Corfu', 4 September '10

Is the Flavour Group Discrete?

Guido Altarelli
Universita' di Roma Tre/CERN

Status and current problems on ν mass and mixing

A recent review: GA, F. Feruglio, ArXiv:1002.0211
(Review of Modern Physics)

Evidence for solar and atmosph. ν oscillatn's confirmed on earth by K2K, KamLAND, MINOS...

Δm^2 values:

Δm^2_{atm} ~ 2.5 10^{-3} eV^2,

Δm^2_{sol} ~ 8 10^{-5} eV^2

and mixing angles measur'd:

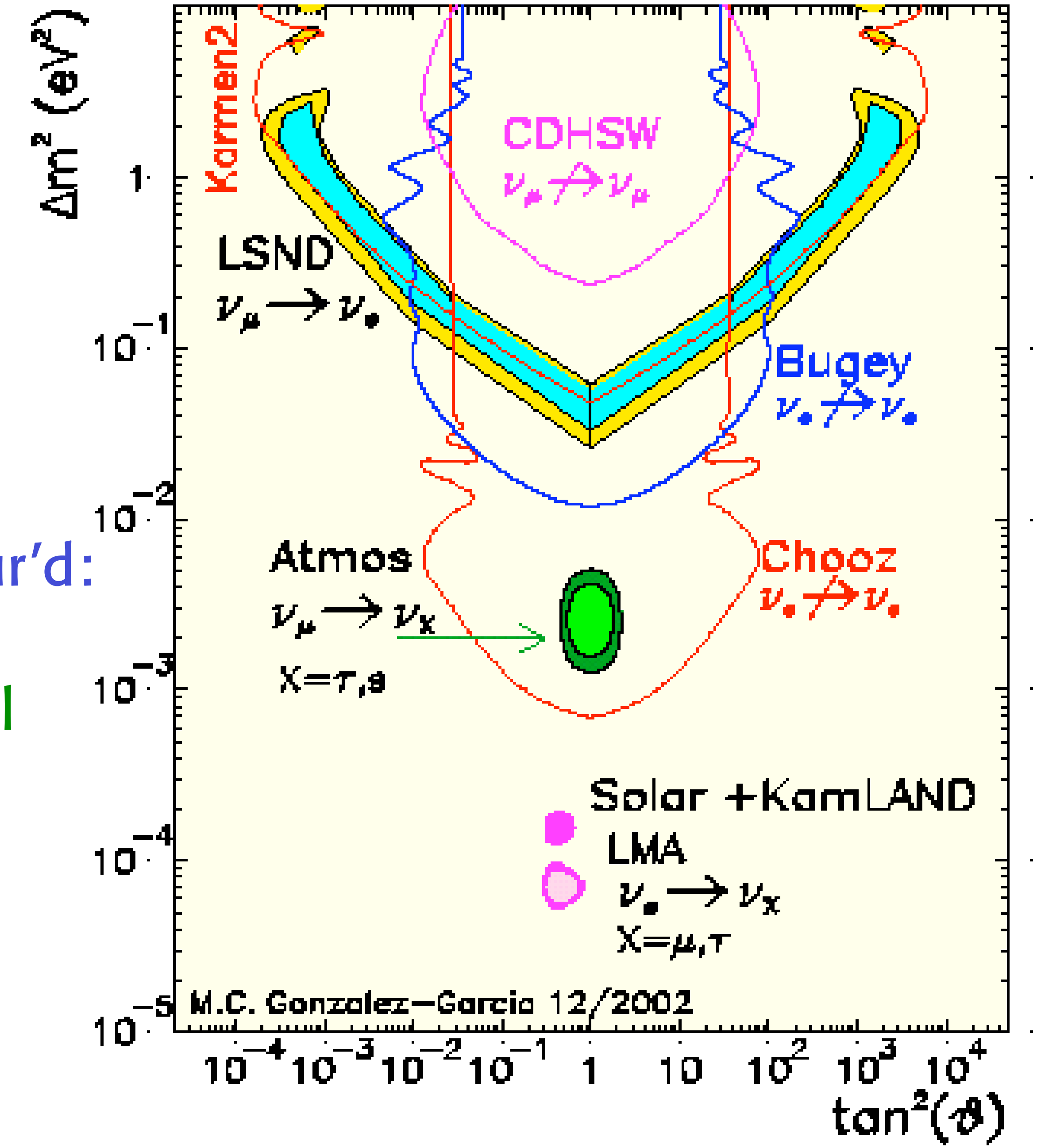
θ_{12} (solar) large

θ_{23} (atm) large,~ maximal

θ_{13} (CHOOZ) small

A 3rd frequency?

