Nonequilibrium transport of fermions through an Anderson quantum dot

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Outline:
Introduction
2PI formalism
s-channel resummation
Effective coupling
Results

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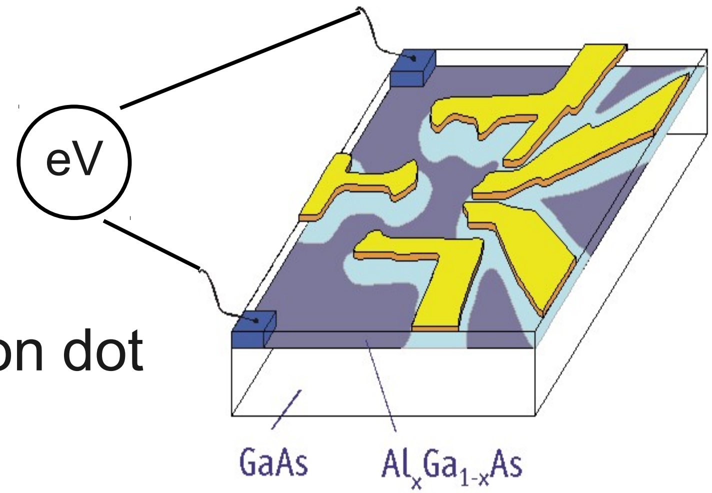
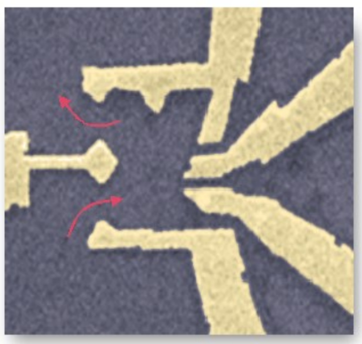
What is a Quantum Dot?

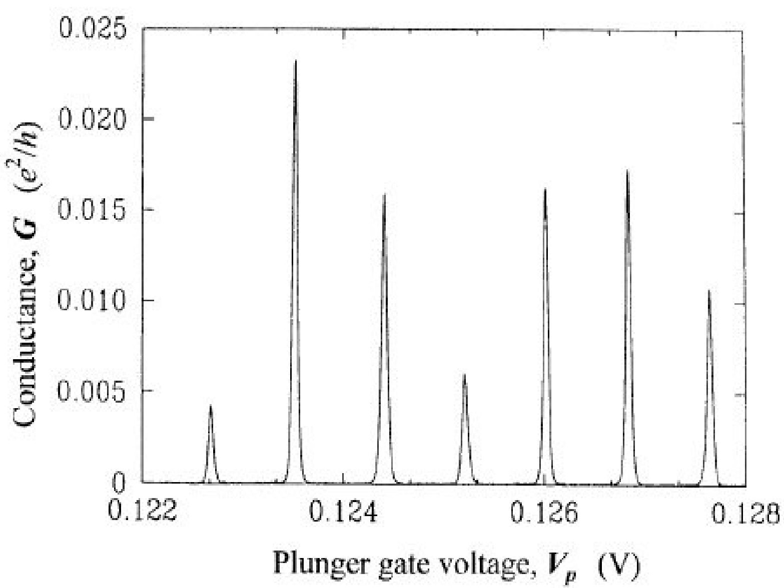


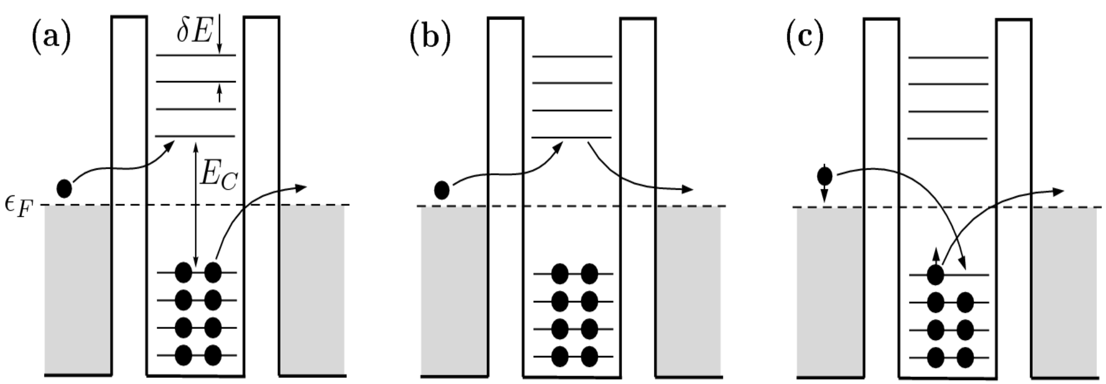


Voltage applied to leads
Gate voltage shifts energy levels on dot
→ G=?

Single electron phenomena

Coulomb blockade
Activationless transport: co-tunneling
Kondo effect





Anderson model

Isolated one level system with coupling to leads
Coulomb interaction on the dot

Non-interacting leads

$$H = H_{dot} + H_{leads} + H_{tunnel}$$

$$H_{dot} = \sum_{\sigma} E_{0\sigma} d_{\sigma}^{+} d_{\sigma} + U n_{\uparrow} n_{\downarrow}$$

$$H_{leads} = \sum_{kp\sigma} \epsilon_{kp\sigma} c_{kp\sigma}^{+} c_{kp\sigma}$$

$$H_{tunnel} = \sum_{kp\sigma} t_p c_{kp\sigma}^{+} d_{\sigma} + t_p^{*} d_{\sigma}^{+} c_{kp\sigma}$$

k: continuous spectrum
p: left or right
σ: up or down

2PI formalism (a.k.a. Kadanoff Beym equations)

Start from Dyson equation: $G^{-1}=G_0^{-1}-\Sigma \quad \rightarrow \quad \delta(x-y)=G_0^{-1}*G-\Sigma*G$

Using a Schwinger-Keldysh contour, decompose

$$G(x,y)=\langle T\, d(x) d^+(y)\rangle = F(x,y)-\frac{i}{2} sign_C(x-y)\rho(x,y)$$

Statistical function: $F(x,y)=\frac{1}{2}[d(x),d^+(y)]$

Spectral function: $\rho(x,y)=i\{d(x),d^+(y)\}$

Self energy also decomposed: $\Sigma(x,y)=\Sigma_F(x,y)-\frac{i}{2} sign_C(x-y)\Sigma_\rho(x,y)$

The Dyson eq. is explicity solvable using memory integrals

$$(i\partial_t-\epsilon)F(t,t')=\int_0^t dz\,\Sigma_\rho(t,z)F(z,t')-\int_0^{t'} dz\,\Sigma_F(t,z)\rho(z,t')$$

$$(i\partial_t-\epsilon)\rho(t,t')=\int_{t'}^t dz\,\Sigma_\rho(t,z)\rho(z,t')$$

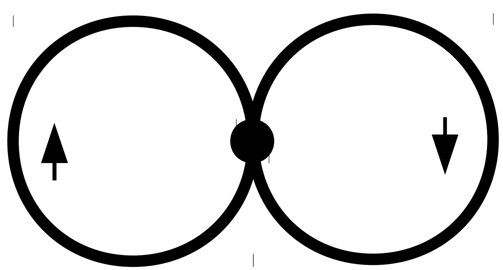
Self energy from a 2PI functional:

