prospects for asymptotically safe gravity

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based on work with:

J Brinckmann, K Falls, S Folkerts, E Gerwick, G Hiller
K Nikolakopoulos, J Pawlowski, T Plehn, C Rahmede

motivation

- **standard model of particle physics**

  theory of the strong, weak, and electro-magnetic interactions

  very successful up to $\sim \mathcal{O}(100)$ GeV

  LHC to explore physics of the electroweak scale, Higgs particle

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- **what's up with gravity?**

  classical:

  Einstein's theory $G_N = 6.7 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

  classical action

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N} \int \sqrt{\det g}(-R(g_{\mu\nu}) + 2\Lambda)$$

  valid on length scales $\sim 10^{-2} - 10^{28}\,\mathrm{cm}$

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  quantum:

Planck length	$\ell_{\rm Pl} = \left(\frac{\hbar G_N}{c^3}\right)^{1/2} \approx 10^{-33}\,{\rm cm}$
Planck mass	$M_{\rm Pl} \approx 10^{19}{\rm GeV}$
Planck time	$t_{\rm Pl} \approx 10^{-44}\,{\rm s}$
Planck temperature	$T_{\rm Pl} \approx 10^{32}\,{\rm K}$

  expect modifications at energy scales $E \approx M_{\rm Pl}$

motivation

- **standard model of particle physics**

  theory of the strong, weak, and electro-magnetic interactions

  very successful up to $\sim \mathcal{O}(100)$ GeV

  LHC to explore physics of the electroweak scale, Higgs particle

- **what's up with gravity?**

  presently, no unified understanding of all fundamental forces

  Planck mass $M_{\rm Pl} \approx 10^{19}{\rm GeV} \gg M_{\rm EW}$

perturbation theory

- **structure of UV divergences**

N-loop Feynman diagram $\sim \int dp\, p^{A-[G]N}$

$[G] > 0 :$ superrenormalisable

$[G] = 0 :$ renormalisable

$[G] < 0 :$ dangerous interactions

gravity: $[g_{\mu\nu}] = 0\,, \quad [\mathrm{Ricci}] = 2\,, \quad [G_N] = 2 - d$

effective expansion parameter: $G_N\, p^2 \sim \frac{p^2}{M_{\mathrm{Pl}}^2}$

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- **perturbative non-renormalisability**

  gravity with matter interactions

  pure gravity (Goroff-Sagnotti term)

perturbation theory

- **effective theory for gravity** (Donoghue '94)

  quantum corrections computable for energies $E^2/M_{\rm Pl}^2 \ll 1$

  knowledge of UV completion not required

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- **higher derivative gravity I** (Stelle '77)

  R^2 gravity perturbatively renormalisable

  unitarity issues at high energies

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- **higher derivative gravity II** (Gomis, Weinberg '96)

  all higher derivative operators

  gravity 'weakly' perturbatively renormalisable

  no unitarity issues at high energies

high energy behaviour

high energy behaviour

- **asymptotic freedom**

  YM theory

high energy behaviour

- **asymptotic freedom**

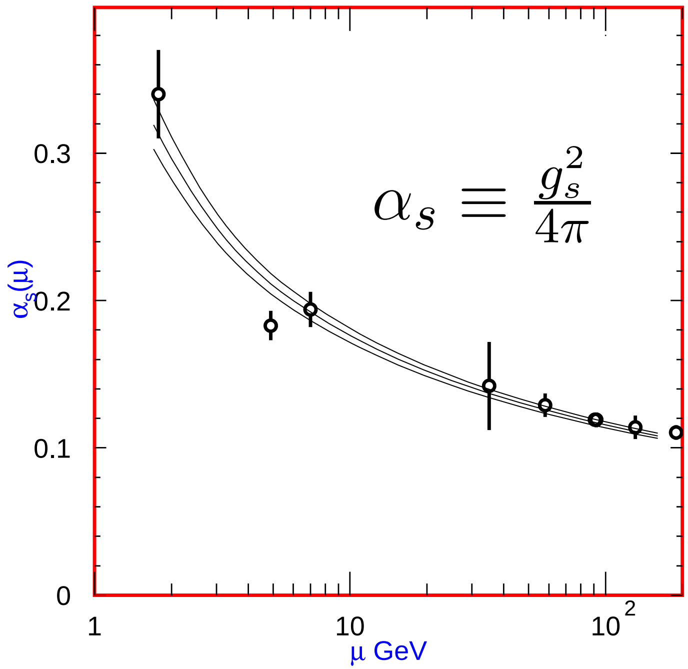
  YM theory

  running coupling

  $$\frac{dg_s}{d\ln\mu} = -\frac{7g_s^3}{16\pi^2}$$

  trivial UV fixed point

  $$g_s = 0$$



high energy behaviour

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  YM theory

- **asymptotic safety** (Weinberg '79)

