Scaling, finite-volume effects and the chiral phase transition in QCD

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ERG 2010, Corfu, Greece, 14 September 2010

[Peter Piasecki, Jens Braun, and B. Klein, arXiv: 1008.2155]

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The chiral phase transition in QCD

- dominated by chiral degrees of freedom: pions
- for two massless quark flavors, expect second-order phase transition in O(4) (d = 3) universality class
- *finite quark mass* explicitly breaks symmetry: *crossover*
- lattice QCD with staggered implementation of fermions: possibly second-order phase transition in O(2) (d = 3) universality class

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→ finite volume cuts off long-range fluctuation and impacts critical scaling behavior!

Scaling behavior in infinite volume

- two relevant couplings: temperature T and symmetry breaking (quark mass) H

$$t = (T - T_c)/T_0, \qquad h = H/H_0$$

- Scaling function for the order parameter **M** (f_π , $\langle\bar{\psi}\psi\rangle$):

$$M(t, h = 0) = (-t)^{\beta} \qquad M(t = 0, h) = h^{1/\delta}$$

$$M(t, h) = h^{1/\delta} f_M(z) \qquad z = t/h^{1/(\beta\delta)}$$

- Scaling function for the susceptibility χ_σ :

$$\chi_\sigma(t, h) = \frac{\partial M}{\partial H}(t, h) = \frac{1}{H_0} h^{1/\delta - 1} f_\chi(z)$$

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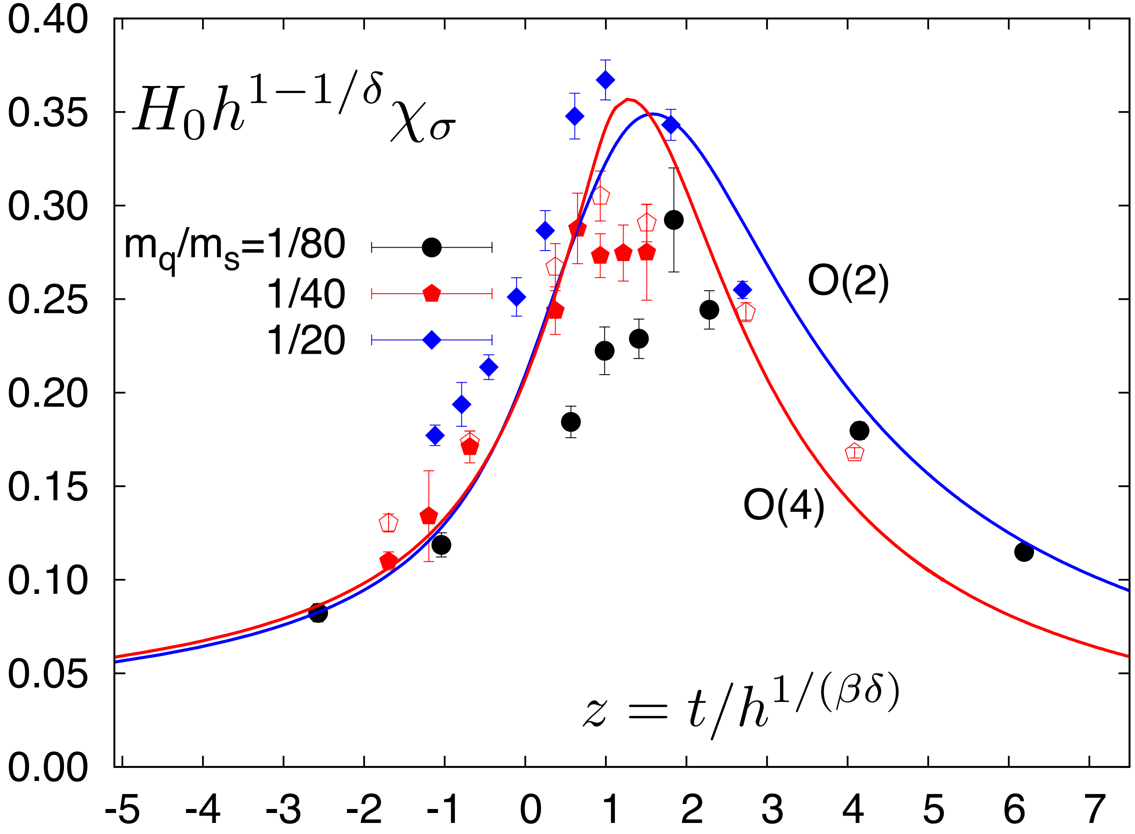
Scaling analysis in lattice QCD