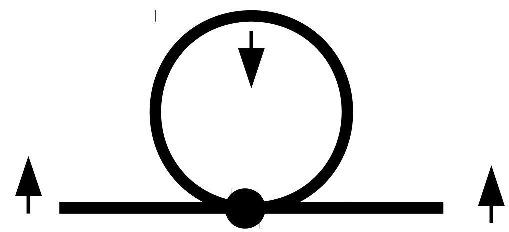
Cornwall, Jackiw, Tombulis (1974)

$$\Sigma(x,y) = -i\,\frac{\delta\Gamma[G]}{\delta G(y,x)}$$

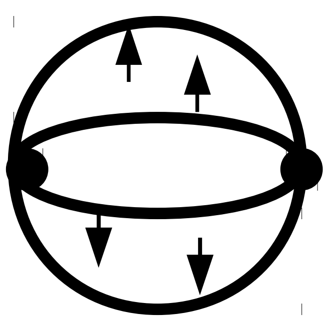
Self energy needs to be 1PI $\quad\rightarrow\quad \Gamma$ is 2 Particle Irreducible

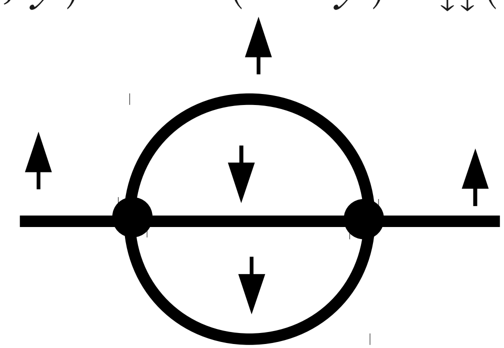
Γ is truncated





$$\Sigma^{(1)}_{\uparrow\uparrow}(x,y) = U\,\delta(x-y)\,G_{\downarrow\downarrow}(x,x)$$





$$\Sigma^{(2)}_{\uparrow\uparrow}(x,y) = U^2\,G_{\uparrow\uparrow}(x,y)\,G_{\downarrow\downarrow}(x,y)\,G_{\downarrow\downarrow}(y,x)$$

Exact solution without leads

Without leads: 2-fermion system
Exactly solvable by diagonalisation

$$H_{dot} = \sum_{\sigma} E_{0\sigma} d_{\sigma}^{+} d_{\sigma} + U n_{\uparrow} n_{\downarrow}$$

Four dimensional
Hilbert space

$$d_{\uparrow} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$d_{\downarrow} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Using initial density matrix corresponding to same initial conditions as 2PI solution

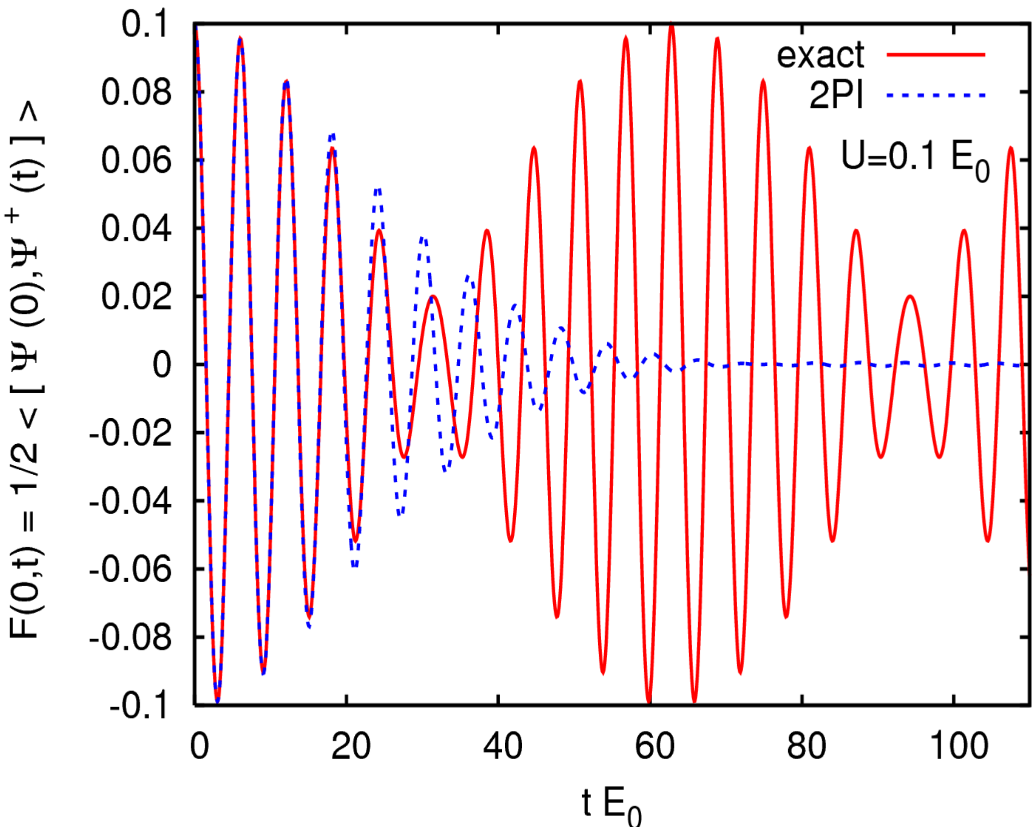
$$Tr(\rho d_{\downarrow} d_{\uparrow}^{+}) = 0, \quad Tr(\rho d_{\downarrow} d_{\uparrow}) = 0,$$
$$Tr(\rho d_{\uparrow}) = 0, \quad Tr(\rho d_{\uparrow} d_{\downarrow}^{+} d_{\downarrow}) = 0,$$
$$Tr(\rho d_{\downarrow}^{+} d_{\downarrow}^{+} d_{\uparrow} d_{\uparrow})_{connected} = 0$$
$$Tr(\rho d_{\uparrow}^{+} d_{\uparrow}) = n_{\uparrow}$$

