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On climbing scalars in String Theory

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Outline

- A climbing scalar in D dimensions
- Connection to String Theory.
- Moduli stabilization, climbing and trapping.
- Climbing and inflation ?
- Conclusions

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List of (some) earlier relevant references :

- Exact solutions, attractors :

Abbott, Wise (1984); Lucchin, Mataresse (1985) ; Halliwell (1987); Dudas, Mourad (2000); Townsend, Wohlfarth; Emparan, Garriga; Russo (2003-2004); Bergshoeff,Collinucci, M. Nielsen et al, (2003-2004)

- Tachyon-free orientifold constructions (Brane SUSY breaking):

Sugimoto (1999); Antoniadis,E.D.,Sagnotti + Angelantonj,d'Appollonio,Mourad; Aldazabal,Uranga (1999)

1. A climbing scalar in D dimensions.

We study low-energy effective actions of the type

$$S \;=\; \frac{1}{2\kappa^2}\int d^Dx\,\sqrt{-g}\left[R-\frac{1}{2}\,(\partial\phi)^2-V(\phi)\,+\,\ldots\right],$$

One study cosmological solutions with the ansatz

$$ds^2 \,=\, -\,e^{\,2B(t)}\,dt^2\,+\,e^{\,2A(t)}\,d\mathbf{x}\cdot d\mathbf{x} \qquad\qquad \phi=\phi(t)\,.$$

A very convenient gauge choice is

$$V\,e^{2B} \;=\; M^2\,.$$

M is the mass scale of potential V, allows to find exact solutions of the field equations in a "parametric" time

t, related to the cosmological time η by $d\eta = e^{B}\, dt$. Let

$$\beta = \sqrt{\frac{D-1}{D-2}}\,, \quad \tau = M\,\beta\,t\,, \quad \varphi = \frac{\beta\,\phi}{\sqrt{2}}\,, \quad a = (D{-}1)\,A\,.$$

Denote all τ derivatives by "dots".

The independent eqs take the convenient form

$$\begin{aligned} &\dot{a}^2 - \dot{\varphi}^{\,2} = 1 \\ &\ddot{\varphi} + \dot{a}\;\dot{\varphi} + \left(1 + \dot{\varphi}^{\,2}\right)\,\frac{1}{2V}\,\frac{\partial V}{\partial \varphi} = 0\,. \end{aligned}$$

For an expanding phase ($\dot{a} > 0$) and exponential potentials $V = M^2\, e^{2\gamma\varphi}$ field eqs. combine to

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1 + \dot{\varphi}^{\,2}} - \gamma\left(1 + \dot{\varphi}^{\,2}\right) = 0\ .$$

Claim :

- For $\gamma < 1$, the scalar φ can descend or climb the potential after the big-bang .
- For $\gamma \geq 1$, the scalar φ is forced to climb immediately after the big-bang .

We begin by considering the “critical” case, $\gamma = 1$; one can solve simply field eqs. by letting

$$\dot{a} = \cosh f \ , \qquad \dot{\varphi} = \sinh f \ .$$

One gets

$$\dot{f} + e^{\,f} = 0 \ .$$

The general solution is

$$\dot{\varphi} = \frac{1}{2\,(\tau - \tau_0)} - \frac{1}{2}\,(\tau - \tau_0)\,,$$
$$\dot{a} = \frac{1}{2\,(\tau - \tau_0)} + \frac{1}{2}\,(\tau - \tau_0)\,.$$

The complete solution ($\tau_0 = 0$) is

$$ds^2 \;=\; e^{\frac{2a_0}{D-1}}\,|\tau|^{\frac{1}{D-1}}\,e^{\frac{\tau^2}{2(D-1)}}\,d\mathbf{x}\cdot d\mathbf{x} \;-\; e^{-\,2\,\varphi_0}\,|\tau|^{-\,1}\,e^{\frac{\tau^2}{2}}\,\left(\frac{d\tau}{M\beta}\right)^2 ,$$
$$e^{\varphi} \;=\; e^{\varphi_0}\,|\tau|^{\frac{1}{2}}\,e^{-\frac{\tau^2}{4}}\,.$$

(E.D. and J. Mourad, 2000).

Upshot: τ_0 defines the Big Bang, other integration constants fix φ and a at a later reference time.

$\rightarrow$ φ *can only emerge from the Big Bang climbing up the potential !*

- The motion will revert at a time τ^* ($\tau^* - \tau_0 = 1$), then φ will begin to slide down the potential.

