# Asymptotic Helicity Conservation in SUSY (an update).

G.J. Gounaris (with J. Layssac and F.M. Renard)

**Introduction**

For processes $a_{\lambda_1} b_{\lambda_2} \to c_{\lambda_3} d_{\lambda_4}$ ,

in the limit $s \gg M_{SUSY}{}^2$ TeV$^2$ and s/t= fixed ,

only the helicity conserving amplitudes, satisfying $\lambda_1+\lambda_2=\lambda_3+\lambda_4$ , survive;
( $\lambda_j$ are the helicities).

All others must vanish.

Essentially, Helicity Conservation is
a **SUSY limit** property ↔ A Coleman-Mandula remnant.

Consequences may be visible at LHC, if **$M_{SUSY}$ ∼1 TeV**, …

## Summary

•The all order proof of Helicity Conservation (HCns) in **MSSM**, neglects all dimensional couplings.

•Detail 1loop EW calculations are needed, to see whether mass-ratios jeopardize HCns, and how the limit is approached.

•In **SM,** HCns is approximately correct, if $F_{Born} \neq 0$.
If $F_{Born}=0$, no general statement exists; 1loop EW examples have been found, where it is either approximately correct or strongly violated.

• Here, I present the 1loop EW amplitudes for $gg \to \gamma\gamma$, $\gamma Z$, $ZZ$, $W^+W^-$

in MSSM and SM, as well the 1loop amplitudes for

$$gg \to \tilde{\chi}_i^0 \tilde{\chi}_j^0 \ , \ \tilde{\chi}_i^+ \tilde{\chi}_j^-$$

•Asymptotic relations among the cross sections for these processes will be shown, which may be testable at LHC.

## **Sketch of the general HCns proof**

Renard+G: PRL 94:131601(2005), PR D73:097301(2006)

- All dimensional parameters are neglected: soft terms and the $\mu$ term.
- Proof straightforward for **2-to-2** processes, with external **fermions or scalars only.**

•Since SUSY transformations, projected on the one-particle states, relate

gauge ↔ gaugino with helicities of the **same sign**

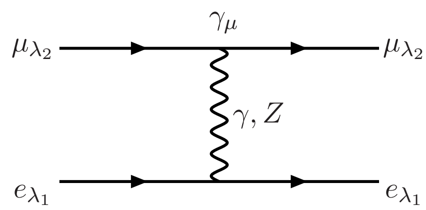
HCns obtains for **all 2-to-2** processes.

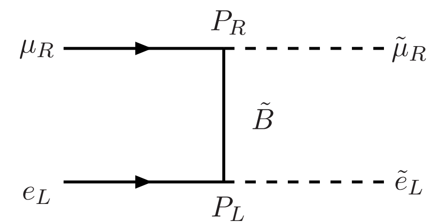
•Could mass-ratios jeopardize the proof? What happens in SM?
This is where 1loop detail calculations are needed.

# HCns at the Born level

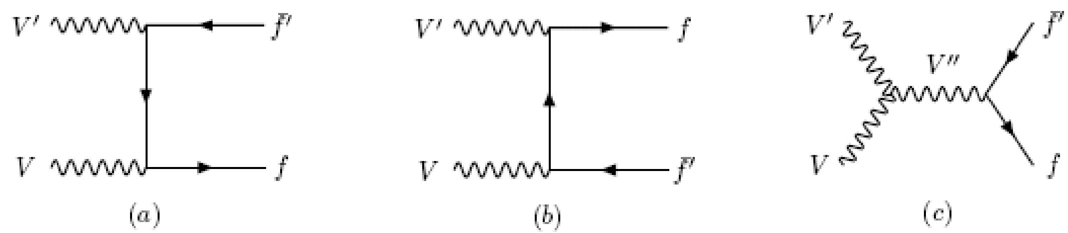
$(s, |t|, |u|) \gg$ all masses,
s/t=fixed

If no external gauge bosons appear, the validity of HCns is trivial.




In the case of external gauge bosons, large gauge cancellations among the diagrams are needed.
The couplings must be the standard **renormalizable** ones for **HCns** to be true.




**At Born level, HCns is valid, in both SM and MSSM**

# 1loop EW processes

$(s, |t|, |u|) \gg$ all masses,
s/t=fixed

•In the past we have looked in $\gamma\gamma \to \gamma\gamma$, $\gamma Z$, $ZZ$ where HCns is valid exactly in MSSM and approximately in SM (Layssac, Porfyriadis, Renard, Diakonidis)

•Then looked at $ug \to dW^+$ and $ug \to \tilde{d}_L \tilde{\chi}_i^+$ where the HC amplitudes were used to construct asymptotic relations among cross sections, which may be at the LHC energy range also.

•Last year I talked about $gg \to VH$, $HH'$ , $V=W, Z, \gamma$
where examples of strong violation of HCns in SM were identified.

•Now I present $gg \to \gamma\gamma$, $\gamma Z$, $ZZ$, $W^+W^-$ and $gg \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$ , $\tilde{\chi}_i^+ \tilde{\chi}_j^-$
among which asymptotic cross section relations are derived.
These may be valid even at LHC type energies….

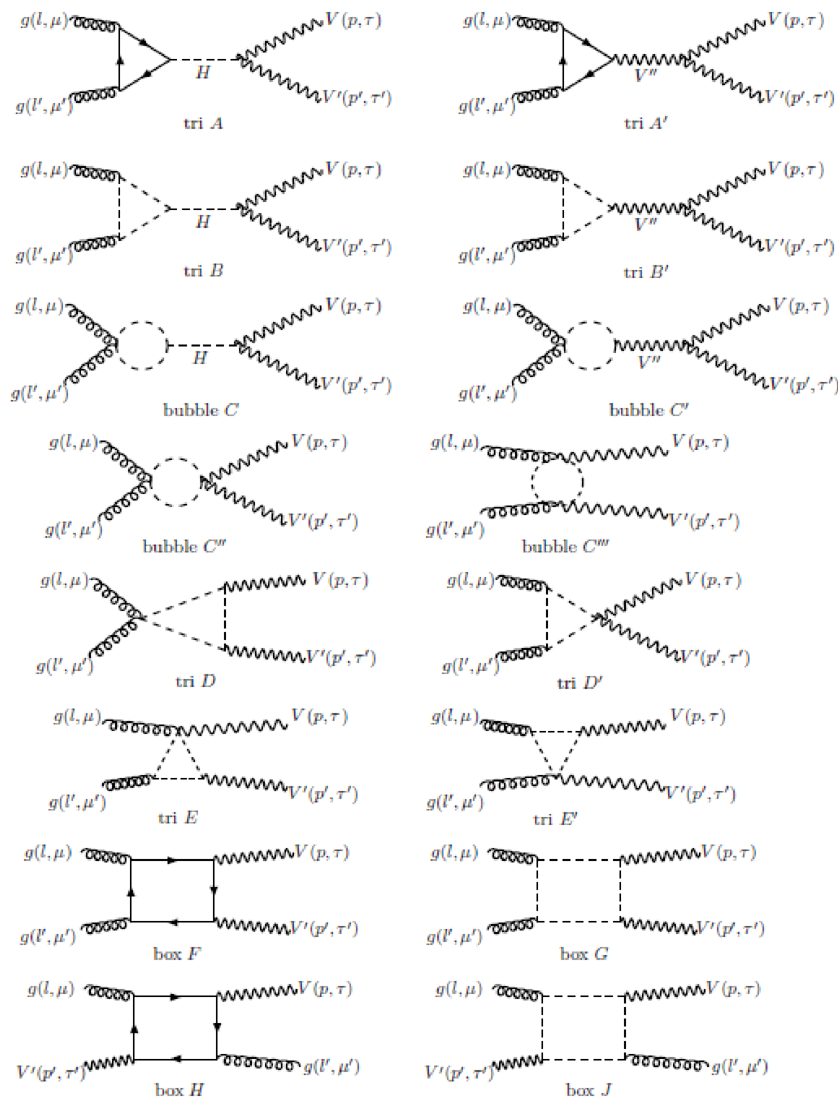
$$g(\mu)+g(\mu')\to V(\tau)+V'(\tau') \Rightarrow F_{\mu\mu'\tau\tau'}, \quad \text{HC} \Rightarrow \mu+\mu'=\tau+\tau'$$

1loop EW diagrams for
**gg → γγ, γZ, ZZ, $W^+W^-$**
in SM and MSSM.