- Scaling for 2+1 staggered flavors:
  susceptibility χ_σ from light quark condensate

$$\chi_\sigma \sim \frac{\partial}{\partial m}\langle\bar{\psi}\psi\rangle$$

$m_\pi > 75$ MeV

filled symbols: L = 32 a
(8 fm)
open symbols: L = 16 a
(4 fm)



[S. Ejiri, F. Karsch et al., arXiv:0909.5122]

Scaling analysis with the functional RG

advantages:

- both chiral limit and large quark masses accessible
- infinite-volume limit and small volumes accessible

previous work: (not exhaustive)

- scaling in the O(N) model [N. Tetradis and C. Wetterich, Nucl. Phys. B422 (1994) 541; O. Bohr, B.J. Schaefer, and J. Wambach, Int. J. Mod. Phys. A16 (2001) 3823; D. F. Litim and J. M. Pawlowski, Phys. Lett. B 516 (2001) 197; J. Braun and B. Klein, Phys. Rev. D 77 (2008) 096008, EPJ C 63 (2009) 443.]
    - $d = 3$ dimensions, no field-theoretical temperature
    - analysis of the scaling region only (small symmetry breaking)
- scaling in the quark-meson model [B. Stokic, B. Friman, K. Redlich, arXiv:0904.0466]
    - not for realistic pion masses or lattice volumes

Questions

- What happens for realistic pion masses?
- What happens for current lattice volumes?
- Are there additional finite-size effects?

The quark-meson model

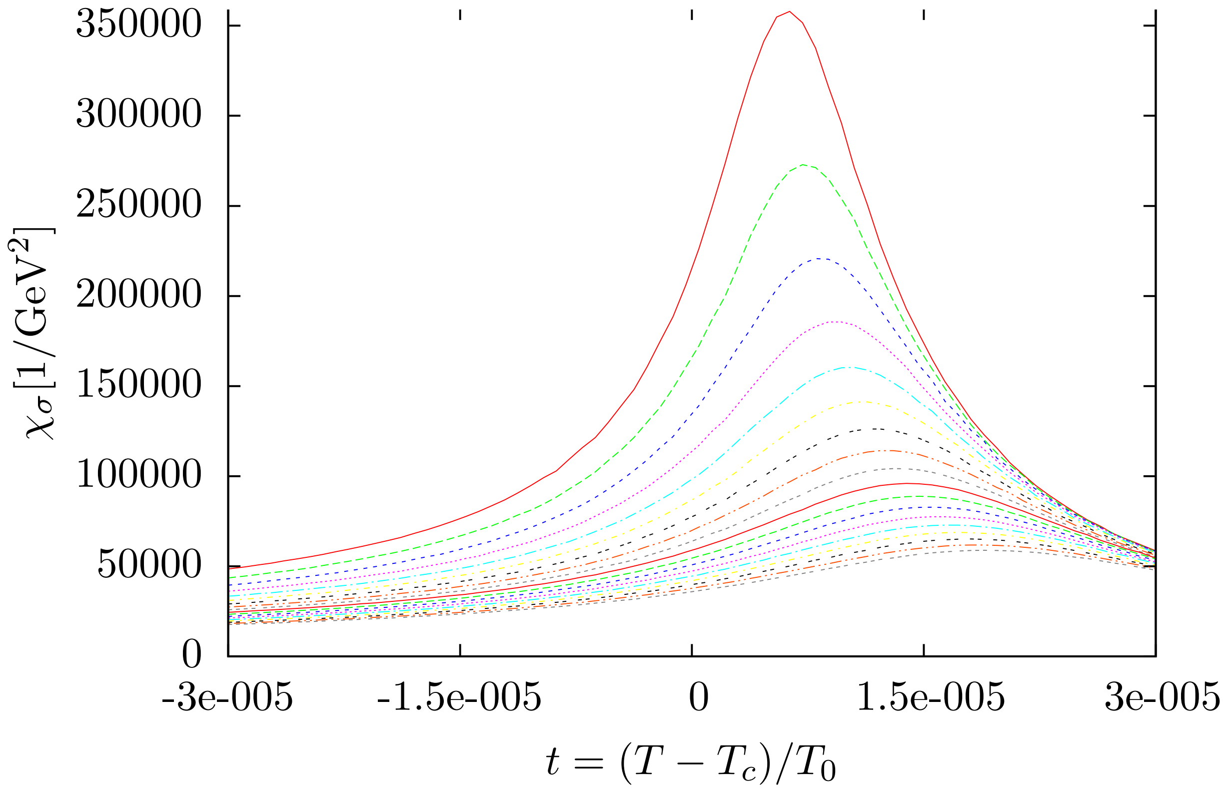
- a model for chiral symmetry breaking
- no gauge degrees of freedom