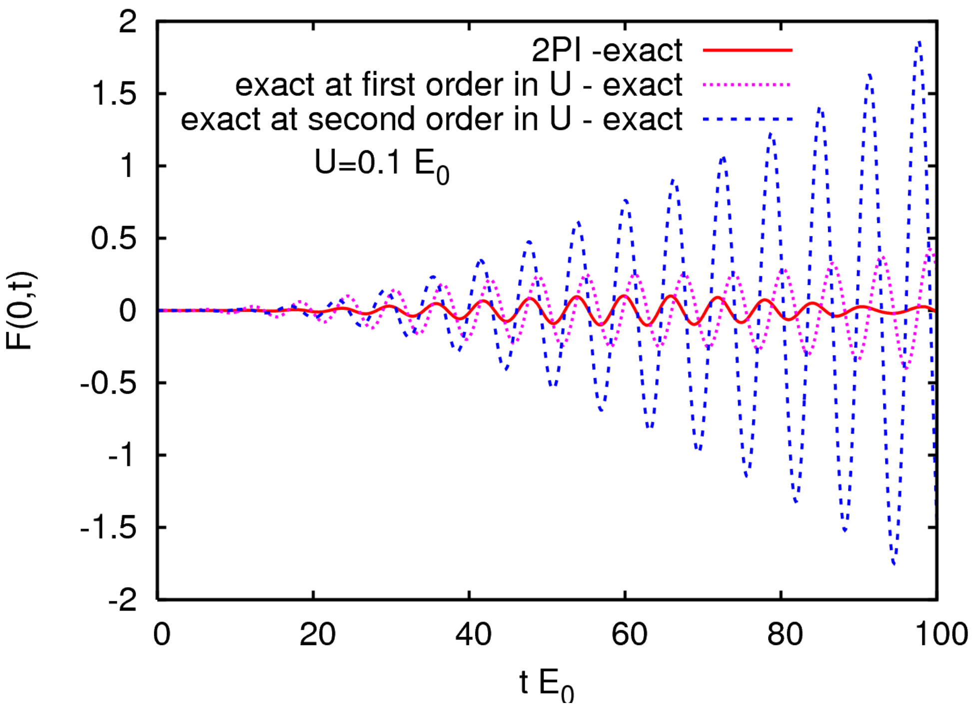
$$\rho = \begin{pmatrix} (1-n_{\uparrow})(1-d_{\downarrow}) & 0 & 0 & 0 \\ 0 & (1-n_{\uparrow}) n_{\downarrow} & 0 & 0 \\ 0 & 0 & n_{\uparrow}(1-n_{\downarrow}) & 0 \\ 0 & 0 & 0 & n_{\uparrow} n_{\downarrow} \end{pmatrix}$$

Comparison with exact results

Without leads: 2-fermion system
Exactly solvable by diagonalisation

$$H_{dot} = \sum_{\sigma} E_{0\sigma} d_{\sigma}^{+} d_{\sigma} + U n_{\uparrow} n_{\downarrow}$$





2PI is non-secular
Works good for damped systems
Coupling to leads introduces damping

Perturbation theory is secular
Breaks down eventually

Leads in the 2PI formalism

$$H_{leads} = \sum_{kp\sigma} \epsilon_{kp\sigma} c^{+}_{kp\sigma} c_{kp\sigma}$$

$$H_{tunnel} = \sum_{kp\sigma} t_p c^{+}_{kp\sigma} d_\sigma + t^{*}_{p} d^{+}_{\sigma} c_{kp\sigma}$$

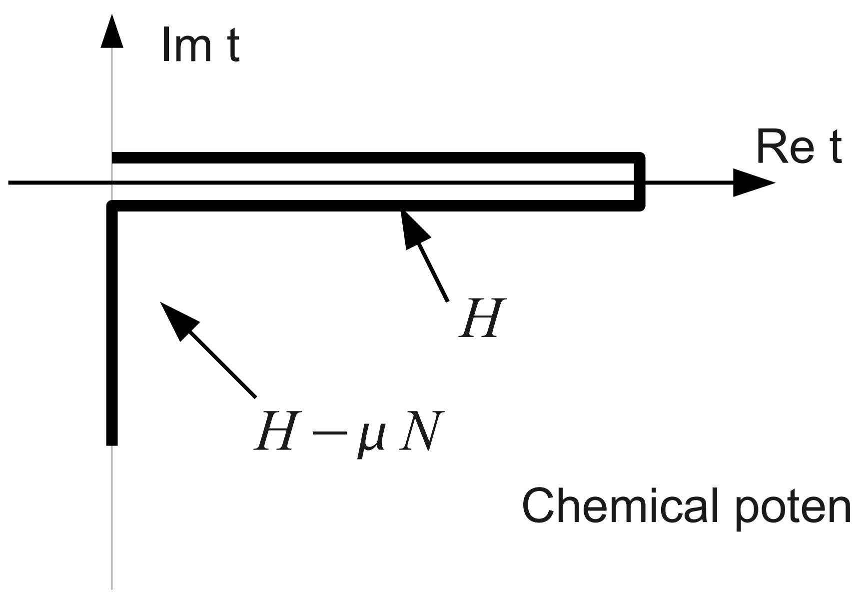
Non interacting leads can be integrated

Voltage ⟶ chemical potential

Using path integral formalism: $\int D\Psi D\bar{\Psi} e^{\bar{\Psi} M \Psi + \bar{\Psi} J + \bar{J} \Psi} = N e^{-\bar{J} M^{-1} J}$

Need to invert M(t,t') $M(t,t') = \delta(t,t')(i\partial_t - \epsilon_0)$

On a Schwinger Keldysh countour



Chemical potential needed!

Leads in the 2PI formalism

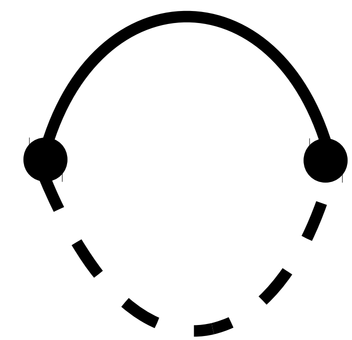
$$H_{leads} = \sum_{kp\sigma} \epsilon_{kp\sigma} c^{+}_{kp\sigma} c_{kp\sigma}$$

$$H_{tunnel} = \sum_{kp\sigma} t_p c^{+}_{kp\sigma} d_{\sigma} + t^{*}_{p} d^{+}_{\sigma} c_{kp\sigma}$$

Other point of view:

Tunneling term is a 2-vertex

Contribution to 2PI functional:



Contribution to fermion self energy:

$$\Sigma_{lead}(x,y) = |t_p|^2 G_{lead}(x,y)$$

Where $G_{lead}(x,y)$ is to be calculated in grand canonical ensemble

Agrees with the first calculation

Leads in the 2PI formalism

$$H_{leads} = \sum_{kp\sigma} \epsilon_{kp\sigma} c^+_{kp\sigma} c_{kp\sigma}$$
$$H_{tunnel} = \sum_{kp\sigma} t_p c^+_{kp\sigma} d_\sigma + t^*_p d^+_\sigma c_{kp\sigma}$$

Contribution of one mode to self-energy of the dot electron:

$$\Sigma_{Fp}(x,y) = -|t_p|^2 \left(\frac{1}{2} - f(\epsilon-\mu_p)\right) e^{i\epsilon(x-y)}$$
$$\Sigma_{\rho p}(x,y) = -i|t_p|^2 e^{i\epsilon(x-y)}$$

Using infinite band limit with constant level density $\lim_{D\to\infty} \int_{-D}^{D} d\epsilon\, \nu(\epsilon)$