• However for small γ, the dilaton should be able to also emerge from the Big Bang by going down (almost flat potential).

Indeed for $\gamma < 1$ the system does admit both kinds of solutions.

The first describes again a climbing scalar

$$\dot{\varphi} = \frac{1}{2}\ \left[\sqrt{\frac{1-\gamma}{1+\gamma}}\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\ -\ \sqrt{\frac{1+\gamma}{1-\gamma}}\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

$$\dot{a} = \frac{1}{2}\ \left[\sqrt{\frac{1-\gamma}{1+\gamma}}\ \coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\ +\ \sqrt{\frac{1+\gamma}{1-\gamma}}\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

This motion reverts at a time $\tau_\star$ such that

$$\tanh\left(\frac{\tau_\star}{2}\sqrt{1-\gamma^2}\right) = \sqrt{\frac{1-\gamma}{1+\gamma}}\ .$$

In contrast with the "critical" case $\gamma = 1$ now the dilaton approaches, again in the "parametric" time τ, the limiting speed

$$\dot{\varphi}_{\rm lim} = -\ \frac{\gamma}{\sqrt{1-\gamma^{\,2}}}\ .$$

This limiting speed diverges when γ approaches one, the critical value. The second type of solution, that becomes singular and thus disappears altogether in the critical case, for $\gamma < 1$ reads

$$\dot{\varphi} = \frac{1}{2}\ \left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

$$\dot{a} = \frac{1}{2}\ \left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) + \sqrt{\frac{1+\gamma}{1-\gamma}}\ \coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right] .$$

The dilaton now emerges from the Big Bang while going down the potential, at a speed larger than the limiting value, but decreases as a result of cosmological friction.

Mechanical analogy

- non-relativistic particle moving in a viscous medium under a constant force f.

$$m \, \dot{v} \; + \; b \, v \; = \; f$$

The solution is

$$v(t) \; = \; (v_0 - v_l) \; e^{-bt/m} \; + \; v_l \; ,$$

where $lim_{t \to \infty} \, v(t) \; = \; v_l \; = \; f/b$.

There are two branches of initial conditions :

$v_0 > v_l$ and $v_0 < v_l$.

The string coupling is

$$e^{\phi} \ = \left[sh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{\frac{\beta\sqrt{2}}{1+\gamma}} \left[ch\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{-\frac{\beta\sqrt{2}}{1-\gamma}}$$

for the climbing scalar and

$$e^{\phi} = \left[ch\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]^{\frac{\beta\sqrt{2}}{1+\gamma}} \left| sh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right|^{-\frac{\beta\sqrt{2}}{1-\gamma}}$$

for the descending scalar.

• The climbing solutions are perturbative near the big-bang, $g_s = e^{\Phi} \to 0$ for $\tau \to 0$.

• Large-τ behaviors of these solutions correspond to the attractor solution (Lucchin, Matarrese), which shows that for $\gamma < 1/\sqrt{D-1}$ we get power-like inflation.

• At the “critical” value $\gamma = 1$ the attractor disappears.

In the supercritical regime $\gamma > 1$ the solutions are

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{\gamma-1}{\gamma+1}}\ \cot\left(\frac{\tau}{2}\sqrt{\gamma^2-1}\right) - \sqrt{\frac{\gamma+1}{\gamma-1}}\ \tan\left(\frac{\tau}{2}\sqrt{\gamma^2-1}\right)\right]$$

$$\dot{a} = \frac{1}{2}\left[\sqrt{\frac{\gamma-1}{\gamma+1}}\ \cot\left(\frac{\tau}{2}\sqrt{\gamma^2-1}\right) + \sqrt{\frac{\gamma+1}{\gamma-1}}\ \tan\left(\frac{\tau}{2}\sqrt{\gamma^2-1}\right)\right]$$

where $\tau \in (0, \frac{\pi}{\sqrt{\gamma^2-1}})$. At the same time, the descending solution ceases to exist. Therefore, for $\gamma \geq 1$, close to the Big Bang singularity at $\tau = 0$ the dilaton is forced to climb up the potential.

