NC GUTS: A STATUS REPORT

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PLAN

10 years of the Enveloping-algebra formalism

- It is already 10 years since the publishing of

  ◇ "Gauge theory on noncommutative spaces",
  Madore, Schraml, Schupp & Wess, EPJC16(2000)16,

  ◇ "Noncommutative gauge theory for Poisson manifolds,
  Jurco,Schupp & Wess, NPB584(2000)784,

  ◇"Enveloping algebra-valued gauge transformations for non-abelian gauge goups on non-commutative spaces",
  Jurco, Schraml, Schupp & Wess, EPJC17(2000)521,

- where it was put forward a formalism –THE ENVELOPING-ALGEBRA FORMALISM–, which led to

  ◇"Non-commutative standard model",
  Calmet, Jurco, Schupp, Wess & Wohlgenannt, EPJC23(2002)363,

  ◇"Noncommutative GUTs, standard model and C,P,T.",
  Aschieri, Jurco, Schupp & Wess, NPB 651(2003)45.

There is an excellent recent review by Blaschke, Kronberger, Sedmik & Wohlgenannt, SIGMA 6(2010) 063.

NC fields as ordinary-field SW map images

In the ENVELOPING-ALGEBRA FORMALISM:

- The noncommutative fields are functions of the ordinary fields –no change in the no. of d.o.f.– such that ordinary gauge orbits are mapped into noncommutative gauge orbits:

$$A_\mu[a_\mu,\psi,\theta] + s_{NC}\, A_\mu[a_\mu,\psi,\theta]A = A_\mu[a_\mu + s\, a_\mu, \psi + s\,\psi, \theta],$$
$$\Psi[a_\mu,\psi,\theta] + s_{NC}\,\Psi[a_\mu,\psi,\theta] = \Psi[a_\mu + s\, a_\mu, \psi + s\,\psi, \theta],$$
$$s_{NC}\Lambda[\lambda,\lambda,\psi,\theta] = s\Lambda[\lambda,\lambda,\psi,\theta],$$

$$A_\mu[a_\mu,\psi,\theta=0] = a_\mu, \Psi[a_\mu,\psi,\theta=0] = \psi, \Lambda[\lambda,\lambda,\psi,\theta=0] = \lambda$$

$$s_{NC}\, A_\mu = \partial_\mu\Lambda - i[A_\mu,\Lambda]_\star, s_{NC}\,\Psi = i\Lambda\star\Psi, s_{NC}\,\Lambda = i\Lambda\star\Lambda,$$
$$s\, a_\mu = \partial_\mu\lambda - i[a_\mu,\lambda], s\,\psi = i\lambda\,\psi, s\,\lambda = i\lambda\,\lambda,$$

- By standard SW map eqs., we mean: Λ acts from the left!.
- a_μ and λ take values on the Lie algebra, $\mathfrak{g}$, of a compact Lie group, G $\Longrightarrow$ A_μ and Λ take values on the universal enveloping algebra of $\mathfrak{g}$.

NC gauge theories for any compact Lie group

- The action, S, for a (nonsusy) NC GUT (-inspired) theory for a compact Lie group, G, reads

$$S = S_{gauge} + S_{fermionic} + S_{Higgs} + S_{Yukawa},$$

$$S_{gauge} \int d^4x \; -\tfrac{1}{2}\sum_{\mathcal{R}} c_{\mathcal{R}} \mathrm{Tr}_{\mathcal{R}} F_{\mu\nu}[\mathcal{R}(A)] \star F^{\mu\nu}[\mathcal{R}(A)],$$
$$S_{fermionic} = \int d^4x \; \bar{\Psi}_L i \not{D}[\rho_\psi(A)]\Psi_L,$$
S_{Higgs} and S_{Yukawa} shall be dropped in the quantum theory,

$$F_{\mu\nu}[\mathcal{R}(A)] = \partial_\mu \mathcal{R}(A)_\nu - \partial_\nu \mathcal{R}(A)_\mu - i[\mathcal{R}(A)_\mu, \mathcal{R}(A)_\nu]_\star,$$
$$D_\mu[\rho_\psi(A)]\psi_L = \partial_\mu \Psi_L - i\rho_\psi(A_\mu) \star \Psi_L,$$

- $\Psi_L[\theta^{\mu\nu}, \rho_\psi(a), \psi_L]$ is the NC left-handed spinor multiplet which is the NC counterpart of the ordinary left-handed spinor multiplet ψ_L. ψ_L carries an arbitrary unitary representation, ρ_ψ, of $\mathfrak{g}$.
- $\mathcal{R}$ labels the unitary IRREPS –typically the adjoint and matter irreps– of $\mathfrak{g}$ and $\sum_{\mathcal{R}} c_{\mathcal{R}} \mathrm{Tr}_{\mathcal{R}} \mathcal{R}(T_l^a)\mathcal{R}(T_l^a) = 1/g_l^2$, G=$\bigotimes_l$ G$_l$.
- Hybrid SW maps (left-right NC gauge trans.) needed for S_{Yukawa}, S_{Higgs}.

Quantising

- The QUANTUM version of the classical field theory defined above is obtained by integrating over the ordinary fields in the path-integral with Boltzmann factor
$$e^{iS}.$$
S is the action above, which we shall understand as a formal power series in $\theta^{\mu\nu}$.
- Caveat: This expansion in θ will not yield the right Physics at
$$\text{Energies} > 1/\sqrt{\theta}.$$

Pause and look back

- After those 10 years, it is advisable that we pause to look back and assess what has been achieved as regards the quantum properties of those GUTs.
- I will not cover all that has been done so far, but focus on
    - gauge anomalies,
    - renormalisability (when there are no Higgs and no Yukawa sectors),
    - construction of Yukawa terms and
    - existence Supersymmetric versions.

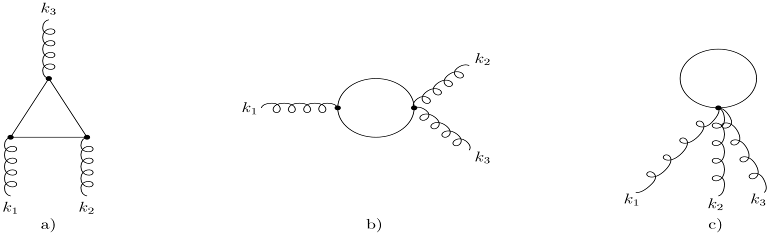
Gauge anomalies

- When quantizing a chiral gauge theory the first problem one has to face is that of gauge anomalies.
- The chiral vertices acquire θ-dependent terms, which can give rise to new θ-dependent anomalous contributions to the famous, already anomalous, ordinary triangle diagrams:

$$S_{fermionic} = \int d^4x \; \bar{\psi} i \partial\!\!\!/ \psi + \bar{\psi}\{ a\!\!\!/ - \frac{1}{2}\theta^{\alpha\beta}[\frac{1}{2} f_{\alpha\beta} i D\!\!\!\!/ \,(a) + \gamma^{\rho} f_{\rho\alpha} i D_{\beta}\,(a)]\}\, \mathrm{P}_L \psi + o(\theta^2).$$

- So, I started long ago the computation of the following three types of one-loop 3point diagrams giving would-be θ-dependent anomalies: $\rightarrow\rightarrow\rightarrow$

Would-be anomalous 3pt diagrams



Wrong guess!