Dimensionful parameter: $\Gamma_p = 2\pi |t_p|^2 \nu$

Zero temperature contribution integrated analytically

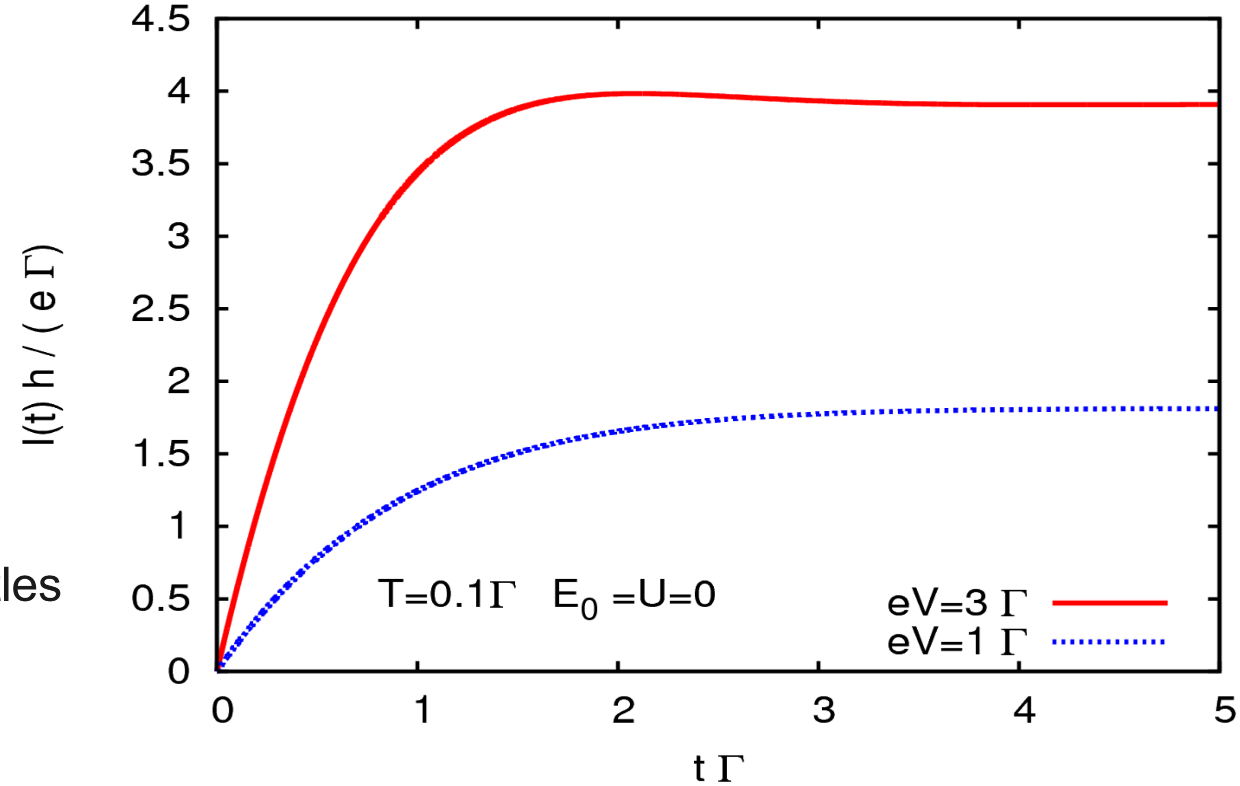
$$\Sigma_{Fp}(x,y) = i\frac{\Gamma_p}{2\pi} \wp \frac{e^{-i\mu(x-y)}}{x-y} +$$ finite temperature contribution

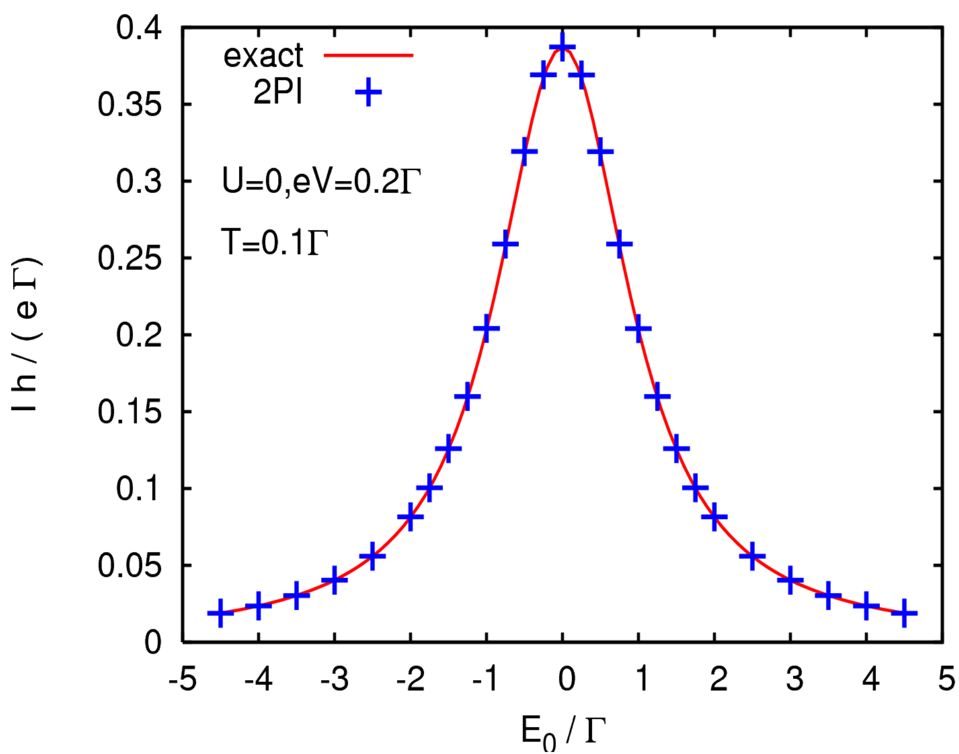
$$\Sigma_{\rho p}(x,y) = -i\frac{\Gamma_p}{2}\delta(x-y)$$

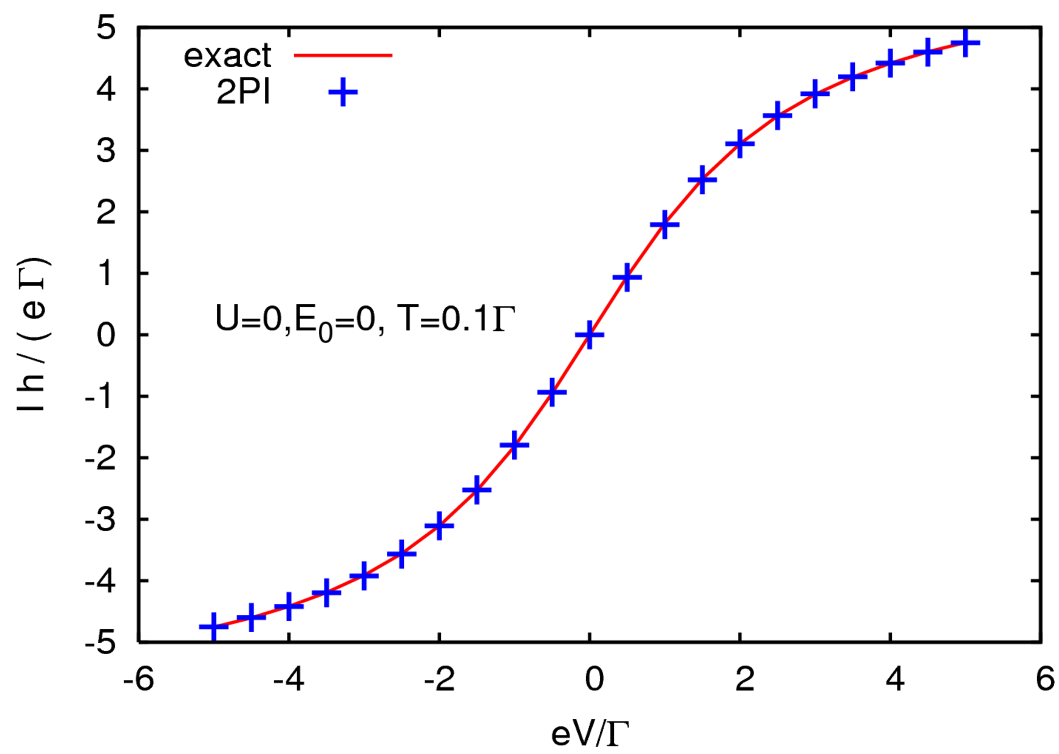
U=0 case

Without interaction dot degrees of freedom only quadratic also integrated analytically