  non-trivial UV fixed point for gravity

  well-defined continuum limit

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    critical trajectory

        stable, marginal, unstable directions

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  YM theory

- **asymptotic safety** (Weinberg '79)

  non-trivial UV fixed point for gravity

  well-defined continuum limit

  critical trajectory

  stable, marginal, unstable directions

  predictive power

  finite number of unstable directions

asymptotic safety

- **RG scaling of gravitational coupling** (DL '06,Niedermaier '06)

dimensionless coupling $g(\mu) = Z_N(\mu)^{-1} \cdot G_N \cdot \mu^{D-2}$

anomalous dimension $\eta_N = -\frac{\mathrm{d} \ln Z_N}{\mathrm{d} \ln \mu}$

RG running $\frac{\mathrm{d} g}{\mathrm{d} \ln \mu} = (D - 2 + \eta_N)\, g$

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- **fixed points**

  Gaussian: $g = 0$ **classical general relativity**

  non-Gaussian: $\eta_N = 2 - D$ **strong quantum effects**

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- **fixed points**

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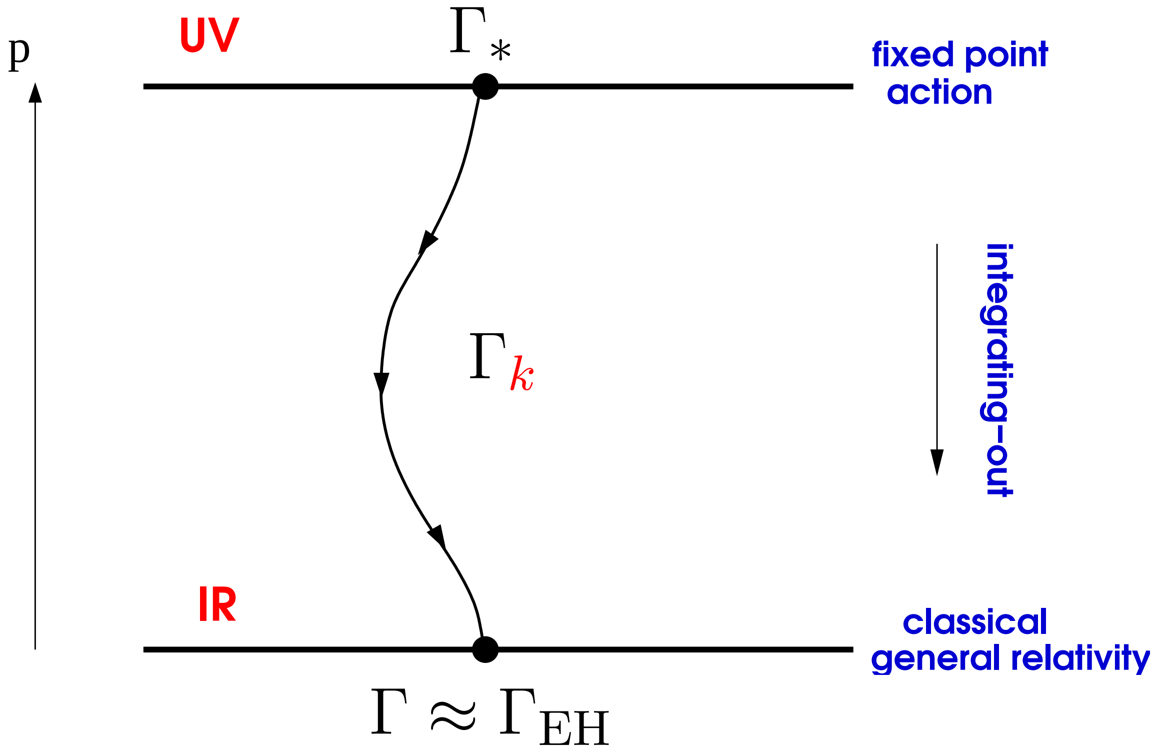
  non-Gaussian: $\eta_N = 2 - D$ **strong quantum effects**

  UV fixed point implies **weakly coupled gravity** at high energies

$$\mu \to \infty : \qquad G(\mu) \to g_* \mu^{2-D} \ll G_N$$

renormalisation group

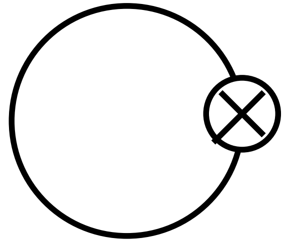
- **for quantum gravity:** **“bottom-up”**



renormalisation group

- **Callan-Symanzik equation** (Symanzik ’70)

