Non-$f(R)$ terms in asymptotically safe gravity

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DB, P. F. Machado, F. Saueressig - Mod. Phys. Lett. A 24 (2009) 2233-2241 [arXiv:0901.2984],
Nucl. Phys. B 824 (2010) 168-191 [0902.4630]

same + K. Groh - ongoing, see Frank's talk

Outline

- Motivations
- Setup
- Results

The perturbative story

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{g}(2\Lambda - R)$$

- 1-loop (pure) quantum gravity ['t Hooft, Veltman ($\int E = \Lambda = 0$); Christensen, Duff]

$$\Gamma_{(1)\text{div}} = \frac{1}{180(4\pi)^2}\frac{1}{\epsilon}\int d^4x\sqrt{g}(212E - 2088\Lambda^2 + X_{\mu\nu}\frac{\delta S}{\delta g_{\mu\nu}})$$

(where E is the integrand of the Gauss-Bonnett theorem)

on-shell: $$= \frac{1}{180(4\pi)^2}\frac{1}{\epsilon}\int d^4x\sqrt{g}(212E - 2088\Lambda^2)$$

$\Rightarrow$ divergences can be removed by field redefinition,
plus renormalization of Λ and topological term
(which can be added to S_{EH} without affecting e.o.m.)

The perturbative story

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{g}(2\Lambda - R)$$

- 2-loop [Goroff, Sagnotti; van de Ven]

$$\Gamma_{(2)\text{div}} = \frac{209}{2880(4\pi)^2}\frac{1}{\epsilon}\int d^4x\sqrt{g}(R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta} + Y_{\mu\nu}\frac{\delta S}{\delta g_{\mu\nu}})$$

$\Rightarrow$ need non-trivial counterterm, not of the type found in the bare action

$\Rightarrow$ the (in-)famous non-renormalizability of gravity!

The perturbative story

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{g}(2\Lambda - R) + \frac{1}{2}\int d^4x\sqrt{g}\; g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi$$

- 1-loop quantum gravity coupled to matter ['t Hooft, Veltman; Deser, van Nieuwenhuizen; Kamenshchik, Karmazin et al.; ...]

$$\Delta\Gamma^{\rm div} = \frac{1}{8\pi^2\epsilon}\int d^4x\sqrt{g}\Big[\frac{31}{18}R^2 + \frac{213}{180}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{463}{20}R\Lambda + \frac{463}{10}\Lambda^2 + Z_{\mu\nu}\frac{\delta S}{\delta g_{\mu\nu}}\Big]$$

$\Rightarrow$ non-renormalizability already at 1-loop

The non-perturbative story

- Include all the terms needed for renormalizability:
generically possible $\Rightarrow$ obtain an Effective Field Theory

  – "Effective" $=$ valid only up to a certain scale $E_{\text{new physics}}$

- Weinberg's proposal of Asymptotic Safety:

  – if a non-trivial fixed point of the RG exists, then such theory is valid up to arbitrarily high energy

  – if there are a finite number of relevant directions at such a fixed point, then the theory is as predictive as a renormalizable one

  $\Rightarrow$ non-triviality requires non-perturbative approximation schemes:
  – lattice methods (dynamical triangulations, Regge calculus)
  – truncations of Functional RG Equation (FRGE)

Truncations

- The FRGE for gravity has been studied within several truncations, mainly of the polynomial $f(R)$ type:

$$\Gamma_k^{\rm gr} = \sum_n u_n(k) \int d^4x \sqrt{g} R^n \ , \qquad 0 \leq n \leq 8(10)$$

  - a non-trivial fixed point with 3 relevant directions was always found

    [Reuter,Lauscher; Litim; ...; Codello,Percacci,Rahmede; Machado,Saueressig; Bonanno,Contillo,Percacci]

But what about the other invariants which play an essential role for the non-renormalizability ?

other reasons for looking at other invariants:
– complete the derivative expansion
– understand about degrees of freedom and unitarity

Taming C^2 (plus matter)

The FRGE for gravity in a nutshell

- Background field method: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

  – Obtain a diffeomorphism invariant effective action

  – Low/high modes separation is achieved in terms of eigenvalues of the (generalized) Laplacian for the background metric

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- Take a truncation ansatz for $\Gamma_k[\Phi]$ of the form

$$\Gamma_k[g, \bar{g}, ...] = \Gamma_k^{\text{gr}}[g] + S^{\text{gf}}[g, \bar{g}] + S^{\text{gh}}[g, \bar{g}, \text{ghosts}] + \Gamma^{\text{matter}}[g, \phi]$$

– $\Gamma_k^{\text{gr}} = \sum_n^N u_n(k)\, I_n[g]$: the gravitational part; $I_n[g]$ =geometric invariant.
– S^{gf}, S^{gh} : background-gauge-fixing term, and the corresponding ghost action

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- Plug ansatz into the FRGE (Wetterich equation)

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- Heat Kernel expansion

  – The traces on the RHS are then performed via the heat kernel expansion, which is an expansion in geometric invariants.