$$\Gamma_\Lambda[\bar\psi,\psi,\sigma,\vec\pi] = \int \mathrm{d}^4x \Big\{ \bar\psi(\mathrm{i}\,\partial\!\!\!/\,)\psi + g\bar\psi(\sigma + \mathrm{i}\gamma_5\vec\tau\cdot\vec\pi)\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec\pi\,)^2 + U_\Lambda(\sigma,\sigma^2+\vec\pi^2) \Big\}$$

- chiral symmetry breaking: SU(2) × SU(2) → SU(2)
  as O(4) → O(3) (meson sector) $\langle\sigma\rangle \neq 0$
- specify effective action for the model at initial scale Λ
- use functional Renormalization Group
  (Wetterich equation) to obtain effective action
  [C.Wetterich, Phys. Lett. B 301 (1993) 90.]

Longitudinal Susceptibility χ_σ

- susceptibility χ_σ for small values of $m_\pi < 0.9$ MeV



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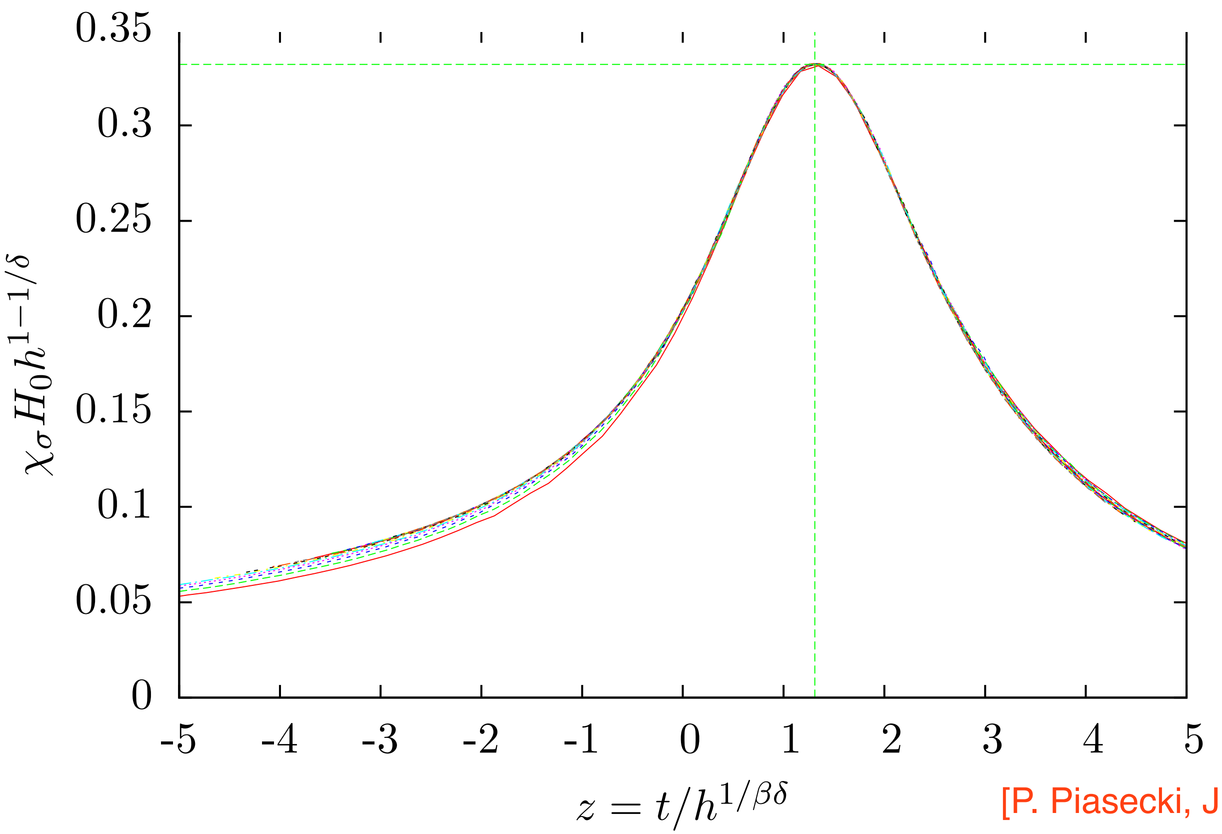
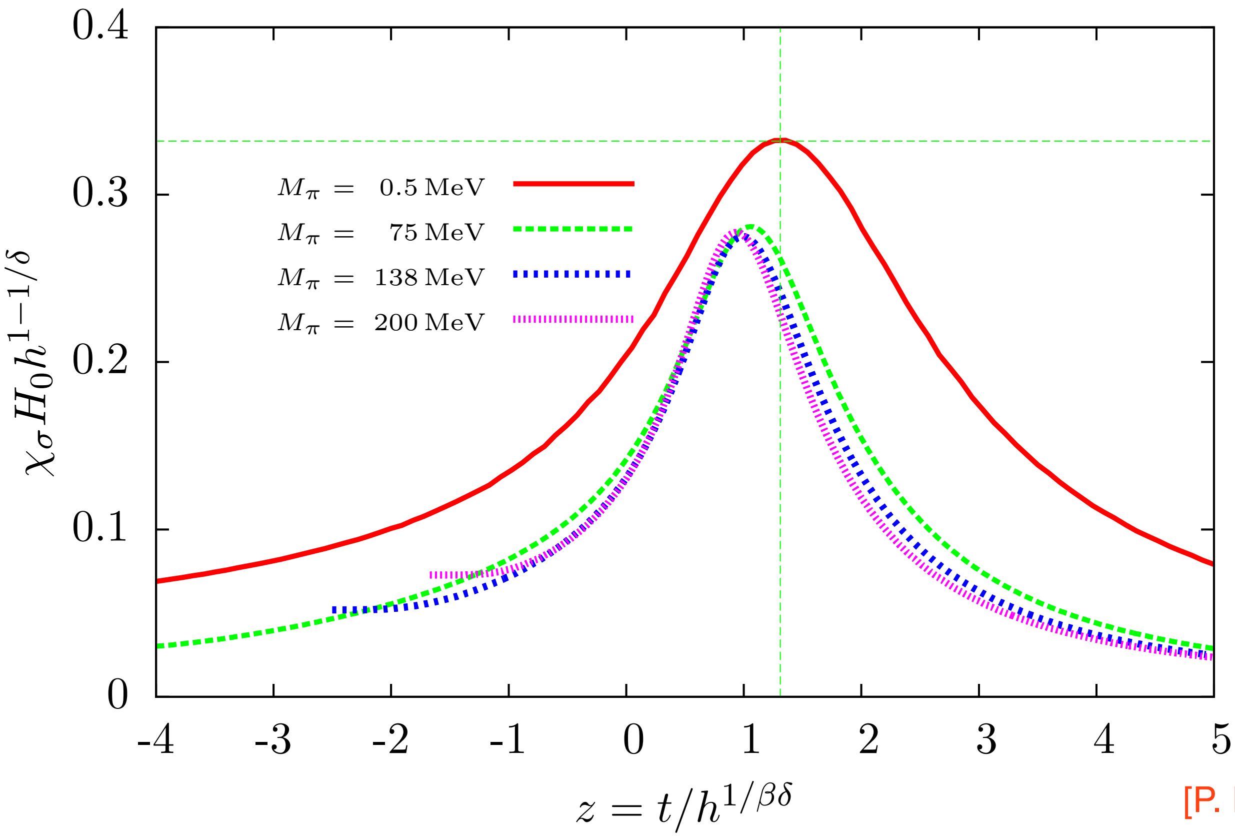