A persisting confusion: LSND+MiniBooNE



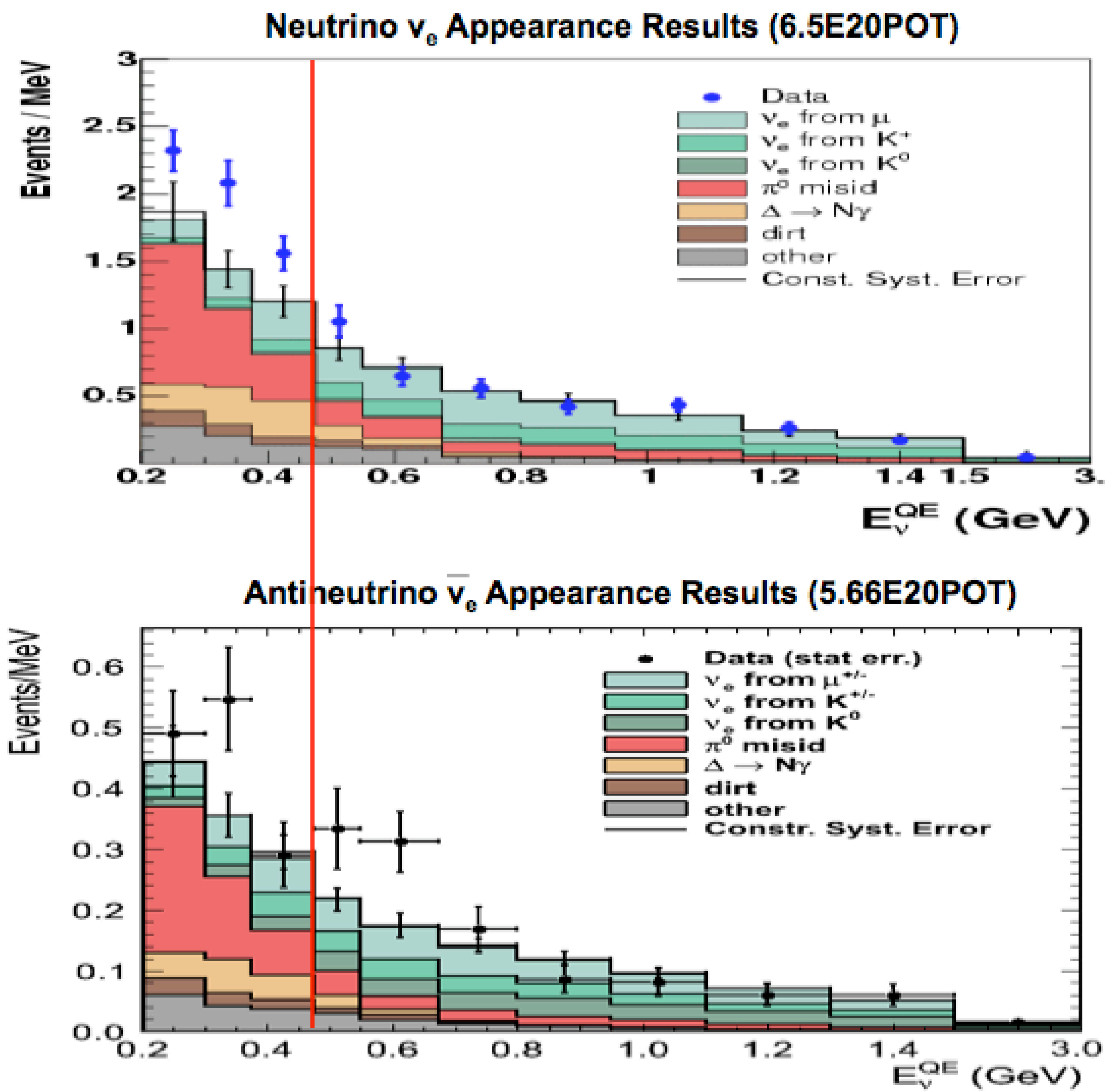
A persisting confusion: LSND/MiniBooNE

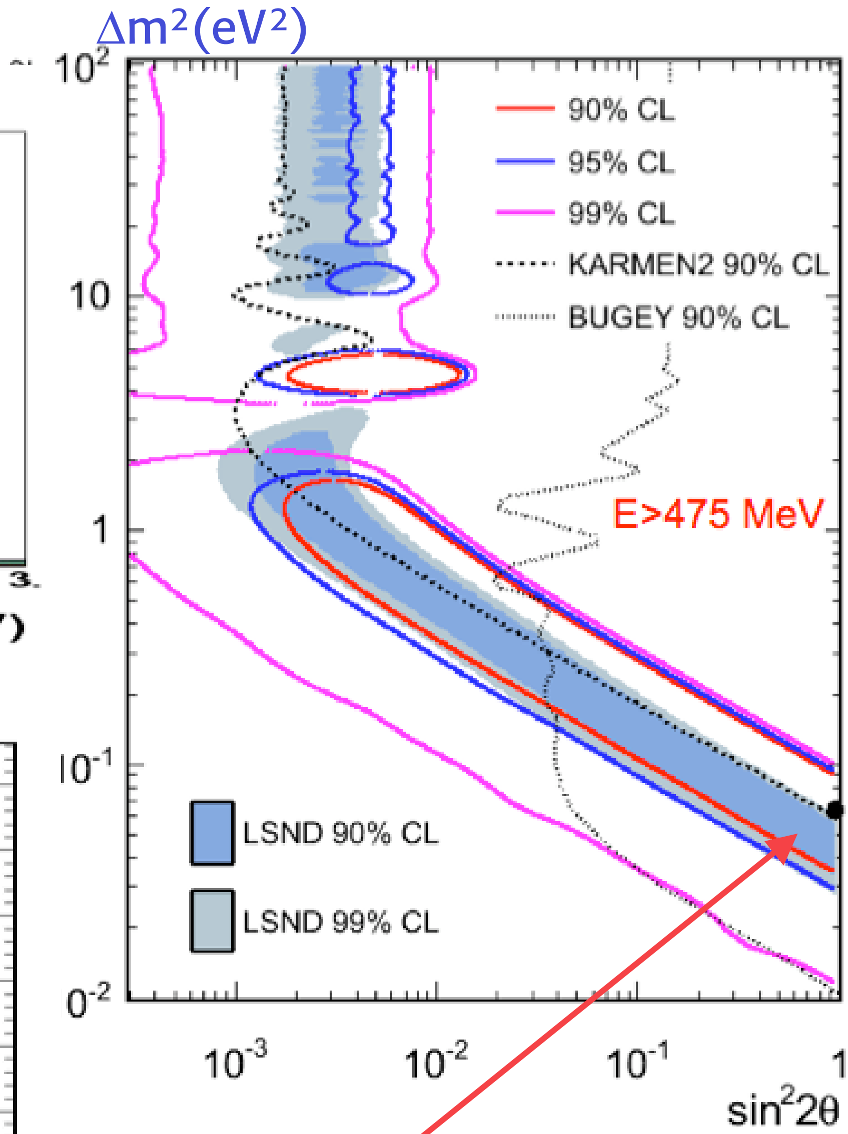
MiniBooNE: LSND not confirmed in ν's (but an excess at low E)
LSND not excluded in antiν's

(LSND claimed a signal in anti ν's)

In presence of a 3rd frequency one needs more than 3 ν's or/and CPT non-conservation (so that ν and anti-ν masses would be different)

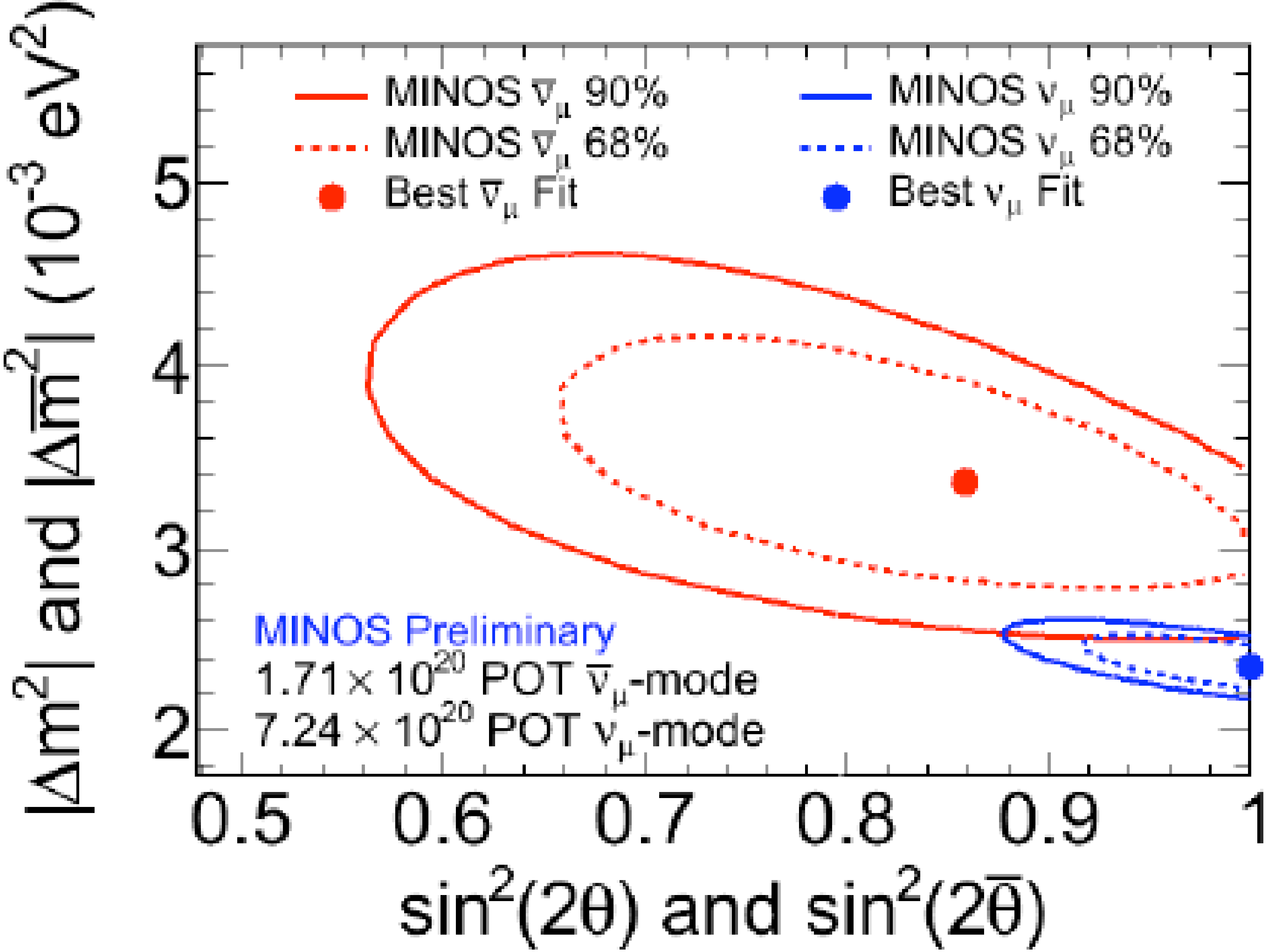
MiniBooNE





Best fit point in Bugey excluded area

A not yet significant hint of difference between ν's and anti-ν's is also reported by MINOS



A persisting confusion: LSND/MiniBooNE

MiniBooNE: LSND not confirmed in ν's (but an excess at low E)
LSND not excluded in antiν's

No oscillation hypothesis can fit all data, even adding sterile ν's: tensions between low/high E, ν's/antiν's, appearance/ disappearance, ... Can be mitigated by invoking CPT violation.

More data and better experiments needed

Here, we do not rush to add new neutrinos:
e.g. sterile neutrinos
We assume 3 light neutrinos are enough
Also, we continue to assume CPT invariance

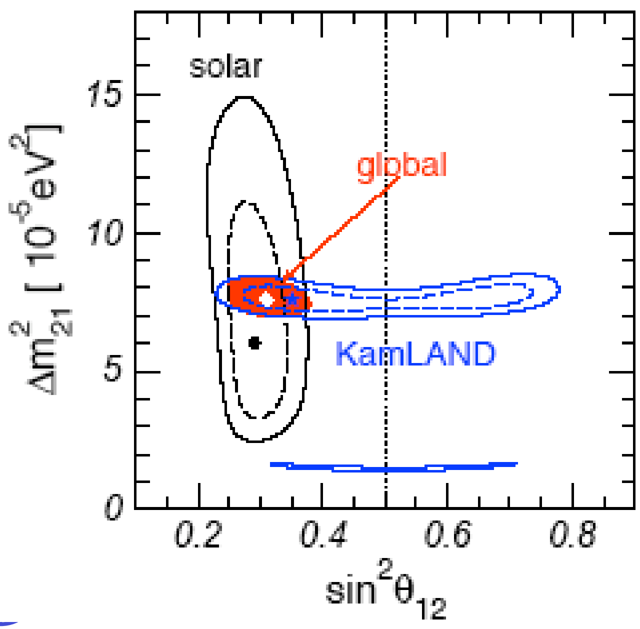
3-Neutrino oscillation parameters

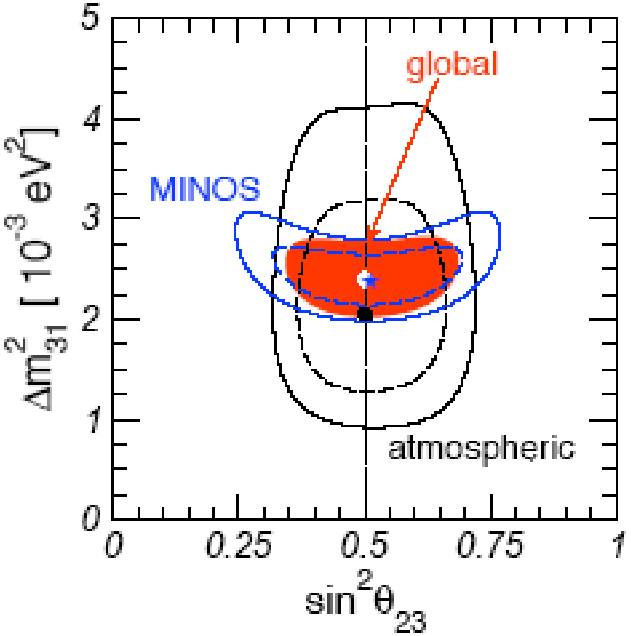
- 2 distinct frequencies
- 2 large angles, 1 small