Asymptotic results depend **only** on gauge couplings. Benchmark independent in MSSM

Strong HCns violation in SM:
all asym HV amplitudes ≠0.

But in MSSM
all asym HV amplitudes vanish
→ sqarks = −quarks



# Asymptotic HC gg→VV' amplitudes in MSSM (all HV vanish!)

$$F(gg\to ZZ)^{\rm as}_{\mu\mu'\tau\tau'} = \alpha\alpha_s\frac{(9-18s_W^2+20s_W^4)}{24s_W^2c_W^2}\delta_{\mu\mu'\tau\tau'} ,$$

$$F(gg\to \gamma Z)^{\rm as}_{\mu\mu'\tau\tau'} = \alpha\alpha_s\frac{(9-20s_W^2)}{24s_Wc_W}\delta_{\mu\mu'\tau\tau'} ,$$

$$F(gg\to \gamma\gamma)^{\rm as}_{\mu\mu'\tau\tau'} = \alpha\alpha_s\frac{5}{6}\delta_{\mu\mu'\tau\tau'} ,$$

$$F(gg\to W^+W^-)^{\rm as}_{\mu\mu'\tau\tau'} = \alpha\alpha_s\frac{3}{8s_W^2}\delta_{\mu\mu'\tau\tau'} ,$$

$\tau\tau'\neq 0$

In SM analogous expressions hold for HC and $HV_{TT}$ , $HV_{LL}$ amplitudes.
Only $HV_{TL}$ vanish.

$$F(gg\to ZZ)^{\rm as}_{+-00} = F(gg\to ZZ)^{\rm as}_{-+00} = \alpha\alpha_s\frac{(m_t^2+m_b^2)}{16s_W^2m_W^2}\left\{\frac{\delta^t(1-\cos\theta)}{1+\cos\theta}+\frac{\delta^u(1+\cos\theta)}{1-\cos\theta}\right\}$$

$$F(gg\to W^+W^-)^{\rm as}_{+-00} = \frac{\alpha\alpha_s}{8s_W^2m_W^2}\left\{\frac{m_b^2\delta^t(1-\cos\theta)}{1+\cos\theta}+\frac{m_t^2\delta^u(1+\cos\theta)}{1-\cos\theta}\right\} ,$$

$$F(gg\to W^+W^-)^{\rm as}_{-+00} = \frac{\alpha\alpha_s}{8s_W^2m_W^2}\left\{\frac{m_b^2\delta^u(1+\cos\theta)}{1-\cos\theta}+\frac{m_t^2\delta^t(1-\cos\theta)}{1+\cos\theta}\right\} . \quad (\text{2}$$

$$\delta^t = \delta_{+-+-} = \delta_{-+-+} = \tilde{\delta}\left(\frac{t+i\epsilon}{s+i\epsilon}\right)$$
$$= -4\left[\ln^2\left(\frac{2}{1-\cos\theta}\right) - i2\pi\ln\left(\frac{2}{1-\cos\theta}\right)\right] \quad ,$$
$$\delta^u = \delta_{+--+} = \delta_{-++-} = \tilde{\delta}\left(\frac{u+i\epsilon}{s+i\epsilon}\right)$$
$$= -4\left[\ln^2\left(\frac{2}{1+\cos\theta}\right) - i2\pi\ln\left(\frac{2}{1+\cos\theta}\right)\right] \quad ,$$
$$\delta_{++++} = \delta_{----} = \tilde{\delta}\left(\frac{t+i\epsilon}{u+i\epsilon}\right) = -4\left[\ln^2\left(\frac{1+\cos\theta}{1-\cos\theta}\right) + \pi^2\right] \quad ,$$

$$\tilde{\delta}(x/y) = -4\left[\ln^2\left(\frac{x}{y}\right) + \pi^2\right] \quad ,$$

dimensionless cross sections

$$\tilde{\sigma}^{as} = \frac{\sum_{HC}\left|F_{\mu\mu'\tau\tau'}\right|^2}{\alpha^2\alpha_s^2}$$

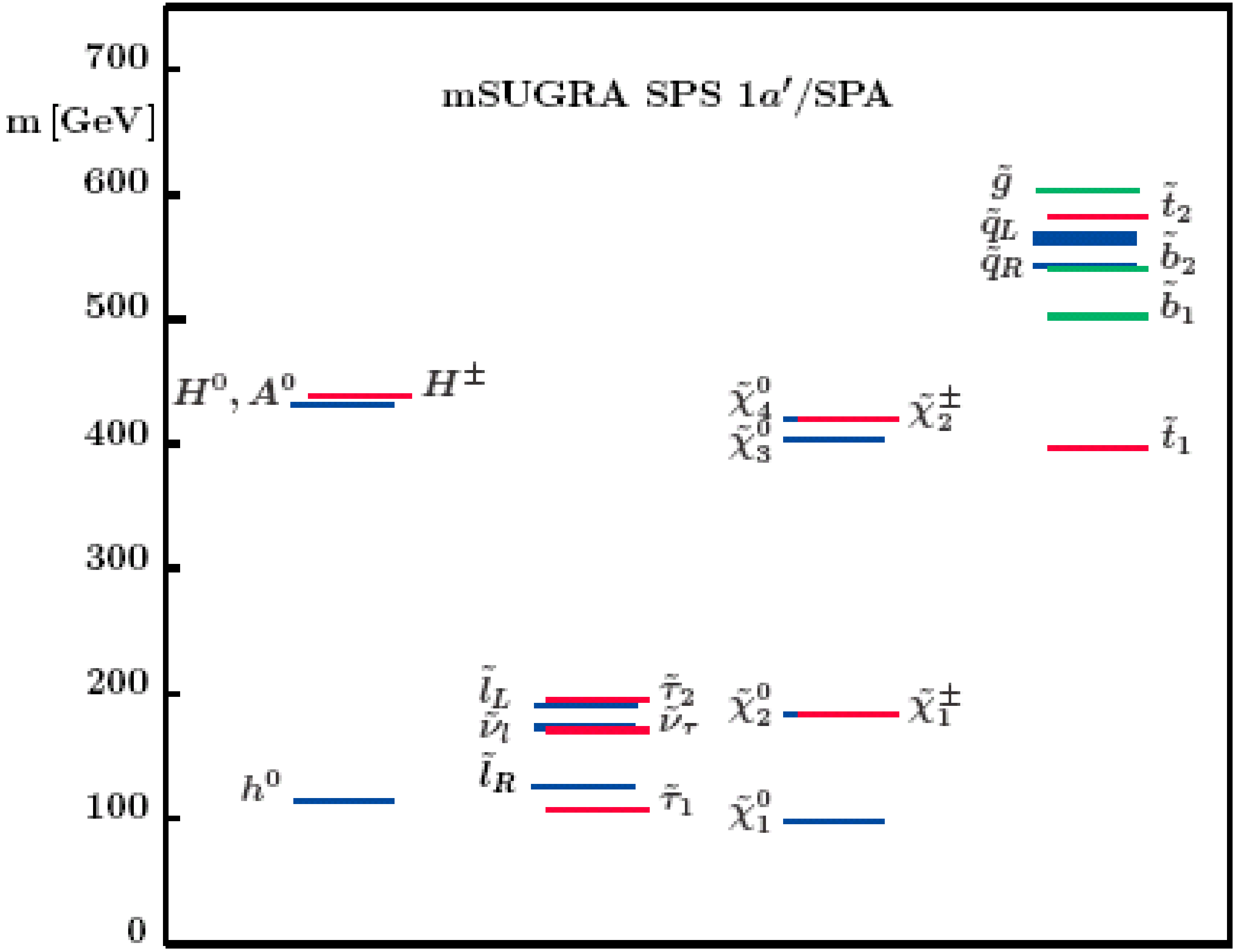
$$\tilde{\sigma} = \frac{\sum_{\mu\mu'\tau\tau'}\left|F_{\mu\mu'\tau\tau'}\right|^2}{\alpha^2\alpha_s^2}$$