- Actually, I was completely sure that they would give rise to new θ-dependent anomalous terms, which would lead to extra anomaly cancellation conditions, which in turn would make most –NC SM, NC GUTS..– of these theories meaningless at the quantum level.
- Couldn't be more wrong! I was very surprised to find that the θ-dependent anomalous contributions to the effective action, Γ, were BRS-exact, i.e., they were not truly anomalous terms:

$$s\Gamma[A[a,\theta],\theta] = -\frac{i}{24\pi^2}\int d^4x\ \varepsilon^{\mu_1\mu_2\mu_3\mu_4}\,\mathrm{Tr}\big(\partial_{\mu_1}\lambda\, a_{\mu_2}\partial_{\mu_3}a_{\mu_4}\big)$$
$$+s\left[\frac{1}{48\pi^2}\int d^4x\ \varepsilon^{\mu_1\mu_2\mu_3\mu_4}\,\theta^{\alpha\beta}\,\mathrm{Tr}\left(\partial_\alpha\partial_{\mu_1}a_{\mu_2}\partial_{\mu_3}a_{\mu_4}a_\beta\right)\right] + o(a^3) + o(\theta^2).$$

- The computations were carried out by using DIM. REG. with a nonanticommuting γ_5. More details in CPM, NPB 652(2003)72.
- When I did the computations back in 2002, I was unaware of the results –obtained using cohomological techniques– by Barnich, Henneaux and Brandt [PR 338 (2000) 439] on the lack of non BARDEEN anomalies for semisimple Lie algebras.

Would-be anomaly at any order in θ

- The next challenge was to show -at one-loop– that there were no θ-dependent gauge anomalies at any $O(\theta)$ and for any number of a_μ's.
- We did so [F.Brandt, CPM & F. Ruiz, JHEP 07(2003)068] by using a mixture of explicit DIM. REG. computations, brute force solution of BRS equations and BRS techniques:
- By taking advantage of the fact that in DIM REG. the Jacobian of $\mathbb{I} + M$ –an operator which enters the SW map for fermions

$$\Psi_{\alpha I} = \left(\delta_{IJ}\, \delta_{\alpha\beta} + M[a, \partial, \gamma, \gamma_5; \theta]_{\alpha\beta\ IJ}\right) \psi_{\beta J}{--}$$

is TRIVIAL, we were able to obtain the complete gauge anomaly candidate:

$$\mathcal{A}[A, \Lambda, \theta] = -\frac{i}{24\pi^2} \int d^4x\, \epsilon^{\mu_1\mu_2\mu_3\mu_4} \mathbf{Tr}\, \Lambda\star\partial_{\mu_1}\left(A_{\mu_2} \star \partial_{\mu_3} A_{\mu_4} + \tfrac{1}{2} A_{\mu_2}\star A_{\mu_3}\star A_{\mu_4}\right)$$

$$A_\mu = A[a, \theta]_\mu, \quad \Lambda = \Lambda[\lambda, \theta]$$

- Then,

$\longrightarrow\longrightarrow\longrightarrow\rightarrow\ \longrightarrow\longrightarrow$

Would-be anomaly at any order in θ, cont'

- by carrying out brute force computations and by using cohomological techniques, we obtained $\mathcal{B}[A^{(a,t\theta)}, t\theta]$ such that

$$t\,\frac{d}{dt}\,\mathcal{A}[A(a,t\theta),\Lambda(\lambda,t\theta),t\theta] = s_{NC}\,\mathcal{B}\left[A^{(a,t\theta)},t\theta\right].$$

- and, hence,

$$\mathcal{A}[A(a,\theta),\Lambda(\lambda,\theta),\theta] = \mathcal{A}^{\text{Bardeen}} - s\int_0^1 \frac{dt}{t}\,\mathcal{B}[A(a,t\theta),t\theta]$$

- THE θ-DEPENDENT TERMS ARE COHOMOLOGICALLY TRIVIAL $\Longrightarrow$ THEY ARE NOT ANOMALOUS CONTRIBUTIONS!

FUJIKAWA'S METHOD

- Another way to obtain the gauge anomaly is Fujikawa's method: the gauge anomaly shows that the fermionic measure is not invariant under chiral gauge transformations. Fujikawa's method helps establish a connection with index theorems.
- As yet, we lack a derivation of the absence of θ-dependent anomalous terms by using Fujikawa's method.
- Within Fujikawa's formalism, the ordinary gauge anomaly comes in two guises, related by local redefinitions of the corresponding currents: the consistent form, $\mathcal{A}_{con}$, and the covariant form, $\mathcal{A}_{cov}$
  - $\mathcal{A}_{con}$ verifies the WZ consistency conditions and involves lengthy and tedious algebra. It is not gauge covariant.
  - $\mathcal{A}_{cov}$ does not verify the WZ conditions, it is gauge covariant and, as a result, the algebraic computations that lead to it are simpler than in the "consistent" case.

The covariant form of the gauge anomaly. I

- A few moths ago I decided to work out the covariant form of the gauge anomaly in the U(1) case –non-trivial from the Cohomological viewpoint: Barnich, Brandt & Henneaux, Phys. Rept. 338 (2000) 439–, up to first order in θ:

$$Z[a,\theta] \;\equiv\; \int d\bar{\psi} d\,\psi \quad e^{-\int d^4x\,\bar{\psi} i\mathcal{D}\psi}$$
$$\mathcal{D} = \hat{\not{D}} + \hat{\not{R}}, \quad \hat{\not{D}} = \not{\partial} - i\not{a}P_L$$
$$\hat{\not{R}} = -[\tfrac{1}{4}\theta^{\alpha\beta}\, f_{\alpha\beta}\gamma^{\mu} D_{\mu} + \tfrac{1}{2}\,\theta^{\alpha\beta}\,\gamma^{\rho} f_{\rho\alpha} D_{\beta}]P_L$$

Then, following Fujikawa, one introduces two bases of orthonormal eigenfunctions $\{\varphi_m\}$ & $\{\phi_m\}$,

$$\Big(i\mathcal{D}(a)\Big)^{\dagger} i\mathcal{D}(a)\varphi_m \;=\; \lambda_m^2\,\varphi_m, \quad i\mathcal{D}(a)\Big(i\mathcal{D}(a)\Big)^{\dagger}\phi_m \;=\; \lambda_m^2\,\phi_m,$$

and expands

$$\psi = \sum_m a_m\varphi_m, \quad \bar{\psi} = \sum_m \bar{b}_m\phi_m^{\dagger} \Longrightarrow d\bar{\psi} d\,\psi \equiv \prod_m d\bar{b}_m da_m.$$