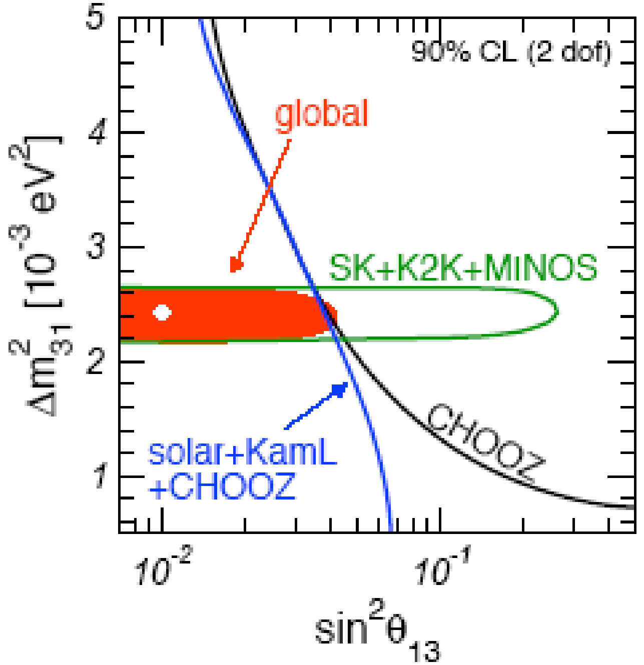
parameter	best fit	2σ	3σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	$7.59^{+0.23}_{-0.18}$	7.22–8.03	7.03–8.27
$\|\Delta m^2_{31}\|$ $[10^{-3}\text{eV}^2]$	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75
$\sin^2\theta_{12}$	$0.318^{+0.019}_{-0.016}$	0.29–0.36	0.27–0.38
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67
$\sin^2\theta_{13}$	$0.013^{+0.013}_{-0.009}$	$\lesssim 0.039$	$\lesssim 0.053$

Schwetz et al '10

Best measured angle





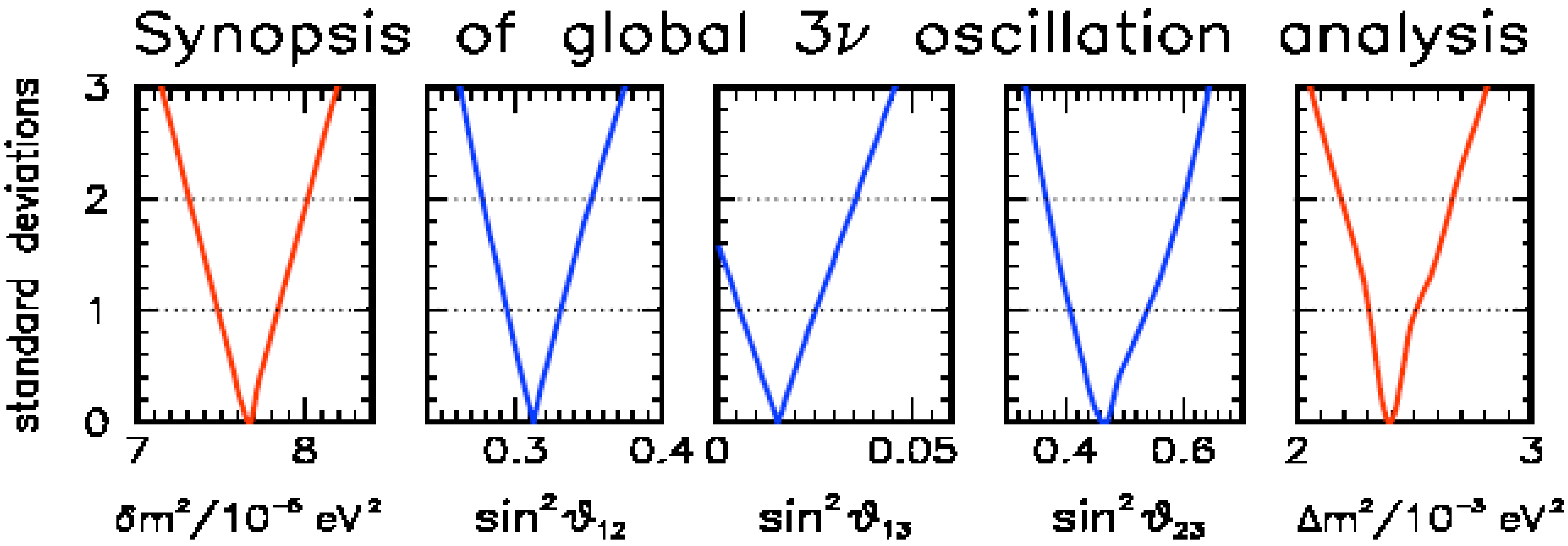


Different fits of the data agree

Fogli et al '08

Table 1: Global 3ν oscillation analysis (2008): best-fit values and allowed n_σ ranges, from Ref. [4].

Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.67	0.312	0.016	0.466	2.39
1σ range	7.48 – 7.83	0.294 – 0.331	0.006 – 0.026	0.408 – 0.539	2.31 – 2.50
2σ range	7.31 – 8.01	0.278 – 0.352	< 0.036	0.366 – 0.602	2.19 – 2.66
3σ range	7.14 – 8.19	0.263 – 0.375	< 0.046	0.331 – 0.644	2.06 – 2.81

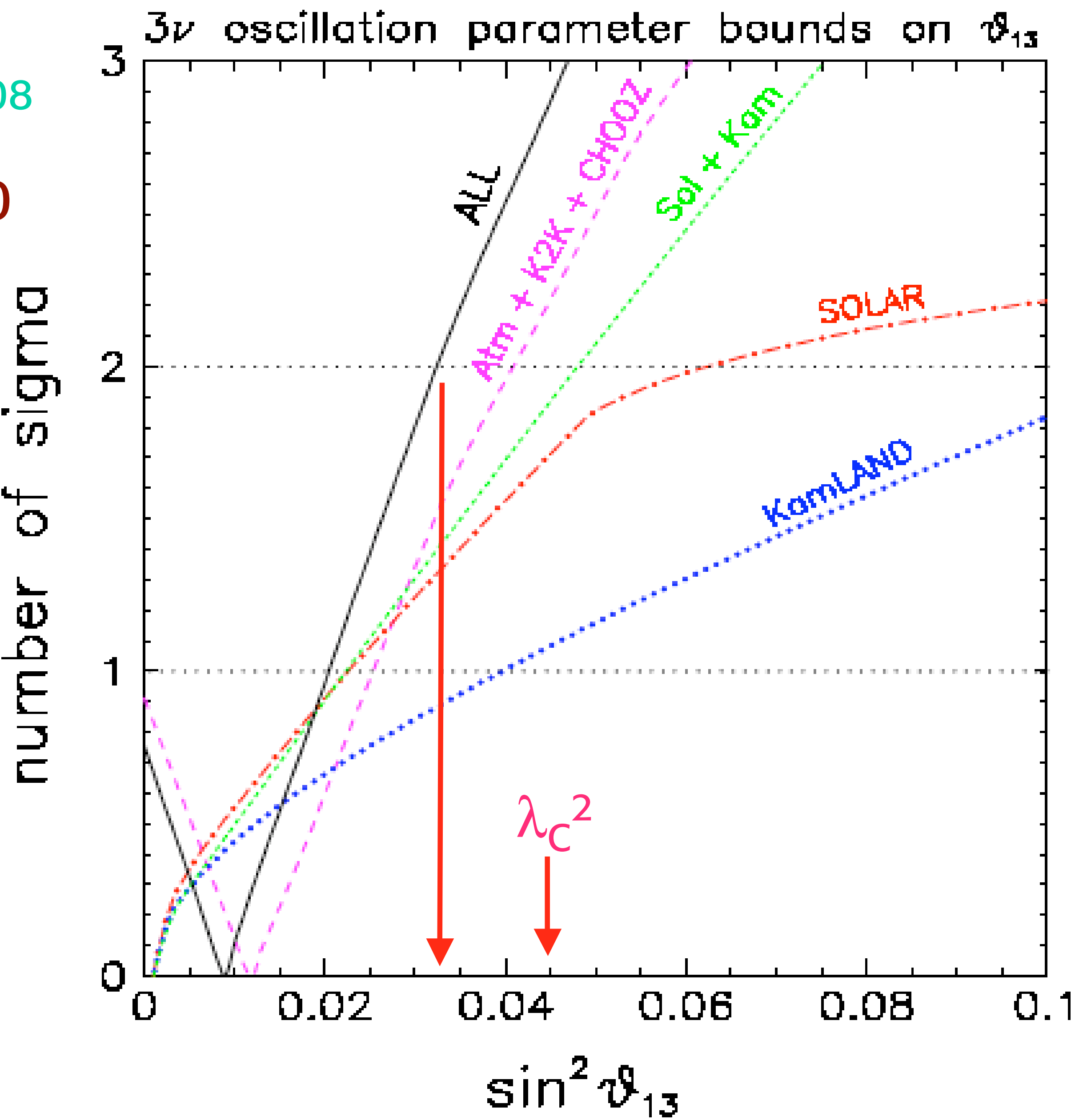


θ_{13} bounds

Fogli et al '08

$\sin^2\theta_{13}=0.016\pm0.010$

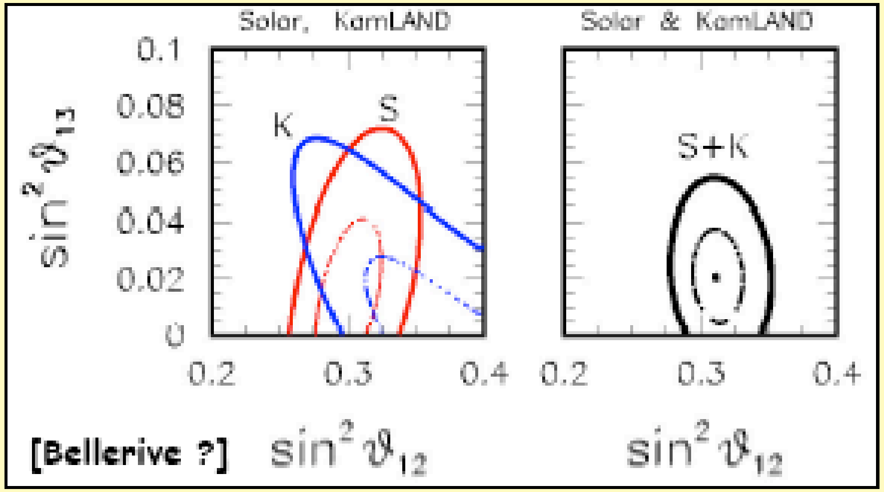
The 95% upper bound on $\sin\theta_{13}$ is close to $\lambda_C = \sin\theta_C$