  – Invariants on LHS and RHS are matched to extract beta functions.

The fourth-order truncation for gravity

- Our ansatz for the gravitational part is

$$\Gamma_k^{\rm gr}[g] = \int d^4x\sqrt{g}\left[\frac{1}{16\pi G}(2\Lambda - R) + \frac{1}{2\lambda}C^2 - \frac{\omega}{3\lambda}R^2 + \frac{\theta}{\lambda}E\right]$$

  - $C^2 \equiv C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$ is the square of the Weyl tensor
  - $E \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}\,R^{\mu\nu\rho\sigma}$ is the Gauss-Bonnet term
  - A complete set of invariants at this order would also include the boundary term $\nabla^2 R$, but we assume no boundary

- Matter:

$$\Gamma^{\rm matter}[g,\phi] = \tfrac{1}{2}\int d^4x\sqrt{g}\; g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi$$

The choice of a background

- Beta functions are background independent
- Exploit background independence to choose a class of backgrounds
  - simple enough to obtain only minimal Laplacians in the Hessian,
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- Exploit background independence to choose a class of backgrounds
  - simple enough to obtain only minimal Laplacians in the Hessian,
  - generic enough to distinguish different invariants
- Usual choice: maximally symmetric (standard sphere)

$\rightarrow$ only possible to distinguish different powers of the Ricci scalar.

In particular $C^2 = 0$ and $E \sim R^2 \Rightarrow$ not good for our truncation

The choice of a background

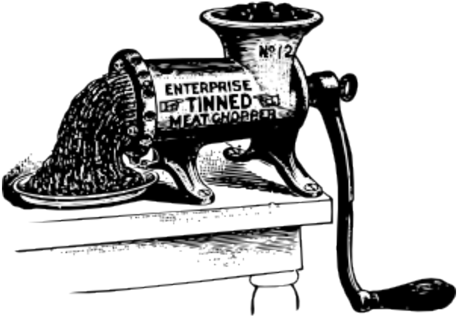
- Beta functions are background independent
- Exploit background independence to choose a class of backgrounds
  - simple enough to obtain only minimal Laplacians in the Hessian,
  - generic enough to distinguish different invariants
- A generic Einstein background satisfying

$$\bar{R}_{\mu\nu} = \frac{\bar{R}}{4}\,\bar{g}_{\mu\nu}$$

is sufficient to partially meet the two criteria.

– We indeed found that the operators appearing in the functional traces can be casted in the form of Lichnerowicz Laplacians, and that the heat kernel methods can be applied smoothly.

Crank the Laplacian-mincer...



The non-Gaussian fixed point

- Our background choice allows us to determine the non-perturbative β-functions of the linear combinations

$$u_2 = -\frac{\omega}{3\lambda} + \frac{\theta}{6\lambda}\,, \qquad u_3 = \frac{1}{2\lambda} + \frac{\theta}{\lambda}$$

(along with $u_0 = \Lambda/(8\pi G)$, $u_1 = -1/(16\pi G)$).

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- Switch to the dimensionless couplings $g_i = k^{-d_i} u_i$ and consider the β-functions $\partial_t g_i = \beta_i$.

- We find that the β-functions for the couplings contained in our truncation indeed give rise to a NGFP very close to the ones obtained within other truncations

$$g_0^* = 0.00442\,, \qquad g_1^* = -0.0101\,,$$
$$g_2^* = 0.00754\,, \qquad g_3^* = -0.00501\,.$$

Stability

- Linearized RG flow around the NGFP: $\partial_t g_i = \mathbf{B}_{ij}(g_j - g_j^*)$
  stability matrix: $\mathbf{B}_{ij} \equiv \partial_j \beta_i|_*$

- Stability coefficients (negative eigenvalues of $\mathbf{B}$):

$$\theta_0 = 2.51\,, \qquad \theta_1 = 1.69\,, \qquad \theta_2 = 8.40\,, \qquad \theta_3 = -2.11\,.$$

- The negative stability coefficient θ_3 thereby indicates that the corresponding eigendirection ($V_3 = [0.07,\, -0.21,\, 0.97,\, -0.09]$) is IR-attractive.

  $\Rightarrow$ 3 relevant directions

- Coefficients are all real, no more spirals... a sign of the physical importance of these non-$f(R)$ terms

Conclusions and outlook

- We have studied an "$R^2 + C^2$" (plus matter) truncation using an Einstein background
- We found a NGFP with very similar properties to what observed in different truncations
  - The value of $(G\Lambda)^*$ is very close to that found before
  - The number of relevant directions is still three
- The asymptotic safety results obtained with the FRGE seem to be pretty robust against the "menace of the counterterms"
- Nonetheless they have a non-trivial effect on stability coefficients, so we should keep an eye on them

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Outlook:

- Studying larger truncations in the polynomial $f(R)$ class has become just a matter of computing power.

  How much can we generalize this statement for generic local truncations?

  For further work in this direction see Frank's talk at 19:00!