The covariant form of the gauge anomaly. II

- The gauge anomaly equation in covariant disguise reads

$$\int d^4x\,\mathrm{Tr}\,\omega(x)\big(D^\mu[a]\mathcal{J}_\mu^{(cov)}\big)(x) = -\delta J \equiv \mathcal{A}[\omega, a, \theta]_{cov},$$

  where

$$\delta J = d\bar\psi' d\,\psi' - d\bar\psi d\,\psi \quad \psi' = \psi + i\omega P_L\psi, \bar\psi' = \bar\psi - i\bar\psi P_R\omega$$
$$\delta J = \lim_{\Lambda\to\infty}\int d^4x\,\sum_m\{\phi_m^\dagger\omega e^{-\lambda_m^2/\Lambda^2}P_R\phi_m - \varphi_m^\dagger\omega e^{-\lambda_m^2/\Lambda^2}P_L\varphi_m\}$$
$$\mathcal{J}_\mu^{a,\,(cov)}(x) = \tfrac{1}{Z[a,\theta]}\int d\bar\psi d\,\psi\,\frac{\delta S_{fermionic}}{\delta a_\mu^a(x)}\,e^{-S_{fermionic}},\ S_{fermionic} = \int d^4x\,\bar\psi i\mathcal{D}\psi$$

- By changing to a plane wave basis, one gets

$$\mathcal{A}[\omega, a, \theta]_{cov} = \lim_{\Lambda\to\infty}\ -\int d^4x\,\mathrm{Tr}\,\omega(x)\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\left\{\left(\gamma_5\,e^{-ipx}\,e^{-\frac{(i\not{D}^{(\theta)}(a))^2}{\Lambda^2}}\,e^{ipx}\right)\right\},$$
$$\not{D}^{(\theta)}(a) = \not{D} + \not{R},\ \not{R} = -[\tfrac{1}{4}\theta^{\alpha\beta}\,f_{\alpha\beta}\gamma^\mu D_\mu + \tfrac{1}{2}\,\theta^{\alpha\beta}\,\gamma^\rho f_{\rho\alpha}D_\beta].$$

The covariant form of the gauge anomaly. III

- By expanding in powers of θ and removing the terms that vanish as $\Lambda \to \infty$, one gets

$$\mathcal{A}[\omega, a, \theta]_{cov} = \lim_{\Lambda\to\infty} \; -\int d^4x \,\mathrm{Tr}\,\omega \int \frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\left\{\left(\gamma_5\, e^{-ipx}\, e^{-\frac{(i\not D^{(\theta)}(a))^2}{\Lambda^2}}\, e^{ipx}\right)\right\} = \mathcal{A}^{(ordinary)}[\omega, a] + \mathcal{A}^{(1)}[\omega, a, \theta] + o(\theta^2)$$

$$\mathcal{A}[\omega, a]^{(ordinary)} = -\frac{1}{32\pi^2}\int d^4x\,\mathrm{Tr}\omega\epsilon^{\mu\nu\rho\sigma} f_{\mu\nu} f_{\rho\sigma}\,,$$

$$\mathcal{A}^{(1)}[\omega, a, \theta] = \int d^4x\,\mathrm{Tr}\omega(x)[\mathcal{A}_1(x) + \mathcal{A}_2(x) + \mathcal{A}_3(x)]$$

$$\mathcal{A}_1 = -\sum_{l=0}^{1} \lim_{\Lambda\to\infty} 2i\int\frac{d^4q}{(2\pi)^4}\; e^{-q^2}\,\frac{1}{2}\mathrm{tr}\gamma_5 \not\mathcal{D}^{2l}(\Lambda q)\left\{\not D(\Lambda q), \not R(\Lambda q)\right\}\not\mathcal{D}^{2(1-l)}(\Lambda q)\mathbb{I},$$

$$\mathcal{A}_2 = -\sum_{l=0}^{2} \lim_{\Lambda\to\infty} 2i\int\frac{d^4q}{(2\pi)^4}\; e^{-q^2}\,\frac{1}{3!\Lambda^2}\mathrm{tr}\gamma_5 \not\mathcal{D}^{2l}(\Lambda q)\left\{\not D(\Lambda q), \not R(\Lambda q)\right\}\not\mathcal{D}^{2(2-l)}(\Lambda q)\mathbb{I},$$

$$\mathcal{A}_3 = -\sum_{l=0}^{3} \lim_{\Lambda\to\infty} 2i\int\frac{d^4q}{(2\pi)^4}\; e^{-q^2}\,\frac{1}{4!\Lambda^4}\mathrm{tr}\gamma_5 \not\mathcal{D}^{2l}(\Lambda q)\left\{\not D(\Lambda q), \not R(\Lambda q)\right\}\not\mathcal{D}^{2(3-l)}(\Lambda q)\mathbb{I}.$$

The covariant form of the gauge anomaly. IV

- Some lengthy algebra and the fact that the a_μ's commute –U(1) case– lead to

$$\begin{aligned}\mathcal{A}_1 &= -\frac{i}{8\pi^2}\theta^{\alpha\beta}\epsilon^{\mu\nu\rho\sigma}(-\tfrac{1}{2}f_{\alpha\beta}f_{\mu\nu}f_{\rho\sigma} - f_{\nu\alpha}f_{\mu\beta}f_{\rho\sigma})\\ &\quad+\frac{1}{16\pi^2}\theta^{\alpha\beta}\epsilon^{\mu\nu\rho\sigma}\big[f_{\mu\nu}(\partial_\rho f_{\sigma\alpha}D_\beta\mathbb{I} + \tfrac{1}{2}\partial_\rho f_{\alpha\beta}D_\sigma\mathbb{I}) + \partial_\mu f_{\nu\alpha}f_{\rho\sigma}D_\beta\mathbb{I}\\ &\qquad+\tfrac{1}{2}\partial_\mu f_{\alpha\beta}f_{\rho\sigma}D_\nu\mathbb{I}\big],\\ \mathcal{A}_2 &= -\frac{i}{2(4\pi)^2}\theta^{\alpha\beta}\epsilon^{\mu\nu\rho\sigma}f_{\alpha\beta}f_{\mu\nu}f_{\rho\sigma}+\\ &\quad-\frac{1}{16\pi^2}\theta^{\alpha\beta}\epsilon^{\mu\nu\rho\sigma}[f_{\mu\nu}\partial_\rho f_{\sigma\alpha}D_\beta\mathbb{I} + \partial_\mu f_{\nu\alpha}f_{\rho\sigma}D_\beta\mathbb{I} + \tfrac{1}{2}(f_{\mu\nu}\partial_\rho f_{\alpha\beta}D_\sigma\mathbb{I}\\ &\qquad+\partial_\mu f_{\alpha\beta}f_{\rho\sigma}D_\nu\mathbb{I})],\\ \mathcal{A}_3 &= 0.\end{aligned}$$

- Then,

$$\mathcal{A}^{(1)}[\omega, a, \theta] = \int d^4x\, \mathrm{Tr}\omega(x)[\mathcal{A}_1(x) + \mathcal{A}_2(x) + \mathcal{A}_3(x)] = \frac{i}{32\pi^2}\theta^{\alpha\beta}\epsilon^{\mu\nu\rho\sigma}\int d^4x\, \mathrm{Tr}\omega(f_{\alpha\beta}f_{\mu\nu}f_{\rho\sigma} + 4\, f_{\nu\alpha}f_{\mu\beta}f_{\rho\sigma}) = 0.$$

The covariant form of the gauge anomaly. IV

- In summary,

$$\mathcal{A}[\omega, a, \theta]_{cov} = \mathcal{A}^{(ordinary)}[\omega, a] + o(\theta^2)$$

- NO FIRST-ORDER-IN-θ-CORRECTIONS TO THE ORDINARY GAUGE ANOMALY IN THE U(1) CASE: AGREEMENT WITH DIM. REG.
- NONABELIAN CASE SHOULD BE WORKED OUT!