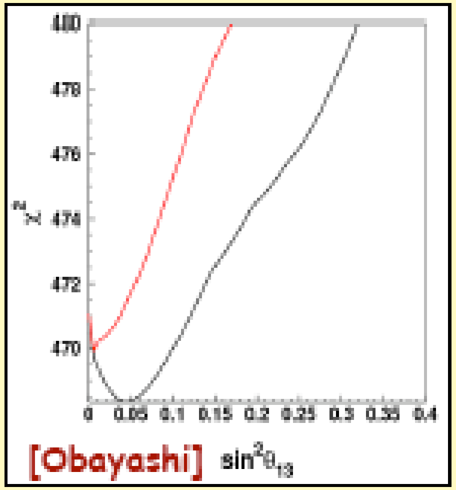
Lisi, ICHEP'10

Hints of $\theta_{13}>0$? [Fogli, EL, Marrone, Palazzo, Rotunno.] **Current status:**

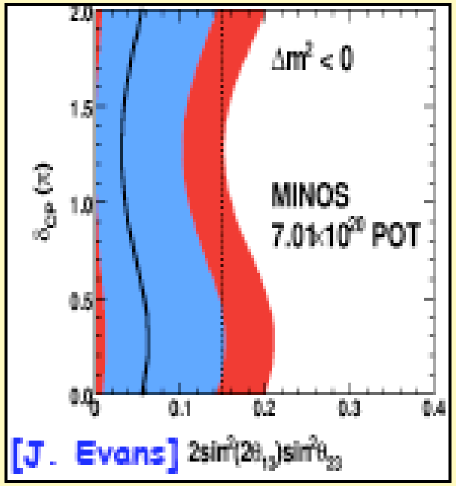
Solar & KamLAND: ~1.5σ



SK atmos.: ~1.5σ

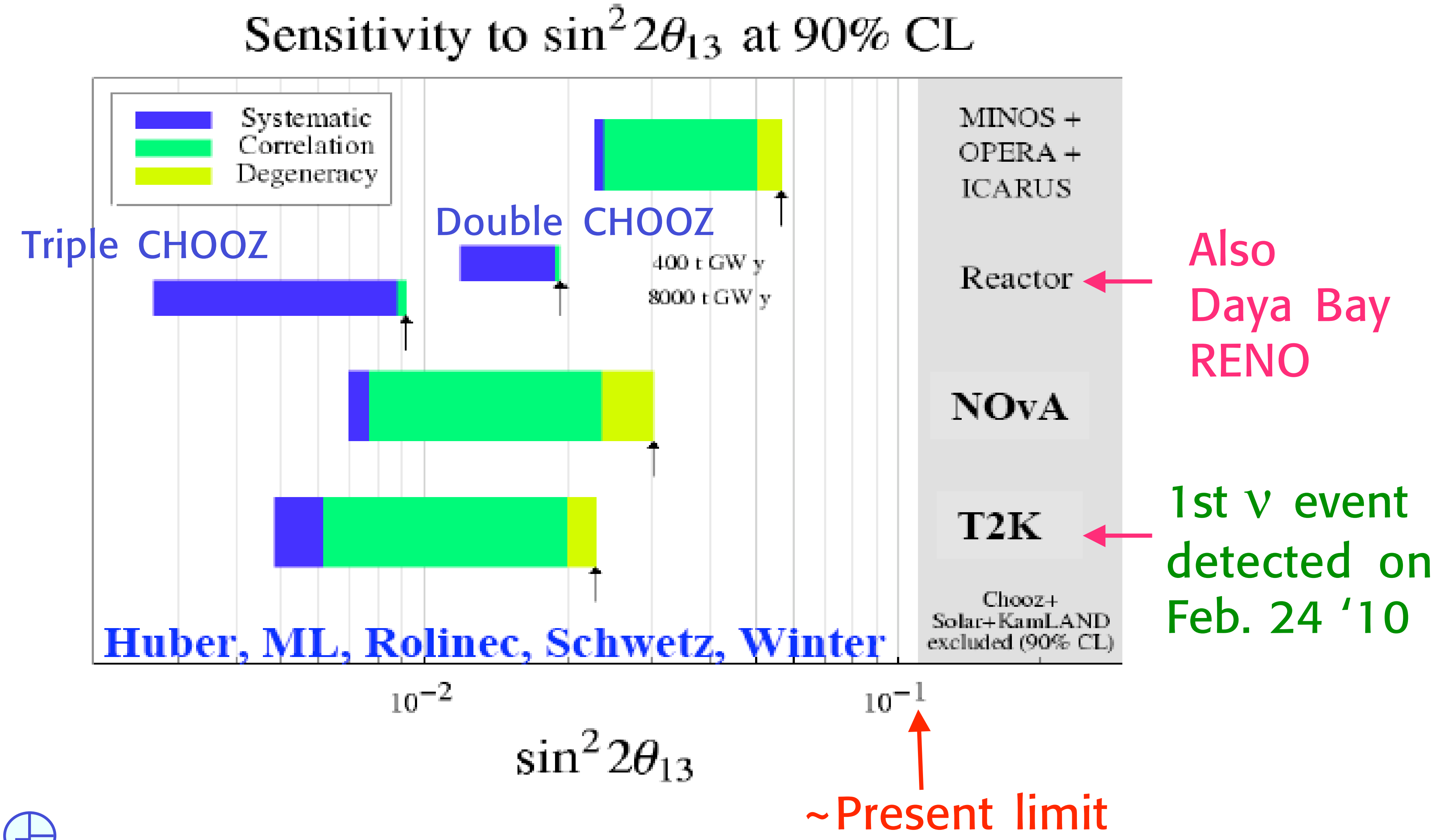


MINOS: ~0.7σ



Overall significance close to ~2σ. Intriguing, but still weak.

Measuring θ_{13} is crucial for future ν-oscill. physics (eg CP violation)



ν oscillations measure Δm^2. What is m^2?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2 = (0.05\ eV)^2$; $\Delta m^2_{sun} \sim 8\ 10^{-5}\ eV^2 = (0.009\ eV)^2$

- Direct limits

$m_{"\nu e"} < 2.2$ eV

$m_{"\nu\mu"} < 170$ KeV

$m_{"\nu\tau"} < 18.2$ MeV

End-point tritium β decay (Mainz, Troitsk)
Future: Katrin
0.2 eV sensitivity
(Karsruhe)

$m_{ee} = |\Sigma\, U_{ei}^2\, m_i|$

- $0\nu\beta\beta$ $m_{ee} < 0.2 - 0.7 - ?$ eV (nucl. matrix elmnts)