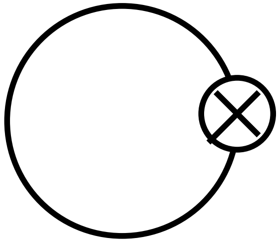
$$k\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + k^2\right)^{-1} k\frac{\mathrm{d}k^2}{\mathrm{d}k}\right]_{\mathrm{ren.}} = \frac{1}{2}$$



renormalisation group

- **functional RG equation** (Wetterich '93)

$$k\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right] \quad = \frac{1}{2}$$



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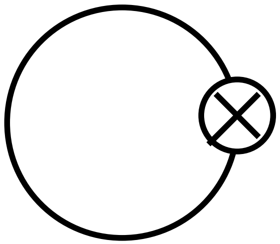
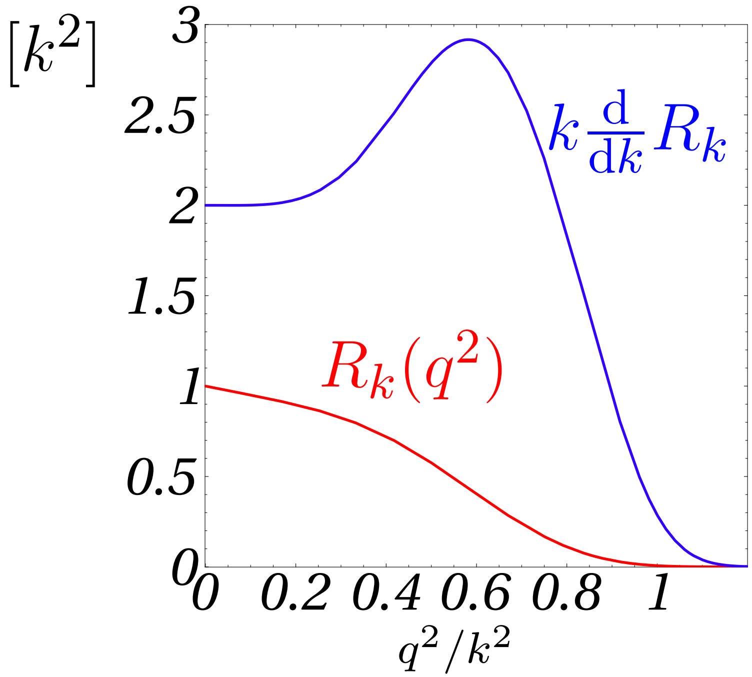


- **IR momentum cutoff**



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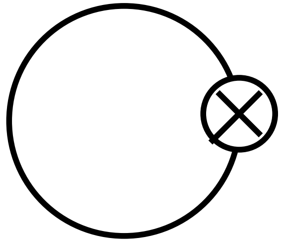

- **definition of the theory**

  finite initial (boundary) condition at $k = \Lambda$: Γ_Λ ,
  and finite flow equation $k\partial_k\Gamma_k$, regulator function R_k ,
  altogether:

$$\Gamma = \Gamma_\Lambda \ + \ \frac{1}{2}\int_\Lambda^0 \mathrm{d}k\,\partial_k\Gamma_k[\Gamma_k^{(2)}; R_k]$$

renormalisation group

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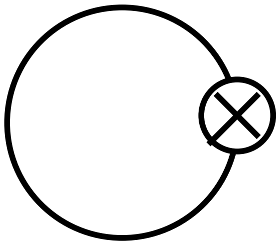

- **renormalisability**

  if $\Gamma_{\Lambda\to\infty} = \Gamma_*$ exists, Γ_* qualifies as **fundamental theory.**

  - perturbatively renormalisable theories: $\Gamma_* = S_{\mathrm{cl}}$ (e.g. QCD)
  - non-perturbatively renormalisable: $\Gamma_* =$ **non-trivial**
  - non-renormalisable: $\Gamma_{\Lambda\to\infty}$ **does not exist**

renormalisation group

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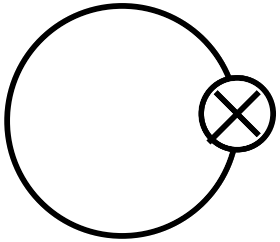

- **symmetries**

  global vs local

  if regulator respects symmetry: ok

  if not: **(modified) Ward identities** ensure that the physical theory $\Gamma_{k=0}$ respects the symmetry

renormalisation group

- **for quantum gravity** (Reuter '96)

$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma_k[g_{\mu\nu};\bar{g}_{\mu\nu}] = \frac{1}{2}\,\mathrm{Tr}\,\left[\left(\Gamma_k^{(2)}[g_{\mu\nu};\bar{g}_{\mu\nu}] + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right]$$

renormalisation group

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- **effective action**

$$\Gamma_k = \frac{1}{16\pi G_k}\int\sqrt{g}\,(-R + 2\Lambda_k + \cdots) + S_{\mathrm{matter},k} + S_{\mathrm{gf},k} + S_{\mathrm{ghosts},k}$$