Renormalisability and the enveloping-algebra formalism. I

- The issue of the renormalisability of NC theories formulated within the enveloping-algebra formalism started off splendidly, for it was shown by Bichl, Grimstrup, Grosse, Popp. Schweda and Wulkenhaar –JHEP 0106(2001)013– that the photon 2pt function is renormalisable at any order in θ.
- Unfortunately, Wulkenhaar –JHEP 0203(2002)024– showed that this θ-expanded QED was not renormalisable mainly due to the infamous 4pt fermionic divergence:

$$\frac{c}{\epsilon}\theta^{\alpha\beta}\epsilon_{\mu\nu\rho\sigma}\int d^4x\,\bar{\psi}\gamma_5\gamma^\rho\psi\bar{\psi}\gamma^\sigma\psi$$

- 4 years after Wulkenhaar's paper, there came along the encouraging results by Buric, Latas and Radovanovic –JHEP 02(2006)040– & Buric, Radovanovic and Trampetic –JHEP 03(2007) 030– that the gauge sector of SU(N) and the NC SM were one-loop renormalizable at first order in θ.

Renormalisability and the enveloping-algebra formalism: Matter sector

- However, due to the infamous 4pt fermionic divergence above, the construction of theories with a renormalisable one-loop and first-order-in-θ matter sector remained an open issue.
- Then, it came along the paper by Buric, Latas, Radovanovic and Trampetic –PRD 77 (2008) 045031–, where they showed that the divergence of the 4pt fermionic function vanishes for a NC SU(2) chiral theory with the matter sector being an SU(2)-doublet of NC LH fermions.
- This result was later generalized –CPM & C. Tamarit, PRD 80 (2009) 065023– to any NC GUT inspired theory with only fermions as matter fields.♢ NC GUT inspired theories: gauge theories whose noncommutative fermions are left-handed multiplets.
- Thus, one of the obstacles –what about the renormalisability of the other 1PI functions?– to achieve one-loop and first-order-in-θ renormalisability had been removed by selecting Grand Unification as a guiding principle.

One-loop & $o(\theta)$ ren. GUT inspired models

- The absence of the infamous 4pt fermionic divergence opened up the possibility of building NC theories with massless fermionic NC chiral matter that are one-loop renormalisable at first in θ.
- Actually, Wulkenhaar had already pointed out in his non-renormalizability-of-θ-expanded-noncommutative-QED paper that, in the massless case, the theory is (off-shell) one-loop renormmalisable, at first order in θ, if one forgets about the fermionic 4pt function.
- At long last, it was shown –CPM & C. Tamarit, JHEP 12(2009)042– that NC GUT inspired theories, with a matter sector made out of fermions and no scalars, were, on-shell, one-loop-and-first-order-in-θ renormalisable for any anomaly safe compact simple gauge group, if, and only if, all the flavour fermionic multiplets carry irreps with the same quadratic Casimir [ie, RENORMALISABILITY is very partial to FAMILY UNIFICATION]. (SO(10), E_6, not SU(5) –See C.TAMARIT, PRD81 (2010) 025006.

The on-shell renormalisability

- NC GUT inspired models:

$$S = \int d^4x \; -\frac{1}{2g^2} \mathrm{Tr} F_{\mu\nu} \star F^{\mu\nu} + \bar{\Psi}_L i \not{D} \Psi_L,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]_\star, \quad D_\mu \psi_L = \partial_\mu \Psi_L - i\rho_\Psi(A_\mu) \star \Psi_L,$$

ρ_ψ denotes an arbitrary unitary representation, which is a direct sum of irreducible representations, $\rho_\psi = \bigoplus_{r=1}^{F} \rho_\psi^r$.

The effective action divergent part

- Lengthy computations led to the following result:

Once ψ^r_L, g and θ have been renormalised as follows

$$\psi^r = (Z^r_\psi)^{1/2}\psi^r_R, g = \mu^{-\epsilon} Z_g g_R, \theta^{\mu\nu} = Z_\theta \theta^{\mu\nu}_R,$$
$$Z^r_\psi = 1 + \frac{g^2 C_2(r)}{16\pi^2\epsilon}, Z_g = 1 + \frac{g^2}{16\pi^2\epsilon}\left[\frac{11}{6}C_2(G) - \frac{2}{3}\sum_r c_2(r)\right],$$
$$Z_\theta = -Z^r_\psi - \frac{g^2}{48\pi^2\epsilon}(13C_2(r) - 4C_2(G)),$$

the UV divergences –one-loop and first order in θ– which remain in the background-field effective action are

$$S^{\rm ct} = \int d^4x\, \frac{\delta S}{\delta a^a_\mu(x)} F^a_\mu[a,\psi] + \Big(\sum_r \frac{\delta S}{\delta\psi^r(x)} G_r[a,\psi] + \text{c.c.}\Big),$$

—VANISHING ON-SHELL!–, where

F and G functions

$$F_\mu = y_1\theta^{\alpha\beta}\mathcal{D}_\mu f_{\alpha\beta} + y_2\theta_\mu{}^\alpha D^\nu f_{\nu\alpha} + \sum_r y_3^r\theta_\mu{}^\alpha(\bar\psi_r\gamma_\alpha P_L T^a\psi^r)T^a$$
$$+ i\sum_r y_4^r\theta^{\alpha\beta}(\bar\psi_r\gamma_{\mu\alpha\beta}P_L T^a\psi^r)T^a + y_5\tilde\theta_\mu{}^\beta D^\nu f_{\nu\beta},$$
$$G_{r,L} = k_1^r\theta^{\alpha\beta}f_{\alpha\beta}P_L\psi^r + k_r^2\theta^{\alpha\beta}\gamma_{\alpha\mu}P_L f_\beta{}^\mu\psi^r$$
$$+ k_3^r\theta^{\alpha\beta}\gamma_{\alpha\mu}P_L D_\beta D^\mu\psi^r + k_4^{r\,\alpha\beta}\gamma_{\alpha\beta}P_L D^2\psi^r$$
$$+ k_5^r\tilde\theta^{\alpha\beta}\gamma_5 P_L f_{\alpha\beta}\psi^r;\quad y_i\in\mathbb{R},\ k_i\in\mathbb{C},$$

with

$$y_1 = \mathrm{Im}k_1^r,\ y_3^r = 2g^2 y_2,$$
$$y_4^r = -y_5 g^2 - \frac{g^4}{384\pi^2}(16C_2(r) - 13C_2(G)),$$
$$\mathrm{Re}k_1^r = -\frac{1}{2}\mathrm{Im}k_3^r - \frac{g^2}{384\pi^2\epsilon}(13C_2(r) - 8C_2(G)),$$
$$\mathrm{Im}k_5^r = -\frac{g^2}{384\pi^2\epsilon}(11C_2(r) - 8C_2(G)),$$
$$\mathrm{Im}k_4^r = \frac{g^2C_2(r)}{384\pi^2\epsilon},\ \mathrm{Re}k_2^r = -\frac{5g^2}{192\pi^2\epsilon}(2C_2(r) - C_2(G)),$$
$$\mathrm{Im}k_2^r = \mathrm{Re}k_3^r = 2\mathrm{Re}k_5^r = -2\mathrm{Re}k_4^r.$$