Evidence of signal? Klapdor-Kleingrothaus

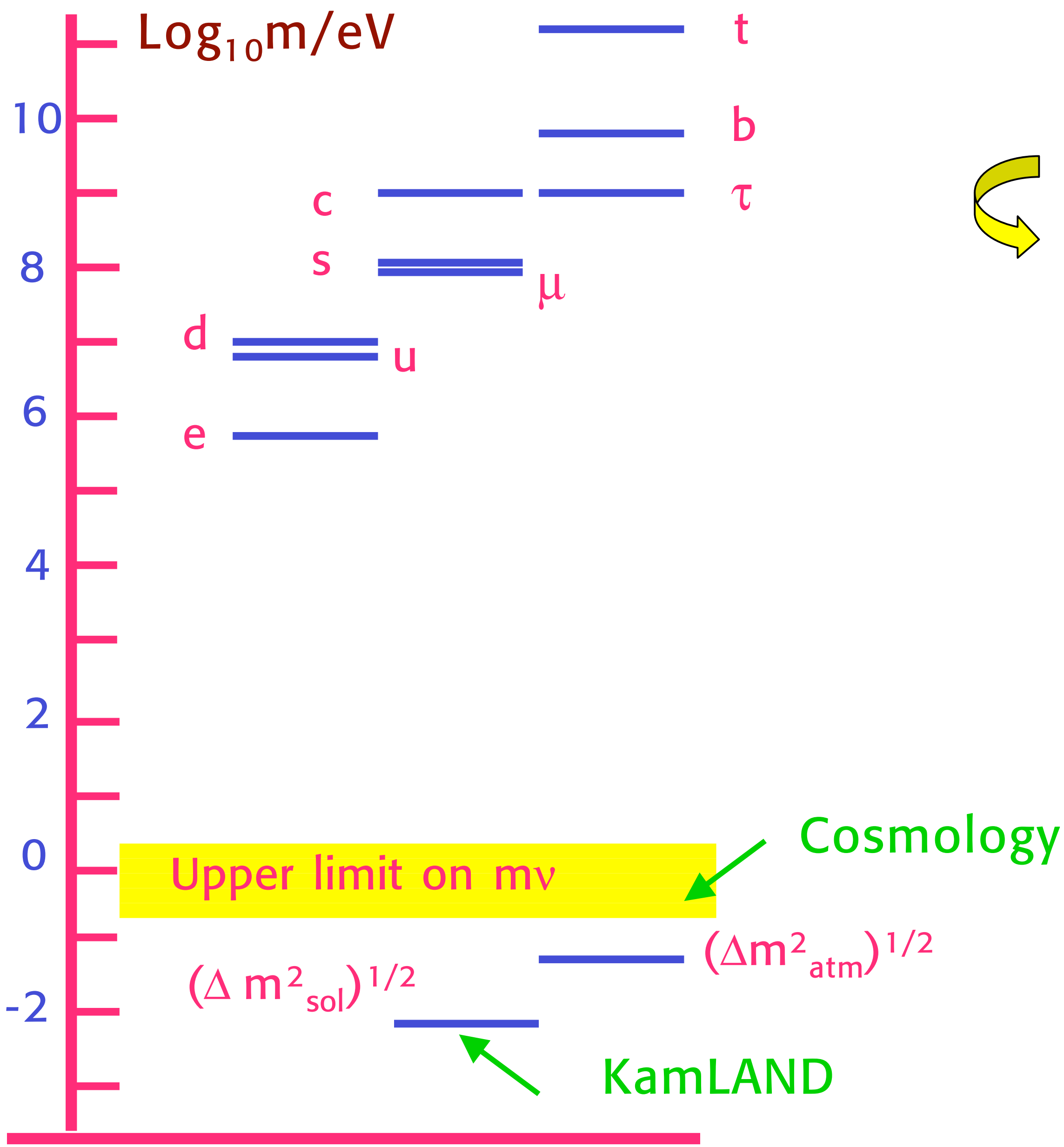
- Cosmology $\Omega_\nu\, h^2 \sim \Sigma_i m_i\ /94eV$ ($h^2 \sim 1/2$)

$\Sigma_i m_i < 0.2\text{-}0.7$ eV (dep. on data&priors)

WMAP, SDSS, 2dFGRS, Ly-α

→ Any ν mass $< 0.06 - 0.23 - \sim 1$ eV

depending on your weight on cosmology



Neutrino masses are really special!

$m_t/(\Delta m^2_{atm})^{1/2} \sim 10^{12}$

Massless ν's?

- no ν_R
- L conserved

Small ν masses?

- ν_R very heavy
- L not conserved

A very natural and appealing explanation:

ν's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale M (the scale of ν_{RH} Majorana mass)

$$m_\nu \sim \frac{m^2}{M}$$

m: $\leq m_t \sim v \sim 200$ GeV

M: scale of L non cons.

Note:

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.05$ eV

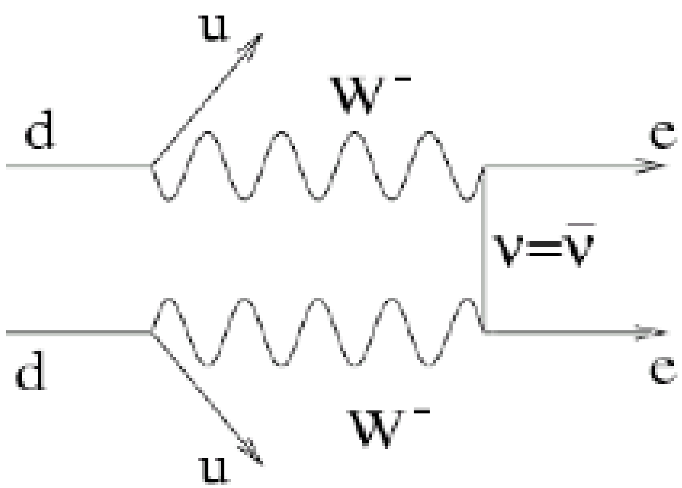
$m \sim v \sim 200$ GeV

$M \sim 10^{15}$ GeV

Neutrino masses are a probe of physics at M_{GUT} !

All we know from experiment on ν masses strongly indicates that ν's are Majorana particles and that L is not conserved (but a direct proof still does not exist).

Detection of 0νββ would be a proof of L non conservation. Thus a big effort is devoted to improving present limits and possibly to find a signal.



Heidelberg-Moscow
IGEX
Cuoricino-Cuore
Nemo
Sokotvina
Lucifero
.....

$0\nu\beta\beta$ = dd -> uue^-e^-

Baryogenesis by decay of heavy Majorana ν's

BG via Leptogenesis near the GUT scale

$T \sim 10^{12\pm3}$ GeV (after inflation)

Buchmuller, Yanagida, Plumacher, Ellis, Lola, Giudice et al, Fujii et al …..

Only survives if Δ(B-L) is not zero
(otherwise is washed out at T_{ew} by instantons)

Main candidate: decay of lightest ν_R ($M \sim 10^{12}$ GeV)

L non conserv. in ν_R out-of-equilibrium decay:
B-L excess survives at T_{ew} and gives the obs. B asymmetry.

Quantitative studies confirm that the range of m_i from ν oscill's is compatible with BG via (thermal) LG

In particular the bound was derived for hierarchy

$m_i < 10^{-1}$ eV

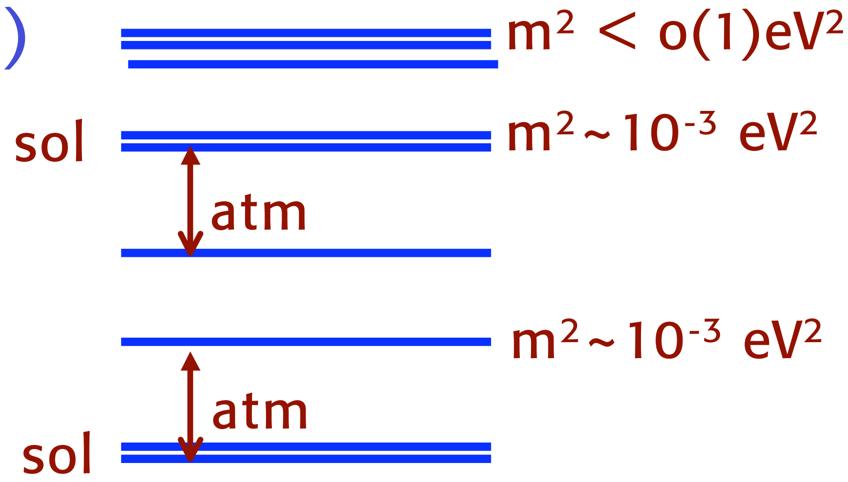
Buchmuller, Di Bari, Plumacher; Giudice et al; Pilaftsis et al; Hambye et al Hagedorn et al

Can be relaxed for degenerate neutrinos
So fully compatible with oscill'n data!!

The current experimental situation on ν masses and mixings has much improved but is still incomplete

- what is the absolute scale of ν masses?
- value of θ_{13}......
- pattern of spectrum (sign of Δm^2_{atm})

- Degenerate ($m^2 >> \Delta m^2$)
- Inverse hierarchy
- Normal hierarchy



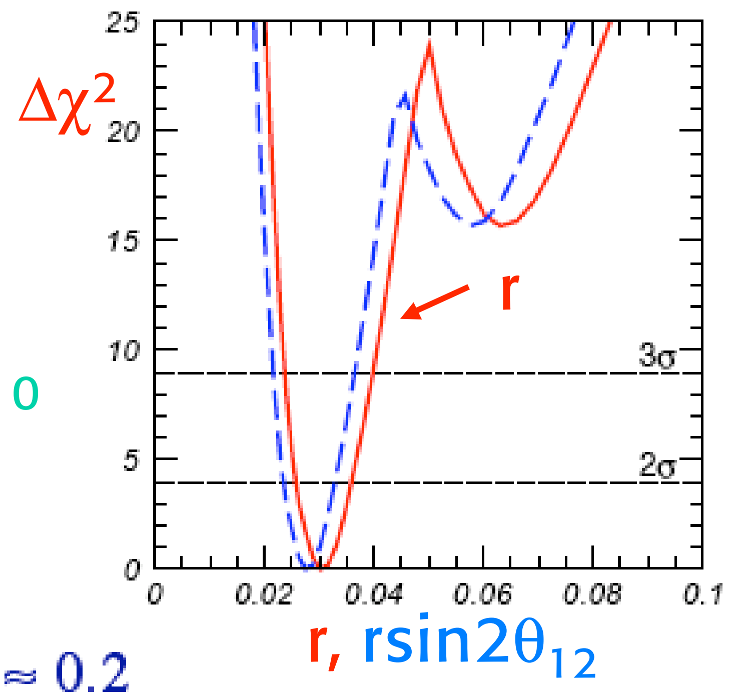
- no detection of $0\nu\beta\beta$ (i.e. no proof that ν's are Majorana) see-saw?
- are 3 light ν's OK? (MiniBooNE)

→ Different classes of models are still possible

General remarks

- After KamLAND, SNO .... and Cosmology not too much hierarchy is found in ν masses:

$$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$$



Only a few years ago could be as small as 10^{-8}!

Precisely at 3σ: $0.025 < r < 0.039$

Schwetz et al '10

or

$m_{heaviest} < 0.2$ - 0.7 eV
$m_{next} > \sim 8\ 10^{-3}$ eV

For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to $\lambda_C = \sin\theta_C$: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν
(small powers of λ_C) → e.g. θ_{13} not too small!

- θ_{13} not necessarily too small
  probably accessible to exp.

