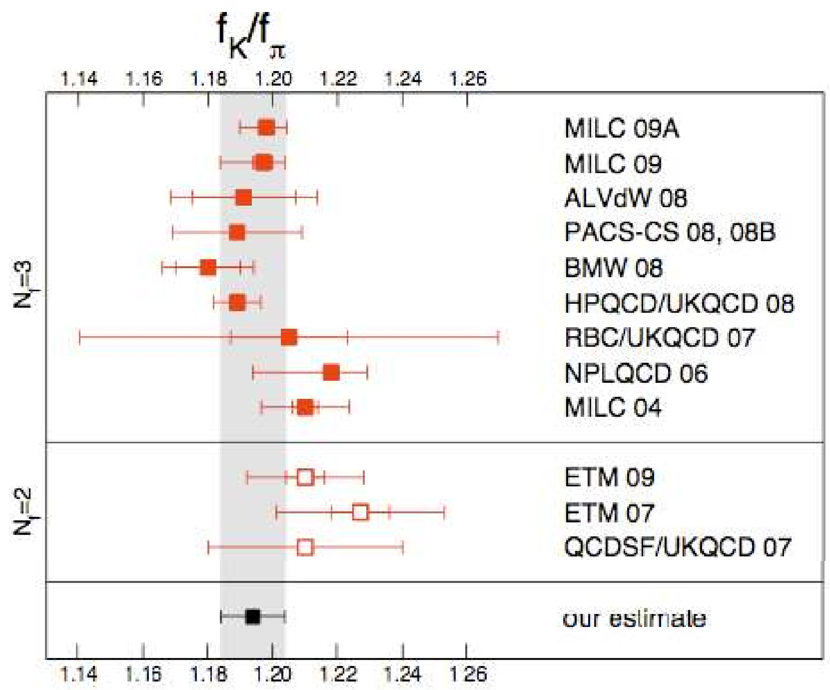
most systematics OK

Collaboration	N_f	publication status	chiral extrapolation	finite volume errors	continuum extrapolation	f_K/f_π
MILC 09A	2+1	C	★	★	★	$1.198(2)(^{+6}_{-8})$
MILC 09	2+1	P	★	★	★	$1.197(3)(^{+6}_{-13})$
ALVdW 08	2+1	C	★	●	●	1.191(16)(17)
PACS-CS 08, 08B	2+1	A	★	■	■	1.189(20)
BMW 08	2+1	C	★	★	★	1.18(1)(1)
HPQCD/UKQCD 08	2+1	A	★	●	★	1.189(2)(7)
RBC/UKQCD 08	2+1	A	●	★	■	1.205(18)(62)
NPLQCD 06	2+1	A	●	■	■	$1.218(2)(^{+11}_{-24})$
ETM 09	2	A	●	●	★	1.210(6)(15)(9)
QCDSF/UKQCD 07	2	C	●	★	●	1.21(3)

Table 1: Colour code for the data on f_K/f_π.

Form factor, decay constants and unitarity





- lattice agrees with nuclear β decay
- disagrees with semi-inclusive τ decay
- "our estimate" explained later
- from χPT:

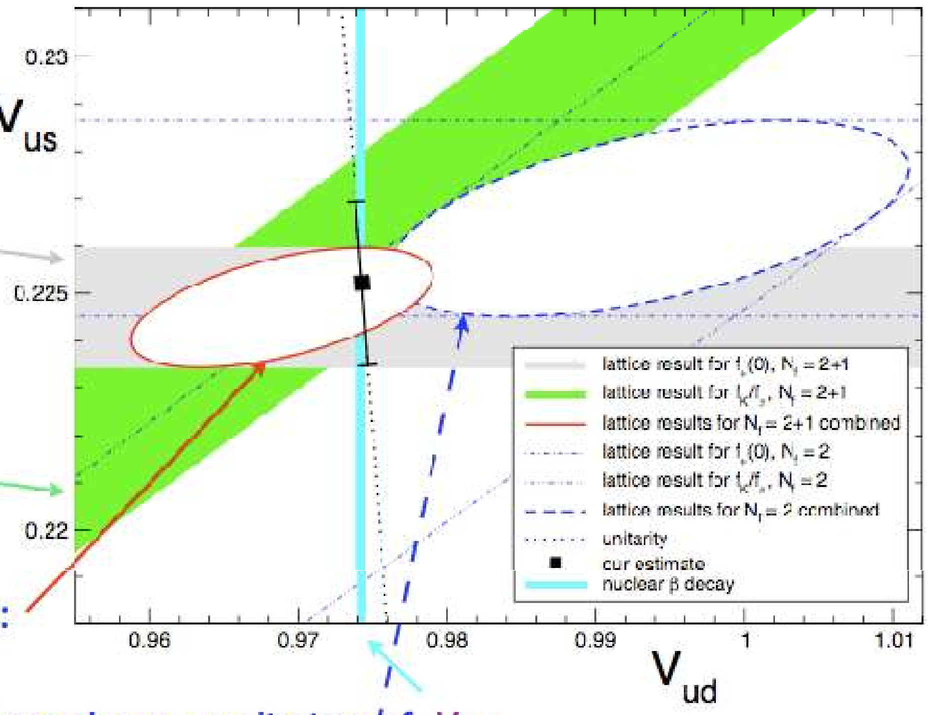
$$\Delta f = f_+(0) - 1 - f_2 = f_+(0) - 0.977$$

- lattice suggests $\Delta f < 0$
- results from various model estimates vary; Δf sign unclear

Form factor, decay constants and unitarity

- use: $|V_{us}|\ f_+(0) \ = \ 0.21661\ (47)$
- $N_f = 3$ result of $f_+(0)$ gives:
- use: $\left|\dfrac{V_{us} f_K}{V_{ud} f_\pi}\right| \ = \ 0.27599\ (59)$
- $N_f = 3$ result of f_K/f_π gives:

treating these two results as independent measurements gives the 68% likelihood contour:



- $N_f = 3$ lattice data consistent with nuclear beta decay prediction of V_{ud}:
- $N_f = 2$ lattice data consistent with $N_f = 3$ data within errors (just!!):

note the scale of the errors:
this is really precision physics.

Unquenched calculations, nf=2
at smaller quark masses and
more accurate continuum limit.

Form factor, decay constants and unitarity

- Test of Standard Model: relax unitarity constraint and test it!

- from Kaon decays we have:

$$|V_{us}|\ f_+(0)\ =\ 0.21661\ (47) \qquad \left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.27599\ (59)$$

- which combine with $N_f = 3$ lattice results of $f_+(0)$ and f_K/f_π to give $|V_{us}|$ and $|V_{ud}|$
- take $|V_{ub}|$ from experiment; the unitarity constraint is well satisfied:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.989\ (20) \qquad N_f = 2+1$$

- $N_f = 2$ 1.038(35) - OKish

- now use V_{ud} from β decays and $f_+(0)$ from $N_f = 3$ lattice:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9997\ (7)$$

- $N_f = 2$ 1.0005(10) - OK

- now use V_{ud} from β decays and f_K / f_π from $N_f = 3$ lattice:

- $N_f = 2$ 0.9986(16) - OK

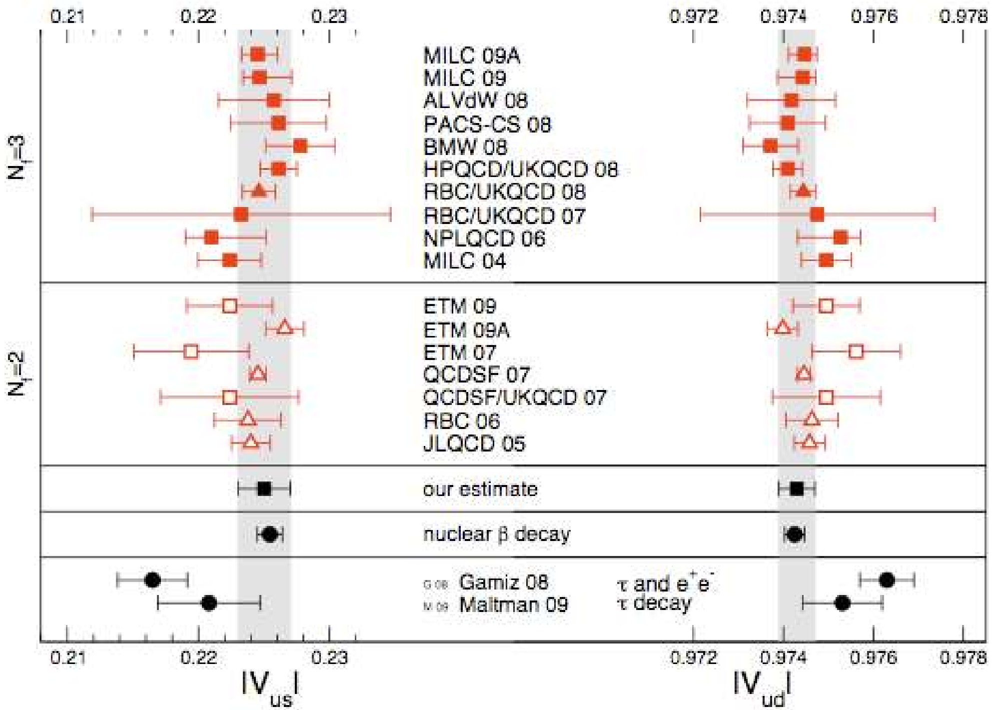
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0002\ (10)$$

Form factor, decay constants and unitarity

- Analysis based on Standard Model:

	$\|V_{us}\|$	$\|V_{ud}\|$	$f_+(0)$	f_K/f_π
$N_f = 2+1$	0.2251(11)	0.97433(24)	0.9626(43)	1.1944(61)
$N_f = 2$	0.2253(17)	0.97428(40)	0.9608(73)	1.1934(98)
our estimate	0.225(2)	0.9743(4)	0.962(8)	1.194(10)

Table 1: Final results for the analysis of the lattice data within the Standard Model



- combine data from direct f_K/f_π measurements with f_K/f_π results obtained from direct $f_+(0)$ measurements, to get **best f_K/f_π result** at a given N_f

- vice versus get **best f_K/f_π result**

- extremely close agreement between N_f=2 and N_f=2+1 results; take biggest uncertainty into account to obtain "our estimate"

Heavy-Light Semileptonic Decays

D -> K,K* DECAYS PROBE LATTICE (or model) RESULTS BY COMPARISON WITH EXPERIMENTAL DATA:

Γ(D -> K) = known constant $|V_{cs}|^2$ $|A|^2$

Also $\Gamma(D \to K^*)_L / \Gamma(D \to K)_T$

$$f_+^{D\to\pi}(0) = 0.64(3)(6) \quad f_+^{D\to K}(0) = 0.73(3)(7) \quad \frac{f_+^{D\to\pi}(0)}{f_+^{D\to K}(0)} = 0.87(3)(9)$$

theory

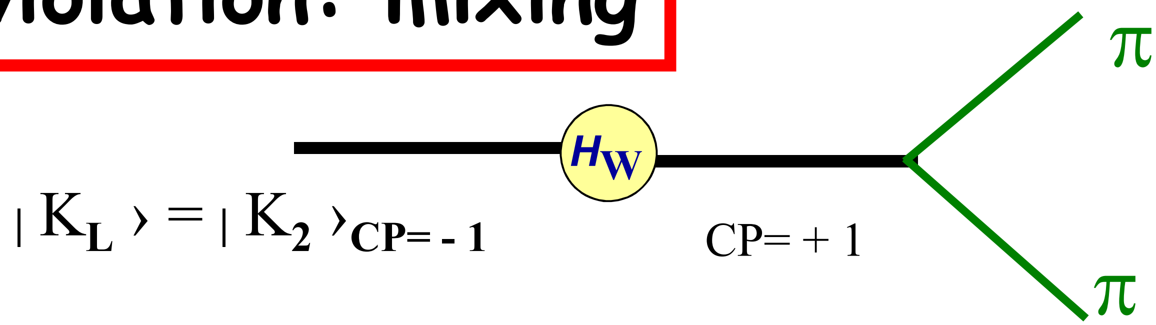
$$f_+^{D\to\pi}(0) = 0.73(15) \quad f_+^{D\to K}(0) = 0.78(5) \quad \frac{f_+^{D\to\pi}(0)}{f_+^{D\to K}(0)} = 0.86(9)$$