Table 1: Asymptotic TT and LL Helicity Amplitudes divided by $\alpha\alpha_s$, and asymptotic $\tilde{\sigma}(gg \to VV')$, in MSSM and SM, at $\theta = 60^o$.

| $gg \to \gamma\gamma$ | | | $gg \to \gamma Z$ | | |
|---|---|---|---|---|---|
| | MSSM | SM | | MSSM | SM |
| $F_{++++}(\theta)$ | $-37.$ | $-26.$ | $F_{++++}(\theta)$ | $-19.$ | $-13.5$ |
| $F_{+-+-}(\theta)$ | $-6.4 + i29$ | $-3.4 + i20$ | $F_{+-+-}(\theta)$ | $-3.3 + i15.$ | $-1.7 + i10.3$ |
| $F_{+--+}(\theta)$ | $-0.28 + i6.0$ | $-0.14 + i4.0$ | $F_{+--+}(\theta)$ | $-0.14 + i3.1$ | $-0.07 + i2.1$ |
| $F_{++--}(\theta)$ | 0 | 6.7 | $F_{++--}(\theta)$ | 0 | 3.4 |
| $F_{+++-}(\theta)$ | 0 | 6.7 | $F_{+++-}(\theta)$ | 0 | 3.4 |
| | | | $F_{++-+}(\theta)$ | 0 | 3.4 |
| | | | $F_{+-++}(\theta)$ | 0 | 3.4 |
| $\tilde{\sigma}(gg \to \gamma\gamma)$ | 4567 | 2654 | $\tilde{\sigma}(gg \to \gamma Z)$ | 1220 | 709 |

| $gg \to ZZ$ | | | $gg \to W^+W^-$ | | |
|---|---|---|---|---|---|
| | MSSM | SM | | MSSM | SM |
| $F_{++++}(\theta)$ | $-61$ | $-43$ | $F_{++++}(\theta)$ | $-72.$ | $-51.$ |
| $F_{+-+-}(\theta)$ | $-10.6 + i48.1$ | $-5.6 + i33.$ | $F_{+-+-}(\theta)$ | $-12. + i56.$ | $-6.5 + i39$ |
| $F_{+--+}(\theta)$ | $-0.46 + i10.$ | $-0.23 + i6.7$ | $F_{+--+}(\theta)$ | $-0.5 + i12.$ | $-0.27 + i7.8$ |
| $F_{++--}(\theta)$ | 0 | 11. | $F_{++--}(\theta)$ | 0 | 12.9 |
| $F_{+++-}(\theta)$ | 0 | 11. | $F_{+++-}(\theta)$ | 0 | 12.9 |
| $F_{+-++}(\theta)$ | 0 | 11. | $F_{+-++}(\theta)$ | 0 | 12.9 |
| $F_{+-00}(\theta)$ | $-4.6 + i43.$ | $-4.6 + i43.$ | $F_{+-00}(\theta)$ | $-2.6 + i56.$ | $-2.6 + i56.$ |
| $F_{++00}(\theta)$ | 0 | $-20.5$ | $F_{++00}(\theta)$ | 0 | $-20.5$ |
| $\tilde{\sigma}(gg \to ZZ)$ | 16226 | 11820 | $\tilde{\sigma}(gg \to W^+W^-)$ | 21232 | 14873 |

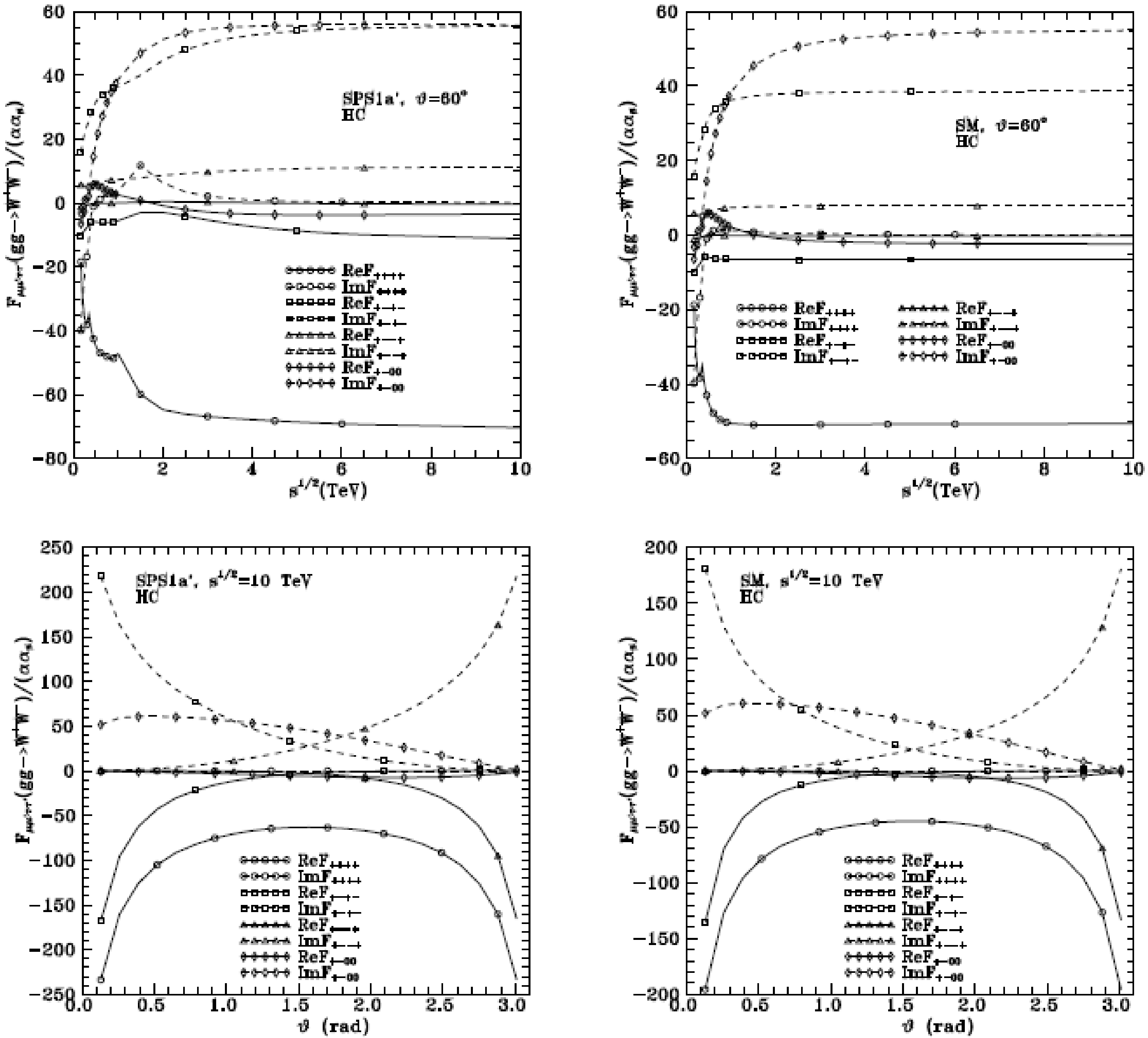
# In all SUSY applications we use SPS1a'