Very small θ_{13} theoretically hard [typically $\theta_{13} > 0.01$]

- Still large space for non maximal 23 mixing

2-σ interval $0.39 < \sin^2\theta_{23} < 0.63$ Schwetz et al '10

Maximal θ_{23} theoretically hard

- θ_{12} is at present the best measured angle

$\Delta\sin^2\theta_{12}/\sin^2\theta_{12} \sim 6\%$

For constructing models we need the data but also to decide which feature of the data is really relevant

Examples:

Is Tri-Bimaximal (TB) mixing really a significant feature or just an accident?

Is lepton-quark complementarity (LQC) a significant feature or just an accident?

TB Mixing

$$U=\begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

A coincidence or a hint?

TB mixing agrees with data at ~ 1σ

At 1σ:

Schwetz et al '10

$\sin^2\theta_{12} = 1/3$: 0.302-0.337

$\sin^2\theta_{23} = 1/2$: 0.44-0.57

$\sin^2\theta_{13} = 0$: < ~0.026

Called:
Tri-Bimaximal mixing

Harrison, Perkins, Scott '02

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

LQC: Lepton Quark Complementarity

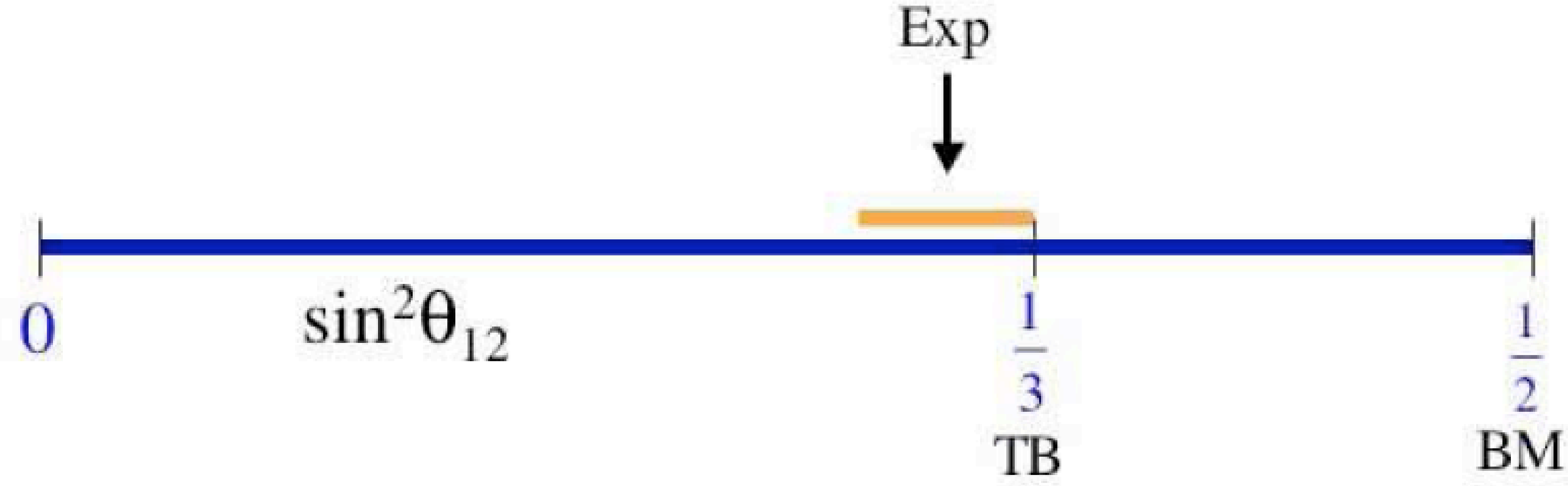
$$\theta_{12} + \theta_C = (47.0 \pm 1.2)^o \sim \pi/4$$

Raidal'04

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Suggests Bimaximal mixing corrected by diagonalisation of charged leptons

A coincidence or a hint?



Suggests that deviations from BM mixing arise from charged lepton diagonalisation

For the corrections from the charged lepton sector, typically $|\sin\theta_{13}| \sim (1-\tan^2\theta_{12})/4\cos\delta \sim 0.15$

GA, Feruglio, Masina
Frampton et al
King
Antusch et al........

Needs $|\sin\theta_{13}|$ near the present bound!

$$\bar{U}_{12} = -\frac{e^{-i(\alpha_1+\alpha_2)}}{\sqrt{2}} + \frac{s^e_{12}e^{-i\alpha_2} + s^e_{13}e^{i\delta_e}}{2}$$

$$\bar{U}_{13} = \frac{s^e_{12}e^{-i\alpha_2} - s^e_{13}e^{i\delta_e}}{\sqrt{2}}$$

$$\bar{U}_{23} = -e^{-i\alpha_2}\frac{1 + s^e_{23}e^{i\alpha_2}}{\sqrt{2}}$$

$\theta_{12} + \theta_C \sim \pi/4$

difficult to get. Rather:

$\theta_{12} + o(\theta_C) \sim \pi/4$

"weak" LQC

Corr.'s from s^e_{12}, s^e_{13} to U_{12} and U_{13} are of first order (2nd order to U_{23})

BM mixing can also be derived from discrete flavour symmetry

One can construct a model, based on S4, where BM mixing holds in 1st approximation and is then corrected by terms $o(\lambda_C)$

G.A., Feruglio, Merlo '09

In our model BM mixing is exact at LO

For the special flavon content chosen, only θ_{12} and θ_{13} are corrected from the charged lepton sector by terms of $o(\lambda_C)$ (large correction!) while θ_{23} gets smaller corrections (great!) [for a generic flavon content also $\delta\theta_{23} \sim o(\lambda_C)$]

An experimental indication for this model would be that θ_{13} is found near its present bound at T2K

⊕ We leave aside LQC here and restrict to TB mixing

For constructing models we need the data but also to decide which feature of the data is really relevant

Examples:

Is Tri-Bimaximal (TB) mixing really a significant feature or just an accident?

Is lepton-quark complementarity (LQC) a significant feature or just an accident?

Here we already see 3 different classes of models that can fit the data:
TB & LQC are accidents or TB is relevant or LQC is relevant

Accidents: a wide spectrum of (mostly old) models
Anarchy, Anarchy in 2-3 sector, Lopsided models, $U(1)_{FN}$, ...
GUT versions exist [SU(5), SO(10)]

Typically there are free parameters fitted to the angles

First, consider models with $\theta_{13}= 0$ and θ_{23} maximal and θ_{12} generic [includes both BM and TB]

The most general mass matrix is given by (after ch. lepton diagonalization!!!) and it is 2-3 or $\mu-\tau$ symmetric

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

Inspired models based on $\mu-\tau$ symmetry

Grimus, Lavoura..., Ma,....
Mohapatra, Nasri, Hai-Bo Yu ....

Neglecting Majorana phases it depends on 4 real parameters (3 mass eigenvalues and 1 mixing angle: θ_{12})

But actually θ_{12} is the best measured angle (after KamLAND, SNO....). And it is directly compatible with TB mixing.

TB mixing

By adding $\sin^2\theta_{12} \sim 1/3$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

Tribimaximal Mixing

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$m_{11} + m_{12} = m_{22} + m_{23}$

$m_1 = x - y$
$m_2 = x + 2y$
$m_3 = x - y + 2v$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

$= 8/9$ for TB

The 3 remaining parameters are the mass eigenvalues

TB mixing

Harrison, Perkins, Scott

A simple mixing matrix compatible with all present data

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U\text{diag}(m_1, m_2, m_3)U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $m_2 \rightarrow \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $m_1 \rightarrow \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

Compare with quark mixings $\lambda_C \sim (m_d/m_s)^{1/2}$

TB Mixing naturally leads to discrete flavour groups

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

This is a particular rotation matrix with specified fixed angles

- For the TB mixing matrix all mixing angles are fixed to particularly symmetric values

Sparked interest in constructing models that can naturally produce this highly ordered structure --> discrete flavour groups

A recent review: GA, Feruglio 1002.0211

Models based on the A4 discrete symmetry (even permutations of 1234) offer a minimal solution

Ma...; GA, Feruglio, GA, Feruglio, Lin; GA, Feruglio, Hagedorn; Y. Lin; Csaki et al; Hirsch et al, GA, Meloni.......

Larger finite groups: S4, T', $PSL_2(7)$.... have also been studied

Feruglio et al; Chen, Mahanthappa; Frampton, Kephart; Lam; Bazzocchi et al , King et al

Alternative models based on $SU(3)_F$ or $SO(3)_F$ or their finite subgroups

Verzielas, G. Ross .... King ....

Discrete symmetries coupled with Sequential Dominance or Form Dominance

King ...., Chen, King......

A4 A4 is the discrete group of even perm's of 4 objects. (the inv. group of a tetrahedron). It has $4!/2 = 12$ elements.

A4 transformations can be written in terms of S and T with: $S^2 = T^3 = (ST)^3 = 1$ as:

$1, T, S, ST, TS, T^2, TST, STS, ST^2, T^2S, T^2ST, TST^2$

An element is abcd which means 1234 --> abcd

C_1: $1 = 1234$

C_2: $T = 2314$ $ST = 4132$ $TS = 3241$ $STS = 1423$

C_3: $T^2 = 3124$ $ST^2 = 4213$ $T^2S = 2431$ $TST = 1342$

C_4: $S = 4321$ $T^2ST = 3412$ $TST^2 = 2143$

C_1, C_2, C_3, C_4 are equivalence classes

x, x′ in same class if $[x' \sim gxg^{-1}]$ g: group element

Irr. reprent'ns 1, 1′, 1″, 3

L: lepton doublet ~ 3

$e^c, \mu^c, \tau^c \sim 1, 1'', 1'$

A4 has 4 inequivalent irreducible representations:
a triplet and 3 different singlets

$$3,\ 1,\ 1',\ 1''$$

(promising for 3 generations!)

