Exact RG for few-body systems:application to dimer-dimer scattering

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Corfu, September, 2010

Motivation and general definitions

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- The exact relation $a_B = 0.6a_0$ was established by Petrov et al (Phys. Rev. Lett. 93, 090404 (2004)). However, it seems to be difficult to extend this approach to many-body physics of dilute Fermi gas.
- We calculate a_B/a_0 ration in the framework of functional RG (with many-body applications in mind)

dimer-dimer scattering with two-body forces

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- It is convenient to use dimer field describing a pair of correlated fermions
- The effective action is

$$\Gamma[\psi,\psi^\dagger,\phi,\phi^\dagger,k] = \int d^4x' \int d^4x \; \Big[\phi^\dagger \Pi(x,x',k)\phi$$
$$+\psi^\dagger \; (i\partial_t + \frac{1}{2M}\nabla^2)\psi - \frac{i}{2}g\left(\psi^{\mathrm{T}}\sigma_2\psi\phi^\dagger - \psi \dagger \sigma_2\psi^{\dagger\mathrm{T}}\phi\right)$$

Here $\Pi(x,x',k)$ is the scale dependent boson self-energy

dimer-dimer scattering with two-body forces

The evolution of the action is given by

$$\partial_k \Gamma = \frac{i}{2}\, Tr\left[(\partial_k R_F)\,(\Gamma^{(2)} - R)^{-1}_{FF}\right] - \frac{i}{2}\, Tr\left[(\partial_k R_B)\,(\Gamma^{(2)} - R)^{-1}_{BB}\right].$$

The self-energy flow is defined as

$$\partial_k \Pi(k) = \frac{\delta^2}{\delta\phi\delta\phi^\dagger}\partial_k \Gamma|_{\phi=0}.$$

The ERG equation has one-loop structure with cutoff R acting as an infrared regulator which goes to zero at vanishing scale, where physics is defined.

Initial conditions are defined at large scale, where theory is simple.

Convenient tool to provide a link between vacuum and in-medium physics.

Many body physics: Normal Fermi liquid, pairing, symmetry breaking, finite temperature effects, etc

Few body physics: (composite) boson-boson and boson-fermion scattering

dimer-dimer scattering with two-body forces

Using the evolution equation for Γ one can get

$$\partial_k \Pi(k) = iTr\left[(\partial_k R_F)(\Gamma^{(2)}_{FF} - R_F)^{-1}\Gamma^{(3)}_{FF\phi}(\Gamma^{(2)}_{FF} - R_F)^{-1}\Gamma^{(3)}_{FF\phi}(\Gamma^{(2)}_{FF} - R_F)^{-1}\right]$$

Here $\Gamma^{(3)}_{FF\phi}$ is the derivative of $\Gamma^{(2)}_{FF}$ with respect to field ϕ.
After the pole integration one can get the 1-loop expression for the $\Pi(k) - \Pi(0)$ containing the fermion propagators modified with cutoff $R_F(q, k)$

dimer-dimer scattering with two-body forces

We used the type of the regulator (optimised cut-off) which cancels the momentum dependence in the fermion propagator

$$R_F(\vec{q}, k) = \frac{k^2 - q^2}{2M}\theta(k - q)$$

The on-shell fermion-fermion scattering amplitude in the physical limit ($k \rightarrow 0$) is given by

$$\frac{1}{T(p)} = \frac{1}{g^2}\Pi(P_0, P, 0),$$

whereP_0, P and $p = (MP_0 - P^2/4)^{1/2}$ are the total energy, total and relative momenta of two fermions. Integrating the ERG equation and using the effective range expansion one can find the self-energy at running scale k

$$\Pi(P_0, P, k) = \frac{g^2 M}{4\pi^2}\left[-\frac{4}{3}k + \frac{\pi}{a} + \left(\frac{8}{3k} - \frac{\pi}{2}r_e\right)(MP_0 - \frac{P^2}{4}) - \frac{P^3}{24k^2} + \ldots\right] \quad (1)$$

dimer-dimer scattering with two-body forces

- Dimer wave-function and mass renormalisation factors are

$$Z_\phi(k) = \frac{\partial}{\partial P_0}\Pi(P_0, P, k), \qquad \frac{1}{4M}Z_m(k) = -\frac{\partial}{\partial P^2}\Pi(P_0, P, k)$$