Notice that y_1, y_2, y_5 and Z_θ must be flavour independent, and so must be y_3, y_4: $C_2(r)$, the same for all irreps $\longrightarrow$ family unification.

Ordinary SO(10), E_6 Yukawa terms

- Yukama terms for (the most promising 4D GUTS) SO(10) and E_6:

$$\mathcal{Y}^{(\mathrm{ord})} = \int d^4x \; \mathcal{Y}_{ff'} \; \mathcal{C}_{AiB} \; \widetilde{\psi}^{\alpha}_{Af} \; \psi_{\alpha Bf'} \; \phi_i,$$

- $\widetilde{\psi}^{\alpha}_f \equiv (\psi^{\alpha}_f)^t$, $\quad \mathcal{Y}_{ff'}$ Yukawa coefficients.
- Fermionic multiplets: $\psi_{\alpha f'} \sim \mathbf{16}$ for SO(10), $\psi_{\alpha f'} \sim \mathbf{27}$ for E_6.
- Higgs multiplets: $\phi_i \sim \mathbf{10}, \mathbf{120}, \overline{\mathbf{126}}$ for SO(10), $\phi_i \sim \mathbf{27}, \mathbf{351}', \mathbf{351}$ for E_6.

$$16 \bigotimes 16 = (10 \bigoplus 126)_s \bigoplus 120_{as}, 27 \bigotimes 27 = (\overline{27} \bigoplus \overline{351'})_s \bigoplus \overline{351}_{as}$$

- $\mathcal{C}_{AiB}$ is an invariant tensor:

$$\widetilde{\Sigma}^a_{AC} \, \mathcal{C}_{CiB} + \mathcal{C}_{AjB} M^a_{ji} + \mathcal{C}_{AjC} \, \Sigma^a_{CB} = 0$$

$\widetilde{\Sigma}^a, M^a, \Sigma^a$ group generators in irreps by $\widetilde{\psi}^{\alpha}_{Af}, \phi_i$ and $\psi^{\alpha}_{Bf'}$, *respectively*.

NC SO(10), E_6 Yukawa terms: naive

- A naive NC version of the ordinary Yukawa term would not do

$$\mathcal{Y}^{(NC)}_{(naive)} = \int d^4x\ \mathcal{Y}_{ff'}\ \mathscr{C}_{AiB}\ \widetilde{\Psi}^{\alpha}_{Af}\ \star\ \Psi_{\alpha Bf'}\ \star\ \Phi_i,$$

$\widetilde{\Psi}^{\alpha}_{Af},\ \Psi_{\alpha Bf'},\ \Phi_i$ denote the NC fermionic and Higgs fields defined by standard SW maps,ie, solutions to $s_{NC}(NCField) \equiv i\Lambda \star (NCField) = s(NCField)$.

Indeed, $0 \neq s_{NC}\mathcal{Y}^{(NC)}_{(naive)} =$

$$\int d^4x\ \mathcal{Y}_{ff'}\ \mathscr{C}_{AiB}\ (i\widetilde{\Lambda}_{AC} \star \widetilde{\Psi}^{\alpha}_{Cf}\ \star\ \Psi_{\alpha Bf'}\ \star\ \Phi_i + \widetilde{\Psi}^{\alpha}_{Af}\ \star\ i\Lambda_{BC} \star \Psi_{\alpha Cf'}\ \star\ \Phi_i + \widetilde{\Psi}^{\alpha}_{Af}\ \star\ \Psi_{\alpha Bf'}\ \star\ i\Lambda_{ij} \star \Phi_j),$$

for

- $\star$ is not commutative &
- $\mathscr{C}_{AiB}$ not invariant for enveloping-algebra valued Λ's.

NC SO(10), E_6 Yukawa terms: tensor rep. fields

- To carry over the properties of $\mathscr{C}_{AiB}$ to the NC theory in a consistent way, one first combines $\mathscr{C}_{AiB}$ with the ordinary fields $\widetilde{\psi}^{\alpha}_{Af}$, $\psi_{\alpha Bf'}$ and ϕ_i and, then, defines new ordinary fields that transform under tensor products of ordinary irreps of the gauge group, but carry the very same NUMBER OF DEGREES of freedom as $\widetilde{\psi}^{\alpha}_{Af}$, $\psi_{\alpha Bf'}$ and ϕ_i:

$$\phi_{AB} = \mathscr{C}_{AiB}\,\phi_i, \quad \widetilde{\psi}^{\alpha}_{iBf} = \widetilde{\psi}^{\alpha}_{Af}\,\mathscr{C}_{AiB}, \quad \psi_{\alpha Aif'} = \mathscr{C}_{AiB}\,\psi_{\alpha Bif'}.$$

- Their BRS transformations read:

$$\begin{aligned} s\phi_{AB} &= -i\,\widetilde{\lambda}^{(\psi)}_{AC}\,\phi_{CB} - i\,\phi_{AC}\,\lambda^{(\psi)}_{CB}, \\ s\widetilde{\psi}^{\alpha}_{iBf} &= -i\,\widetilde{\lambda}^{(\phi)}_{ij}\,\widetilde{\psi}^{\alpha}_{jBf} - i\,\widetilde{\psi}^{\alpha}_{iCf}\,\lambda^{(\psi)}_{CB}, \\ s\psi_{\alpha Aif'} &= -i\,\widetilde{\lambda}^{(\psi)}_{AC}\,\psi_{\alpha Cif'} - i\,\psi_{\alpha Ajf'}\,\lambda^{(\phi)}_{ji}. \end{aligned}$$