Note:

as many representations as equivalence classes 4

$\sum d_i^2$ = # of group elements =12 9+1+1+1=12

true for all finite groups

Three singlet inequivalent represent'ns:

$$\omega = \exp i\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$
$$\omega^3 = 1$$
$$1 + \omega + \omega^2 = 0$$
$$\omega^2 = \omega^*$$

Recall:
$S^2 = T^3 = (ST)^3 = 1$

$$\begin{cases} 1\text{:} & S=1,\ T=1 \\ 1'\text{:} & S=1,\ T=\omega \\ 1''\text{:} & S=1,\ T=\omega^2 \end{cases}$$

The only irreducible 3-dim represent'n is obtained by:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(S-diag basis)

An equivalent form:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

(T-diag basis)

Cabibbo '78

Under A4 the most common classification is:

lepton doublets $l \sim 3$, (in see-saw models $\nu^c \sim 3$)

$e^c, \mu^c, \tau^c \sim 1, 1'', 1'$ respectively

A4 breaking gauge singlet flavons $\phi_S, \phi_T, \xi \sim 3, 3, 1$

For SUSY version: driving fields $\phi_{0S}, \phi_{0T}, \xi_0 \sim 3, 3, 1$

with the alignment:

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

!!! In a serious model the alignment must follow from the symmetries

In all versions there are additional symmetries:

e.g. a broken $U(1)_F$ symmetry and/or discrete symmetries Z_n to ensure hierarchy of charged lepton masses and to restrict allowed couplings

Structure of the model (a 4-dim SUSY version)

GA, Feruglio, hep-ph/0512103

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + (x_a \xi + \tilde{x}_a \tilde{\xi})(ll) + x_b(\varphi_S ll) + h.c. + \ldots$$

shorthand: Higgs and cut-off scale Λ omitted, e.g.:

$$y_e e^c(\varphi l) \sim y_e e^c(\varphi l) h_d/\Lambda, \qquad x_a \xi(ll) \sim x_a \xi(l h_u l h_u)/\Lambda^2$$

In T-diag basis:
with this alignment:

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

Ch. leptons are diagonal

$$m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

ν's are tri-bimaximal

$$m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a + 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a - b/3 \\ -b/3 & a - b/3 & 2b/3 \end{pmatrix}$$

$$a \equiv x_a \frac{u}{\Lambda} \qquad b \equiv x_b \frac{v_T}{\Lambda}$$

recall:

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

So, at LO TB mixing is exact

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm}$

The only modest fine-tuning needed is to account for $r^{1/2} \sim 0.2$

[In most A4 models one would expect $r \sim o(1)$ as $l, \nu^c \sim 3$]

When NLO corrections are included from operators of higher dimension in the superpotential each mixing angle receives generically corrections of the same order $\delta\theta_{ij} \sim o(VEV/\Lambda)$

As the maximum allowed corrections to θ_{12} (and also to θ_{23}) are $o(\lambda_C^2)$, we need $VEV/\Lambda \sim o(\lambda_C^2)$ and we expect:

$\theta_{13} \sim o(\lambda_C^2)$ measurable in next run of exp's

(T2K started at the beginning of '10)

Predictions on the ν spectrum

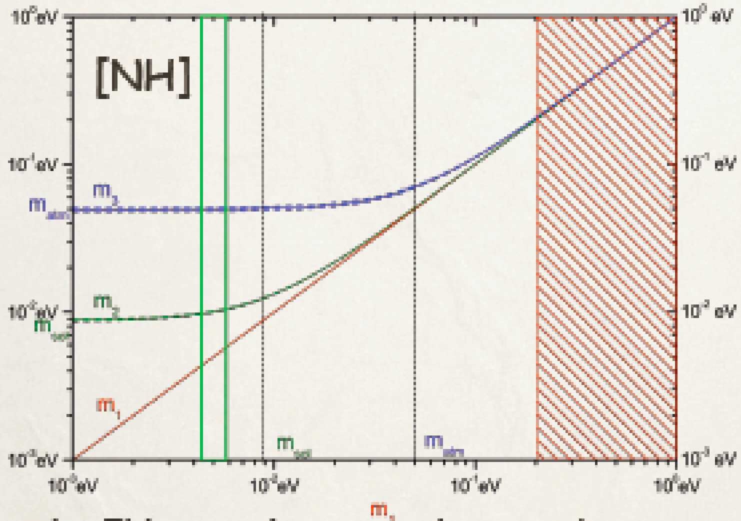
An example based on $G_f = A_4 \times Z_3 \times U(1)_{FN}$ [+ SUSY + SEE-SAW]

lepton mixing is TB, by construction, plus NLO corrections of order $0.005 < u < 0.05$
at the LO neutrino mass spectrum depends on two complex parameters
there is a sum rule among (complex) mass eigenvalues $m_{1,2,3}$

Feruglio, ICHEP'10

$$\frac{1}{m_3} = \frac{1}{m_1} - \frac{2}{m_2}$$

both normal [NH] and inverted [IH] hierarchy are allowed



in the NH case the sum rule completely determines the spectrum

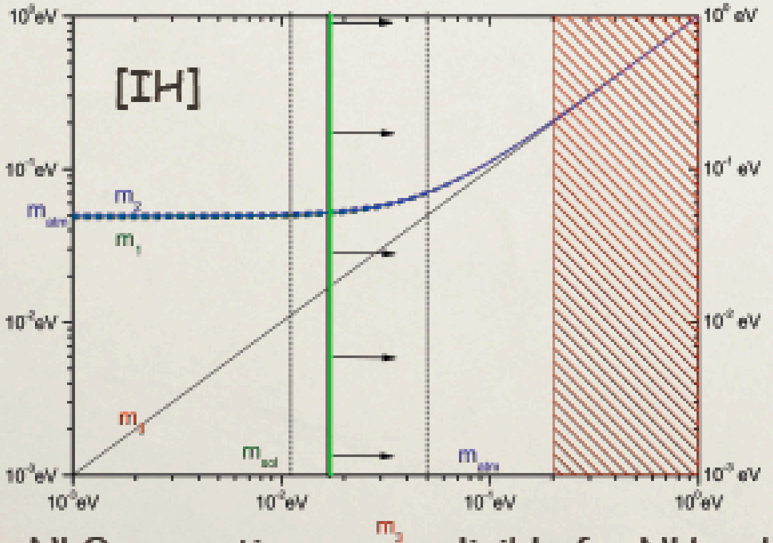
$$m_1 \approx 0.005\ eV \quad m_2 \approx 0.01\ eV \quad m_3 \approx 0.05\ eV$$

$$|m_{ee}| \approx 0.007\ eV$$

in the IH case the sum rule provides a lower bound on m_3

$$m_3 \geq 0.017\ eV$$

$$|m_{ee}| \geq 0.017\ eV$$



NLO corrections are negligible for NH and for IH close to the lower bound

Why and how discrete groups, in particular A4, work?

TB mixing corresponds to m in the basis where charged leptons are diagonal

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

Crucial point 1:

m is the most general matrix invariant under $SmS = m$ and $A_{23} m A_{23} = m$ with:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3 symmetry

$S^2 = A_{23}^2 = 1$

Crucial point 2:

Charged lepton masses:

a generic diagonal matrix

is defined by invariance under T

(or ηT with η a phase):

$$m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$m_l^+ m_l = T^+ m_l^+ m_l T$$

a possible T is

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$\omega^3=1$ --> $T^3=1$

An essential observation is that

S, T and A_{23} are all contained in S4

$S^4=T^3=(ST^2)^2=1$ define S4

Thus S4 is the reference group for TB mixing

Lam

A4 is a subgroup of S4

$S^2=T^3=(ST)^3=1$ define A4

Invariance under S and T is automatic in A4 while
A_{23} is not contained in A4 (2<->3 exchange is an odd perm.)
But 2-3 symmetry happens in A4 if 1′ and 1″ symm. breaking flavons are absent or have equal VEV's [2 of S4 = 1′ + 1″ of A4].

Note:

For $\mu-\tau$ symmetry only invariance under T and A_{23} is required

T and A_{23} are contained in S3 [$A_{23}{}^2=T^3=(A_{23}T)^2=1$ define S3]

Thus S3 is the reference group for $\mu-\tau$ symmetry

S3 has no triplets but only 2 , 1, 1′
TB mixing demands a 3!