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$$Z_\phi(k) = Z_m(k) = \frac{g^2}{4\pi^2}\left(\frac{8}{3k} - \frac{\pi}{2}r_e\right) \qquad (2)$$

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$$-\frac{2}{(2\pi)^4}\partial_k u_2(P_0, k) = \frac{\delta^2}{\delta\phi^2(P_0/2)\delta\phi^{\dagger 2}(P_0/2)}\partial_k\Gamma|_{\phi=0}.$$

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- Let's first look at mean field results (D-D rescattering is neglected).

dimer-dimer scattering with two-body forces

The evolution of u_2 is given by

- $$\partial_k u_2 = -\frac{3g^4}{4} \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{\partial_k R_F}{[(E_{FR}(\vec{q},k) - P_0/4 - i\epsilon)]^4} .$$

  Where $E_{FR}(\vec{q},k) = \frac{1}{2M} q^2 + R_F(q,k)$ and P_0 is twice the binding energy of a pair $P_0 = -2/M/a^2$ with a being fermion scattering length

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- Explicit calculations give $u_2(0) = \frac{1}{16\pi}M^3 g^4 a^3$ and $Z_\phi(0) = \frac{1}{8\pi}M^2 g^2 a$

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$$T_{BB} = \frac{8\pi}{m}a_{BB} = \frac{2u_2(0)}{Z_\phi^2} = \frac{8\pi a}{M} \rightarrow a_{BB} = 2a.$$

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- This is well known MF result which is far away from the exact value $a_{BB} = 0.6a$ (D.S.Petrov et al Phys. Rev. Lett. 93, 090404 (2004))

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$$\partial_k u_2|_B = \frac{u_2^2(k)}{2Z_\phi^3(k)} \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{\partial_k R_B}{[(E_{BR}(\vec{q},k) - P_0/2 - i\epsilon)]^4},$$

where

$$E_{BR}(\vec{q},k) = \frac{1}{4M} q^2 + \frac{u_1(k)}{Z_\phi(k)} + \frac{R_B(q,k)}{Z_\phi(k)},$$

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$$R_B(\vec{q},k) = Z_\phi(k) \frac{(c_B k)^2 - q^2}{4M} \theta(c_B k - q)$$

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- c_B sets the relative scale of the fermion and boson regulators. Using $c_B = 1$ gives $a_{BB} = 1.13a$ and taking $c_B = (2)^{1/2}$ gives $a_{BB} = 0.75a$ (same as in S.Diehl et al, Phys. Rev. A 76, 021602(R) (2007))

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- First guess - three body forces in simplest possible form (with applications to many-body physics in mind)
- The ERG studies of few body physics have just began (Diehl et al., Moroz et al., Birse et al)

adding three-body forces

- We add the three body vertex with energy/momentum independent coupling to the effective action

$$-\lambda\phi^\dagger\phi\psi^\dagger\psi$$

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- The modified equation for u_2 is

$$\partial_k u_2 = -\frac{3g^4}{4}\int\frac{d^3\vec{q}}{(2\pi)^3}\frac{\partial_k R_F}{E_{FR}^4} - 2\lambda g^2\int\frac{d^3\vec{q}}{(2\pi)^3}\frac{\partial_k R_F}{E_{FR}^3} + \frac{u_2^2}{2Z_\phi}\int\frac{d^3\vec{q}}{(2\pi)^3}\frac{\partial_k R_B}{E_{BR}^2}$$