The 1loop EW MSSM and SM HC amplitudes for $gg \to W^+W^-$

In SPS1a' asym values reached at $s^{1/2} \gtrsim 4$ TeV for $\theta = 60^o$

Similar but smaller in SM



The 1loop EW MSSM and SM $HV_{TT}$, $HV_{LL}$ amplitudes for $gg \to W^+W^-$

For $s^{1/2} \gtrsim 4$ TeV

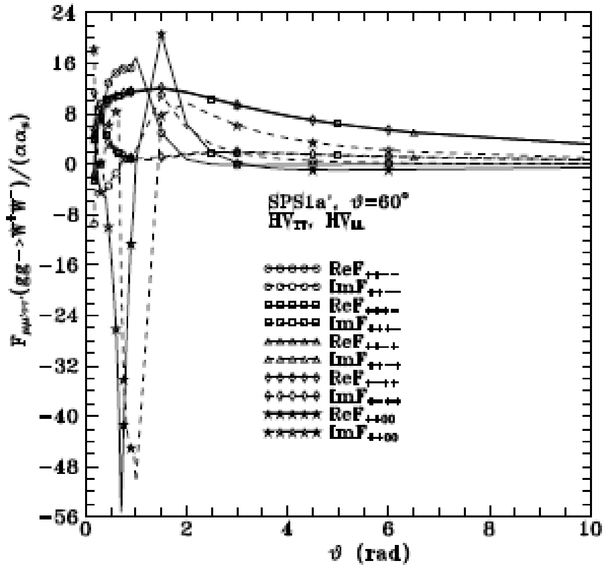
In SM
$|HC| \gtrsim |HV_{LL}| \sim |HV_{TT}|$

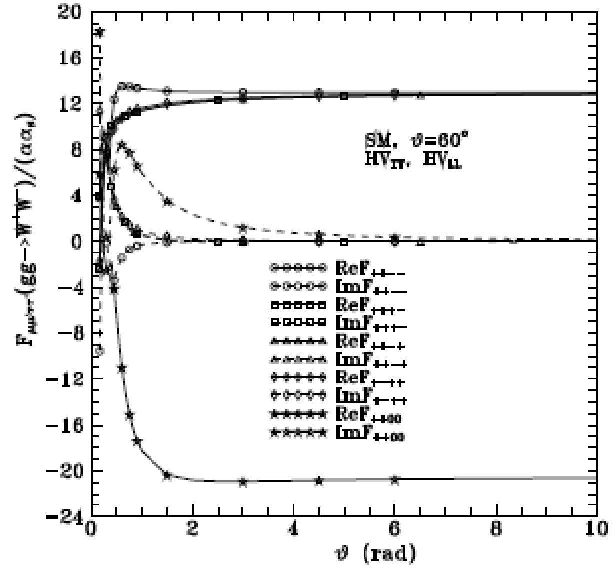
In MSSM (SPS1a')