[P. Piasecki, J. Braun, and B. Klein (2010)]

Longitudinal Susceptibility χ_σ

- rescaled susceptibility $\chi_\sigma H_0 h^{1\text{-}1/\delta}$ for realistic values of m_π



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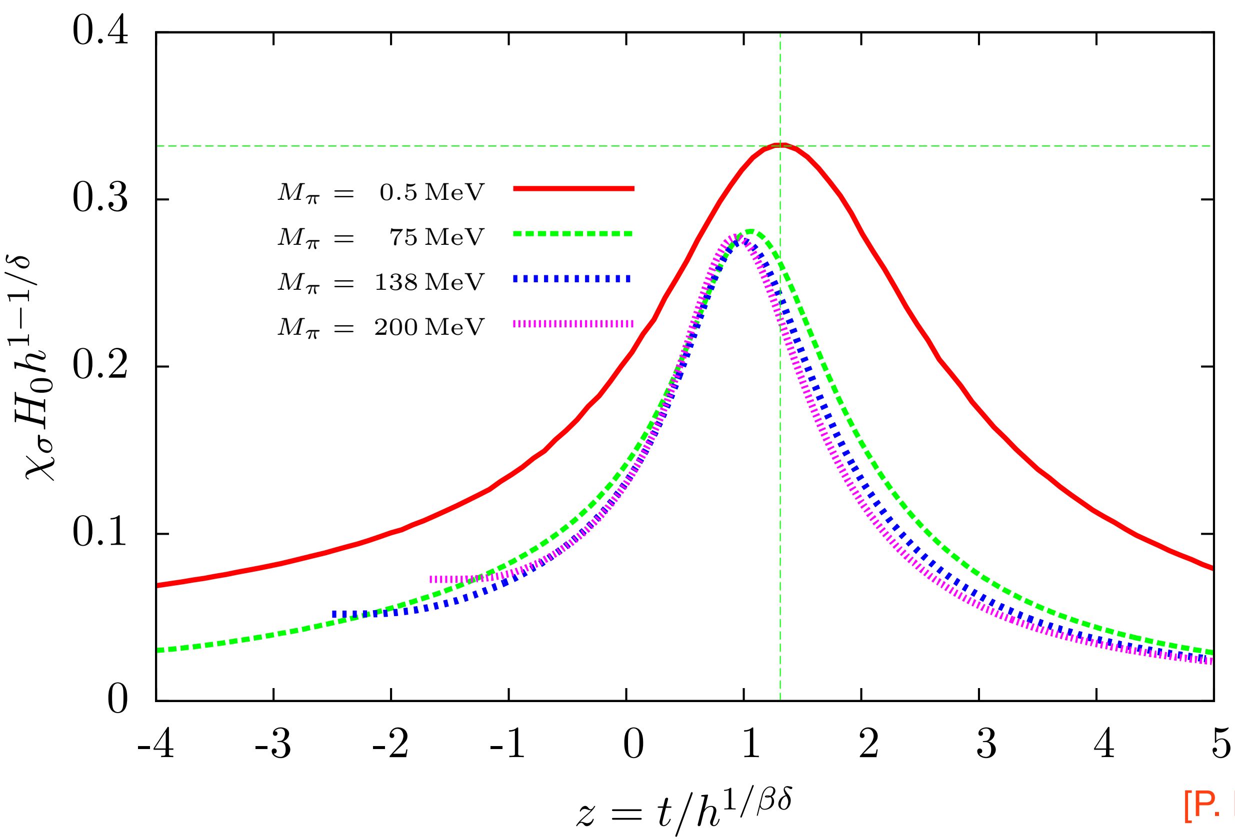