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- **running couplings**

  projection of $k\partial_k\Gamma_k$ onto $\sqrt{g},\ \sqrt{g}R,\ \sqrt{g}R^2, \cdots$

- **optimisation** (DL '00, '01, '02, Pawlowski '05)

  choice of regulator function R_k

  stability $\leftrightarrow$ convergence $\leftrightarrow$ control of approximations

Yang-Mills + gravity

- **does asymptotic freedom persist?**

  1-loop / effective theory

Robinson, WIlczek ('05)

Pietrykowski ('06)

Toms ('07)

Ebert, Plefka, Rodigast ('08)

result: asymptotic freedom persists

$$\beta_{\mathsf{YM}}\Big|_{\mathrm{grav}} = -\frac{6\,I}{\pi}\, g_{\mathsf{YM}}\, G_N\, E^2 \leq 0$$

Yang-Mills + gravity

- **background field flow** S. Folkerts, DL, JM. Pawlowski (in preparation)

  ansatz

$$\Gamma_k = \int \sqrt{g}\left[\frac{Z_{N,k}}{16\pi G_N}\left(-R(g_{\mu\nu}) + 2\bar{\Lambda}_k\right) + \frac{Z_{A,k}}{4g^2} F^a_{\mu\nu} F_a^{\mu\nu}\right]$$

$$R_k[\bar{\phi}] = \Gamma_k^{(2)}[\bar{\phi}]\, r[\bar{\phi}]$$

$$r^{gg} = r^{gg}(-\Delta_{\bar{g}}) \quad r^{\bar{\eta}\eta} = -r^{\eta\bar{\eta}} = r^{\bar{\eta}\eta}(-\Delta_{\bar{g}})$$

$$r^{AA} = r^{AA}(-\Delta_{\bar{g}}(A) \quad r^{\bar{C}C} = -r^{C\bar{C}} = r^{\bar{C}C}(-\Delta_{\bar{g}}(A))$$

Yang-Mills + gravity

- **background field flow** S. Folkerts, DL, JM. Pawlowski (in preparation)

  flow

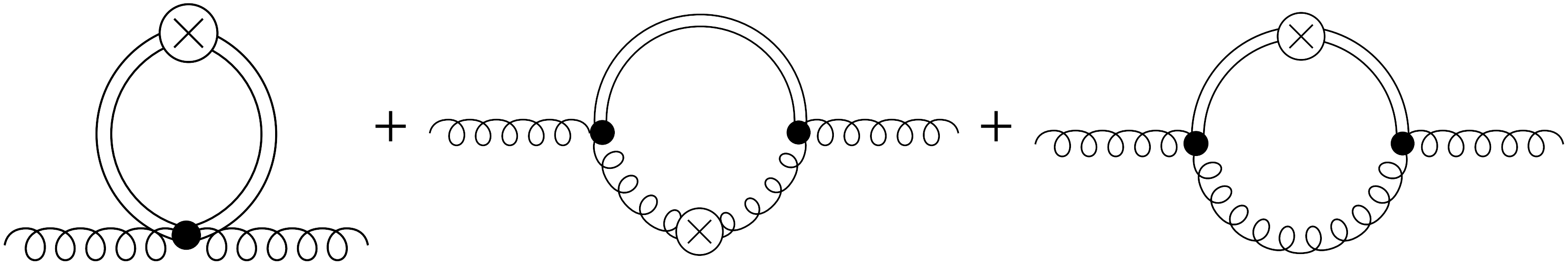
$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \frac{1}{1+r[\phi]} \partial_t r[\phi] + \mathrm{Tr}\, \frac{\partial_t \Gamma_k^{(2)}[\phi,\phi]}{\Gamma_k^{(2)}[\phi,\phi]} \frac{r[\phi]}{1+r[\phi]}$$

  result: **no graviton contribution at one-loop**

$$\beta_g\big|_{1-\mathrm{loop}} = \beta_{g,\mathrm{YM}}\big|_{1-\mathrm{loop}}$$