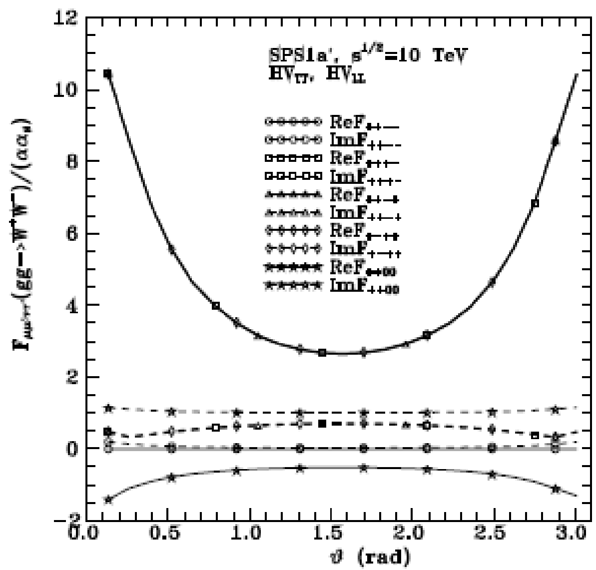
$|HC| \gg |HV_{TT}| \sim |HV_{LL}|$

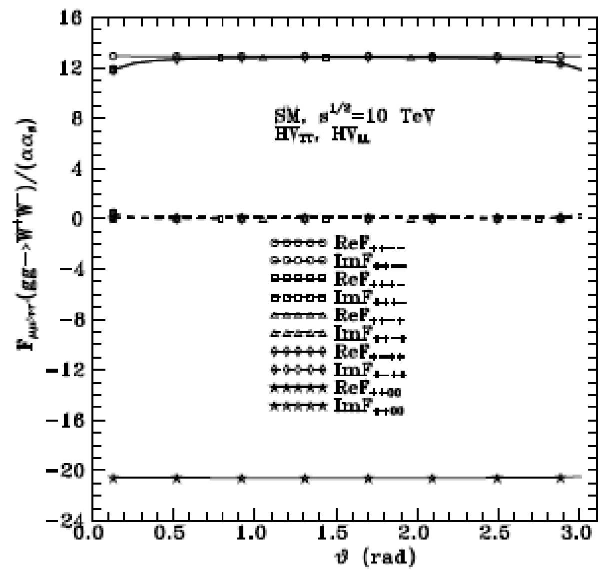
Overall picture →

$|HV_{SM}| \gg |HV_{MSSM}|$



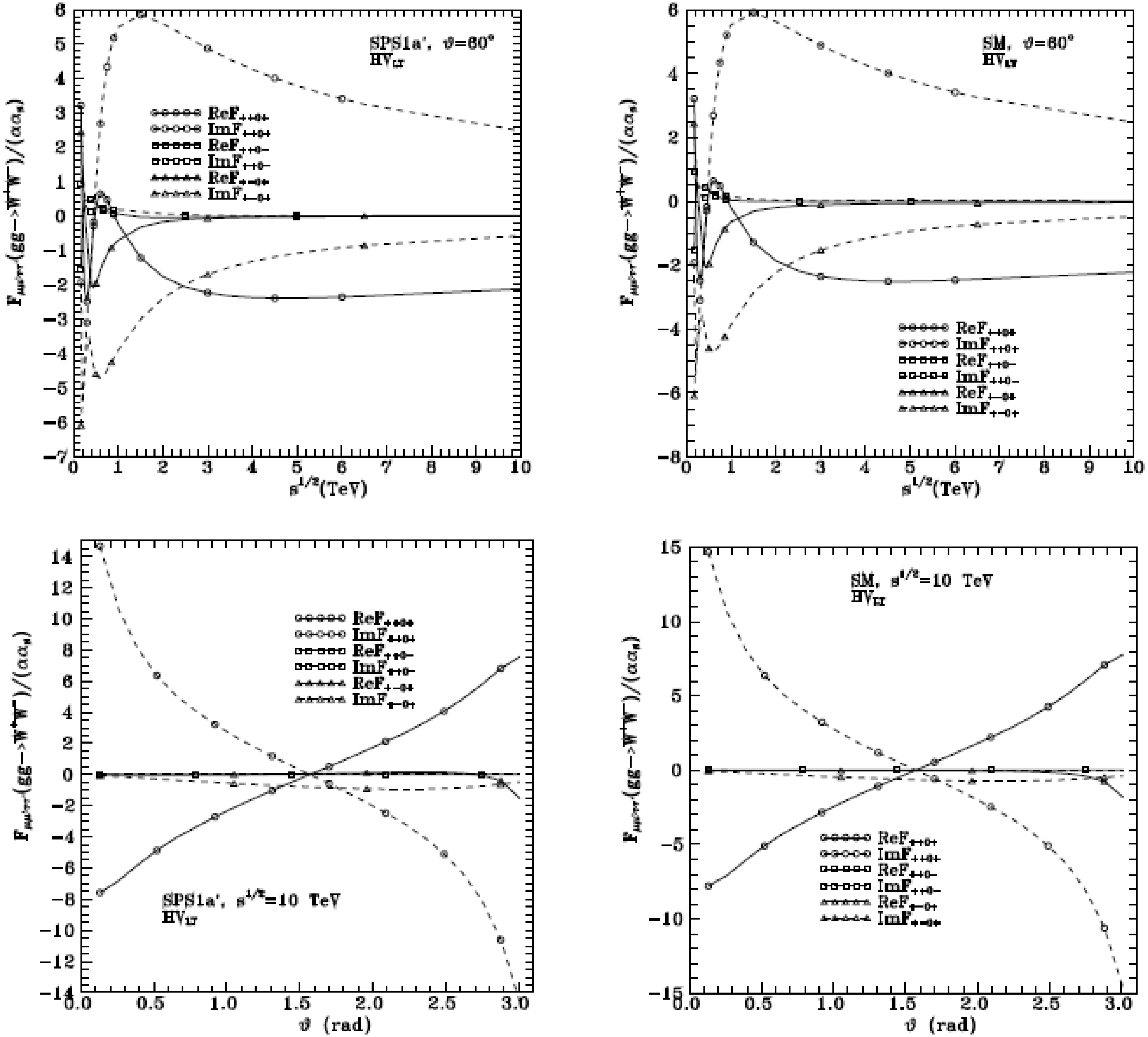



For $s^{1/2} \gtrsim 4$ TeV

$HV_{TL} \sim$ small for $gg \to W^+W^-$

in both MSSM and SM




Similarly for all other $gg \to VV'$

Overall picture for

1loop EW for **$gg \to \gamma\gamma$, $\gamma Z$, $ZZ$, $W^+W^-$** in SM and MSSM.

- Mass-ratios never jeopardize HCns

•For MSSM(SPS1a') or SM, at $\theta=60^o$ ,
the limiting values are reached at $s^{1/2} \gtrsim 4$ TeV

•Limits $\to$ benchmark independent

•In SM $|HC| \gtrsim |HV_{LL}| \sim |HV_{TT}|$

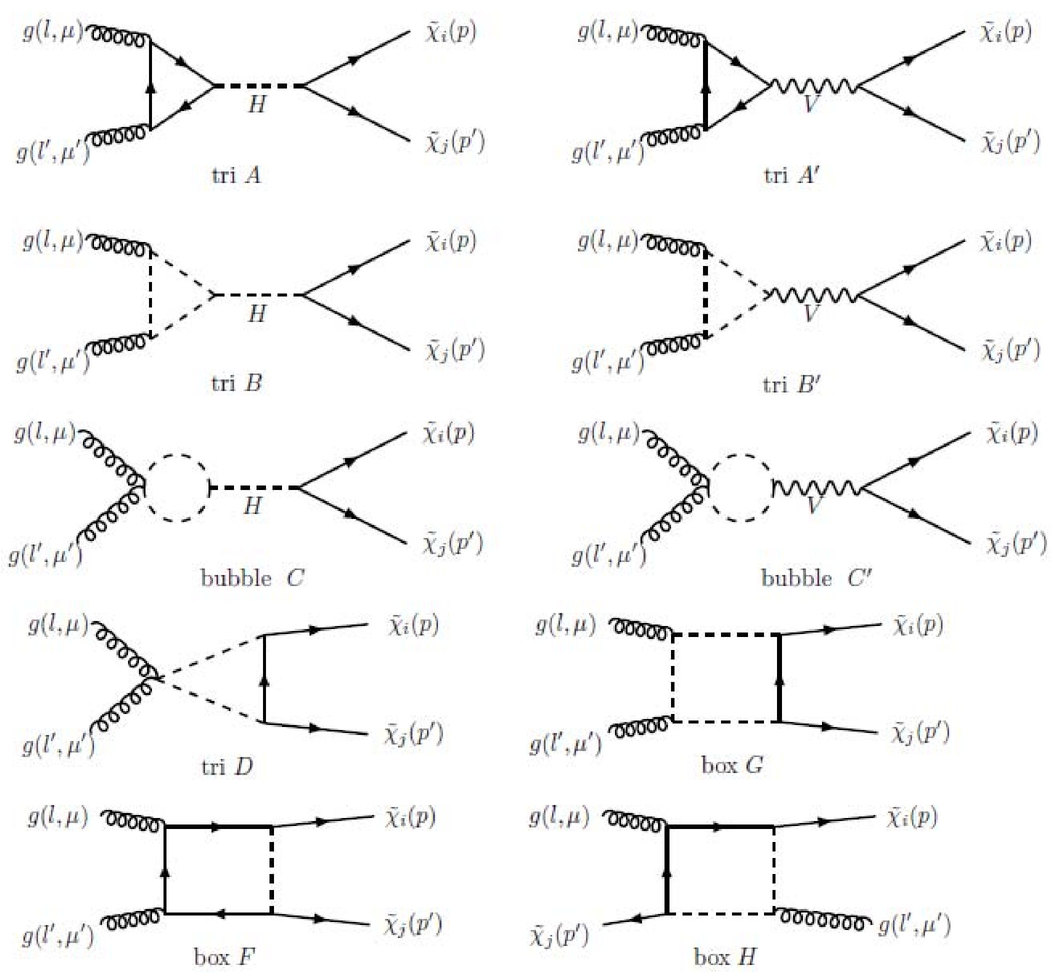
•In MSSM (SPS1a') $|HC| \gg |HV_{TT}| \sim |HV_{LL}|$

$$g(\mu)+g(\mu')\rightarrow\tilde{\chi}_i(\tau)+\tilde{\chi}_j(\tau')\;\Rightarrow\;F_{\mu\mu'\tau\tau'}\,,\qquad \text{HC}\;\Rightarrow\;\mu+\mu'=\tau+\tau'=0$$

Charginos or neutralinos.

Asym results respect HCNs and depend on

$\alpha$, $\alpha_s$, $\beta$ , $Z^+$, $Z^-$ , $Z^N$



## Asymptotic HC amplitudes in MSSM(SPS1a') (all HV vanish!)

$$F(gg \to \tilde{\chi}_i^+ \tilde{\chi}_j^-)^{as}_{-+-+} = \frac{\alpha\alpha_s \delta^t (1-\cos\theta)}{8s_W^2 \quad \sin\theta} \left\{ 3Z^+_{1i} Z^{+*}_{1j} + Z^+_{2i} Z^{+*}_{2j} \frac{m_t^2}{m_W^2 \sin^2\beta} \right\} ,$$

$$F(gg \to \tilde{\chi}_i^+ \tilde{\chi}_j^-)^{as}_{+-+-} = -\frac{\alpha\alpha_s \delta^t (1-\cos\theta)}{8s_W^2 \quad \sin\theta} \left\{ 3Z^-_{1i} Z^{-*}_{1j} + Z^-_{2i} Z^{-*}_{2j} \frac{m_b^2}{m_W^2 \cos^2\beta} \right\} ,$$

$$F(gg \to \tilde{\chi}_i^+ \tilde{\chi}_j^-)^{as}_{+--+} = -\frac{\alpha\alpha_s \delta^u (1+\cos\theta)}{8s_W^2 \quad \sin\theta} \left\{ 3Z^+_{1i} Z^{+*}_{1j} + Z^+_{2i} Z^{+*}_{2j} \frac{m_t^2}{m_W^2 \sin^2\beta} \right\} ,$$

$$F(gg \to \tilde{\chi}_i^+ \tilde{\chi}_j^-)^{as}_{-++-} = \frac{\alpha\alpha_s \delta^u (1+\cos\theta)}{8s_W^2 \quad \sin\theta} \left\{ 3Z^-_{1i} Z^{-*}_{1j} + Z^-_{2i} Z^{-*}_{2j} \frac{m_b^2}{m_W^2 \cos^2\beta} \right\} ,$$

$$F(gg \to \tilde{\chi}_i^0 \tilde{\chi}_j^0)^{as}_{-+-+} = -F(gg \to \tilde{\chi}_i^0 \tilde{\chi}_j^0)^{as}_{+-+-} = \alpha\alpha_s \frac{\delta^t (1-\cos\theta)}{\sin\theta} \left\{ Z^N_{1i} Z^{N*}_{1j} \frac{11}{24c_W^2} \right.$$
$$\left. + Z^N_{2i} Z^{N*}_{2j} \frac{3}{8s_W^2} + Z^N_{3i} Z^{N*}_{3j} \frac{m_b^2}{8s_W^2 m_W^2 \cos^2\beta} + Z^N_{4i} Z^{N*}_{4j} \frac{m_t^2}{8s_W^2 m_W^2 \sin^2\beta} \right\} ,$$

$$F(gg \to \tilde{\chi}_i^0 \tilde{\chi}_j^0)^{as}_{-++-} = -F(gg \to \tilde{\chi}_i^0 \tilde{\chi}_j^0)^{as}_{+--+} = \alpha\alpha_s \frac{\delta^u (1+\cos\theta)}{\sin\theta} \left\{ Z^N_{1i} Z^{N*}_{1j} \frac{11}{24c_W^2} \right.$$
$$\left. + Z^N_{2i} Z^{N*}_{2j} \frac{3}{8s_W^2} + Z^N_{3i} Z^{N*}_{3j} \frac{m_b^2}{8s_W^2 m_W^2 \cos^2\beta} + Z^N_{4i} Z^{N*}_{4j} \frac{m_t^2}{8s_W^2 m_W^2 \sin^2\beta} \right\} ,$$