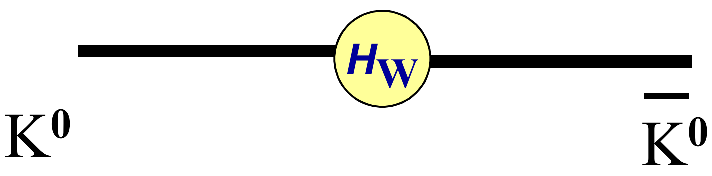
experiment hep-ex/0406028

or provide and independent determination of the CKM matrix elements

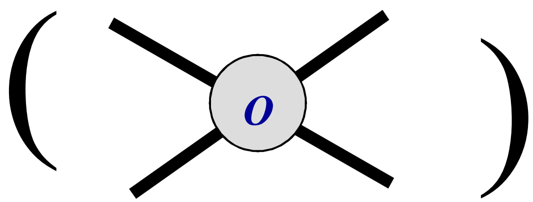
$$|V_{cd}| = 0.239(10)(24)(20) \quad |V_{cs}| = 0.969(39)(94)(24)$$

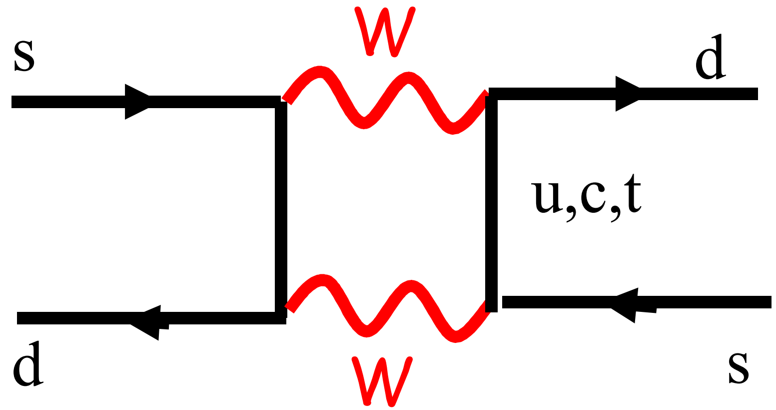
Indirect CP violation: mixing





ΔS=2





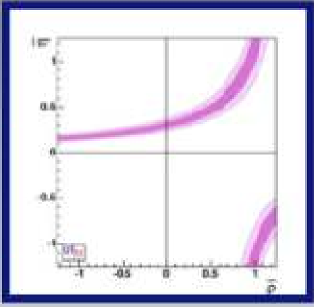
Box diagrams:
They are also responsible
for B^0 - $\bar{B}^0$ mixing
$\Delta m_{d,s}$

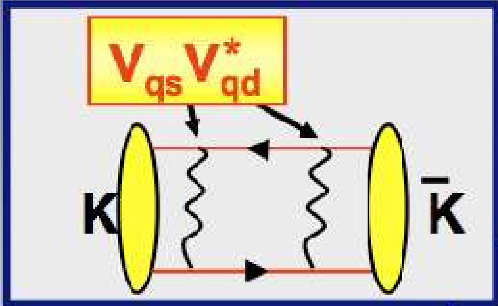
Complex ΔS=2 effective coupling

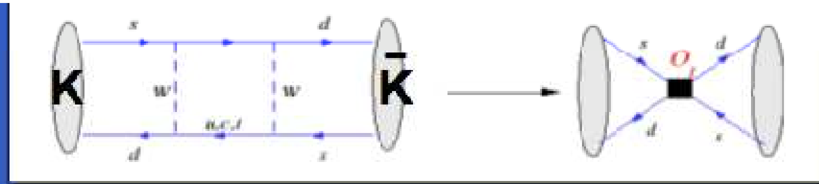
Progresses in the long distance calculation? See N. Christ at Lattice 2010

K^0-$\bar{K}^0$ mixing: B_K

V. Lubicz SuperB meeting nov. 2009







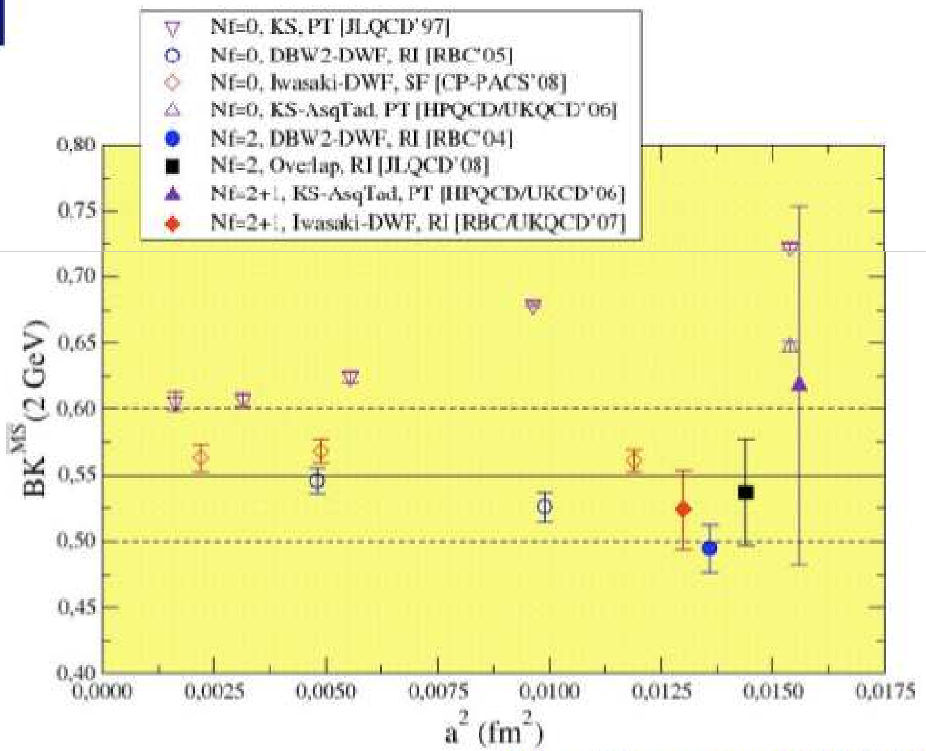
$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

$\hat{B}_K = 0.90 \pm 0.03 \pm 0.15$
S.Sharpe@Latt'96 — 17%

$\hat{B}_K = 0.86 \pm 0.05 \pm 0.14$
L.Lellouch@Latt'00 — 17%

$\hat{B}_K = 0.79 \pm 0.04 \pm 0.08$
C.Dawson@Latt'05 — 11%

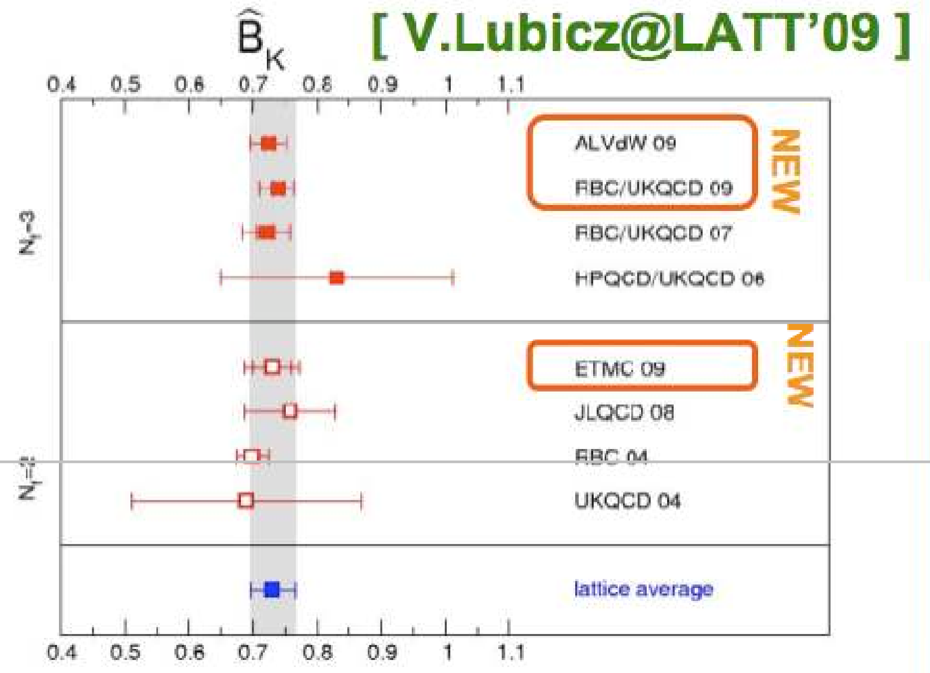
$\hat{B}_K = 0.723 \pm 0.037$
L.Lellouch@Latt'08 — 5%

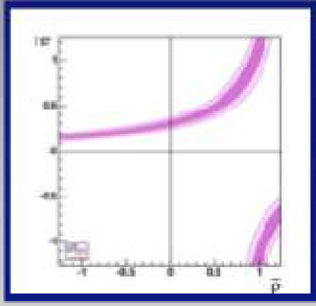


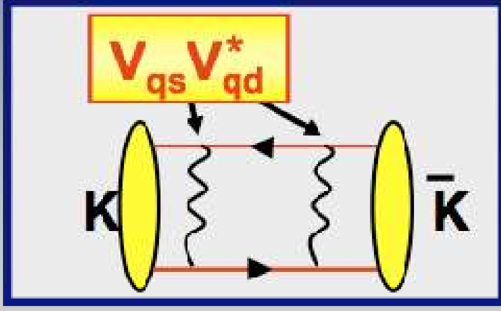
[VL, C.Tarantino 0807.4605]

All unquenched calculations until last year at fixed (and rather large) lattice spacing

K^0-$\bar{K}^0$ mixing: B_K







From the UT fit, assuming the validity of the Standard Model

$\hat{B}_K = 0.81\,(8)$

UTfit

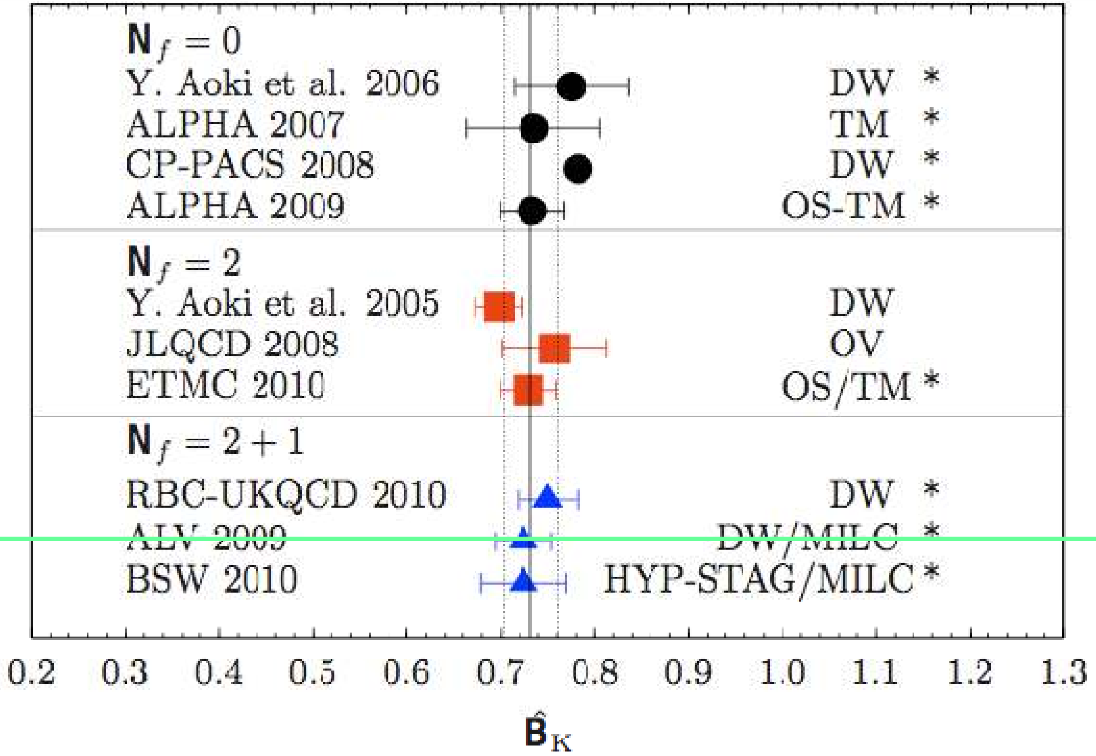
(with $K\varepsilon \approx 0.92$, A.Buras, D.Guadagnoli, 0805.3887)