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- The evolution of lambda is extracted from

$$\partial_k \lambda = -\frac{i}{2} \frac{\delta^4}{\delta\phi^\dagger \delta\phi \delta\psi^\dagger \delta\psi} \left[\partial_k R (\Gamma^{(2)} - R)^{-1} \right].$$

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- The corresponding evolution equations are

$$D_l = \lambda^2 \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{\partial_k(R_F Z_\phi) + \partial_k R_B}{(E_{FR,D} Z_\phi + E_{BR,D})^2},$$

$$D_t^F = g^2\lambda \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{\partial_k R_F (E_{BR,D} + 2 Z_\phi E_{FR,D})}{E_{FR,D}^2 (E_{FR,D} Z_\phi + E_{BR,D})^2},$$

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where $E_{FR,D} = E_{FR} - \mathcal{E}_D/2$ and $E_{BR,D} = E_{BR} - \mathcal{E}_D$.

adding three-body forces

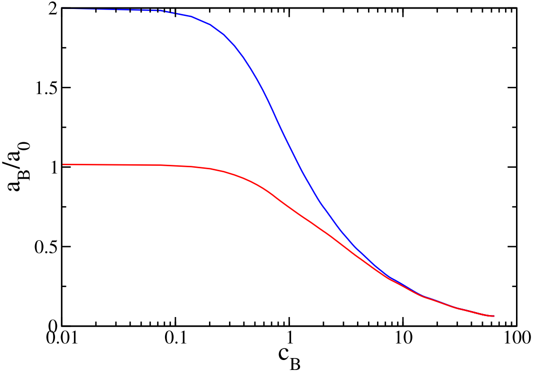

Figure: Dimer-Dimer to Fermion-Fermion scattering lengths ratio as a function of the scale parameter c_B, blue/red curves - results without/with 3 body forces

adding three-body forces

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- Choice $c_B \simeq 1$ gives $a_B/a_0 = 0.75$. The "optimal" choice $c_B \simeq \sqrt(2)$ gives $a_B/a_0 \simeq 0.7$

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- Further terms are needed to get a wider stability window
- Getting completely regulator independent results would require four body forces and complete energy/momentum dependence of the couplings

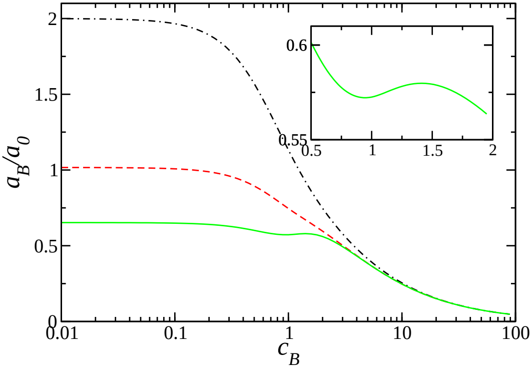
Adding 4-body forces

We add the simplest 4-body terms

$$-(\gamma(\phi^\dagger\phi^\dagger\phi\phi\psi\psi+\phi^\dagger\phi\phi\psi^\dagger\psi^\dagger)+\beta\phi^\dagger\psi^\dagger\psi^\dagger\phi\psi\psi)$$

The 4-body couplings γ, β are assumed to be energy independent. They vanish at the microscopic UV scale and pick up nonzero values in the process of evolution

adding 4-body forces



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- The value of a_B/a_0 is close to the results of exact QM calculations (0.6)

Adding 4-body forces

- The results do not depend on c_B in wide region
- The value of a_B/a_0 is close to the results of exact QM calculations (0.6)
- Some dependence on c_B starting from $c_B \simeq 2$ is still there (integrating out fermions first)

Conclusion

- Using the Effective Action with energy/momentum independent 2, 3 and 4-body forces brings the value of a_B/a_0 to an agreement with the exact QM calculations

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- Things to do include:
- Nuclear few-body physics (inclusion of spin/isospin D.O.F), energy/momentum dependence, 3- and 4- body forces in many-body environment