After short transient the current settles

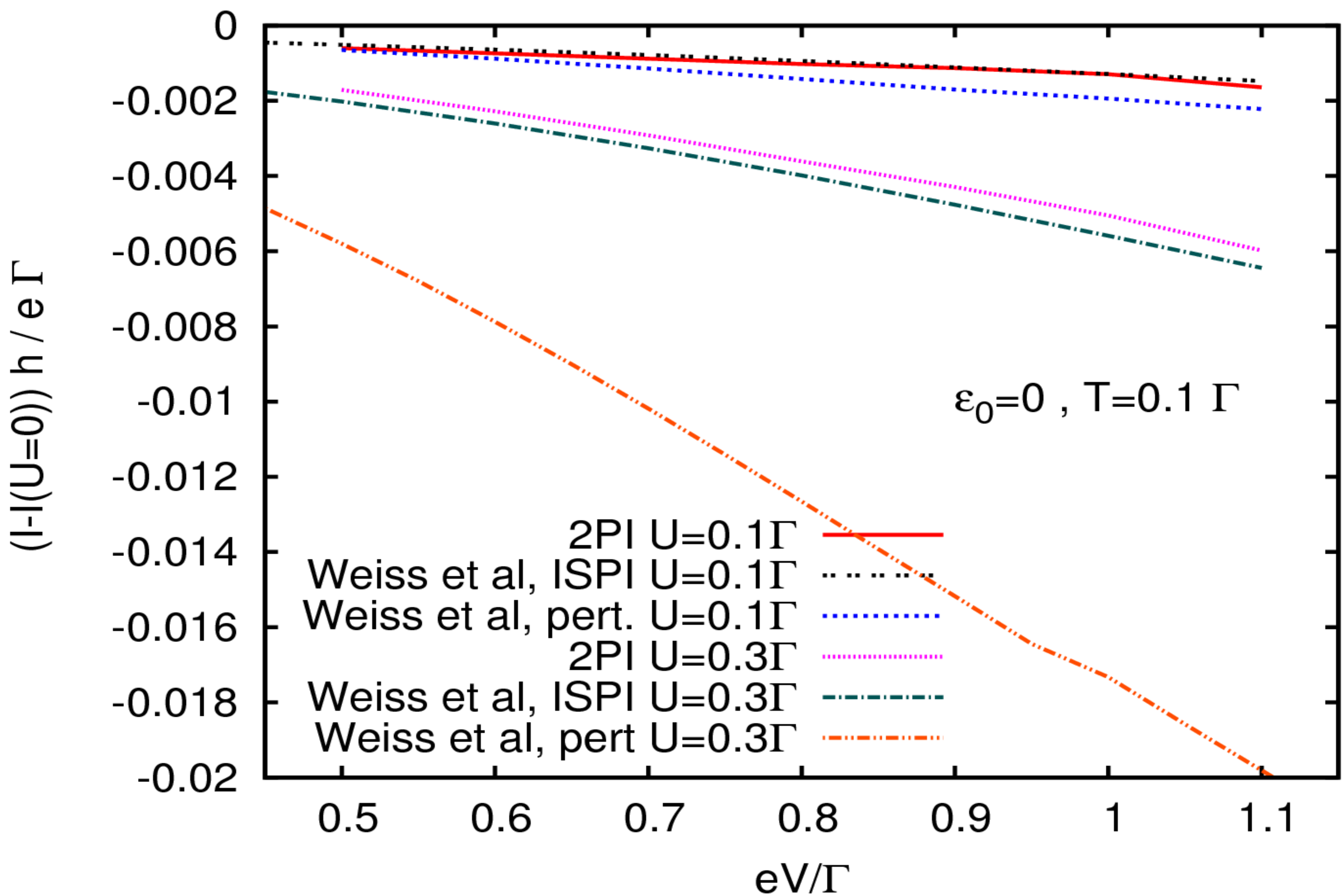






The effect of interactions on the current

Coulomb blockade suppresses current



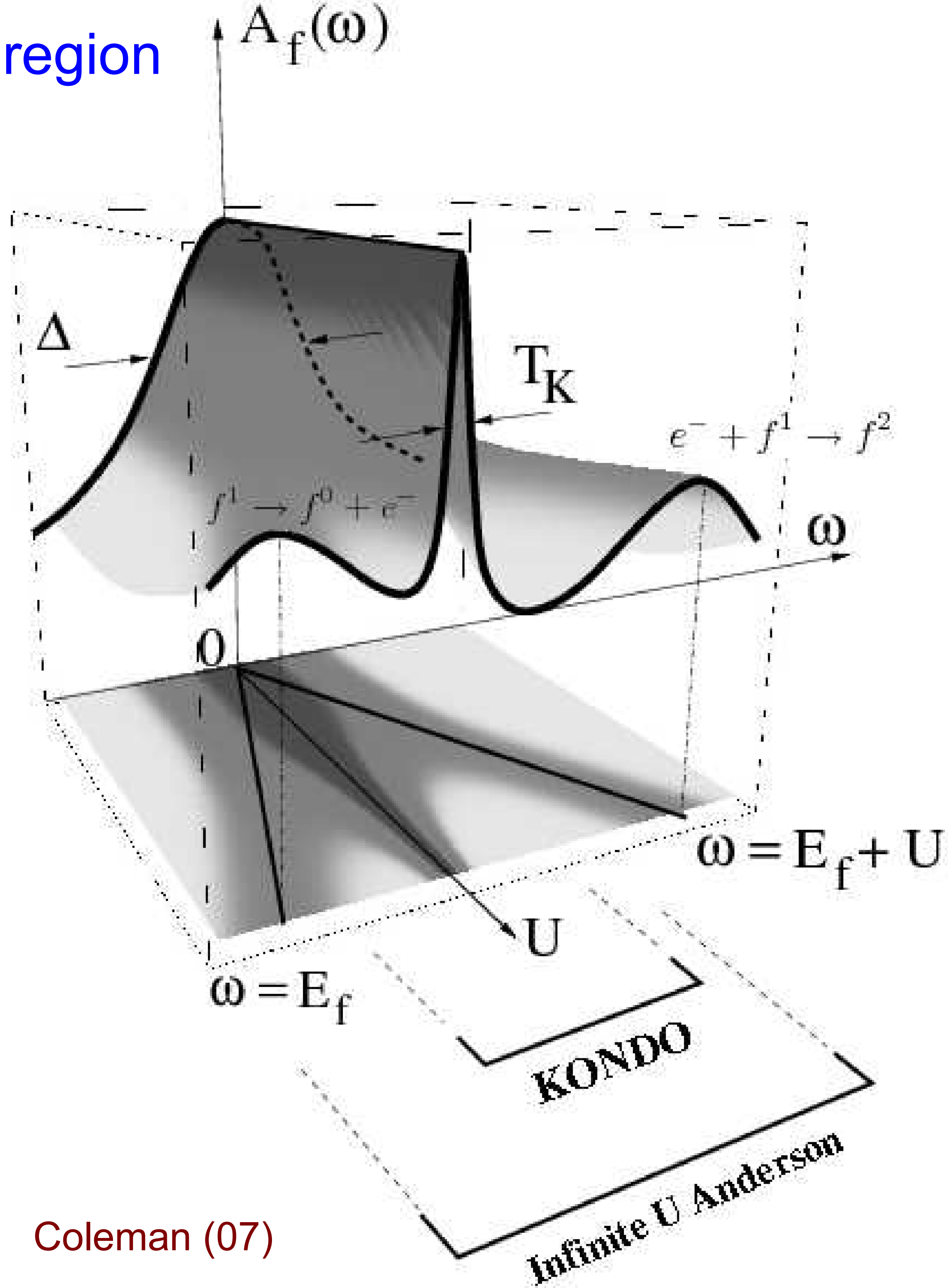
Spectral function at Kondo region

At big coupling,
there's a sharp peak

↓

Effective coupling
should be small

$$T_k \sim e^{-U/8\Gamma}$$

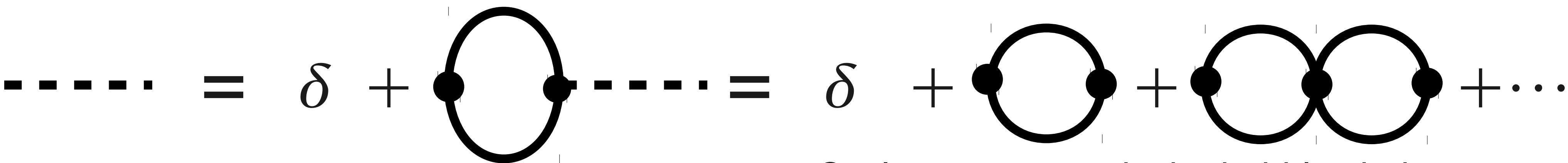


Coleman (07)

s-channel resummation

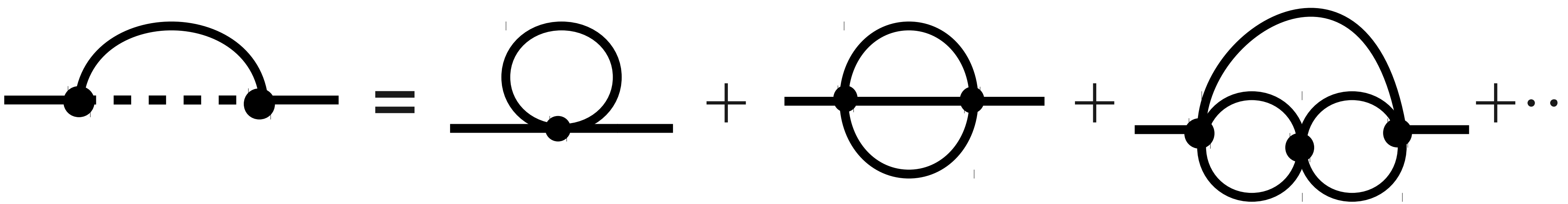
Inspired by 1/N resummation
In this case N=2

Introduce a scalar field

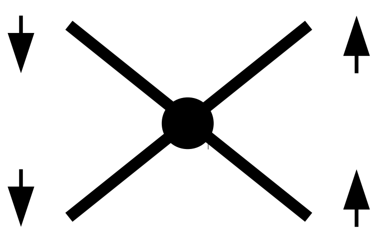


Scalar propagator is the bubble chain sum

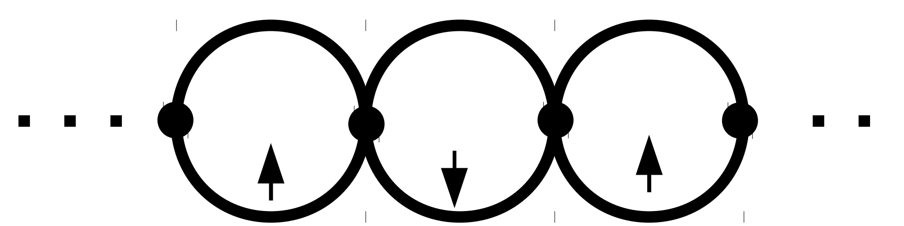
Using this in the fermion selfenergy



Actually, it's more complicated, because the femion vertex is:



One needs to resum bubble chains with alternating spins



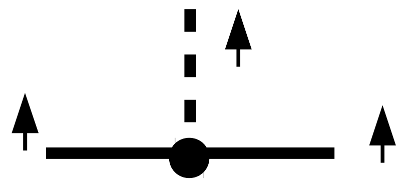
Alternating resummation

Elegant way to do this is: using Hubbard-Stratonovich transformation
Using two scalar fields

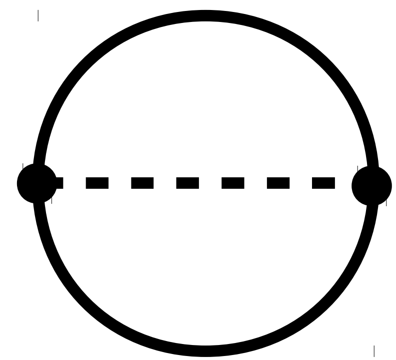
Originally: $$S_{dot} = \int_C dt\, d_\sigma^+ (i\partial_t - E_{0\sigma}) d_\sigma - U d_\uparrow^+ d_\uparrow d_\downarrow^+ d_\downarrow$$

HS transformation: $$U d_\uparrow^+ d_\uparrow d_\downarrow^+ d_\downarrow \quad \rightarrow \quad \frac{1}{U}\chi_\uparrow \chi_\downarrow + d_\uparrow^+ d_\uparrow \chi_\uparrow + d_\downarrow^+ d_\downarrow \chi_\downarrow$$

Three point vertex:



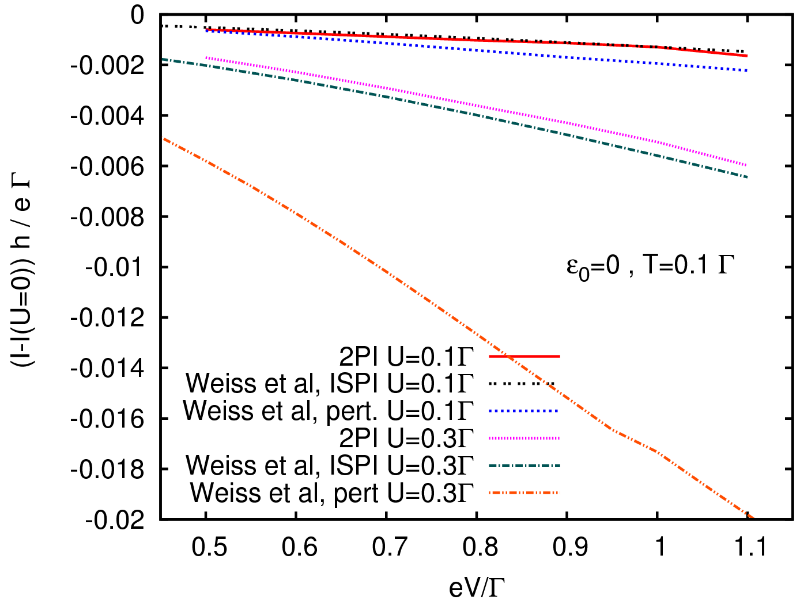
Two 2PI diagrams at the lowest order with $spin = \uparrow, \downarrow$