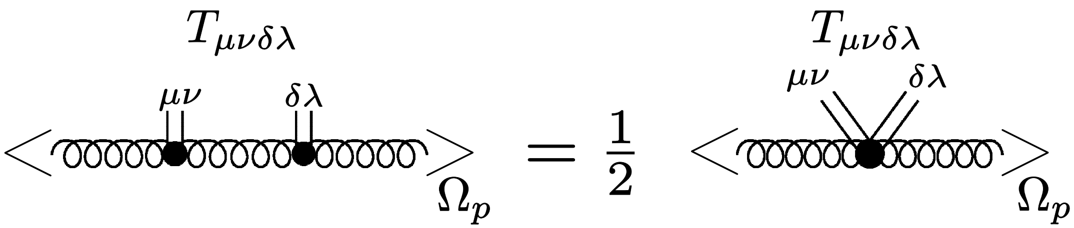
Yang-Mills + gravity

- **flat background**



Yang-Mills + gravity

- **kinematical identity**



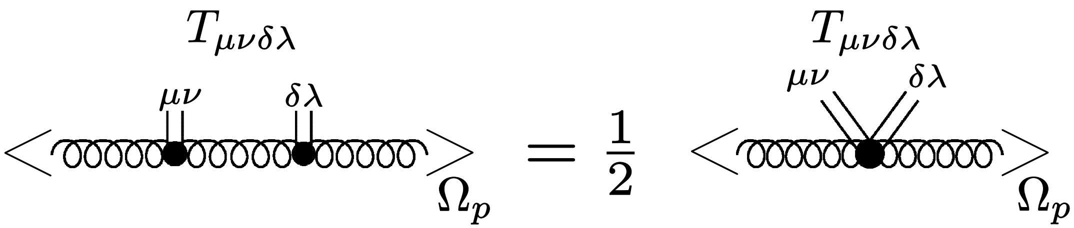
- **1-loop result**

$$\beta_{\mathsf{YM}}\Big|_{\mathrm{grav}} = -\frac{6\,I}{\pi} G_N\, g_{\mathsf{YM}}\, E^2$$

$$I = \int_0^\infty dx\, \frac{1+\alpha}{1+r_g(x)} \left(1 - \frac{1}{1+r_A(x)}\right) \geq 0$$

Yang-Mills + gravity

- **kinematical identity**



- **beyond 1-loop**

$$\beta_{\mathrm{YM}}\Big|_{\mathrm{grav}} \leq 0$$

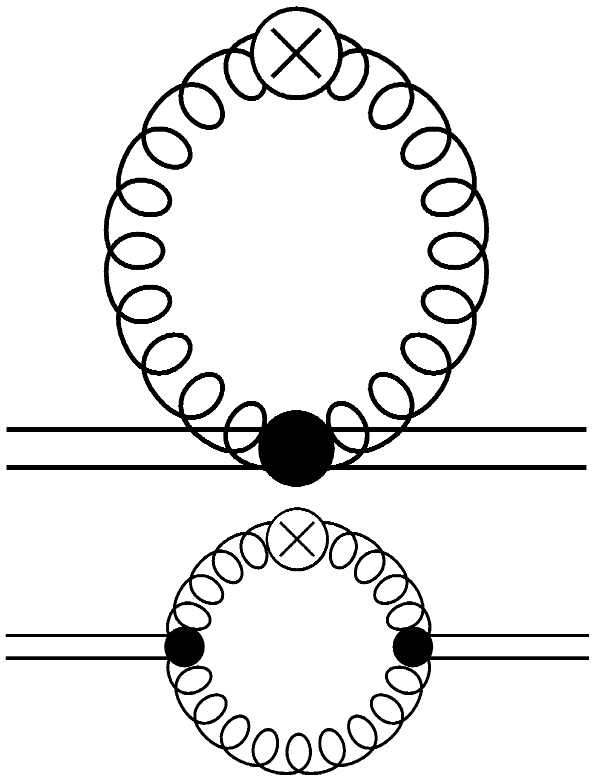
asymptotic freedom persists in presence of gravity FP

Yang-Mills + gravity

S. Folkerts, DL, JM. Pawlowski (in preparation)

- **Yang-Mills contribution to gravity**

  diagrams



Yang-Mills + gravity

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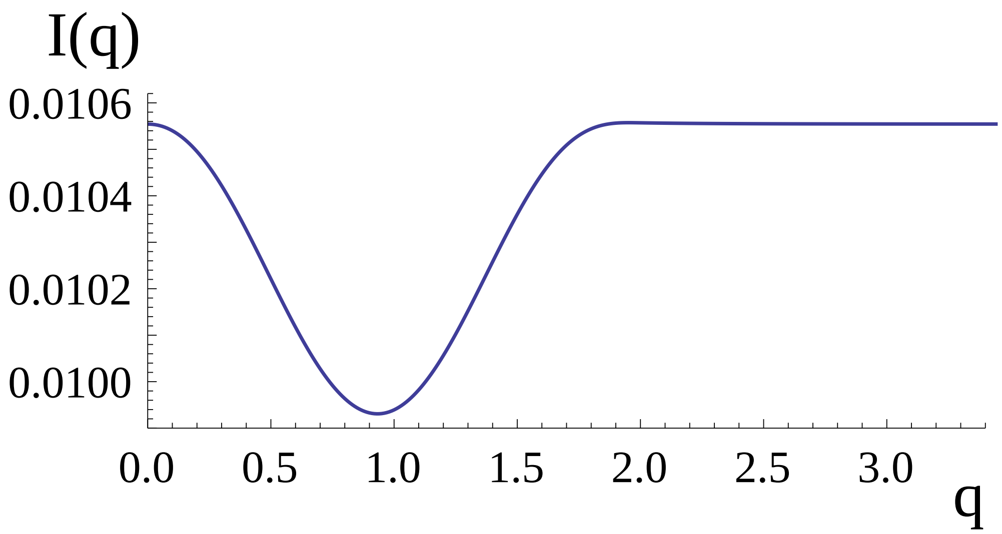
  rhs of flow equation (optimised cutoff)