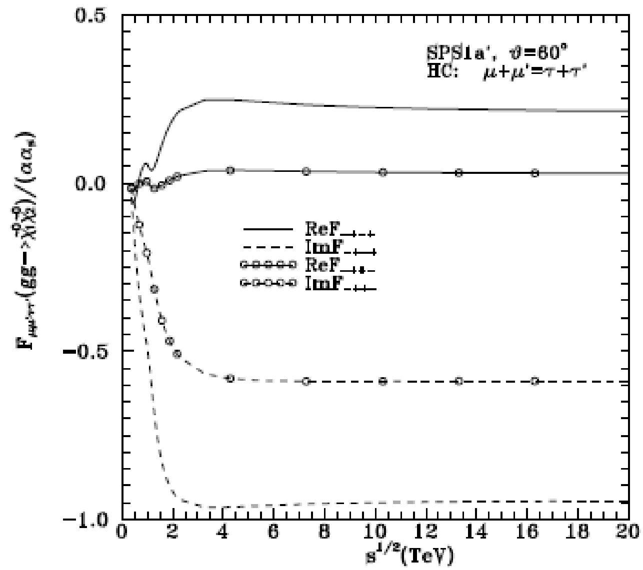
$gg \to \tilde{\chi}_1^0 \tilde{\chi}_2^0$