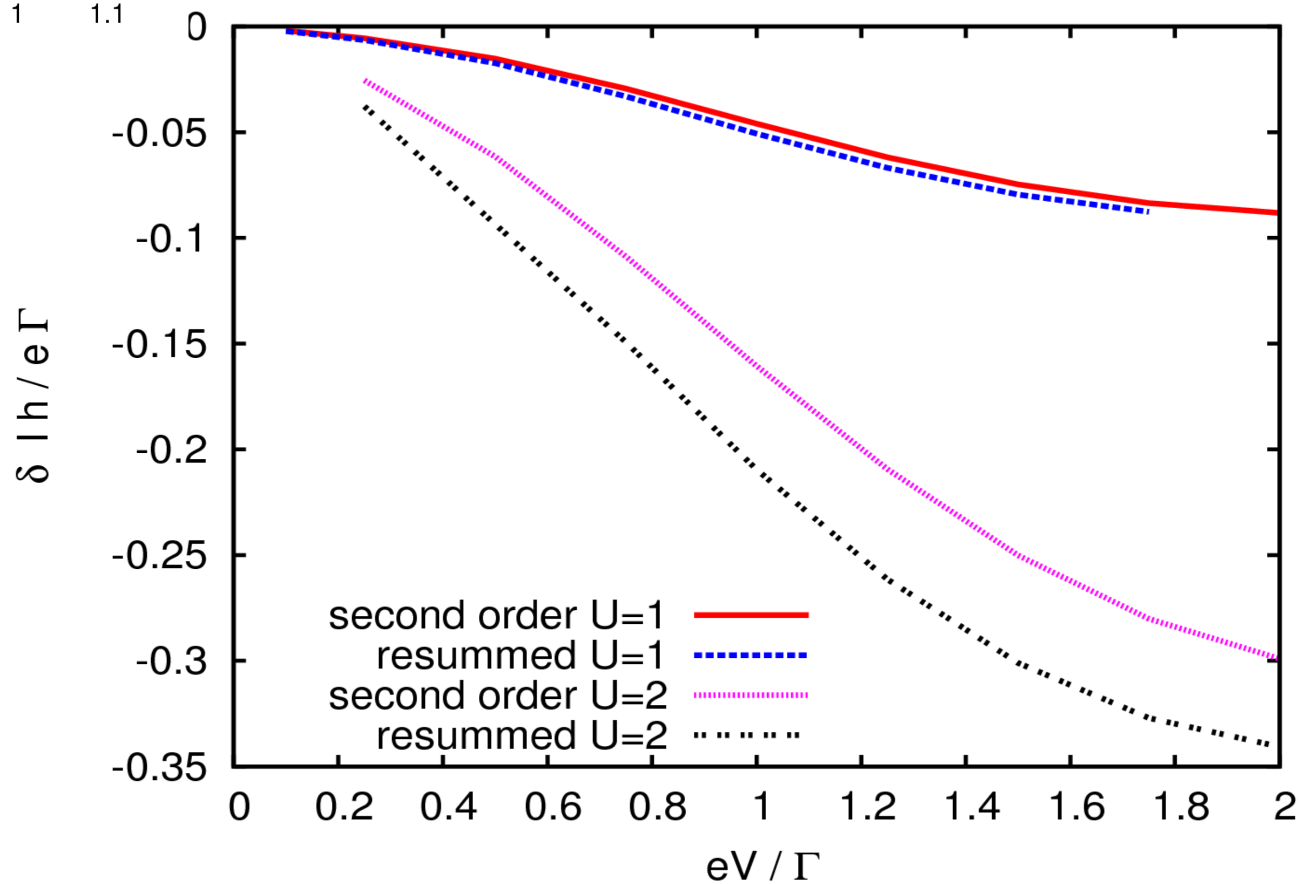
Self energies:

Scalar EOM resums bubble chains with alternating spins

The effect of interactions on the current



At small coupling resummation
Makes no difference



Effective Coupling

Symmetric case $G_{\uparrow}(x,y)=G_{\downarrow}(x,y)$ $\quad G_{\chi 11}(x,y)=G_{\chi 22}(x,y)$

Time translation invariant state (equilibrium) $G(x,y)=G(x-y)$

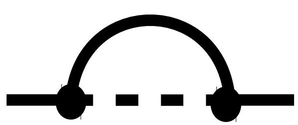
… some algebraic manipulation...

$$G_{\chi,diag}(\omega)=\lambda_{eff}(\omega)U^2\Pi(\omega)$$

$$\lambda_{eff}(\omega)=\frac{1-U^2|\Pi_R(\omega)|^2}{|1-U^2\Pi_R^2(\omega)|^2}$$

$$\Pi_R(t)=\Theta(t)\Pi_\rho(t)$$

Contribution to self energy:



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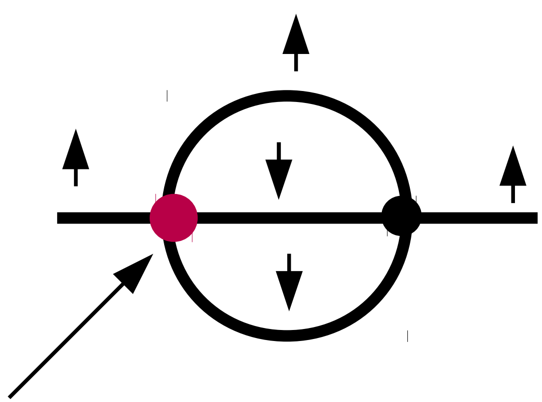