The turning point $\tau^\star$ is determined by the equation

$$\tan\left(\frac{\tau^\star}{2}\sqrt{\gamma^2-1}\right) \; = \; \sqrt{\frac{\gamma-1}{\gamma+1}} \; ,$$

then the scalar goes down the potential, approaching asymptotically an infinite speed in τ. The string coupling for $\gamma > 1$ is

$$e^{\phi} = \left[\sin\left(\frac{\tau}{2}\sqrt{\gamma^2-1}\right)\right]^{\frac{\beta\sqrt{2}}{1+\gamma}} \left[\cos\left(\frac{\tau}{2}\sqrt{\gamma^2-1}\right)\right]^{\frac{\beta\sqrt{2}}{\gamma-1}} \; .$$

2. Connection to String Theory.

The climbing appears in various contexts :

• "Brane supersymmetry breaking" (Antoniadis,E.D.,Sagnotti,99) in the 10d model (Sugimoto, 99):

- gravitational (closed-string) sector is SUSY
- charged (open string) spectrum is non-SUSY (non-linear SUSY) (E.D.,J.Mourad,2000).

The scalar potential is induced at open string tree-level (disc). The 10d model corresponds precisely to the "critical" case $\gamma = 1$.

• KKLT moduli stabilisation : critical behavior in 4d.

3. Moduli stabilization, climbing and trapping.

Moduli stabilization: recent progress from string compactifications with fluxes.

Classical toy model : KKLT. Anticipate main results :

- the KKLT system has also the critical behavior with climbing scalar: instrumental for moduli trapping.

Starting point: 4d effective action described by

$$W \ = \ W_0 \ + \ a \ e^{-bT} \ , \qquad K \ = \ -3 \ \ln(T+\bar{T}) \ .$$

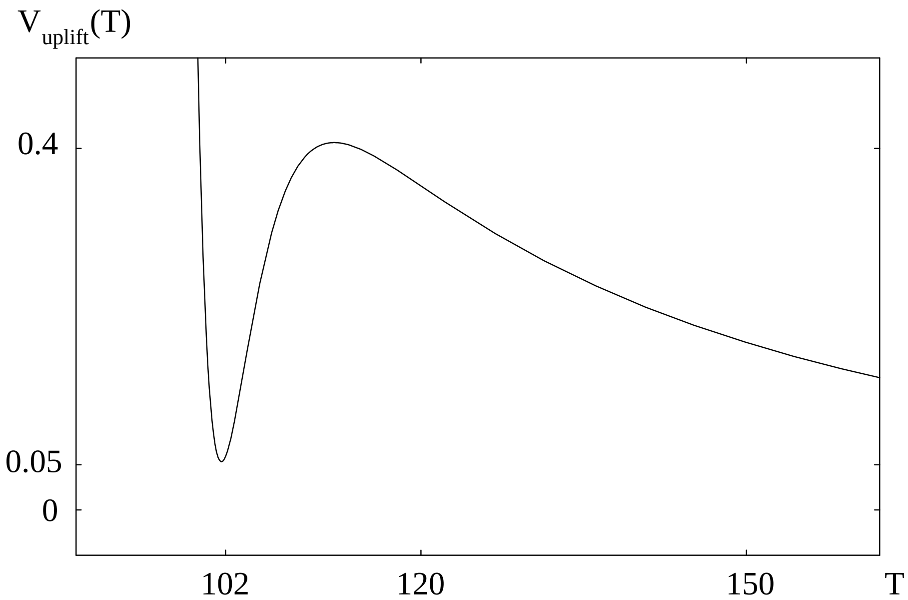
They determine the scalar potential

$$V_F \;=\; e^K \;\left[|D_T W|^2 - 3|W|^2\right] \;=$$
$$\frac{b}{(T+\bar{T})^2} \;\left\{2Re(a\,\bar{W}_0 e^{-bT}) \;+\; \frac{|a|^2}{3}\;[6+b(T+\bar{T})]\; e^{-b(T+\bar{T})}\;\right\}\;.$$

The KKLT potentials contain an "uplift" term,

$$V \;=\; V_F \;+\; \frac{c}{(T+\bar{T})^n}\;,$$

Its (logarithmic) slope corresponds precisely to the critical value for a climbing behavior in four dimensions for $n = 3$. This comes from an F-term uplift of the vacuum energy. In the asymptotic region $Re\; T \to \infty$, the potential is dominated by the uplift.



The complex field T defines the dilaton Φ_T - axion θ KKLT system

$$T \;=\; e^{\frac{\Phi_T}{\sqrt{3}}} \;+\; i\,\frac{\theta}{\sqrt{3}}\;,$$

where Φ_T is a canonically-normalized field.