Yang-Mills + gravity

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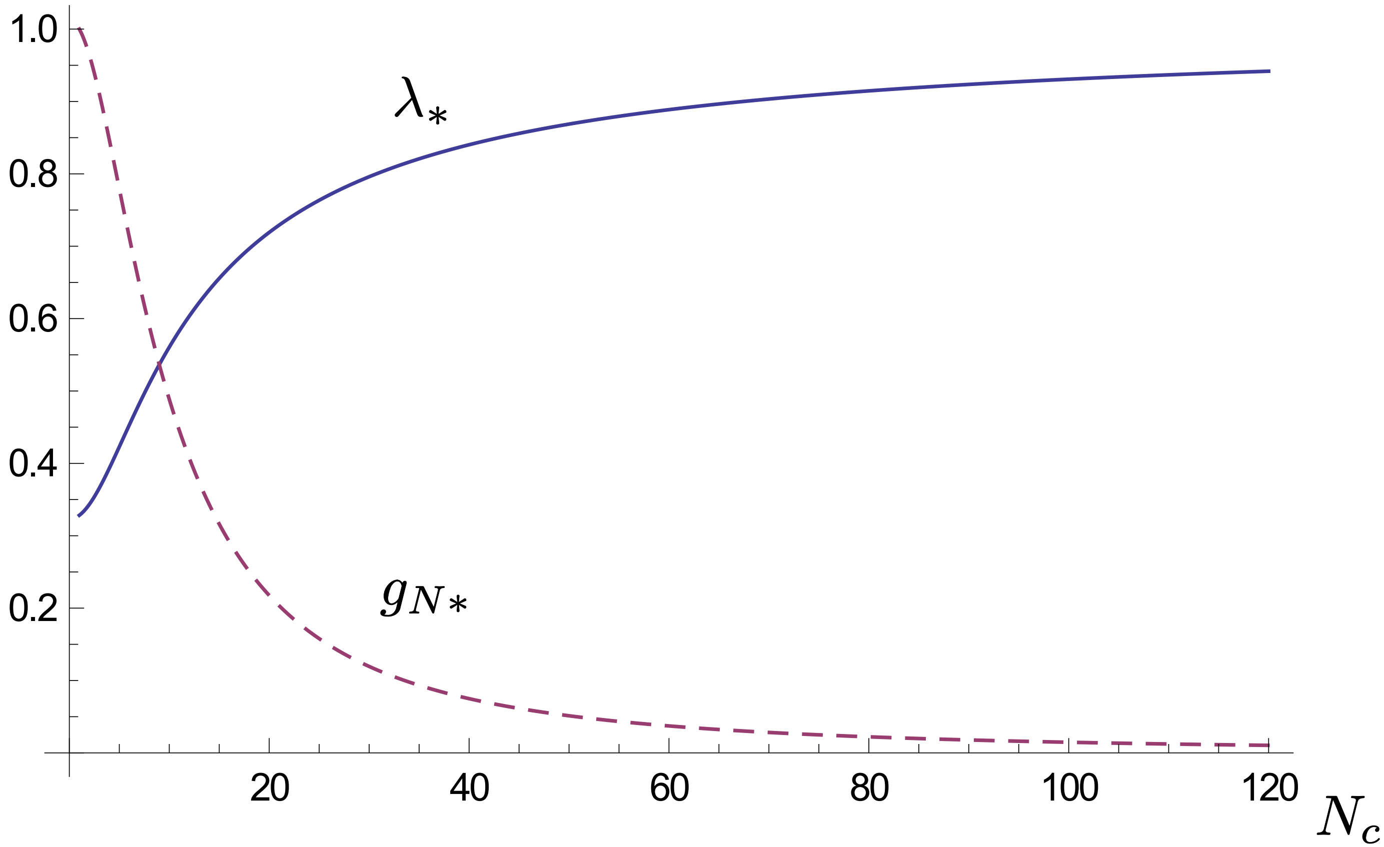
- **Yang-Mills contribution to gravity**

  UV fixed point of coupled system



signatures for asymptotic safety

- **cosmology, early universe, inflation**

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- cosmology, early universe, inflation
- black holes

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- ‘gravitational’ particle scattering