$\hat{B}_K = 0.731\,(7)\,(35)$ 5%

Predicted error with 6 TFlops 5%

Error in 2006: 11%

$\hat{B}_K$

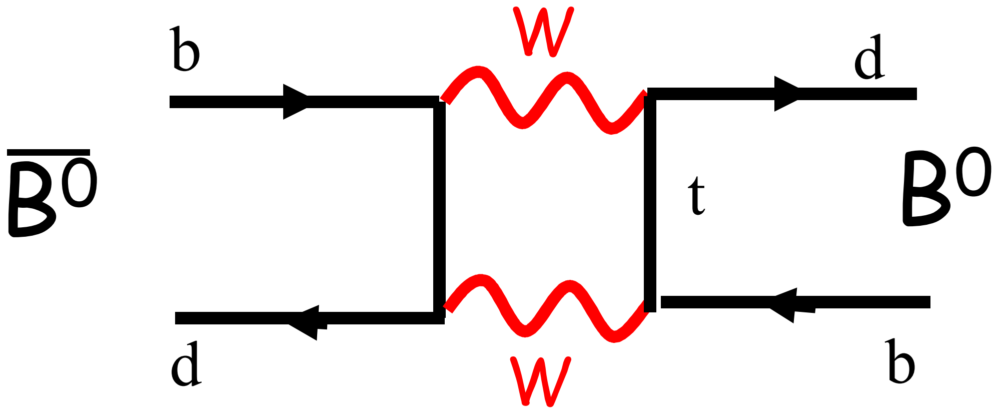


- $*$ $\longrightarrow$ result already in the **CL**.
- Average: $\hat{B}_K^{(N_f=2)}$(ETMC) $= 0.729(30)$; $\hat{B}_K^{(N_f=2+1)} = 0.732(06)(28)$
- No dependence on the strange quark (with the present precision)!
- Difference of less than $\sim 2\,\sigma$ with the most precise quenched result.

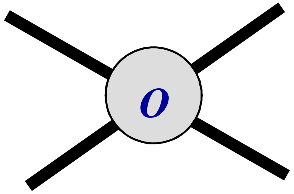
B^0 - B^0 mixing

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

ΔB=2 Transitions



$H_{eff}^{\Delta B=2} =$



$$\propto \left(\overline{\mathbf{d}}\, \gamma_\mu (\mathbf{1} - \gamma_5)\, \mathbf{b} \right)^2$$

Hadronic matrix element

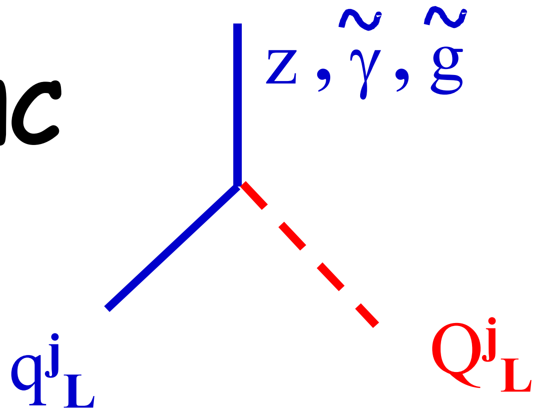
$\boxed{\text{CKM}}$

$$\Delta m_{d,s} = \frac{G_F^2 M_W^2}{16\,\pi^2} A^2 \lambda^6 F_{tt}\left(\frac{m_t^2}{M_W^2}\right) \langle O \rangle$$

In general the mixing mass matrix of the SQuarks (SMM) is not diagonal in flavour space analogously to the quark case We may either

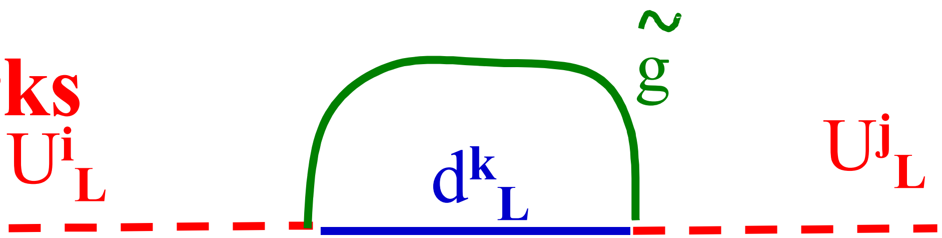
Diagonalize the SMM

FCNC

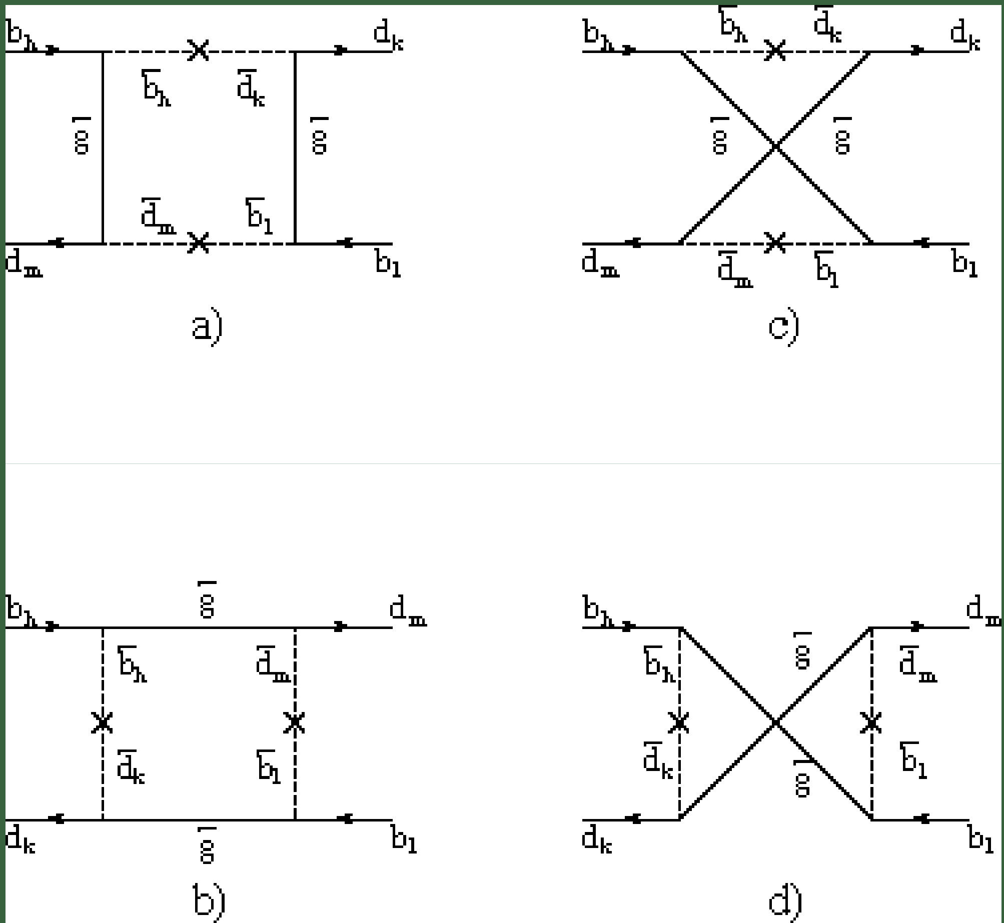


or Rotate by the same matrices the SUSY partners of the u- and d- like quarks

$(Q^j_L)' = U^{ij}_L\, Q^j_L$



In the latter case the Squark Mass Matrix is not diagonal



$(m^2_Q)_{ij} = m^2_{average}\, \mathbf{1}_{ij} + \Delta m_{ij}^2$ $\quad \delta_{ij} = \Delta m_{ij}^2 / m^2_{average}$

New local four-fermion operators are generated

$Q_1 = (\bar{b}_L{}^A \gamma_\mu d_L{}^A)(\bar{b}_L{}^B \gamma_\mu d_L{}^B)$ SM

$Q_2 = (\bar{b}_R{}^A d_L{}^A)(\bar{b}_R{}^B d_L{}^B)$

$Q_3 = (\bar{b}_R{}^A d_L{}^B)(\bar{b}_R{}^B d_L{}^A)$

$Q_4 = (\bar{b}_R{}^A d_L{}^A)(\bar{b}_L{}^B d_R{}^B)$

$Q_5 = (\bar{b}_R{}^A d_L{}^B)(\bar{b}_L{}^B d_R{}^A)$

+ those obtained by $L \leftrightarrow R$

Similarly for the s quark e.g.

$(\bar{s}_R{}^A d_L{}^A)(s_R{}^B d_L{}^B)$

$$\langle \bar{K}^0 | O_1(\mu) | K^0 \rangle = \frac{8}{3} M_K^2 f_K^2 B_1(\mu) ,$$

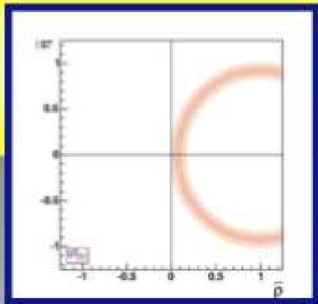
$$\langle \bar{K}^0 | O_2(\mu) | K^0 \rangle = -\frac{5}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_2(\mu) ,$$

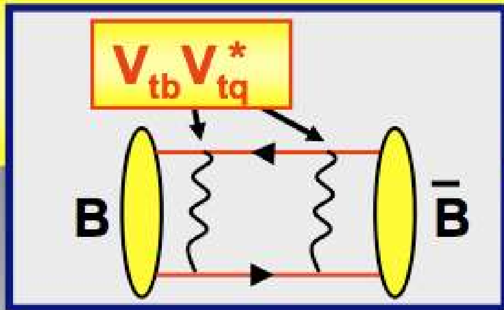
$$\langle \bar{K}^0 | O_3(\mu) | K^0 \rangle = \frac{1}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_3(\mu) ,$$

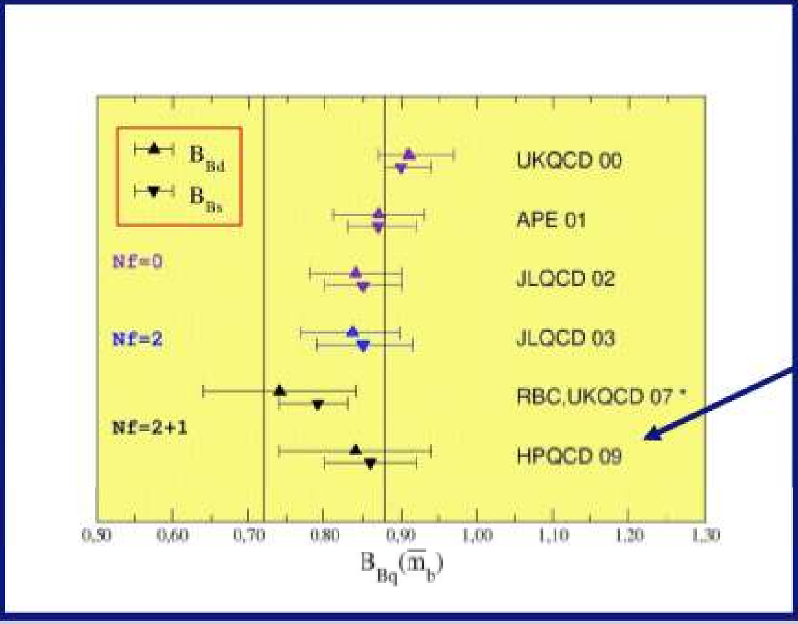
$$\langle \bar{K}^0 | O_4(\mu) | K^0 \rangle = 2 \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_4(\mu) ,$$

$$\langle \bar{K}^0 | O_5(\mu) | K^0 \rangle = \frac{2}{3} \left(\frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K^2 f_K^2 B_5(\mu) ,$$