The effective action is

$$S = \frac{1}{2k_4^2} \int d^4x \sqrt{g} \left[R - \frac{1}{2} \, (\partial \Phi_T)^2 - \frac{1}{2} \, e^{-\frac{2}{\sqrt{3}} \Phi_T} \, (\partial \theta)^2 - V(\Phi_T, \theta) \right] \, .$$

Perform the field redefinitions

$$\Phi_T = \frac{2}{\sqrt{3}} \, x \, , \quad \theta = \frac{2}{\sqrt{3}} \, y \, , \quad \tau = M \sqrt{\frac{3}{2}} \, t \, .$$

The scalar potential becomes

$$V = \frac{c}{8} \, e^{-2x} + \frac{b}{2} \, e^{-\frac{4x}{3} - b \, e^{\frac{2x}{3}}} \Big[Re(\, a\overline{W_0} \, \, e^{\frac{-2iby}{3}}) + \frac{|a|^2}{3} \, \left(3 + b \, e^{\frac{2x}{3}} \right) \, e^{-b \, e^{\frac{2x}{3}}} \Big] \, .$$

The uplift term corresponds precisely to the critical value $\gamma = 1$.

Further field redefinition ($\mu = 4/3$ for KKLT) :

$$\frac{dx}{d\tau} = r\,w \;, \qquad e^{-\,\mu x}\,\frac{dy}{d\tau} = r\sqrt{1-w^2}\;,$$

with $w \in [-1,1]$. By keeping only the uplift in the scalar potential, one finally gets the field eqs.

$$\frac{dr}{d\tau} + \; r\sqrt{1+r^2}\; - \gamma\,\left(1+r^2\right)\,w = 0\;,$$
$$\frac{dw}{d\tau} + (1-w^2)\,\left(\frac{\mu}{2}\,r - \frac{\gamma}{r}\right) = 0\,.$$

The first eq. is strikingly similar to the one-field case but now the parameter γ is now the dynamical quantity

$$\gamma_e(u) = \gamma\,w$$

that can take up any value in the interval $(-\gamma, \gamma)$.

In addition to the LM attractor

$$r_0 \ = \ \pm\frac{\gamma}{1-\gamma^2} \quad , \quad w_0 \ = \ \pm 1 \ ,$$

there are other attractors. Interesting one:

$$r_0 = \sqrt{\frac{2\,\gamma}{\mu}} \ , \ w_0 = \frac{1}{\sqrt{\gamma\,\left(\gamma + \frac{\mu}{2}\right)}} \ ,$$

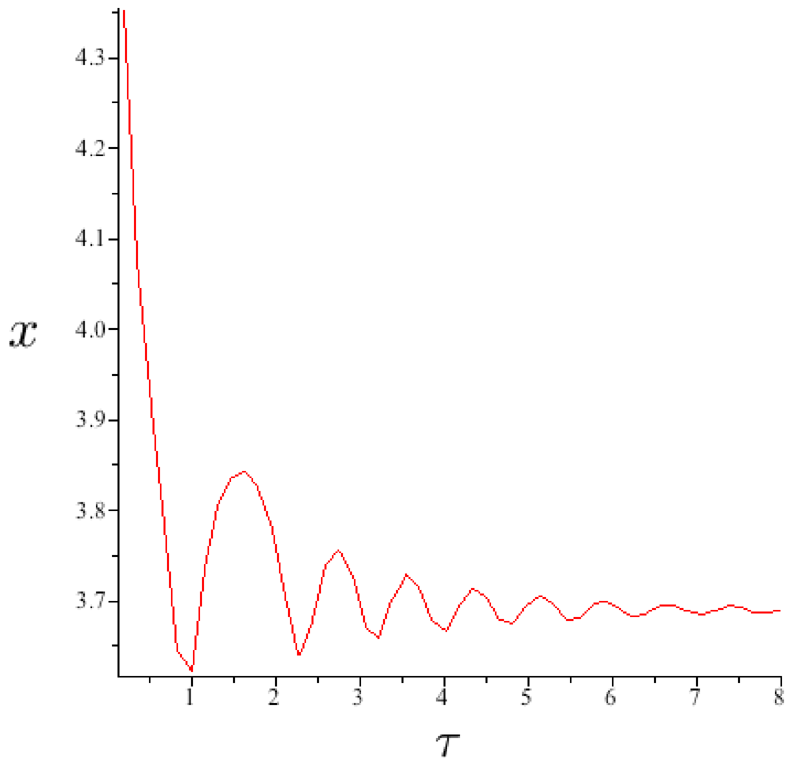
which exists provided

$$\gamma \ \geq \ \gamma_0 = \sqrt{1+\frac{\mu^2}{16}} - \frac{\mu}{4} \ ,$$

$\gamma_0 = 0.7207592197$ for $\mu = 4/3$. Hence, this attractor is available in the KKLT system, where