NC SO(10), E_6 Yukawa terms: hybrid SW NC fields

- To each ordinary ϕ_{AB}, $\widetilde{\psi}^{\alpha}_{iBf}$ and $\psi_{\alpha Aif'}$, we associate a NC counterpart

$$\Phi_{AB}[\phi_{AB}, a^a_\mu, \theta], \quad \widetilde{\Psi}^{\alpha}_{iBf}[\widetilde{\psi}^{\alpha}_{iBf}, a^a_\mu, \theta] \quad \text{and} \quad \Psi_{\alpha Aif'}[\psi_{\alpha Aif'}, a^a_\mu, \theta],$$

  respectively, which are solutions to the Hybrid SW map eqs.:

$$s_{NC}\Phi_{AB} = s\Phi_{AB}, \quad s_{NC}\widetilde{\Psi}^{\alpha}_{iBf} = s\widetilde{\Psi}^{\alpha}_{iBf}, \quad s_{NC}\Psi_{\alpha Aif'} = s\Psi_{\alpha Aif'},$$

  where one defines

$$\begin{aligned}
s_{NC}\Phi_{AB} &\equiv -i\,\widetilde{\Lambda}^{(\psi)}_{AC} \star \Phi_{CB} - i\,\Phi_{AC} \star \Lambda^{(\psi)}_{CB},\\
s_{NC}\widetilde{\Psi}^{\alpha}_{iBf} &\equiv -i\,\widetilde{\Lambda}^{(\phi)}_{ij} \star \widetilde{\Psi}^{\alpha}_{jBf} - i\,\widetilde{\Psi}^{\alpha}_{iCf} \star \Lambda^{(\psi)}_{CB}\\
s_{nc}\Psi_{\alpha Aif'} &\equiv -i\,\widetilde{\Lambda}^{(\psi)}_{AC} \star \Psi_{\alpha Cif'} - i\,\Psi_{\alpha Ajf'} \star \Lambda^{(\phi)}_{ji}
\end{aligned}$$

- The Left-Right action (as opposed to LL, RR actions) of the Λ's is the only choice consistent with $(s_{NC})^2 = 0$!
- The solutions to the eqs. above are SW maps of hybrid type, a notion introduced by Schupp [hep-th/0111038].

NC SO(10), E_6 Yukawa terms: Term 1

- We are now in the position to obtain in a natural(naive) way NC SO(10), E_6 Yukawa terms from their ordinary counterparts:
- In terms of ϕ_{AB}, the ordinary Yukawa term reads:

$$\mathcal{Y}_1^{(\mathrm{ord})} \equiv \mathcal{Y}^{(\mathrm{ord})} = \int d^4x \; \mathcal{Y}_{ff'} \; \widetilde{\psi}^{\alpha}_{Af} \; \phi_{AB} \; \psi_{\alpha Bf'} \,.$$

- Then, its NC counterpart is

$$\mathcal{Y}_1^{(\mathrm{nc})} = \int d^4x \; \mathcal{Y}^{(1)}_{ff'} \; \widetilde{\Psi}^{\alpha}_{Af} \star \Phi_{AB} \star \Psi_{\alpha Bf'} \,,$$

- THE NC YUKAWA TERM IS OBTAINED BY REPLACING EACH ORDINARY FIELD IN $\mathcal{Y}_1^{(\mathrm{ord})}$ WITH ITS NC COUNTERPART AND THE ORDINARY PRODUCT WITH THE $\star$ PRODUCT!

NC SO(10), E_6 Yukawa terms: NC BRS INV. Term 1

- By construction $\mathcal{Y}_1^{(\mathrm{nc})}$ is invariant under the following NC BRS transformations:

$$s_{NC}\widetilde{\Psi}^{\alpha}_{Af} = i\,\widetilde{\Psi}^{\alpha}_{Bf}\star\widetilde{\Lambda}^{(\psi)}_{BA}\,,\quad s_{\rm nc}\Psi_{\alpha Bf'} = i\,\Lambda^{(\psi)}_{BC}\star\Psi_{\alpha Cf'}\,,$$
$$s_{NC}\Phi_{AB} = -i\,\widetilde{\Lambda}^{(\psi)}_{AC}\star\Phi_{CB} - i\,\Phi_{AC}\star\Lambda^{(\psi)}_{CB}\,,$$
$$s_{NC}\widetilde{\Lambda}^{(\psi)}_{BA} = -i\,\widetilde{\Lambda}^{(\psi)}_{BC}\star\widetilde{\Lambda}^{(\psi)}_{CA}\,,\quad s_{\rm nc}\Lambda^{(\psi)}_{BC} = i\,\Lambda^{(\psi)}_{BD}\star\Lambda^{(\psi)}_{DC}\,.$$

- The SW maps which define the NC fields are

$$\widetilde{\Psi}^{\alpha}_{Af} = \widetilde{\psi}^{\alpha}_{Af} - \tfrac{1}{2}\,\theta^{\mu\nu}\,\partial_\mu\widetilde{\psi}^{\alpha}_{Bf}\widetilde{a}^{(\psi)}_{\nu\,BA} + \tfrac{i}{4}\,\theta^{\mu\nu}\,\widetilde{\psi}^{\alpha}_{Cf}\widetilde{a}^{(\psi)}_{\mu\,CB}\widetilde{a}^{(\psi)}_{\nu\,BA} + O(\theta^2),$$
$$\begin{aligned}\Phi_{AB} = \phi_{AB} &+ \tfrac{1}{2}\,\theta^{\mu\nu}\,\widetilde{a}^{(\psi)}_{\mu\,AC}\partial_\nu\phi_{CB} + \tfrac{i}{4}\,\theta^{\mu\nu}\,\widetilde{a}^{(\psi)}_{\mu\,AC}\widetilde{a}^{(\psi)}_{\nu\,CD}\phi_{DB}+\\ &+ \tfrac{1}{2}\,\theta^{\mu\nu}\,\partial_\mu\phi_{AC}a^{(\psi)}_{\nu\,CB} + \tfrac{i}{4}\,\theta^{\mu\nu}\,\phi_{AC}a^{(\psi)}_{\mu\,CD}a^{(\psi)}_{\nu\,DB}\\ &+ \tfrac{i}{2}\,\theta^{\mu\nu}\,\widetilde{a}^{(\psi)}_{\mu\,AC}\phi_{CD}a^{(\psi)}_{\nu\,DB} + O(\theta^2),\end{aligned}$$
$$\Psi_{\alpha Bf'} = \psi_{\alpha Bf'} - \tfrac{1}{2}\,\theta^{\mu\nu}\,a^{(\psi)}_{\mu\,BC}\partial_\mu\psi_{\alpha Cf'} + \tfrac{i}{4}\,\theta^{\mu\nu}\,a^{(\psi)}_{\mu\,BC}a^{(\psi)}_{\nu\,CD}\psi^{\alpha}_{Df'} + O(\theta^2).$$

NC SO(10), E_6 Yukawa terms: Terms 2 & 3

- If we use $\widetilde{\psi}^{\alpha}_{iBf}$ and $\psi_{\alpha Aif'}$ to formulate an ordinary Yukawa term, we obtain the same ordinary Yukawa term:

$$\mathcal{Y}_2^{\text{(ord)}} = \int d^4x \; \mathcal{Y}_{ff'} \; \widetilde{\phi}_i \; \widetilde{\psi}^{\alpha}_{iBf} \; \psi_{\alpha Bf'} \, ,$$
$$\mathcal{Y}_3^{\text{(ord)}} = \int d^4x \; \mathcal{Y}_{ff'} \; \widetilde{\psi}^{\alpha}_{Af} \; \psi_{\alpha Aif'} \; \phi_i ,$$
$$\mathcal{Y}_1^{\text{(ord)}} = \mathcal{Y}_2^{\text{(ord)}} = \mathcal{Y}_3^{\text{(ord)}}$$