$B-\bar{B}$ mixing: $B_{Bd/s}$







Only one modern calculation
HPQCD [0902.1815]

$$\hat{B}_{Bd} = 1.26 \pm 0.11$$

$$\hat{B}_{Bs} = 1.33 \pm 0.06$$

Combining with fB and fBs:

Error in 2006: 13%	Error in 2006: 5%
$f_{Bs}\sqrt{\hat{B}_{Bs}} = 275 \pm 13$ MeV — 5%	$\xi = 1.243 \pm 0.028$ — 2%
Predicted error with 6 TFlops — 4%	Predicted error with 6 TFlops — 3%

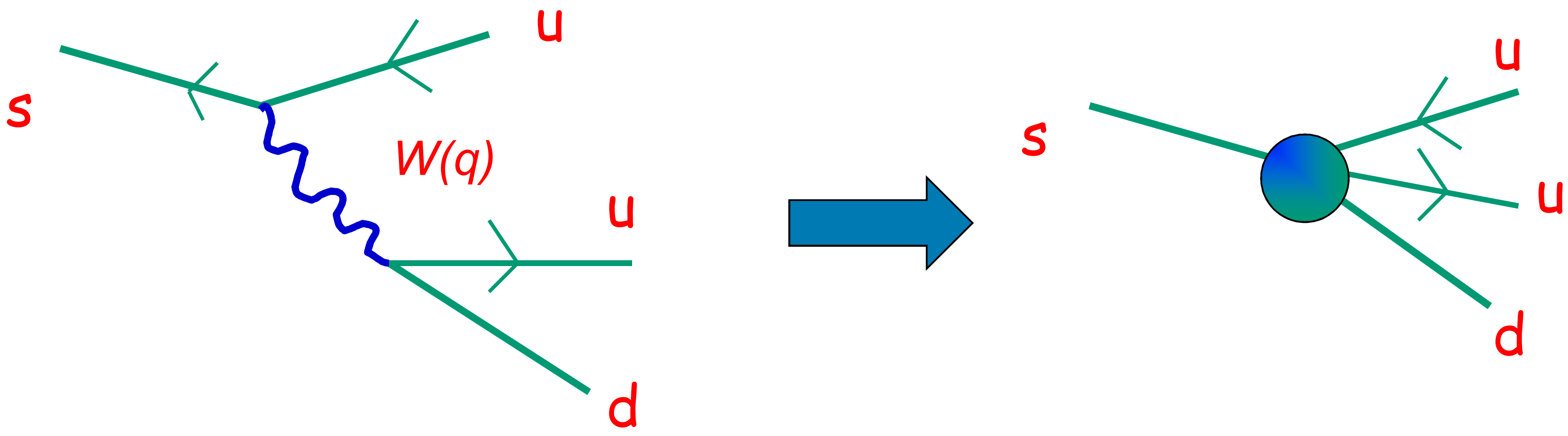
NON LEPTONIC DECAYS

- Theoretical framework
- Present situation e outlook

PROTOTYPE

$$K \longrightarrow \pi\pi$$

The Effective Hamiltonian



$$q \sim m_K \ll M_W$$

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \; (\bar{s}\gamma_\mu(1-\gamma_5)u)\,(\bar{u}\gamma^\mu(1-\gamma_5)d)$$

New local four-fermion operators are generated

$$Q_1 = (\bar{s}_L{}^A \gamma_\mu u_L{}^B)\,(\bar{u}_L{}^B \gamma_\mu d_L{}^A)$$
$$Q_2 = (\bar{s}_L{}^A \gamma_\mu u_L{}^A)\,(\bar{u}_L{}^B \gamma_\mu d_L{}^B)$$

Current-Current

$$Q_{3,5} = (\bar{s}_R{}^A \gamma_\mu d_L{}^A) \sum_q (\bar{q}_{L,R}{}^B \gamma_\mu q_{L,R}{}^B)$$
$$Q_{4,6} = (\bar{s}_R{}^A \gamma_\mu d_L{}^B) \sum_q (\bar{q}_{L,R}{}^B \gamma_\mu q_{L,R}{}^A)$$

Gluon Penguins

$$Q_{7,9} = 3/2(\bar{s}_R{}^A \gamma_\mu d_L{}^A) \sum_q e_q\,(\bar{q}_{R,L}{}^B \gamma_\mu q_{R,L}{}^B)$$
$$Q_{8,10} = 3/2(\bar{s}_R{}^A \gamma_\mu d_L{}^B) \sum_q e_q\,(\bar{q}_{R,L}{}^B \gamma_\mu q_{R,L}{}^A)$$

Electroweak Penguins

+ Chromomagnetic end electromagnetic operators

$$\boxed{\mathcal{A}(K \to \pi\pi) = \sum_i C_W^i(\mu) \langle \pi\pi | O_i(\mu) | K \rangle}$$

3. Direct Calculations of $K \to \pi\pi$ Decay Amplitudes

UNIVERSITY OF Southampton
School of Physics and Astronomy

- We need to be able to calculate $K \to \pi\pi$ matrix elements directly.
- The main theoretical ingredients of the *infrared* problem with two-pions in the s-wave are now understood.
- Two-pion quantization condition in a finite-volume

$$\delta(q^*) + \phi^P(q^*) = n\pi\,,$$

where $E^2 = 4(m_\pi^2 + q^{*2})$, δ is the s-wave $\pi\pi$ phase shift and ϕ^P is a kinematic function. M.Lüscher, 1986, 1991, $\cdots$.

- The relation between the physical $K \to \pi\pi$ amplitude A and the finite-volume matrix element M

$$|A|^2 = 8\pi V^2 \frac{m_K E^2}{q^{*2}} \left\{\delta'(q^*) + \phi^{P\prime}(q^*)\right\} |M|^2\,,$$

where $'$ denotes differentiation w.r.t. q^* .

L.Lellouch and M.Lüscher, hep-lat/0003023; C.h.Kim, CTS and S.Sharpe, hep-lat/0507006; N.H.Christ, C.h.Kim and T.Yamazaki hep-lat/0507009

- Computation of $K \to (\pi\pi)_{I=2}$ matrix elements does not require the subtraction of power divergences or the evaluation of disconnected diagrams.
- In principle, we understand how to calculate the $\Delta I = 3/2$ $K \to \pi\pi$ matrix elements.
- Our aim is to calculate the matrix elements with as good a precision as we can.

$Re(A_0)$ and $Im(A_0)$

Notice that $E_{I=0} = 0.450(17)$ and $E_{I=0vout} = 0.4392(59)$

m_K	F_f	$Re(A_0')(GeV)$	$Re(A_0)(GeV)$	$Im(A_0')(GeV)$	$Im(A_0)(GeV)$
0.4255(6)	40.5	$53.5(2.9)e^{-8}$	$40(11)e^{-8}$	$-87.8(7.4)e^{-12}$	$-30(28)e^{-12}$
0.5070(6)	44.2	$61.5(3.4)e^{-8}$	$50(14)e^{-8}$	$-95.7(7.8)e^{-12}$	$-68(38)e^{-12}$
on shell	-	$54.8(3.0)e^{-8}$	$43(12)e^{-8}$	$-89.2(7.5)e^{-12}$	$-41(31)e^{-12}$

From $t_\pi - t_K = 14$, and fitting range [5:10]

Used Free field normalization of states.

For I=0, it is very difficult to apply Lellouch Luscher factor here given the small volume. Numerically, $\partial\phi(q)/\partial q$ becomes divergent at $q^2 = -0.06639$ which correspond to $E_{I0} = 0.441$. Luscher's derivation requires that the Interaction range $R < L/2$. If we plug in $E_{I0} = 0.450$, we'll get $F = 90 \approx 2F_f$.

Determine physical A_2

(Matthew Lightman and Elaine Goode)

- Recall $\langle \pi\pi(I=2)|\mathcal{L}_W(0)|K\rangle = A_2 e^{i\delta_2}$

$$A_2 = \frac{\sqrt{3}}{2\sqrt{2}} \frac{1}{\pi q_\pi} \sqrt{\frac{\partial\phi}{\partial q_\pi} + \frac{\partial\delta}{\partial q_\pi}} L^{3/2} a^{-3} G_F V_{ud} V_{us} \sqrt{m_K} E_{\pi\pi}$$
$$\times \sum_{i,j} C_i(\mu) Z_{ij}(\mu) \langle \pi\pi|Q_j|K\rangle$$

- $\mathrm{Re}(A_2)$ dominated by single operator $O^{(27,1)}$.

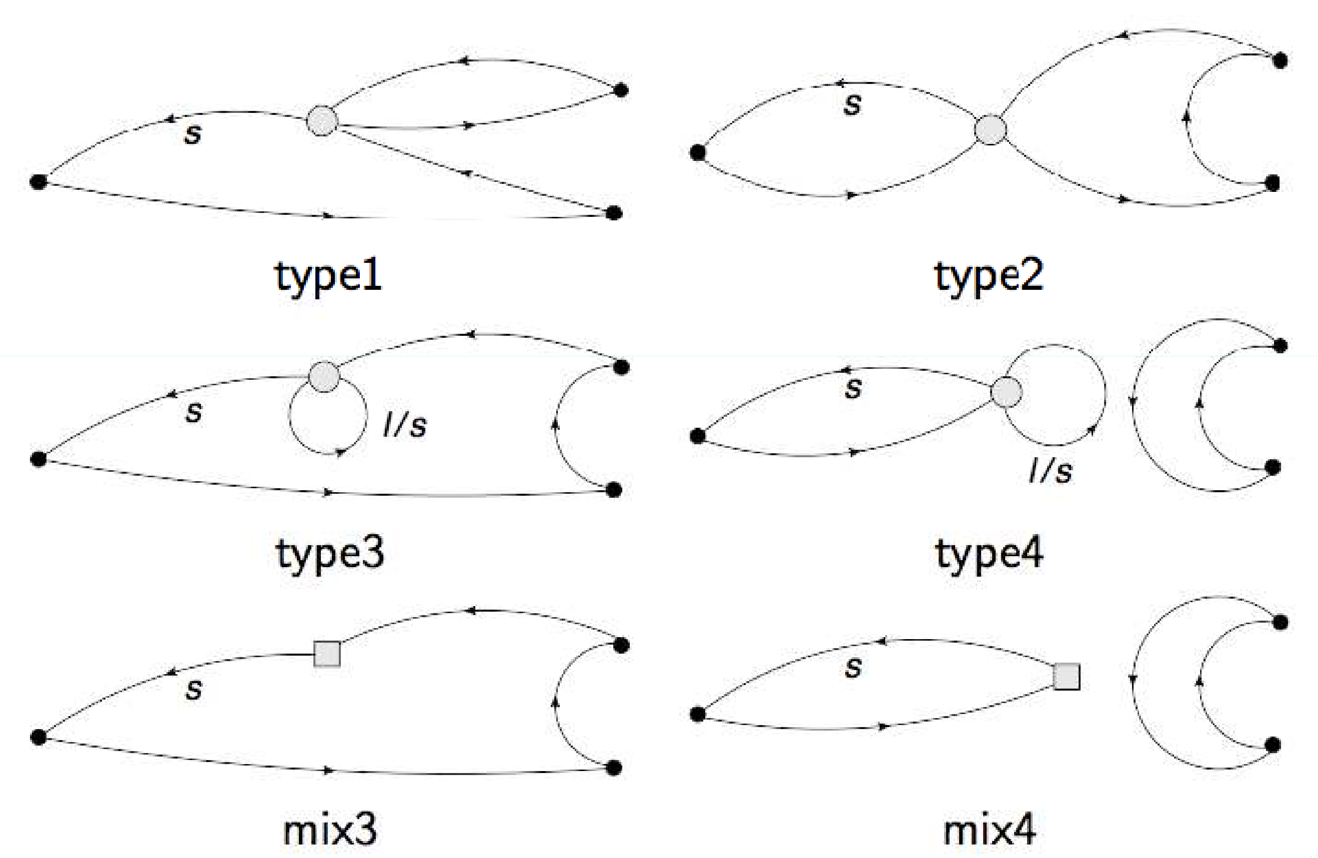
- Determine Lellouch-Luscher factor.