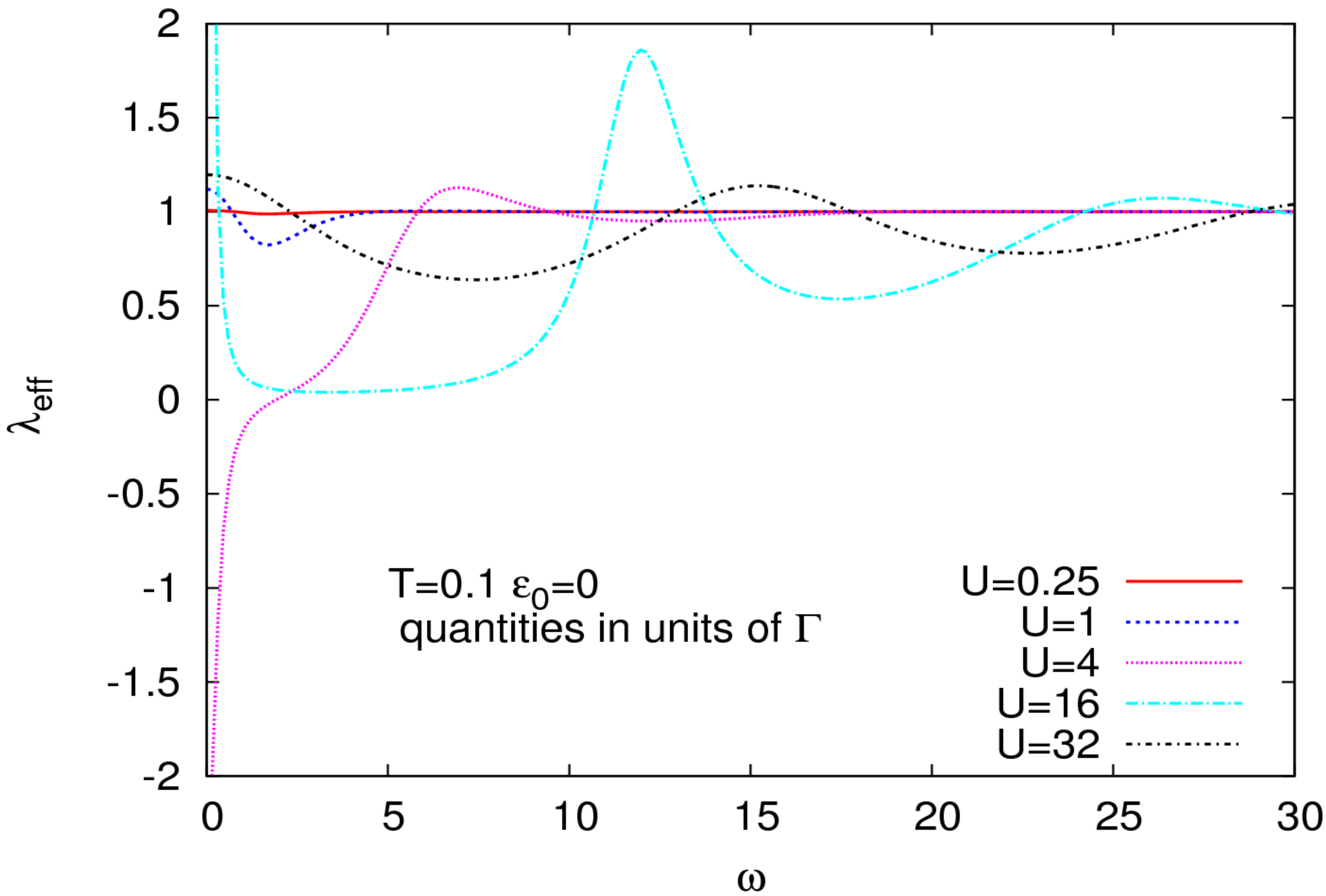
$\lambda_{eff}(\omega)$

Without resummation: sunset diagram

$$\lambda_{eff}=1$$

Effective Coupling

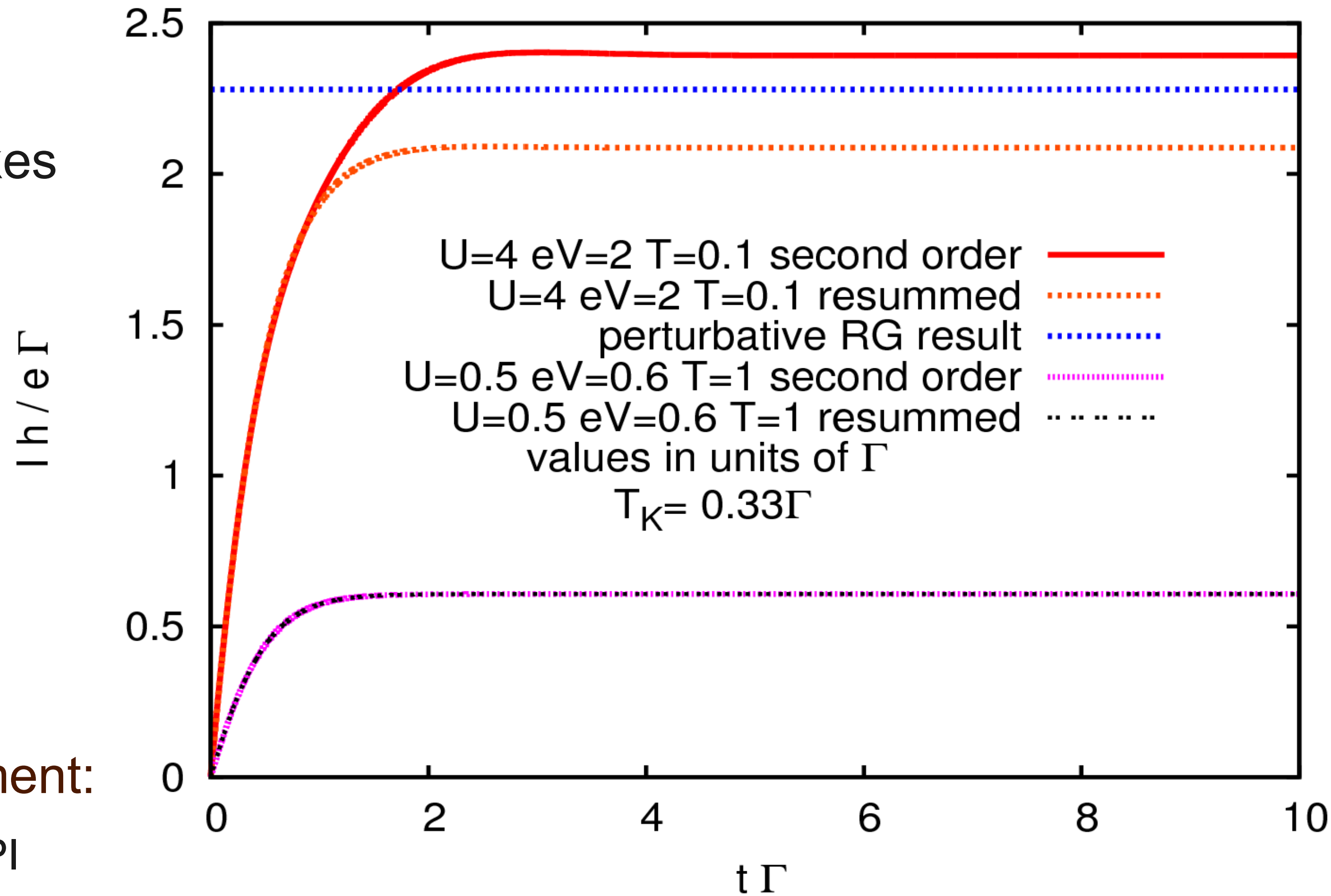
$$\lambda_{eff}(\omega)=\frac{1-U^2|\Pi_R(\omega)|^2}{|1-U^2\Pi_R^2(\omega)|^2}$$



Region of Kondo Physics

$eV \gg T_K$ Perturbative RG results are available Kaminski et. al (2000)

Resummation makes big difference for Kondo region



Ways of improvement:

Higher order 2PI

Non-equilibrium RG: next truncation would be to use 4 point function

Gasenzer, Pawlowski (2008)

Summary

•2PI formalism for Anderson model

•S channel resummation

•Time evolution of the current

•Checking with analytic and perturbative results

•Possible extensions: Higher order 2PI, non-eq RG