- BUT THE NC COUNTERPARTS OF $\mathcal{Y}_2^{\text{(ord)}}$ and $\mathcal{Y}_3^{\text{(ord)}}$ ARE NOT EQUAL:

$$\mathcal{Y}_2^{\text{(nc)}} = \int d^4x \; \mathcal{Y}^{(2)}_{ff'} \; \widetilde{\Phi}_i \star \widetilde{\Psi}^{\alpha}_{iBf} \star \Psi_{\alpha Bf'}$$
$$\mathcal{Y}_3^{\text{(nc)}} = \int d^4x \; \mathcal{Y}^{(3)}_{ff'} \; \mathcal{C}_{AiB} \; \widetilde{\psi}^{\alpha}_{Af} \; \phi_i \; \psi_{\alpha Bf'}$$
$$\mathcal{Y}_1^{\text{(nc)}} \neq \mathcal{Y}_2^{\text{(nc)}} \neq \mathcal{Y}_3^{\text{(nc)}} \neq \mathcal{Y}_1^{\text{(nc)}}$$

NC SO(10), E_6 Yukawa terms: The Yukawa Term

- Since

$$\mathcal{Y}_1^{(\mathrm{nc})} \neq \mathcal{Y}_2^{(\mathrm{nc})} \neq \mathcal{Y}_3^{(\mathrm{nc})} \neq \mathcal{Y}_1^{(\mathrm{nc})}$$

- the NC YUKAWA term reads

$$\mathcal{Y}^{(\mathrm{nc})} = \mathcal{Y}_1^{(\mathrm{nc})} + \mathcal{Y}_2^{(\mathrm{nc})} + \mathcal{Y}_3^{(\mathrm{nc})}.$$

- It can be shown –see CPM, arXiv: 1008.1871— that at first order in θ this is the most general BRS invariant Yukawa-type term:

$$\theta^{\mu\nu} \int d^4x\, \mathcal{Y}_{ff'}\; \psi^{\alpha}_{Af}\; \mathcal{V}^{AiB}_{\mu\nu}[\theta^{\rho\sigma}, \partial_\mu, a^a_\nu]\; \phi_i\; \psi_{\alpha Bf'},$$

  one can write. This Yukawa term is therefore renormalisable at first order in θ.

- FURTHER DETAILS IN CPM, arXiv: 1008.1871.

SUSY

WHAT ABOUT SUSY?

Details in CPM & C. Tamarit JHEP 0811 (2008) 087, JHEP 0911 (2009) 092.

- For $U(N)$ in the fundamental rep., $\mathcal{N}=1$ SYM exists –at least in the WZ gauge– in the enveloping-algebra formalism as a classical theory:

$$S_{NCSYM} = \frac{1}{2g^2}\mathrm{Tr}\int d^4x\,[-\frac{1}{2}\,F^{\mu\nu}\star F_{\mu\nu} - 2i\,\Lambda^{\alpha}\star\sigma^{\mu}_{\alpha\,\dot{\alpha}}D_{\mu}\bar{\Lambda}^{\dot{\alpha}} + D\star D]$$

where

$$A_\mu = A_\mu[a,\lambda_\alpha,d,\theta],\ \Lambda_\alpha[a,\lambda_\alpha,d,\theta]\ \text{and}\ D = D[a,\lambda_\alpha,d,\theta]$$

are SW maps.

S_{NCSYM} is invariant under $\mathcal{N}=1$ SUSY:

- linearly realized in terms of the NC fields,
  (there is a local superfield formulation)
  and
- nonlinearly realized in terms of the ordinary fields.
  (no local superfield formulation exists, but a nonlocal one does, at least for U(1))

Susy transformations

$$A_\mu[\varphi,\theta] \to A_\mu^{(\epsilon)}[\varphi,\theta] = A_\mu[\varphi,\theta] + \delta_\epsilon A_\mu[\varphi,\theta]$$
$$\Lambda_\alpha[\varphi,\theta] \to \Lambda_\alpha^{(\epsilon)}[\varphi,\theta] = \Lambda_\alpha[\varphi,\theta] + \delta_\epsilon \Lambda_\alpha[\varphi,\theta]$$
$$D[\varphi,\theta] \to D^{(\epsilon)}[\varphi,\theta] = D[\varphi,\theta] + \delta_\epsilon D[\varphi,\theta]$$

φ stands for the ordinary fields

$$A_\mu^{(\epsilon)}[\varphi,\theta] = A_\mu[\varphi + \delta_\epsilon\varphi,\theta]$$
$$\Lambda_\alpha^{(\epsilon)}[\varphi,\theta] = \Lambda_\alpha[\varphi + \delta_\epsilon\varphi,\theta]$$
$$D^{(\epsilon)}[\varphi,\theta] = D[\varphi + \delta_\epsilon\varphi,\theta]$$

where

$$\delta_\epsilon A^\mu = i\epsilon^\alpha \sigma^\mu_{\alpha\,\dot\alpha}\bar\Lambda^{\dot\alpha} + i\bar\epsilon^{\dot\alpha}\bar\sigma^\mu_{\dot\alpha\,\alpha}\Lambda^\alpha,$$
$$\delta_\epsilon \Lambda_\alpha = (\sigma^{\mu\nu})_\alpha{}^\beta \epsilon_\beta F_{\mu\nu} + i\epsilon_\alpha D,$$
$$\delta_\epsilon D = -\epsilon^\alpha \sigma^\mu_{\alpha\,\dot\alpha} D_\mu \bar\Lambda^{\dot\alpha} + \bar\epsilon^{\dot\alpha}\bar\sigma^\mu_{\dot\alpha\,\alpha} D_\mu \Lambda^\alpha.$$

and, up to first order in θ, $\longrightarrow\longrightarrow\longrightarrow$

nonlinear susy trans.