$$\frac{\partial\phi}{\partial q_\pi} = 5.141 \qquad \frac{\partial\delta}{\partial q_\pi} = 0.305$$

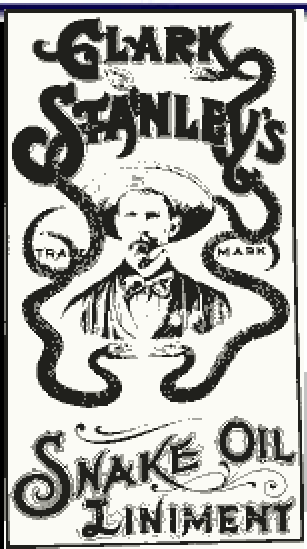
- **$\mathrm{Re}(A_2) = 1.56(7)_{\mathrm{stat}}(25)_{\mathrm{sys}}\ 10^{-8}$ GeV** [Expt: 1.5 10^{-8} GeV]

- $\mathrm{Im}\, A_2 = -9.6(4)(24) \times 10^{-13}$ GeV.
  In addition to lattice artefacts, we are in the process of performing the NPR for the EWP operators $O_{7,8}$ The result above is obtained by taking $Z_{ij} = 0.9(0.18)\delta ij$.
- $\mathrm{Im}\, A_2/\mathrm{Re}\, A_2 = -6.2(0.3)(1.3)\, 10^{-5}$.

$K^0 \to \pi\pi(I = 0)$ contractions



THE COLLABORATION



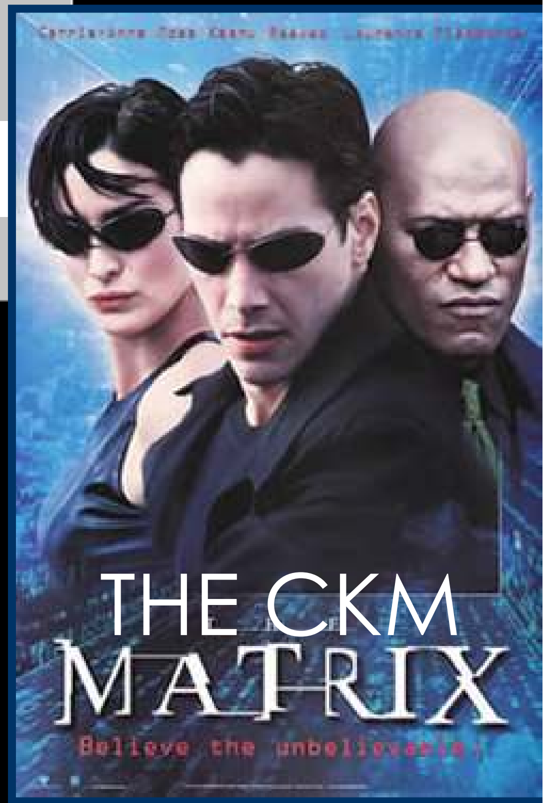
M.Bona, M.Ciuchini, E.Franco, V.Lubicz,
G.Martinelli, F.Parodi,M.Pierini,
P.Roudeau, C.Schiavi,L.Silvestrini,
V. Sordini, A.Stocchi, V.Vagnoni

Cern, Roma, Genova, Orsay, Bologna

2008 (2009) ANALYSES

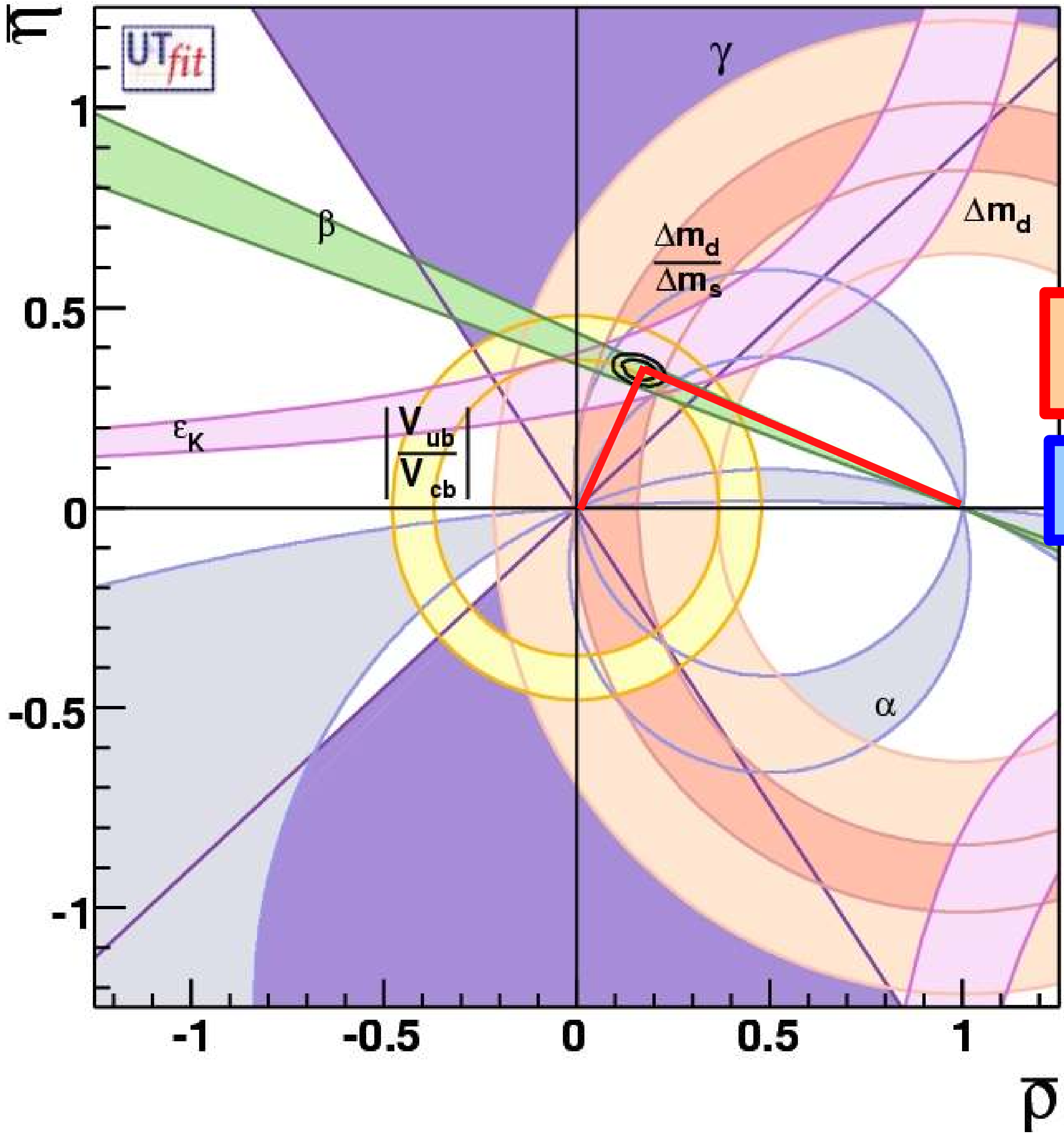
- New quantities included
- Upgraded exp. numbers (after ICHEP '08)
- (CDF) & D0 new measurements

www.utfit.org



Results for ρ and η & related quantities

With the constraint fromΔm_s



contours @ 68% and 95% C.L.

ρ = 0.155 ± 0.022

η = 0.342 ± 0.014

$\alpha = (92.0 \pm\ 3.4)^0$
$\sin 2\beta = 0.695 \pm 0.020$
$\gamma = (65.6 \pm\ 3.3)^0$

A closer look to the analysis:

1) **(some) Predictions vs Postdictions (past)**
2) **Lattice vs angles**
3) **V_{ub} inclusive, V_{ub} exclusive vs sin 2β**
4) **Experimental determination of lattice parameters**

Comparison of $\sin 2\beta$ from direct measurements (Aleph, Opal, Babar, Belle,D0 and CDF) and UT analysis

$$\sin 2\beta_{measured} = \mathbf{0.668 \pm 0.028}$$

$$\sin 2\beta_{UTA} = 0.731 \pm 0.036$$

correlation (tension) with V_{ub}, see later

$$\sin 2\beta_{UTA} = 0.698 \pm 0.066$$

prediction from Ciuchini et al. (2000)