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low-scale quantum gravity models
what if the fundamental Planck scale M_D obeys

$$M_D \approx \mathcal{O}(M_{\rm EW}) \approx \mathcal{O}(1\text{TeV}) \ll M_{\rm Pl}$$

asymptotically safe gravity accessible at colliders

(work with J Brinckmann, K Falls, E Gerwick, G Hiller, T Plehn)

collider signatures of quantum gravity

- **real gravitons**

  graviton production via p p $\longrightarrow$ jet + G

  signature: missing energy

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  lepton production $q\bar{q} \longrightarrow \ell^+\ell^-$ via graviton exchange

  signature: deviations in SM reference processes

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  signature: deviations in SM reference processes

- **mini-black holes**

  black hole production and decay

  signature: spectacular (many body final states)

black holes at the LHC

- **semi-classical vs renormalisation group**

K. Falls, G. Hiller, DL (in prep.)

elastic BH production $pp \longrightarrow$ **BH**

$$\frac{d\sigma}{dM} = \frac{2M}{s} \sum_{i,j} \int_{M^2/s}^{1} \frac{dx}{x} f_i\left(\frac{M^2}{xs}\right) f_j(x)\, \hat{\sigma}(q_i q_j \longrightarrow \mathrm{BH})|_{\hat{s}=M^2}.$$

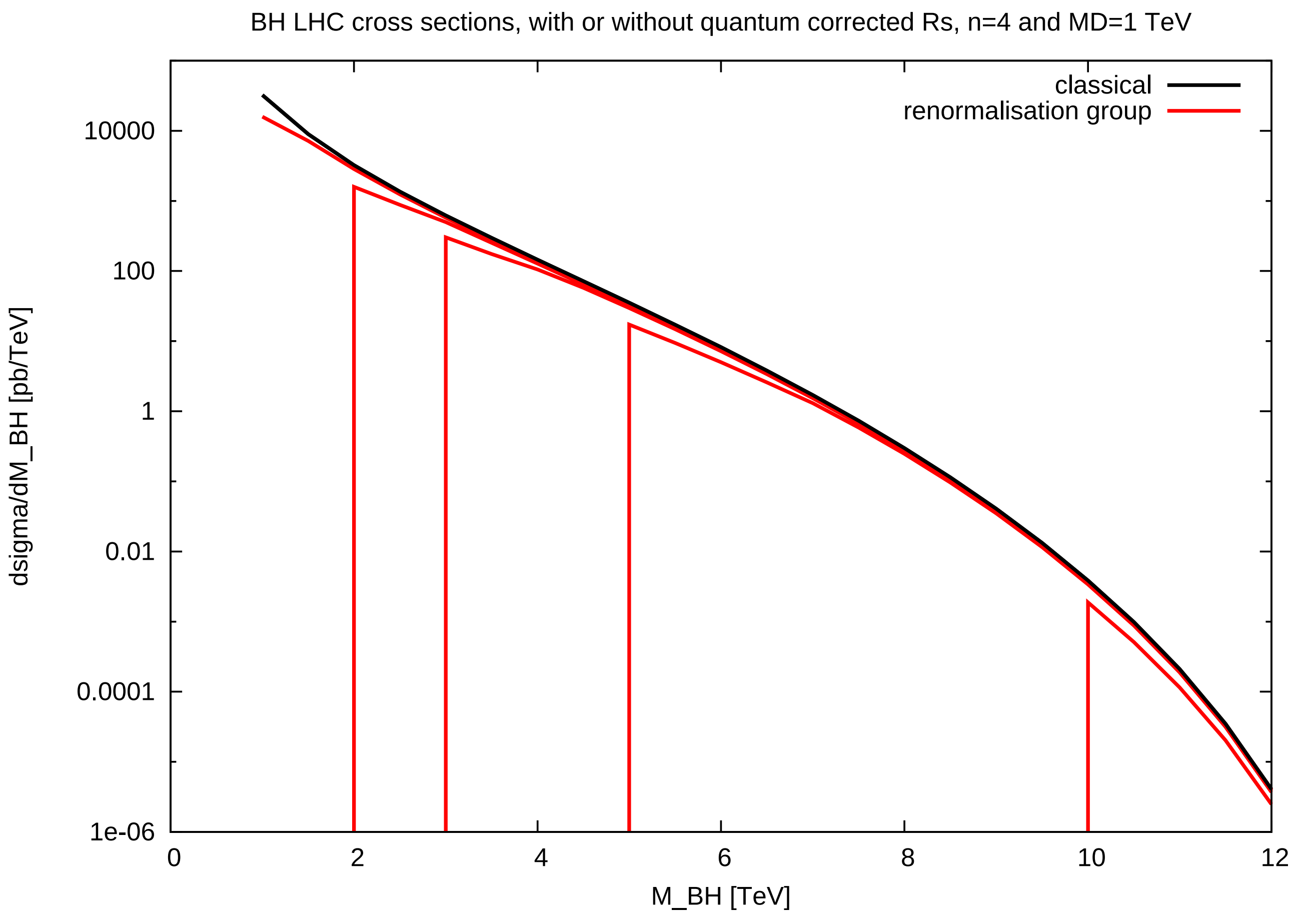
parton distribution functions from **CTEQ61**
evaluated at $Q^2 = M_{\mathrm{BH}}^2$.