Mohapatra, Nasri, Yu
Koide; Kubo et al
Kaneko et al
Caravaglios et al
Morisi; Picariello
Grimus, Lavoura……

Crucial point 3: A4 must be broken: the alignment

Before SSB the model is invariant under the flavour group A4

There are flavons ϕ_T, ϕ_S , ξ... with VEV's that break A4:

ϕ_T breaks A4 down to G_T, the subgroup generated by
1, T, T^2, in the charged lepton sector

ϕ_S , ξ break A4 down to G_S, the subgroup generated by
1, S, in the neutrino sector

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

ϕ_T, ϕ_S ~ 3
ξ ~ 1

The 2-3 symmetry occurs in A4 if 1' and 1" flavons are absent

TB mixing broken by higher dimension operators

Typically $\delta\theta \sim o(\lambda_C^2)$

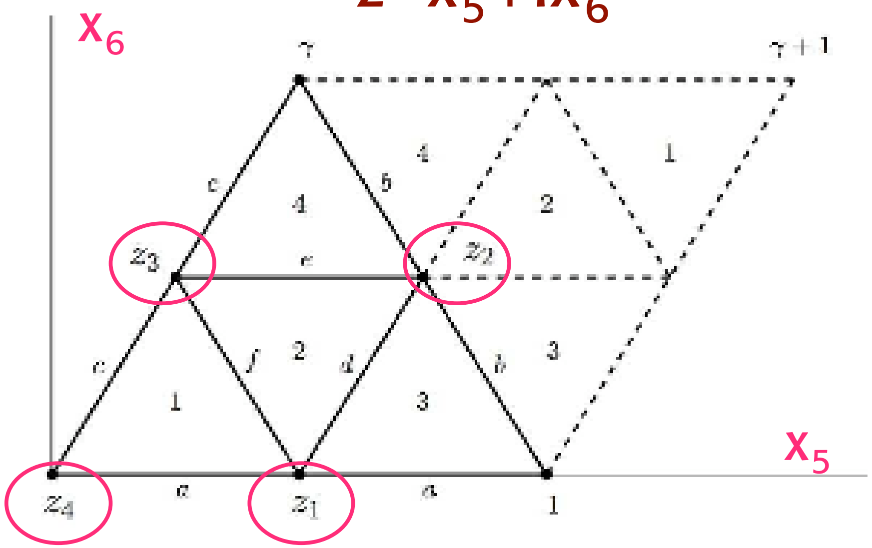
This aligment along subgroups of A4 must naturally occur in a good model

What can be the origin of A4?

A4, S4 (or some other discrete group) could arise from extra dimensions (by orbifolding with fixed points) as a remnant of 6-dim spacetime symmetry:

G.A.,F. Feruglio&Y. Lin, NP B775(2007)31
Adulpravitchai, Blum, Lindner '09

$z = x_5 + i x_6$



A torus with identified points:

$z \to z + 1$

$z \to z + \gamma \quad \gamma = \exp(i\pi/3)$

and a parity $z \to -z$
leads to 4 fixed points
(equivalent to a tethraedron).

There are 4D branes at the fixed points where the SM fields live (additional gauge singlets are in the bulk)

⊕ A4 interchanges the fixed points

Many versions of A4 models exist by now

- with dim-5 effective operators ($\nu_L{}^T\nu_L HH$) or with see-saw
- with SUSY or without SUSY
- in 4 dimensions or in extra dimensions

e.g G.A., Feruglio'05; G.A., Feruglio, Lin '06; Csaki et al '08, Kadosh, Pallante'10.....

- with different solutions to the alignment problem

e.g Hirsch, Morisi, Valle '0,...

- with sequential (or form) dominance

e.g King'07 ; Chen, King '09

- with charged lepton hierarchy also following from a special alignment (no $U(1)_{FN}$) Lin'08; GA, Meloni'09
- extension to quarks, possibly in a GUT context

In lepton sector TB (or BM) mixing point to discrete flavor groups

What about quarks?

A problem for GUT models is how to reconcile the quark with the lepton mixings

quarks: small angles, strongly hierarchical masses
abelian flavour symm. [e.g. $U(1)_{FN}$]
neutrinos: large angles, perhaps TB or BM
non abelian discrete symm. [e.g. A4]

A4: Extension to quarks

If we take all fermion doublets as 3 and all singlets as 1, 1′, 1″ (as for charged leptons):

$$Q_i \sim 3;\ u^c, d^c \sim 1;\ c^c, s^c \sim 1'';\ t^c, b^c \sim 1'$$

Then u and d quark mass matrices, like for charged leptons, are BOTH diagonal in the T-diagonal basis

As a result V_{CKM} is unity: $V_{CKM} = U_u^{+} U_d \sim 1$

So, in first approx. (broken by loops and higher dim operators), ν mixings are TB and quark mixings ~ identity: NOT BAD

BUT the size and hierarchy of q mixing angles is not reproduced by NLO corrections and
the above A4 transf. properties are not compatible with GUT's

From experiment:

a good first approximation for quarks

$\lambda = \sin\theta_C$

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & 0 \\ -\lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + o(\lambda^2)$$

and for neutrinos

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix} + o(\lambda^2)\,?$$

Current research

- Larger discrete flavour groups for quark mixings (no GUT's)

Carr, Frampton
Feruglio et al
Frampton, Kephart
....

- GUT models with approximate TB mixing

  it is indeed possible, also for A4, but not easy!
  [SU(5) less difficult than SO(10)]

Ma, Sawanaka, Tanimoto; Ma; GA, Feruglio, Hagedorn 0802.0090
Morisi, Picarello, Torrente Lujan; Bazzocchi et al;
de Madeiros Verzielas, King, Ross [$\Delta(27)$];
King, Malinsky [$SU(4)_C x SU(2)_L x SU(2)_R$]; Antusch et al;
Chen, Mahanthappa [T']; Bazzocchi et al [$\Delta(27)$];
King, Luhn [$PSL_2(7)$]; Dutta, Mimura, Mohapatra [S4];
....

SUSY-SU(5) GUT with A4 and TB

GA, Feruglio, Hagedorn 0802.0090

A satisfactory ~ realistic model

Key ingredients:

- SUSY

  In general SUSY is crucial for hierarchy, coupling unification and p decay

  Specifically it makes simpler to implement the required alignment

- GUT's in 5 dimensions

  In general GUT's in ED are most natural and effective

  Here also contribute to produce fermion hierarchies

- Extended flavour symmetry: $A4 x U(1) x Z_3 x U(1)_R$

  $U(1)_R$ is a standard ingredient of SUSY GUT's in ED

  Hall-Nomura'01

ED effects contribute to the fermion mass hierarchies

A bulk field is related to its zero mode by: $B = \frac{1}{\sqrt{\pi R}} B^0 + \ldots$

This produces a suppression parameter for couplings with bulk fields $s \equiv \frac{1}{\sqrt{\pi R \Lambda}} < 1$

Λ: UV cutoff

- In bulk: N=2 SUSY Yang-Mills fields + H_5, H_5^{bar}+ T_1, T_2, T_1', T_2'

(doubling of bulk fermions to obtain chiral massless states at y=0)

also crucial to avoid too strict mass relations for 1,2 families:

(b-τ unification only for 3rd family)

- All other fields on brane at y=0 (in particular N, F, T_3)

$$m_u = \begin{pmatrix} s^2t^5t'' + s^2t^2t''^4 & s^2t^4 + s^2tt''^3 & stt''^2 \\ s^2t^4 + s^2tt''^3 & s^2t''^2 & st'' \\ stt''^2 & st'' & 1 \end{pmatrix} sv_u^0 \sim \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \lambda v_u^0$$

Note: all m of rank 1 in LO: only m_{33} ~o(1)!

dots=0 in 1st approx

$$m_d = \begin{pmatrix} st^3 + st''^3 & \dots & \dots \\ st^2t'' & st & \dots \\ stt''^2 & st'' & 1 \end{pmatrix} v_T s v_d^0 \sim \begin{pmatrix} \lambda^4 & \dots & \dots \\ \lambda^4 & \lambda^2 & \dots \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_T \lambda v_d^0$$

$$m_e = \begin{pmatrix} st^3 + st''^3 & st^2t'' & stt''^2 \\ \dots & st & st'' \\ \dots & \dots & 1 \end{pmatrix} v_T s v_d^0 \sim \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \dots & \lambda^2 & \lambda^2 \\ \dots & \dots & 1 \end{pmatrix} v_T \lambda v_d^0$$

with

A4 breaking (over φ_T, φ_S, ξ) — $U(1)_{FN}$ breaking (over θ, θ'')

$$\frac{\langle \varphi_T \rangle}{\Lambda} = (v_T, 0, 0) \quad , \quad \frac{\langle \varphi_S \rangle}{\Lambda} = (v_S, v_S, v_S) \quad , \quad \frac{\langle \xi \rangle}{\Lambda} = u \qquad \frac{\langle \theta \rangle}{\Lambda} = t \quad , \quad \frac{\langle \theta'' \rangle}{\Lambda} = t''$$

$s \sim t \sim t'' \sim \lambda \sim 0.22$ $\qquad$ $v_T \sim \lambda^2 \sim m_b/m_t$ $\qquad$ $v_S,\ u \sim \lambda^2$

Finally:

By taking $s \sim t \sim t'' \sim \lambda \sim 0.22$ $\quad v_T \sim \lambda^2 \sim m_b/m_t$ $\quad v_S, u \sim \lambda^2$

a good description of all quark and lepton masses is obtained.
As for all U(1) models only $o(\lambda^p)$ predictions can be given (modulo o(1) coeff.s)

TB mixing for neutrinos is reproduced in first approximation

Quark hierarchies force corrections to TB mixing to be $o(\lambda^2)$
(in particular we predict $\theta_{13} \sim o(\lambda^2)$, accessible at T2K).

A moderate fine tuning is needed to fix λ_C and r (nominally of $o(\lambda^2)$ and 1 respectively)

Normal or inverse hierarchy are possible, degenerate ν's are excluded

Conclusion

- Majorana ν's, the see-saw mechanism and $M \sim M_{GUT}$ explain the data (we expect L non cons. in GUT's)
    - needs confirmation from $0\nu\beta\beta$ decay
    - ν's support GUT's, baryo- via lepto-genesis
- Different models can accommodate the data on ν mixing
    - e. g. TB mixing accidental or a hint?

Anarchy
Lopsided models
$U(1)_{FN}$,
••••••

discrete groups

Value of θ_{13} important for deciding

no supporting evidence from quarks

- Exp.: θ_{13}, sign Δm^2_{23}, CP phase δ, absolute m^2 scale....

Do we need more than 3 light neutrinos or CPT violation?????