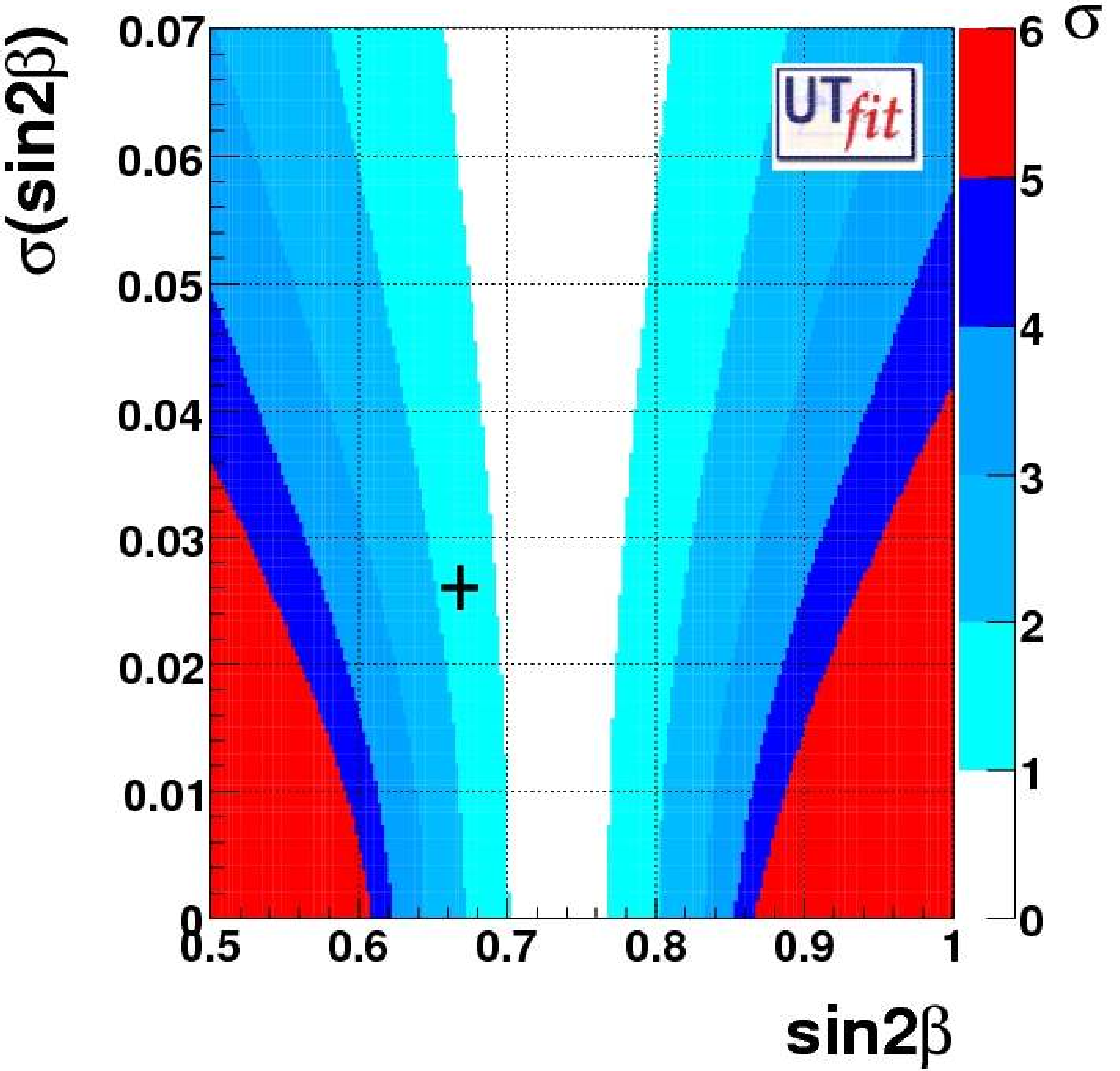
$$\sin 2\beta_{UTA} = 0.65 \pm 0.12$$

Prediction 1995 from Ciuchini,Franco,G.M.,Reina,Silvestrini

$$\sin 2\beta_{tot} = 0.695 \pm 0.020$$

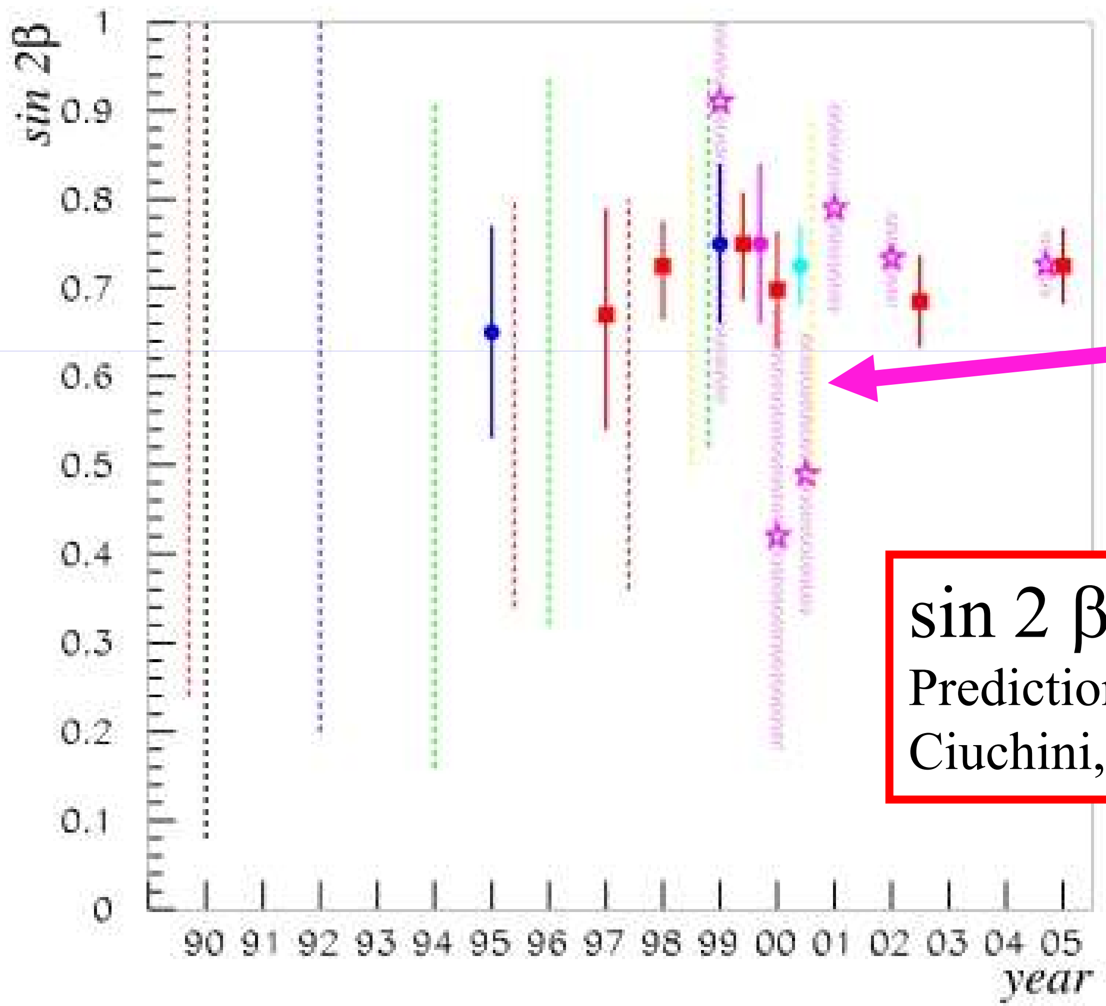
Very good agreement

no much room for physics beyond the SM !!



σ(sin2β)
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.5
0.6
0.7
0.8
0.9
1
sin2β
6
5
4
3
2
1
0
σ
UTfit

Theoretical predictions of Sin 2 β in the years



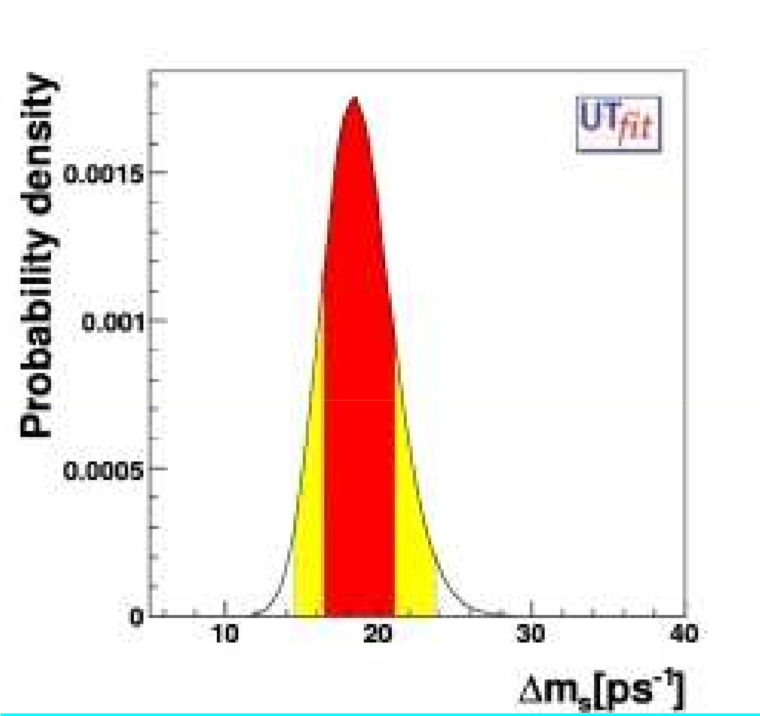
predictions exist since '95

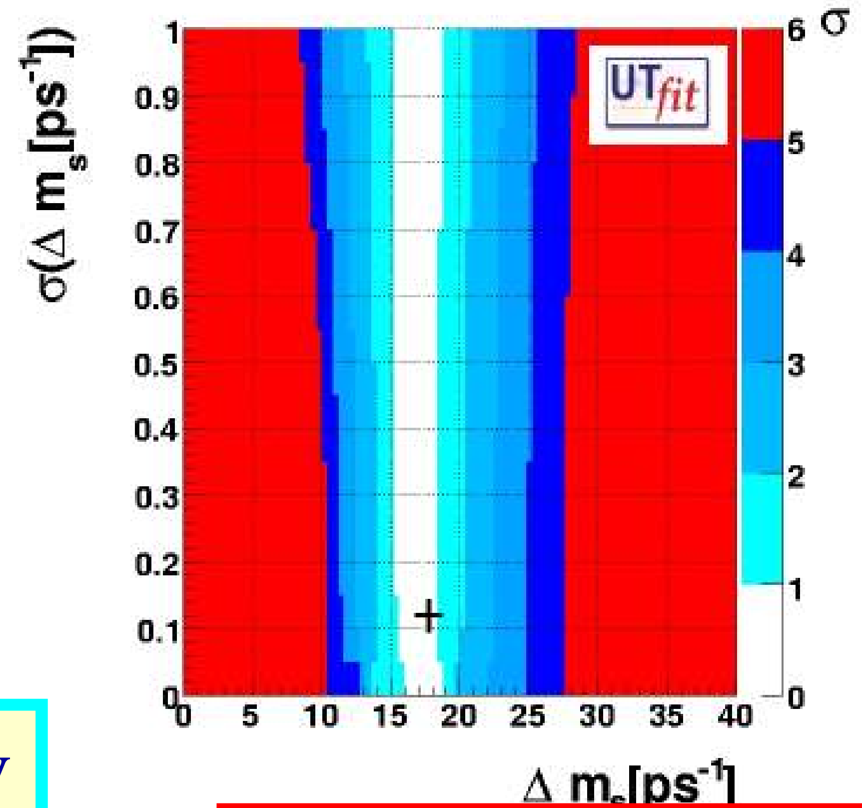
experiments

$\sin 2\beta_{UTA} = 0.65 \pm 0.12$

Prediction 1995 from Ciuchini,Franco,G.M.,Reina,Silvestrini

NEWS from NEWS(Standard Model)
The opening of the B_s era





Δm_s Probability Density

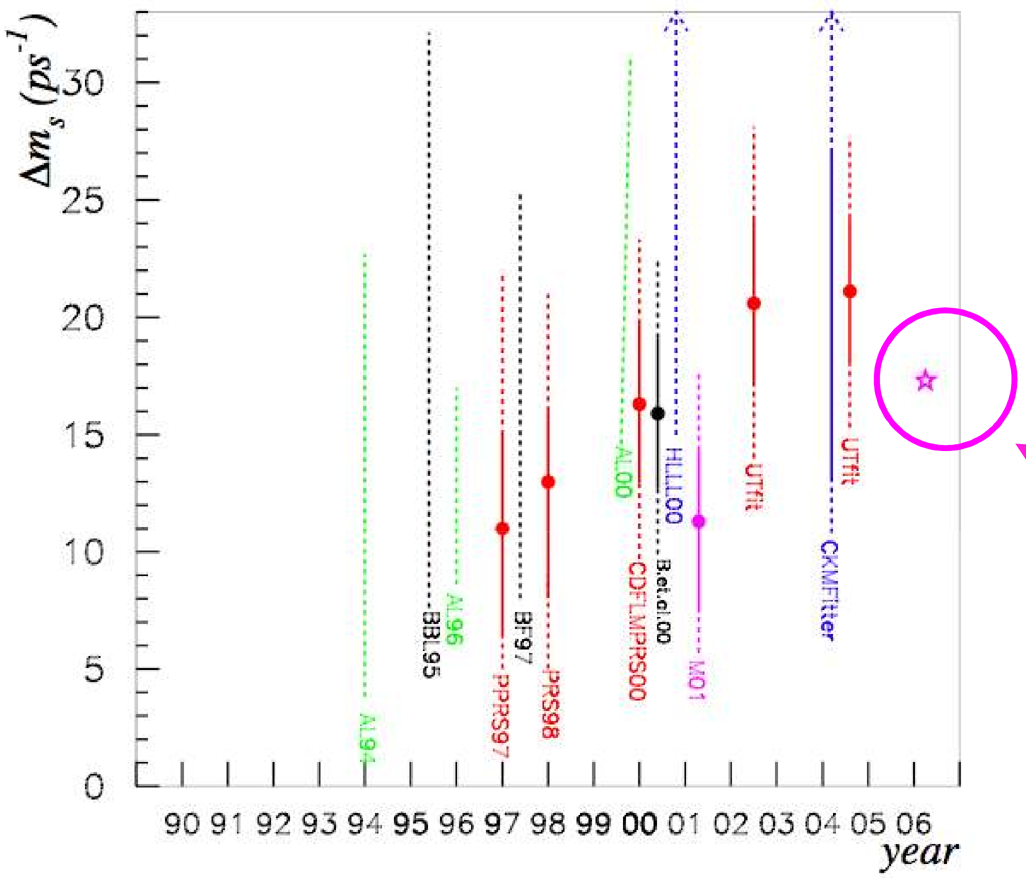
$\Delta m_s = 17.5 \pm 2.1 ps^{-1}$ *INDIRECT*

