The renormalization group flow of gravity and scalar fields and implications for cosmology

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17/09/2010

- Observation indicates that the expansion of the universe has phases of acceleration (inflation in the early universe and late time acceleration)
- Driven by the cosmological constant, a scalar field, or modified versions of gravity
- Influence of quantum gravitational effects
- Gravity with matter is nonrenormalizable at one loop level
- Asymptotic safety
  - Renormalization group flow approaches a UV fixed point
  - The UV critical surface has finite dimension
- Calculate beta functions for gravity coupled to scalar fields

Functional Renormalization Group Equation

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left(\frac{\delta^2\Gamma_k}{\delta\Phi\delta\Phi} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k$$

- Use truncations including gravity-scalar field interactions

$$\Gamma_k[\phi] \quad = \quad \sum_i g_i(k)\mathcal{O}^i[\phi] = \sum_i \tilde{g}_i k^{d_i} \mathcal{O}^i[\phi]$$

Scalar-Tensor theory

$$\Gamma_k[h_{\mu\nu}, c_\mu, \bar{c}_\nu, \phi] = \int d^4x\sqrt{g}\left\{F(\phi^2, R) + \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\,\nabla_\nu\phi\right\} + S_{GF} + S_{gh}$$

$$F(\phi^2, R) = V_0(\phi^2) + V_1(\phi^2)\,R + \cdots + V_p(\phi^2)\,R^p = \sum_{a=0}^{p} V_a(\phi^2)\,R^a$$

- Arbitrary potential terms, analytic and Taylor expandable around $\phi^2 = 0$
- Expand around spherical background

G. Narain, C.R. (2009)

Pure gravity

- $f(R) = \sum_{i=0}^{n} g_i R^i$
  $\Rightarrow$ three dimensional UV critical surface up to $n = 8$

  A. Codello, R. Percacci, C. R. (2007, 2008), P. Machado, F. Saueressig (2007)
- If the minimally coupled theory with $F(\phi^2, R) = f(R)$ has a non-Gaussian fixed point, there exists a fixed point also when matter interactions are included

Gaussian matter fixed point

- At the Gaussian matter fixed point the $\tilde{V}_a$ are $\tilde{\phi}^2$-independent

$$\tilde{V}_a^{(i)}(0) = 0 \text{ for } i \geq 1$$

- Can be shown to exist for arbitrary potential
- For polynomial form

$$V_a(\phi^2)R^a = \sum_{i=0}^{q} \lambda_{2i}^a(k)\phi^{2i}R^a \Rightarrow \tilde{\lambda}_{2i}^a(k) \rightarrow 0 \text{ for } i \geq 1$$

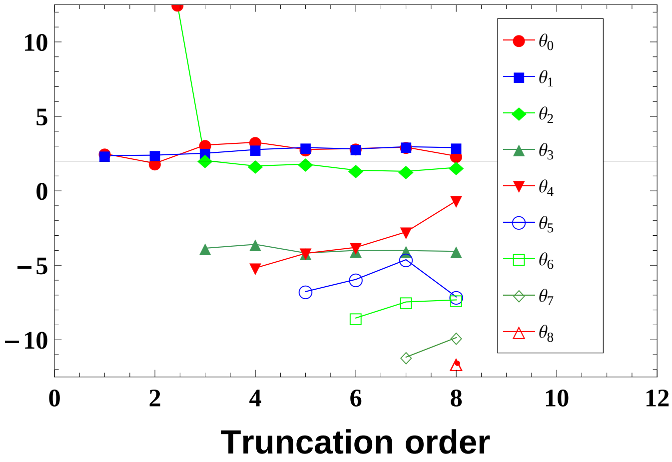
Linearized flow

$$M = \begin{pmatrix} M_{00} & M_{01} & 0 & 0 & \cdots \\ 0 & M_{11} & M_{12} & 0 & \ddots \\ 0 & 0 & M_{22} & M_{23} & \ddots \\ 0 & 0 & 0 & M_{33} & \ddots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \text{ with } M_{ij} = \begin{pmatrix} \frac{\partial \beta^0_{2i}}{\partial \tilde{\lambda}^0_{2j}} & \cdots & \frac{\partial \beta^0_{2i}}{\partial \tilde{\lambda}^p_{2j}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \beta^p_{2i}}{\partial \tilde{\lambda}^0_{2j}} & \cdots & \frac{\partial \beta^p_{2i}}{\partial \tilde{\lambda}^p_{2j}} \end{pmatrix}$$

