# Flow equations of supersymmetric Wess-Zumino models

Franziska Synatschke-Czerwonka

Friedrich-Schiller-University Jena
Theoretisch-Physikalisches Institut

in collaboration with
G. Bergner, J. Braun, T. Fischbacher, H. Gies, A. Wipf

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# Outline

# Motivation

- Supersymmetric models in general not analytically solvable: Approximation schemes necessary
- Successful approximations often break supersymmetry explicitly (e.g. lattice calculations)
    - problem for the investigation of dynamical supersymmetry breaking
- Functional renormalization group:
  A nonperturbative tool that preserves supersymmetry

# The $\mathcal{N}=2$ WZ Model in 2 dimensions

## Field content (on-shell):

- complex scalar field $\phi, \bar{\phi}$
- Dirac fermions $\psi, \bar{\psi}$

## on-shell Lagrangian

complex coordinates: $z = x_1 + ix_2$ and $\partial = \frac{1}{2}(\partial_1 - i\partial_2)$

$$\mathcal{L}_{\text{on}} = 2\bar{\partial}\bar{\phi}\partial\phi + \frac{1}{2}\left|W'(\phi)\right|^2 + \bar{\psi}M\psi$$

fermion matrix: $M = \not{\partial} + W''(\phi)P_+ + \bar{W}''(\bar{\phi})P_-$

superpotential:

$$W(\phi) = \frac{1}{2}m\phi^2 + \frac{1}{3}g\phi^3,\ W'(\phi) = m\phi + g\phi^2,\ W''(\phi) = m + 2g\phi$$

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# The $\mathcal{N} = 2$ WZ Model in 2 dimensions

- The model is obtained by dimensional reduction from four dimensional $\mathcal{N} = 1$ Wess-Zumino model
- The superpotential is a holomorphic function
- The superpotential obeys a non-renormalization theorem: bare couplings receive no quantum corrections
- Model is UV-finite
- Supersymmetry is always unbroken

# The $\mathcal{N}=2$ WZ Model in 2 dimensions

## off-shell Lagrangian

$$\mathcal{L}_{\text{off}} = 2\bar{\partial}\bar{\phi}\partial\phi + \bar{\psi}M\psi - \frac{1}{2}\bar{F}F + \frac{1}{2}FW'(\phi) + \frac{1}{2}\bar{F}\bar{W}'(\bar{\phi})$$

fermion matrix:

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equations of motion for auxiliary fields:

$$F = \bar{W}'(\bar{\phi}), \quad \bar{F} = W'(\phi)$$

plug these equations in:

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# Flow equations in the LPA – Nonrenormalization theorem

superpotential

$$W(\phi) = u(\phi_1,\phi_2) + iv(\phi_1,\phi_2)$$

is holomorphic, Cauchy-Rieman differential equations hold:

$$\frac{\partial u}{\partial \phi_1} = \frac{\partial v}{\partial \phi_2}, \quad \frac{\partial u}{\partial \phi_2} = -\frac{\partial v}{\partial \phi_1}.$$

**non-renormalization theorem**

$$\partial_k W_k = \partial_k \bar{W}_k = 0$$

non-renormalization theorem in four dimensions:
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- Flow equations for wave function renormalization with *full* momentum dependence
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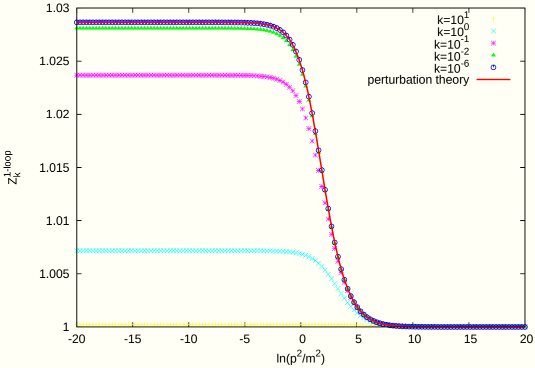

# Renormalized mass

Renormalized mass: pole of the propagator in the complex plane

Propagator

$$G_{\text{bos}}(p) = \frac{1}{p^2 + m^2 + \Sigma(p,m,g)}$$

Propagator in NLO

$$G_{\text{bos}}^{\text{NLO}}(p) = \frac{1}{p^2 Z_{k\to 0}(p^2) + m^2/Z_{k\to 0}(p^2)}$$

Correlator

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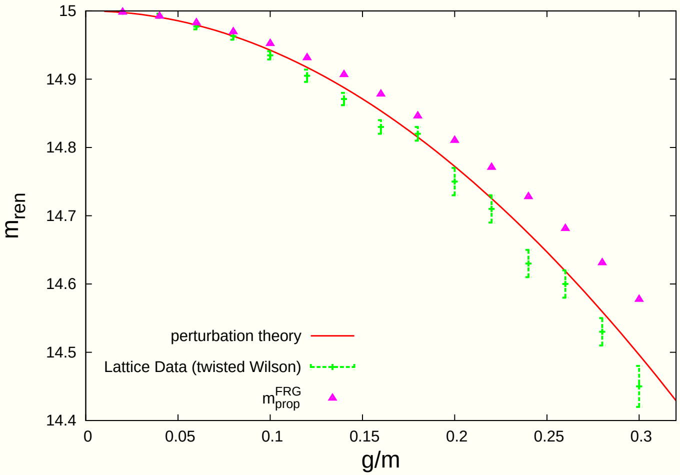