$\Delta m_s = 17.77 \pm 0.12 ps^{-1}$ *DIRECT*

$\Delta m_s = (16.3 \pm 3.4)\ ps^{-1}$

Ciuchini et. al. 2000

Theoretical predictions of Δm_s in the years



predictions exist since '97

CDF

A GREAT SUCCESS OF (QUENCHED) LATTICE QCD CALCULATIONS

A closer look to the analysis:

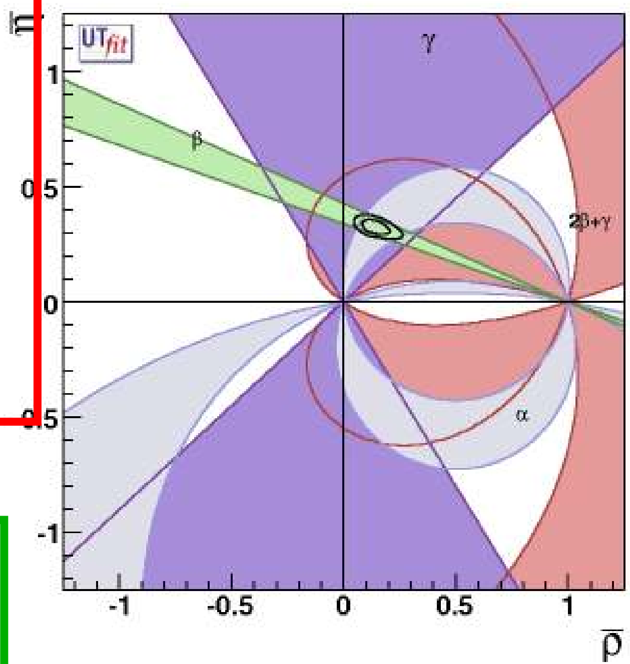
1) **Predictions vs Postdictions**
2) **Lattice vs angles**
3) **V_{ub} inclusive, V_{ub} exclusive vs sin 2β**
4) **Experimental determination of lattice parameters**

The UT-angles fit does not depend on theoretical calculations (treatement of errors is not an issue)

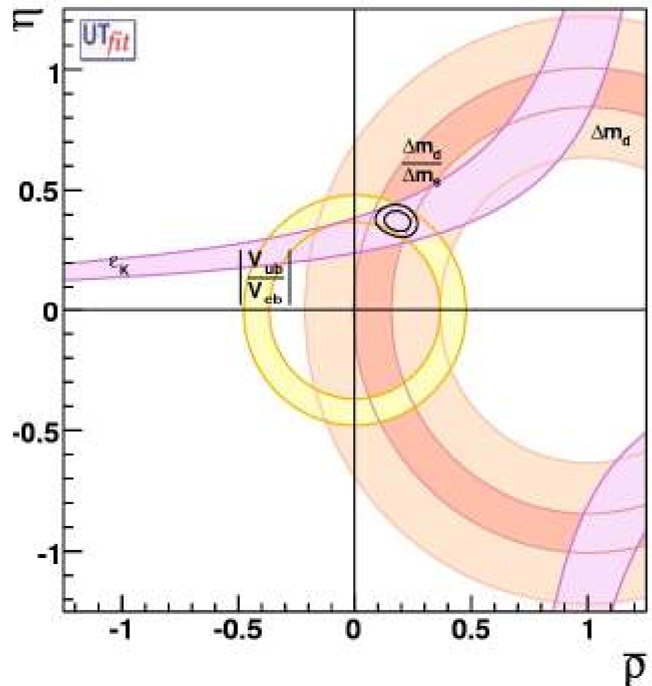
Comparable accuracy due to the precise sin2β value and substantial improvement due to the new Δm_s measurement

<u>Crucial to improve measurements of the angles, in particular γ</u> (tree level NP-free determination)

UT-angles



UT-lattice



$\rho = 0.120 \pm 0.034$

$\eta = 0.335 \pm 0.020$

$\rho = 0.175 \pm 0.027$

$\eta = 0.360 \pm 0.023$

Still imperfect agreement in $\bar{\eta}$ due to sin2β and V_{ub} tension

ANGLES VS LATTICE 2008

A closer look to the analysis:

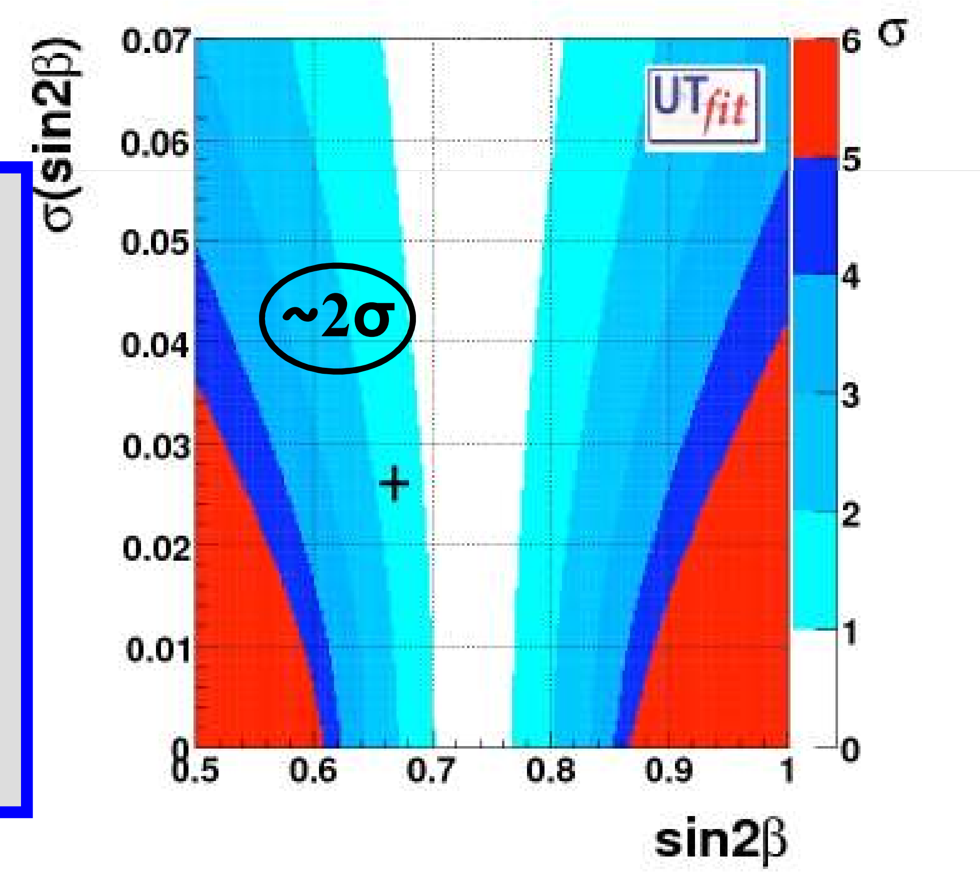
1) **Predictions vs Postdictions**
2) **Lattice vs angles**
3) **V_{ub} inclusive, V_{ub} exclusive vs sin 2β**
4) **Experimental determination of lattice parameters**

Correlation of sin 2 β with V_{ub}

$\sin 2\beta_{measured} = 0.668 \pm 0.028$

$\sin 2\beta_{UTA} = 0.731 \pm 0.036$

Although compatible, these results show that there is a ``tension" . This is due to the correlation of Vub with sin 2 β



V_{UB} PUZZLE

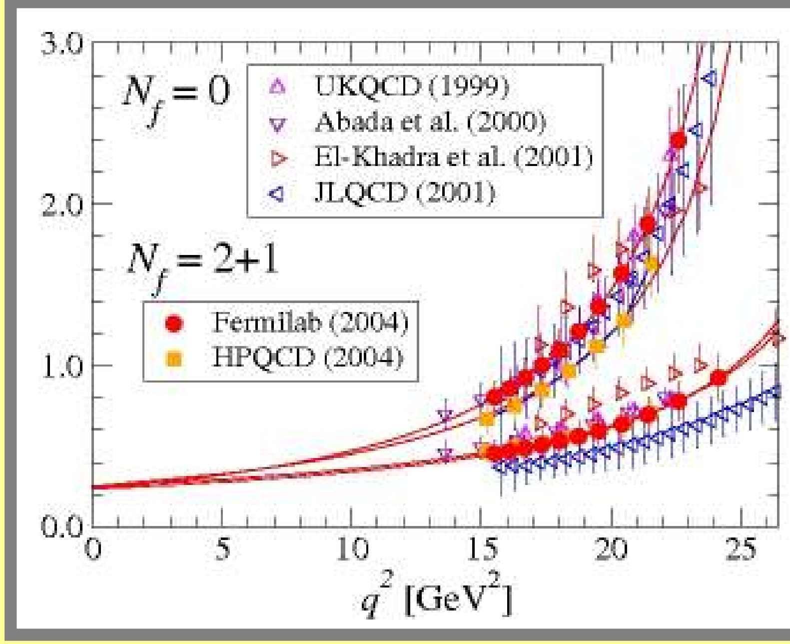
$\lvert V_{ub}\rvert \times 10^4$	excl.	35.0	4.0	Lattice QCDSR
$\lvert V_{ub}\rvert \times 10^4$	incl.	44.9	3.3	HQET+Model
$\lvert V_{ub}\rvert \times 10^4$	average	40.9	2.5	

Inclusive: uses non perturbative parameters most **not** from lattice QCD (fitted from the lepton spectrum)

$$\bar{\Lambda} \quad \lambda_1 \sim \frac{\bar{b}\vec{D}^2 b}{2m_b} \quad \lambda_2 \sim \frac{\bar{b}\sigma_{\mu\nu}G^{\mu\nu}b}{2m_b}$$

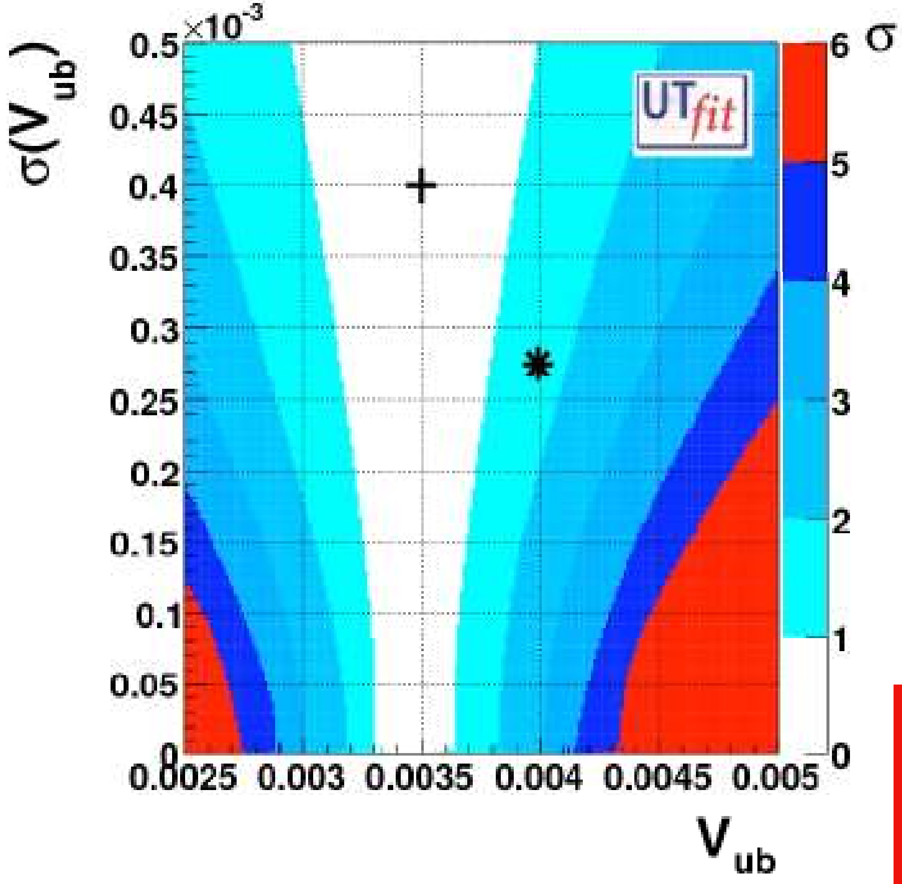
Exclusive: uses non perturbative form factors from LQCD and QCDSR

$$f^+(q^2) \quad V(q^2) \quad A_{1,2}(q^2)$$



S.Hashimoto@ICHEP'04

Tension between inclusive Vub and the rest of the fit



INCLUSIVE $V_{ub} = (43.1 \pm 3.9)\ 10^{-4}$

Model dependent in the threshold region (BLNP, DGE, BLL)