$$M_{ii} = 2i\,\mathbf{1} + M_{00} \;\; ; \qquad M_{i,i+1} = (i+1)(2i+1)M_{01}$$

$$\rho^a_{2i} = \rho^a_0 - 2\,i$$

Critical exponents



Linearized flow

- Convergent behaviour
- Three critical exponents with positive real part
  - Two > 2, one < 2
- Adding scalar field interactions will only give two more positive critical exponents

Scalar-tensor theory in cosmology

- Expect strong renormalization group effects in the early universe
- Correct the equations of motion by inserting scale-dependent couplings
- In the fixed point regime, assume $k \propto H$
- Require that the Bianchi identities are fulfilled and there is no energy transfer to matter

  M. Reuter, F. Saueressig (2005)

Standard cosmology

$$\begin{aligned}\dot{H} &= -\frac{\kappa^2}{2}\left((1+\omega)\rho + \dot{\phi}^2\right) \\ \dot{\rho} &= -3(1+\omega)H\,\rho \\ \ddot{\phi} &= -3H\dot{\phi} - \frac{\mathrm{dV}}{\mathrm{d}\phi} \\ H^2 &= \frac{\kappa^2}{3}\left(\rho + \frac{\dot{\phi}^2}{2} + V\right)\end{aligned}$$

$$\kappa^2 = 8\pi G$$

Dimensionless variables

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H} \; ; \quad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H} \; ; z = \frac{V'}{\kappa V} \; ; \quad \eta = \frac{V''}{\kappa^2 V}$$

$$\begin{aligned}
\frac{dx}{dN} &= 3x(1-x^2) + \sqrt{\frac{3}{2}}y^2 z - \frac{3}{2}x(1+\omega)(1-x^2-y^2), \\
\frac{dy}{dN} &= -xy\left(3x + \sqrt{\frac{3}{2}}z\right) - \frac{3}{2}y(1+\omega)(1-x^2-y^2), \\
\frac{dz}{dN} &= -\sqrt{6}\,x(\eta - z^2), \\
1 &= \frac{\kappa^2\rho}{3H^2} + x^2 + y^2 = \Omega + \Omega_\phi \qquad N = -\ln a
\end{aligned}$$

RG modified cosmology

$$
\begin{aligned}
\frac{dx}{dN} &= 3x\left(1 - A\,x^2\right) + \sqrt{\frac{3}{2}}y^2 z - \frac{3}{2}(1+\omega)\,A\,x(1-x^2-y^2) \\
\frac{dy}{dN} &= -xy\left(3x\,(A+B) + \sqrt{\frac{3}{2}}z\right) - \frac{3}{2}(1+\omega)\,(A+B)\,y(1-x^2-y^2) \\
\frac{dz}{dN} &= -\sqrt{6}x(\eta - z^2) + 3Cx^2 z + \frac{3}{2}(1+\omega)\,C\,z(1-x^2-y^2) \\
1 &= x^2 + y^2 + \Omega
\end{aligned}
$$

- A, B, C are functions of $\frac{d\ln G}{d\ln k}$, $\frac{d\ln V}{d\ln k}$, $\frac{d\ln V'}{d\ln k}$

Fixed point behaviour

- Various fixed points where
  - either the fluid component dominates,
  - or the scalar field dominates (either only the kinetic term or the potential or a mixture of both)
  - or scaling solution (neither fluid nor scalar field dominate)
- At a fixed point the equation of state parameter for the scalar field becomes constant

$$1 + \omega_\phi = 1 + \frac{p_\phi}{\rho_\phi} = \frac{2x^2}{x^2 + y^2}$$

- Attractivity properties of the fixed point can lead naturally to accelerated expansion

Renormalization group modified cosmology

- Fixed points are modified by RG effects
- Coincide with RG fixed point regime
- Distinctive features of asymptotically safe cosmology
- Conditions for inflationary regime at the fixed point

D. Litim, M. Hindmarsh, C.R. (unpublished)