black holes at the LHC

- **semi-classical vs renormalisation group**

K. Falls, G. Hiller, DL (in prep.)

$n = 4$ **extra dimensions**



unitarity bounds

J. Brinkmann, G. Hiller, DL (in prep.)

- **Higgs-Higgs elastic scattering**

  extra dimensions, gravity-mediated, KK modes

  effective theory study X.G. He ('00)
  partial wave decomposition:

$$M(s,t) = 16\pi \sum_J (2J+1)\, a_J(s)\, P_J(\cos\theta), \quad t = t(\cos\theta)$$

$$\sigma \approx 16\pi \frac{|a_0(s)|^2}{s}, \quad a_0(s) = \frac{1}{16\pi} \frac{1}{s - 4m_h^2} \int_{4m_h^2 - s}^{0} dt\, M(s,t)$$

  optical theorem, unitarity bound

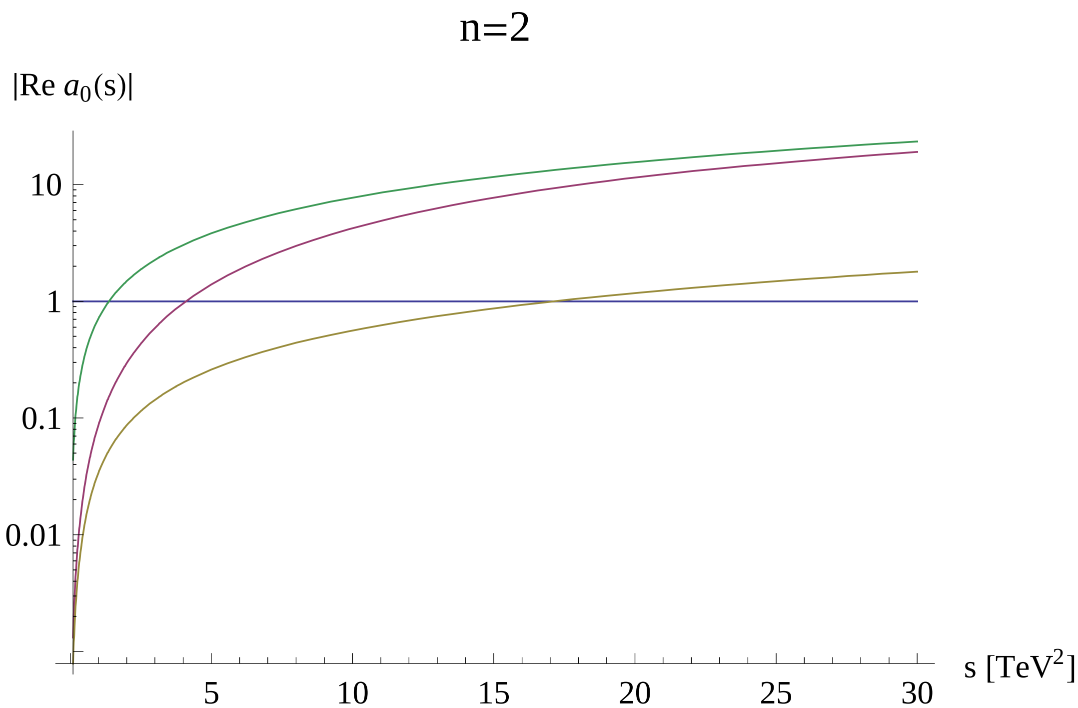
$$|a_0(s)| \leq 1$$

unitarity bounds

- **results** $|a_0(s)| \longrightarrow c_n \frac{s}{M_D}$

effective theory: valid for $s < M_D^2$

RG study: $c_n \ll 1$ J. Brinkmann, G. Hiller, DL (in prep.)



parameter: $m_h = 0, n = 2, M_D = 1\text{TeV}, \Lambda_X = 1, 5, \frac{1}{5}\text{TeV}$

conclusions

- **asymptotic safety for quantum gravity**

  tools are available

  eg. PT / renormalisation group / lattice

  results promising

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- **challenges for theory (and experiment)**

  structure of the UV fixed point?

  predictions for Planck scale physics

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  predictions for Planck scale physics

- **signatures of asymptotic safety**

  cosmology and black holes

  gravitational scattering, phenomenology at colliders