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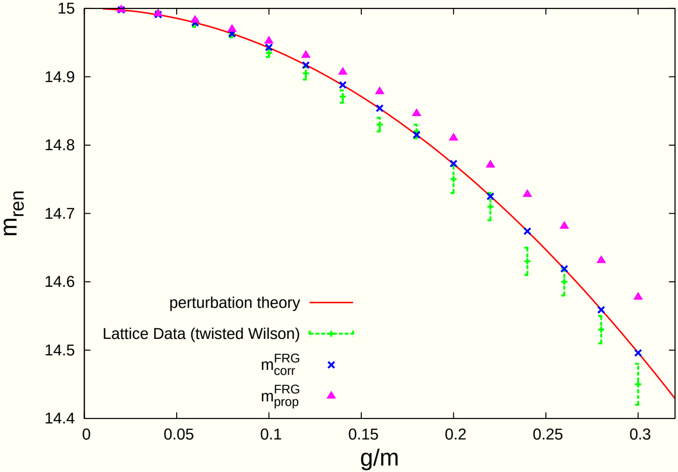

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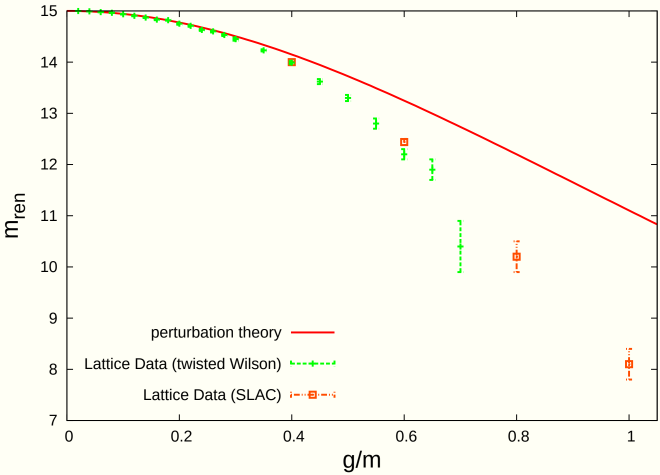

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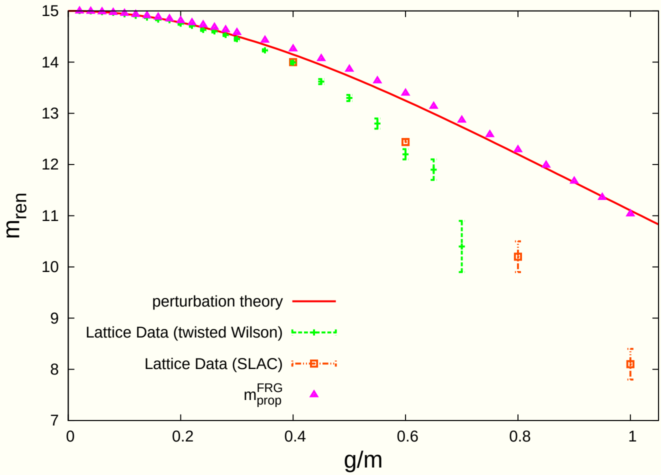

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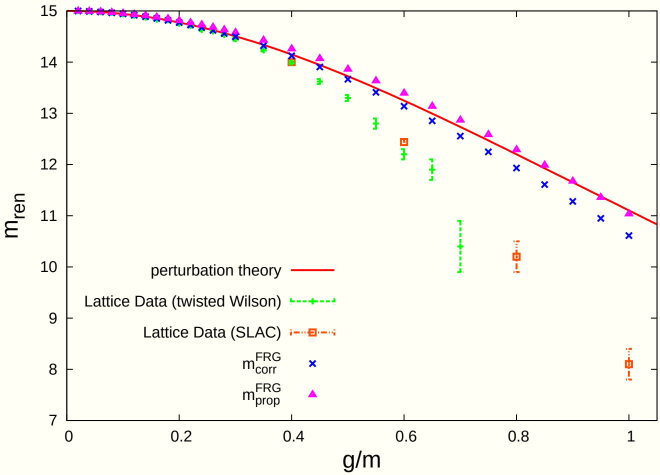

# Summary

- FRG can be extended to supersymmetric theories in a way that preserves supersymmetry
- Provides an approach complementary to lattice calculations

Results for the Wess-Zumino $\mathcal{N}=2$ model in two dimensions:

- Nonrenormalization theorem is recoverd in a very simple form
- Wave function renormalization with full momentum dependence is calculated
- Comparision to lattice results
  - weak coupling: good agreement between lattice and FRG
  - intermediate coupling: wave function renormalization does not suffice to capture all quantum effects, higher order operators are needed