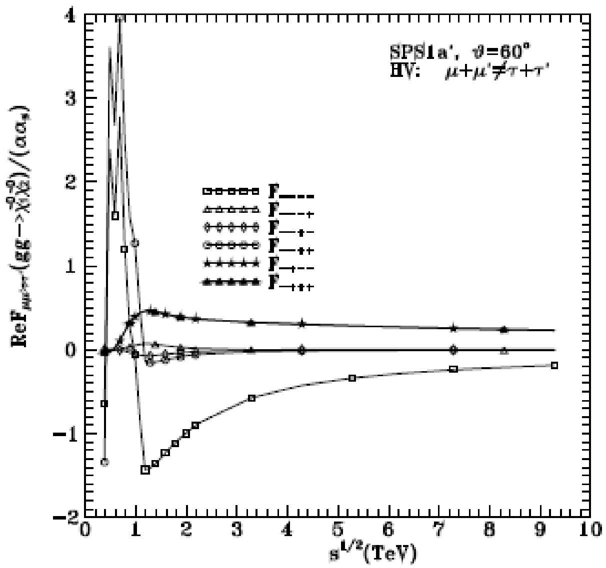
amplitudes

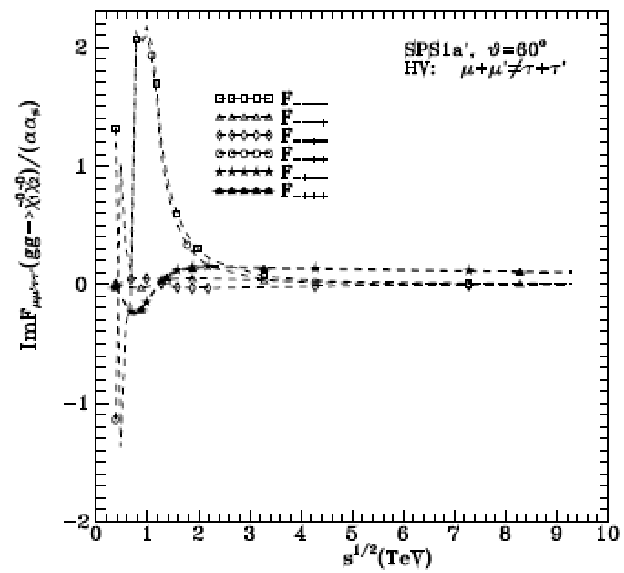
For MSSM(SPS1a') at θ=60°, and i=1, j=2

The benchmark dependent limits are reached at $s^{1/2} \gtrsim$ 10 TeV where

|HC| ≫ |HV|



## $gg \to \tilde{\chi}_1^+ \tilde{\chi}_2^-$

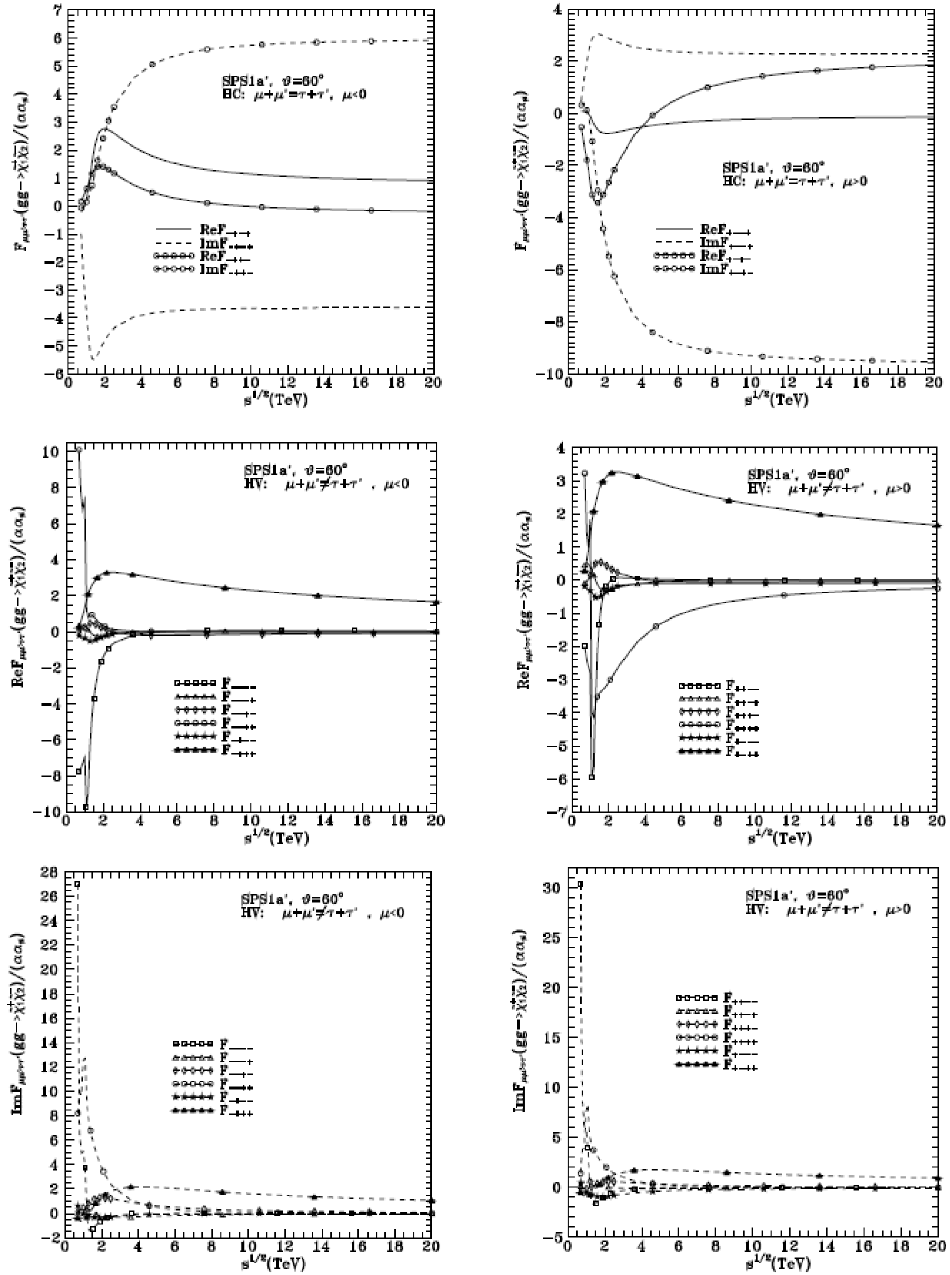
## amplitudes

For MSSM(SPS1a') at θ=60°, and i=1, j=2

The benchmark dependent limits are reached at $s^{1/2} \gtrsim 20$ TeV where

$|HC| \gg |HV|$

Picture somewhat cluttered because of the large number of amplitudes.




G.J. Gounaris, Corfu Sep. 2010

dimensionless cross sections

$$\tilde{\sigma} = \frac{\sum_{\mu\mu'\tau\tau'} \left|F_{\mu\mu'\tau\tau'}\right|^2}{\alpha^2 \alpha_s^2} \ , \ \tilde{\sigma}^{as} = \frac{\sum_{HC} \left|F_{\mu\mu'\tau\tau'}\right|^2}{\alpha^2 \alpha_s^2}$$

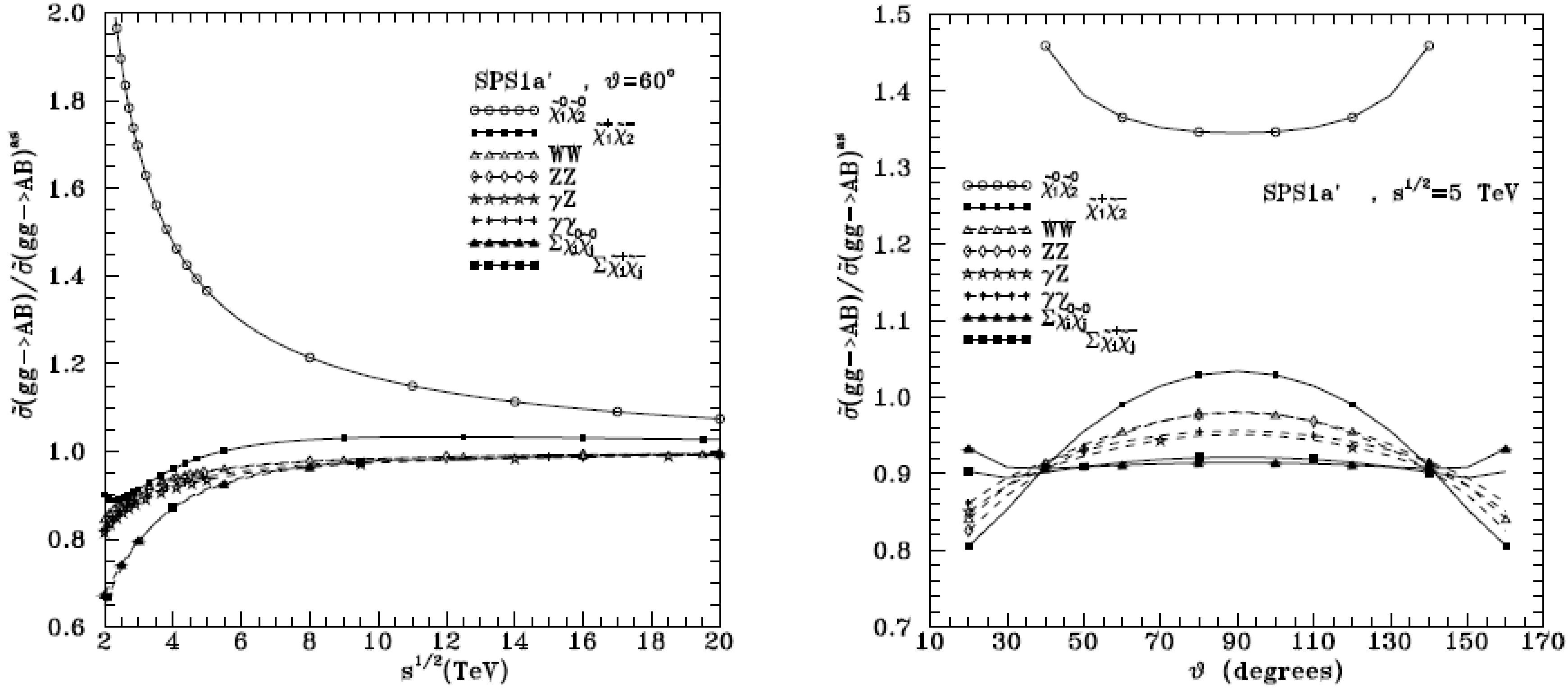
$$\frac{\tilde{\sigma}(gg \to W^+W^-)}{\tilde{\sigma}(gg \to W^+W^-)^{as}} \ , \ \frac{\tilde{\sigma}(gg \to ZZ)}{\tilde{\sigma}(gg \to ZZ)^{as}} \ , \ \frac{\tilde{\sigma}(gg \to \gamma Z)}{\tilde{\sigma}(gg \to \gamma Z)^{as}} \ , \ \frac{\tilde{\sigma}(gg \to \gamma\gamma)}{\tilde{\sigma}(gg \to \gamma\gamma)^{as}} \ ,$$

$$\frac{\sum_{ij} \tilde{\sigma}(gg \to \tilde{\chi}_i^+ \tilde{\chi}_j^-)}{\sum_{ij} \tilde{\sigma}(gg \to \tilde{\chi}_i^+ \tilde{\chi}_j^-)^{as}} \ , \ \frac{\sum_{ij} \tilde{\sigma}(gg \to \tilde{\chi}_i^0 \tilde{\chi}_j^0)}{\sum_{ij} \tilde{\sigma}(gg \to \tilde{\chi}_i^0 \tilde{\chi}_j^0)^{as}} \ ,$$

$$\frac{\tilde{\sigma}(gg \to \tilde{\chi}_i^+ \tilde{\chi}_j^-)}{\tilde{\sigma}(gg \to \tilde{\chi}_i^+ \tilde{\chi}_j^-)^{as}} \ , \ \frac{\tilde{\sigma}(gg \to \tilde{\chi}_i^0 \tilde{\chi}_j^0)}{\tilde{\sigma}(gg \to \tilde{\chi}_i^0 \tilde{\chi}_j^0)^{as}} \ ,$$

- In the numerators we hope to use data. At present just the exact 1loop results.
- In the denominators, only the asymptotic HC amplitudes are used.
- At sufficient energies all these σ-ratios $\to 1$ .