$$\delta_\epsilon a_\mu = \tfrac{1}{4}\epsilon\sigma_\mu\bar\lambda - \tfrac{1}{4}\bar\epsilon\bar\sigma_\mu\lambda + \tfrac{1}{16}\theta^{\nu\rho}\Big[\{a_\nu, 2D_\rho(\epsilon\sigma_\mu\bar\lambda - \bar\epsilon\bar\sigma_\mu\lambda) - i[a_\rho, \epsilon\sigma_\mu\bar\lambda - \bar\epsilon\bar\sigma_\mu\lambda]\}$$
$$- \{\epsilon\sigma_n\bar\lambda - \bar\epsilon\bar\sigma_\nu\lambda, \partial_\rho a_\mu + f_{\rho\mu}\} - \{a_\nu, \partial_\rho(\epsilon\sigma_\mu\bar\lambda - \bar\epsilon\bar\sigma_\mu\lambda) + D_\rho(\epsilon\sigma_\mu\bar\lambda - \bar\epsilon\bar\sigma_m\lambda)$$
$$- D_\mu(\epsilon\sigma_\rho\bar\lambda - \bar\epsilon\bar\sigma_l\lambda)\}\Big] + \theta^2,$$

$$\delta_\epsilon\lambda_\alpha = -\epsilon_\alpha d + 2i\epsilon_\gamma(\sigma^{\mu\nu})^\gamma{}_\alpha f_{\mu\nu} + \tfrac{1}{4}\theta^{\nu\rho}\Big[-\tfrac{1}{4}\{\epsilon\sigma_\nu\bar\lambda - \bar\epsilon\bar\sigma_\nu\lambda, 2D_\rho\lambda_\alpha - i[a_\rho, \lambda_\alpha]\}$$
$$-\{a_\nu, 4iD_\rho(\epsilon_\gamma(\sigma^{\mu\rho})^\gamma{}_\alpha f_{\mu\lambda}) + 2[a_\rho, \epsilon_\gamma(\sigma^{\mu\lambda})^\gamma{}_\alpha f_{\mu\lambda}] + \tfrac{i}{4}[\epsilon\sigma_\rho\bar\lambda - \bar\epsilon\bar\sigma_l\lambda, \lambda_\alpha]\}\Big]$$
$$+ \theta^2$$

$$\delta_\epsilon d = i\bar\epsilon\bar\sigma^\mu D_\mu\lambda + i\epsilon\sigma^\mu D_\mu\bar\lambda + \tfrac{1}{4}\theta^{\nu\rho}\Big[2i\{f_{\mu\nu}, \bar\epsilon\bar\sigma^\mu D_\rho\lambda + \epsilon\sigma^\mu D_\rho\bar\lambda\}$$
$$+ i\{a_\nu, (\partial_\rho + D_\rho)(\bar\epsilon\bar\sigma^\mu D_\mu\lambda + \epsilon\sigma^\mu D_\mu\bar\lambda)\} - \tfrac{1}{4}\{\epsilon\sigma_\nu\bar\lambda - \bar\epsilon\bar\sigma_\nu\lambda, 2D_\rho d - i[a_\rho, d]\}$$
$$-\{a_\nu, 2D_\rho(i\bar\epsilon\bar\sigma^\mu D_\mu\lambda + i\epsilon\sigma^\mu D_\mu\bar\lambda) - i[a_\rho, i\bar\epsilon\bar\sigma^\mu D_\mu\lambda + i\epsilon\sigma^\mu D_\mu\bar\lambda]$$
$$+ \tfrac{i}{4}[\epsilon\sigma_\rho\bar\lambda - \bar\epsilon\bar\sigma_\rho\lambda, d]\}\Big] + \theta^2.$$

Comments

COMMENTS ON THE NONLINEAR SUSY TRANSFORMATIONS

- They are truly SUSY transformations,

  $$[\delta_{\epsilon_2}, \delta_{\epsilon_1}](\text{fields}) = i(\epsilon_2\sigma^\mu\bar{\epsilon}_1 - \epsilon_1\sigma^\mu\bar{\epsilon}_2)\partial_\mu(\text{fields}) + \text{gauge transformations},$$

  due to the fact that the NC fields carry a linear realisation of $\mathcal{N} = 1$ SUSY. This holds at any order in θ –see CPM and C.Tamarit, JHEP 2008.
- $\delta_\epsilon a_\mu$, $\delta_\epsilon \lambda_\alpha$ and $\delta_\epsilon d$ belong to the Lie algebra of the ordinary gauge group only for U(N) in the fundamental rep. and its siblings, i.e.,
- for an arbitrary Lie algebra they take values on the enveloping-algebra: they are not ordinary field variations which are also ordinary fields.

Susy for NC GUTS? I

- For simple gauge groups it still makes sense to consider

$$S = \frac{1}{2g^2}\mathrm{Tr}\int d^4x\,[-\frac{1}{2}\,F^{\mu\nu}\star F_{\mu\nu} - 2i\,\Lambda^{\alpha}\star\sigma^{\mu}_{\alpha\,\dot\alpha}D_\mu\bar\Lambda^{\dot\alpha} + D\star D]$$

where

$$A_\mu = A_\mu[a,\lambda_\alpha,d,\theta],\ \Lambda_\alpha[a,\lambda_\alpha,d,\theta]\ \text{and}\ D = D[a,\lambda_\alpha,d,\theta]$$

are SW maps.

- It looks like as if it were a SUSY invariant NC action, but, the CATCH is that the invariance is under

$$\begin{aligned} &A_\mu[\varphi,\theta]\to A^{(\epsilon)}_\mu[\varphi,\theta] = A_\mu[\varphi,\theta]+\delta_\epsilon A_\mu[\varphi,\theta]\\ &\Lambda_\alpha[\varphi,\theta]\to \Lambda^{(\epsilon)}_\alpha[\varphi,\theta] = \Lambda_\alpha[\varphi,\theta]+\delta_\epsilon \Lambda_\alpha[\varphi,\theta]\\ &D[\varphi,\theta]\to D^{(\epsilon)}[\varphi,\theta] = D[\varphi,\theta]+\delta_\epsilon D[\varphi,\theta]\end{aligned}$$

φ stands for the ordinary fields

and $A^{(\epsilon)}_\mu[\varphi,\theta],\Lambda_\alpha[\varphi,\theta],D^{(\epsilon)}[\varphi,\theta]$ ARE NOT SW MAPS!. $\longrightarrow\longrightarrow\longrightarrow\longrightarrow\longrightarrow\longrightarrow$

Susy for NCGUTS? II

- Those transformations are therefore defined from the space of NC "physical" fields –those defined by the SW map— into the space of general fields taking values on the enveloping algebra.
- The so remaining question is whether this invariance has any physical consequences.
- In this regard, it is worth noticing that –unlike in U(N) case–the "SUSY" NC SU(N) theory thus obtained is one-loop and first-order-in-θ (off-shell) renormalisable. This would be just a lucky chance unless there is a symmetry at work, at first order in θ, that relates the gluon and gluino dynamics –see CPM and C. Tamarit JHEP 0911 (2009) 092.

Open problems

PRESSING PROBLEMS

- For SO(10), E_6, inclusion of a phenomenologically relevant NC Higgs potencial: a non trivial issue as implied by the construction of Yukawa terms.
- Study of the one-loop renormalisability of those GUTS at first order in θ.
- Construction and analysis of the properties of NC SO(10), E_6 "SUSY".
- Study of the phenomenological implications of NC SO(10), E_6 GUTs.
- Gauge anomalies, Fujikawa's method and index theorems.
  Recall that the index theorem in 2n+2 dimensions gives the gauge anomaly in 2n dimensions, that the index of the Dirac operator does not change under small deformations of it and that in our formalism we are considering small deformations of the ordinary Dirac operator $\Longrightarrow$ No θ-dependent anomalous terms.
- Will this NC GUTS eventually find accommodation within F-theory?