$$r_0 = \sqrt{\frac{3}{2}} \ , \ w_0 = \sqrt{\frac{3}{5}} \ .$$

Modulus Φ_T is trapped in the minimum even if it starts on the runaway tail close to big-bang.



Trapped solution in a KKLT potential, due to climbing.

4. Climbing and inflation ?

Accelerating universe in our gauge is obtained if

$$\mathcal{I} = \frac{d^2 A}{dt^2} + \frac{dA}{dt}\left(\frac{dA}{dt} - \frac{dB}{dt}\right) > 0$$

One-field case: using field eqs. we find

$$\mathcal{I} = \left(\frac{M\beta}{D-1}\right)^2 \left[1 - (D-2)\,\dot{\varphi}^2\right]$$

so the “slow-roll” condition is

$$\dot{\varphi}^2 < \frac{1}{D-2} \,.$$

KKLT case: manipulating field eqs. we find

$$\mathcal{I} = \frac{1}{3} \left[1 - 6\,\dot{A}^2\right]$$

Inflation asks then for

$$\dot{A} < \frac{1}{\sqrt{6}} \; .$$

The corresponding bound for the speed

$$r = \sqrt{\dot{x}^2 + e^{-\mu x} \; \dot{y}^2} < \sqrt{\frac{1}{2}}$$

lies below the values accessible to the exact KKLT attractor, since for the actual KKLT system, with $\gamma = 1$ and $\mu = \frac{4}{3}$, $r_0 = \sqrt{\frac{3}{2}}$.

Slow-roll inflation only occurs for $\mu > 4$.

The simplest realistic possibility is to have two exponential potentials

$$V \;=\; \alpha_1 \; e^{2\gamma_1 \varphi} \;+\; \alpha_2 \; e^{2\gamma_2 \varphi} \;,$$

with $\gamma_1 \geq 1$ (climbing) and $\gamma_2 < 1/\sqrt{D-1}$ (slow-roll). This is realized in the 10d Sugimoto model, which has a stable non-BPS D3 brane (E.D.,J. Mourad,99). The effective action contains the D3 tension term

$$S_{D3} = -\alpha_2 \int d^4x \sqrt{-g}\; e^{-\phi} \;\rightarrow\; -\alpha_2 \int d^4x \sqrt{-g} \text{ (Einstein)}$$

where $\alpha_2 = \sqrt{2} T_3$ is the tension of the non-BPS D3-brane. We compactify the theory to 4d while keeping track only of the overall breathing mode

$$g^{(10)}_{IJ} \ = \ e^{\sigma} \delta_{IJ} \ , \quad g^{(10)}_{\mu\nu} \ = \ e^{-3\sigma} g^{(4)}_{\mu\nu}$$

Defining the two real fields

$$s \ = \ e^{3\sigma} e^{\frac{\phi}{2}} \ = \ e^{\phi_S} \qquad , \qquad t \ = \ e^{\sigma} e^{-\frac{\phi}{2}} \ = \ e^{\frac{1}{\sqrt{3}} \ \Phi_T} \ ,$$

the resulting scalar potential takes the form

$$V \ = \ \alpha_1 \ e^{-\sqrt{3}\Phi_T} + \ \alpha_2 \ e^{-\frac{3\phi_s}{2} - \frac{\sqrt{3}\Phi_T}{2}} \ .$$

Assume S is stabilized by fluxes. First term (D9 tadpole) gives the critical exponent $\gamma_1 = 1$. Second term (D3 brane) has $\gamma_2 = 1/2$: it is a slow-roll potential.

Conclusions

• Climbing near big-bang with critical exponent seems generic in string theory.

• The phenomenon plays probably an important role in KKLT moduli trapping .

• Rich set of attractor solutions in KKLT model, combined with climbing.

• Climbing can be followed by inflation.

• Central question: are there possible imprints leftover in the sky in the spectrum of cosmological perturbations ?