How this is realized in SPS1a' ?




$$\frac{\tilde\sigma(gg\to W^+W^-)}{\tilde\sigma(gg\to W^+W^-)^{as}}\simeq\frac{\tilde\sigma(gg\to ZZ)}{\tilde\sigma(gg\to ZZ)^{as}}\simeq\frac{\tilde\sigma(gg\to \gamma Z)}{\tilde\sigma(gg\to \gamma Z)^{as}}\simeq\frac{\tilde\sigma(gg\to \gamma\gamma)}{\tilde\sigma(gg\to \gamma\gamma)^{as}}$$

$$\simeq\frac{\sum_{ij}\tilde\sigma(gg\to\tilde\chi^+_i\tilde\chi^-_j)}{\sum_{ij}\tilde\sigma(gg\to\tilde\chi^+_i\tilde\chi^-_j)^{as}}\simeq\frac{\sum_{ij}\tilde\sigma(gg\to\tilde\chi^0_i\tilde\chi^0_j)}{\sum_{ij}\tilde\sigma(gg\to\tilde\chi^0_i\tilde\chi^0_j)^{as}}\simeq 1\ ,$$

OK for $s^{1/2}\gtrsim 5$ TeV

If $M_{SUSY} \sim 1$ TeV:

•The asymptotic limits of the gauge boson cross sections are benchmark independent, and the approach to them **very fast.**

•When summing over all charginos (neutralinos), the dependence on $Z^{+}$, $Z^{-}$ ($Z^{N}$) **disappears**, in

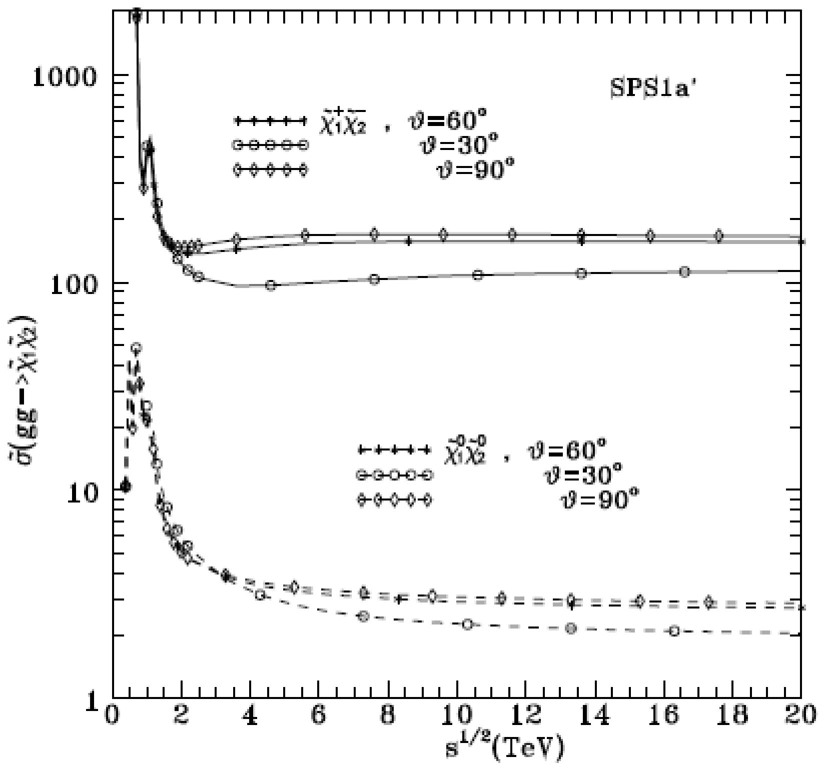
$$\sum_{ij} \tilde{\sigma}(gg \to \tilde{\chi}_i \tilde{\chi}_j)^{as}$$

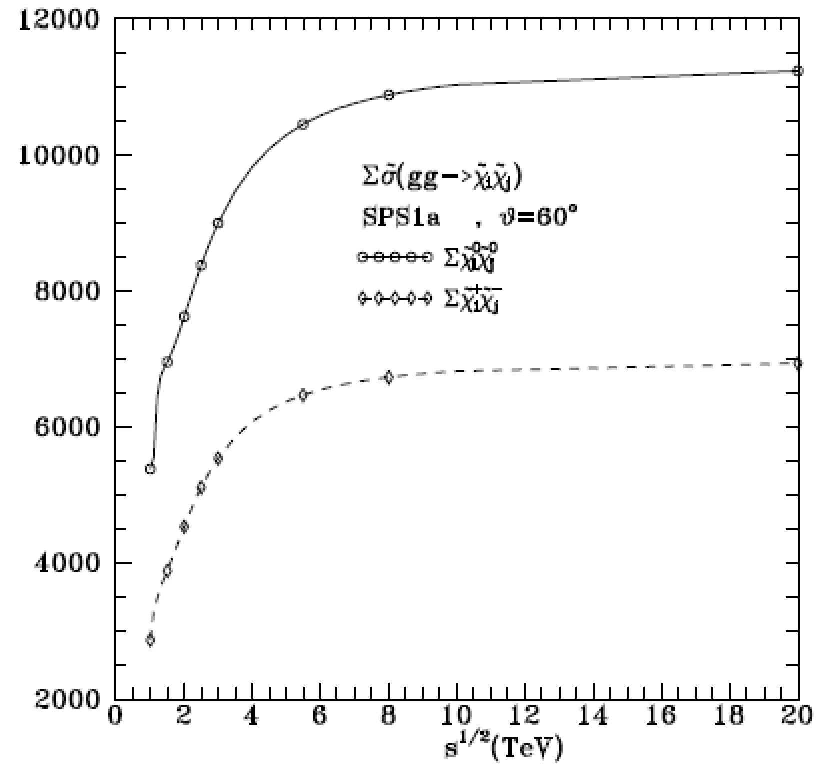
and they only depend on $\beta$, $\alpha$, $\alpha_s$, $m_t$ and $m_b$.
The chargino cross sections approaches their asymptotic values **quickly**.

•In case **no i, j summation** is done, in

$$\frac{\tilde{\sigma}(gg \to \tilde{\chi}_i^{+} \tilde{\chi}_j^{-})}{\tilde{\sigma}(gg \to \tilde{\chi}_i^{+} \tilde{\chi}_j^{-})^{as}}\ , \quad \frac{\tilde{\sigma}(gg \to \tilde{\chi}_i^{0} \tilde{\chi}_j^{0})}{\tilde{\sigma}(gg \to \tilde{\chi}_i^{0} \tilde{\chi}_j^{0})^{as}}\ ,$$

then the benchmark dependence of asymptotic cross sections becomes strong, (due to the Z-matrices) and the approach to asymptopia is **slow**.




$$\sum_{ij} \tilde{\sigma}(gg \to \tilde{\chi}_i^+ \tilde{\chi}_j^-) \gg \tilde{\sigma}(gg \to \tilde{\chi}_1^+ \tilde{\chi}_2^-)$$

Similarly for neutralinos

Relations like

$$\frac{\tilde\sigma(gg\to W^+W^-)}{\tilde\sigma(gg\to W^+W^-)^{as}} \simeq \frac{\tilde\sigma(gg\to ZZ)}{\tilde\sigma(gg\to ZZ)^{as}} \simeq \frac{\tilde\sigma(gg\to \gamma Z)}{\tilde\sigma(gg\to \gamma Z)^{as}} \simeq \frac{\tilde\sigma(gg\to \gamma\gamma)}{\tilde\sigma(gg\to \gamma\gamma)^{as}}$$

$$\simeq \frac{\sum_{ij}\tilde\sigma(gg\to \tilde\chi_i^+\tilde\chi_j^-)}{\sum_{ij}\tilde\sigma(gg\to \tilde\chi_i^+\tilde\chi_j^-)^{as}} \simeq \frac{\sum_{ij}\tilde\sigma(gg\to \tilde\chi_i^0\tilde\chi_j^0)}{\sum_{ij}\tilde\sigma(gg\to \tilde\chi_i^0\tilde\chi_j^0)^{as}} \simeq 1 \quad ,$$

require $s^{1/2} \gtrsim$ 5 TeV in SPS1a' and may be visible at LHC

# Conclusions

• Many or most of the 2-to-2 amplitudes vanish in the SUSY symmetric limit.
Only the helicity conserving ones can survive.
This limit is reached by $s \gg M_{SUSY}^2$ , $t/s$=fixed. .

•HCns provides many asymptotic relations among various process cross sections. If the SUSY scale is not too high, these may be useful for LHC, or a future higher energy machine.

•HCns is a SUSY property, as basic as the gauge coupling unification

• Fortran codes for all 1loop EW amplitudes we have calculated are released in http://users.auth.gr/gounaris/FORTRANcodes