But with a different modelling of the threshold region [U.Aglietti et al., 0711.0860] $V_{ub} = (36.9 \pm 1.3 \pm 3.9)\ 10^{-4}$

EXCLUSIVE $V_{ub} = (34.0 \pm 4.0)\ 10^{-4}$

Form factors from LQCD and QCDSR

V_{UB} PUZZLE

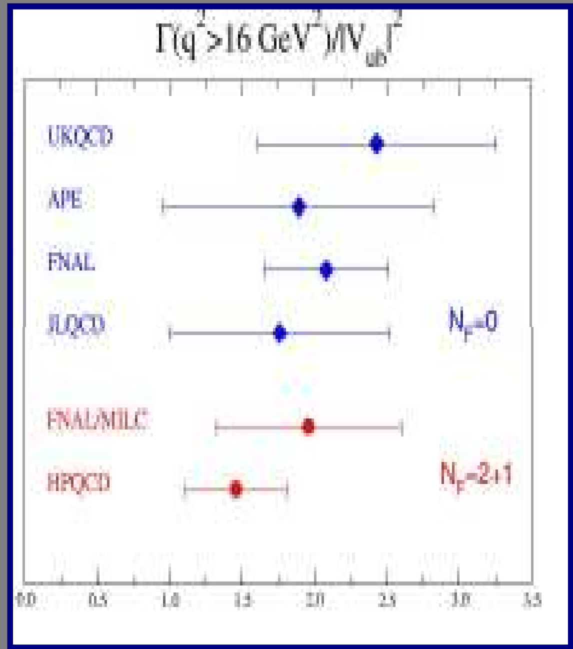
Khodjamirian

Recent $|V_{ub}|$ determinations from $B \to \pi l \nu_l$

[ref.]	$f^+_{B\pi}(q^2)$ calculation	$f^+_{B\pi}(q^2)$ input	$\lvert V_{ub}\rvert \times 10^3$
Okamoto et al.	lattice ($n_f = 3$)	-	3.78+0.25+0.52
HPQCD	lattice ($n_f = 3$)	-	3.55±0.25±0.50
Arnesen et al.	-	lattice⊕SCET	$3.54 + 0.17 + 0.44$
BecherHill	-	lattice	$3.7 \pm 0.2 \pm 0.1$
Flynn et al	-	lattice ⊕ LCSR	$3.47 \pm 0.29 \pm 0.03$
Ball, Zwicky	LCSR	-	$3.5 \pm 0.4 \pm 0.1$
this work	LCSR	-	$3.5 \pm 0.4 \pm 0.2 \pm 0.1$

V_{UB} PUZZLE

LATTICE QCD:
improve V_{ub} excl. to solve the tension



Beneke CERN '08

$|V_{ub}|$ crisis (about to be resolved?)

- $|V_{ub}| f_+^{B\pi}(0) = (9.1 \pm 0.6 \pm 0.3) \times 10^{-4}$ from semileptonic $B \to \pi l \nu$ spectrum + form factor extrapolation (Ball, 2006)
  Also lattice results (HPQCD) tend to small values.
- $|V_{ub}| f_+^{B\pi}(0) = (8.1 \pm 0.4\,(?)) \times 10^{-4}$ from $B \to \pi^+\pi^-, \pi^+\pi^0, \pi\rho, \ldots$ + factorization (MB, Neubert, 2003; Arnesen et al, 2005; MB, Jäger, 2005)

$\Longrightarrow$ $|V_{ub}| \simeq 3.5 \times 10^{-4}$, in contrast to determination from moments of inclusive $b \to u \ell \nu$ decay, which was $|V_{ub}| \simeq (4.5 \pm 0.3) \times 10^{-4}$.

But: according to (Neubert, LP07) $|V_{ub}| \simeq (3.7 \pm 0.3) \times 10^{-4}$ after reevaluation of m_b input and omitting $B \to X_s \gamma$ moments!

Hadronic Parameters From UTfit

1) **Predictions vs Postdictions**
2) **Lattice vs angles**
3) **V_{ub} inclusive, V_{ub} exclusive vs sin 2β**
4) **Experimental determination of lattice parameters**

IMPACT of the NEW MEASUREMENTS on LATTICE HADRONIC PARAMETERS

$$f_{B_s} \hat{B}_{B_s}^{1/2} \qquad \xi \qquad \hat{B}_K$$

Comparison between experiments and theory

exps vs predictions

$f_{Bs}\sqrt{B_{Bs}} = 270 \pm 30$ MeV

(275 ± 13 MeV new)

lattice

$f_{Bs}\sqrt{B_{Bs}} = 265 \pm 4$ MeV

UTA 2% ERROR !!

$\xi = 1.25 \pm 0.06$ **UTA**

$\xi = 1.21 \pm 0.04$

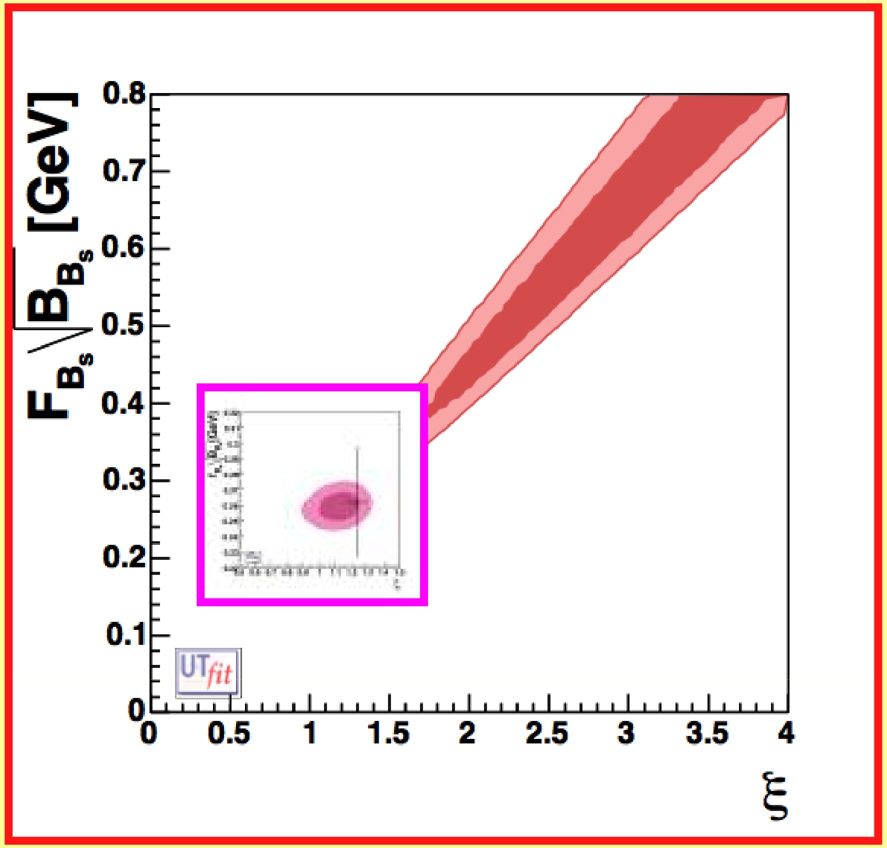
lattice

$B_{\mathbf{K}} = 0.75 \pm 0.07$

$B_{\mathbf{K}} = 0.75 \pm 0.07$

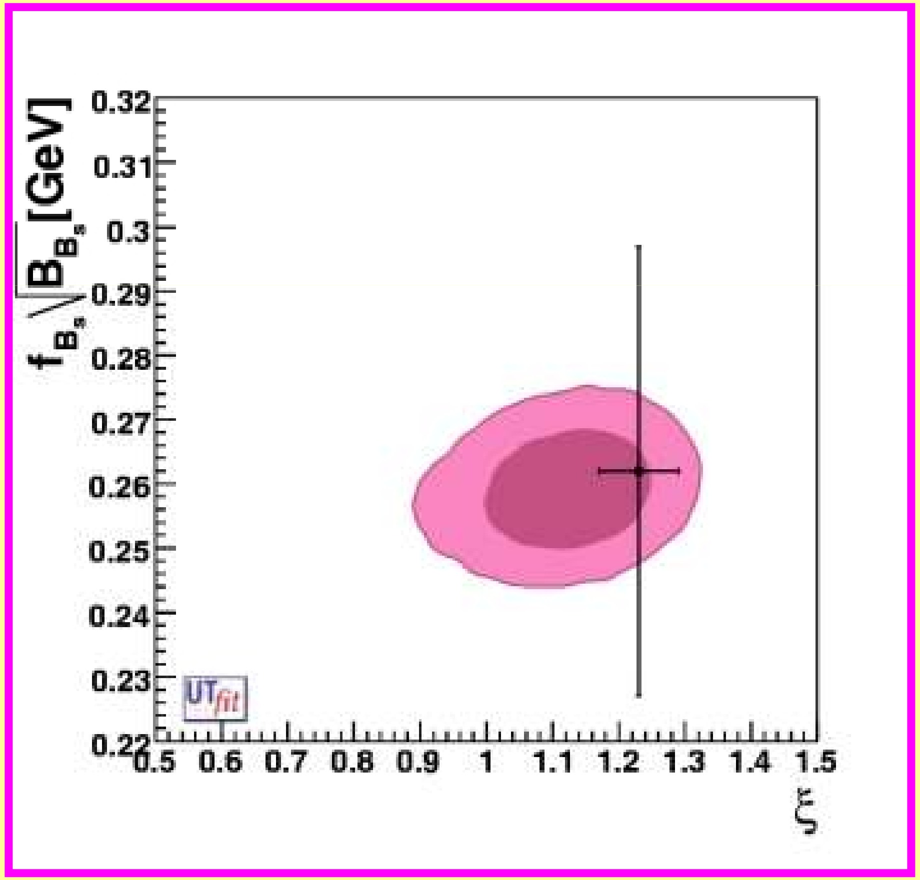
V. Lubicz and
C. Tarantino
0807.4605

SPECTACULAR AGREEMENT
(EVEN WITH QUENCHED
LATTICE QCD)



$F_{B_s}\sqrt{B_{B_s}}$ [GeV]
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
2.5
3
3.5
4
ξ
UTfit

NEW



$f_{B_s}\sqrt{B_{B_s}}$ [GeV]
0.32
0.31
0.3
0.29
0.28
0.27
0.26
0.25
0.24
0.23
0.22
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
ξ
UTfit

OLD

CONCLUSIONS I

For many quantities (quark masses, decay constants, form factors, moments of structure functions, etc.) Lattice QCD is entering the stage of precision calculations, with errors at the level of a few percent and full control of unquenching, discretization, chiral extrapolation and finite volume effects.

CONCLUSIONS II

For non-leptonic decays (particle widths) theoretical and numerical progresses have been made, substantial improvement in the calculation of DI=3/2 amplitudes

It remains open the problem of the decays above the elastic threshold

e.g. $B \rightarrow \pi\pi$