- apparent scaling for large quark masses

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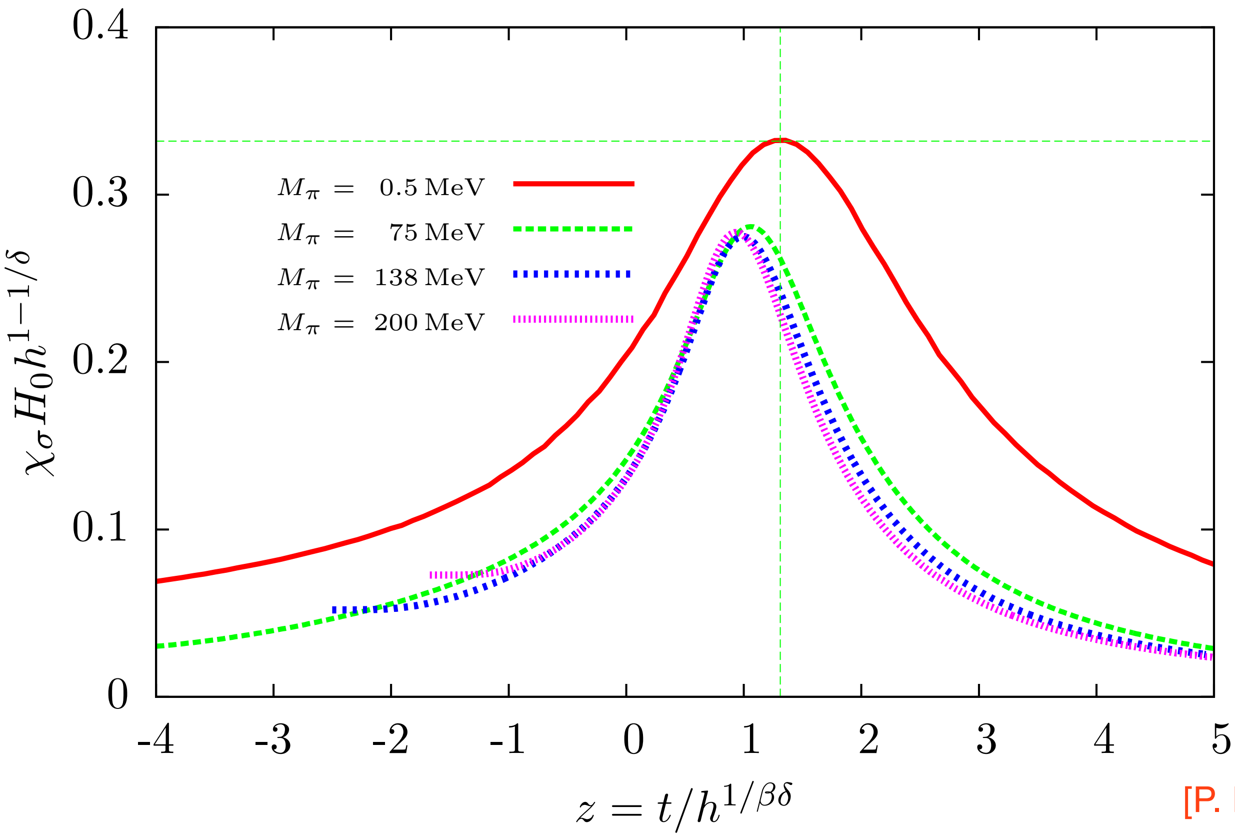

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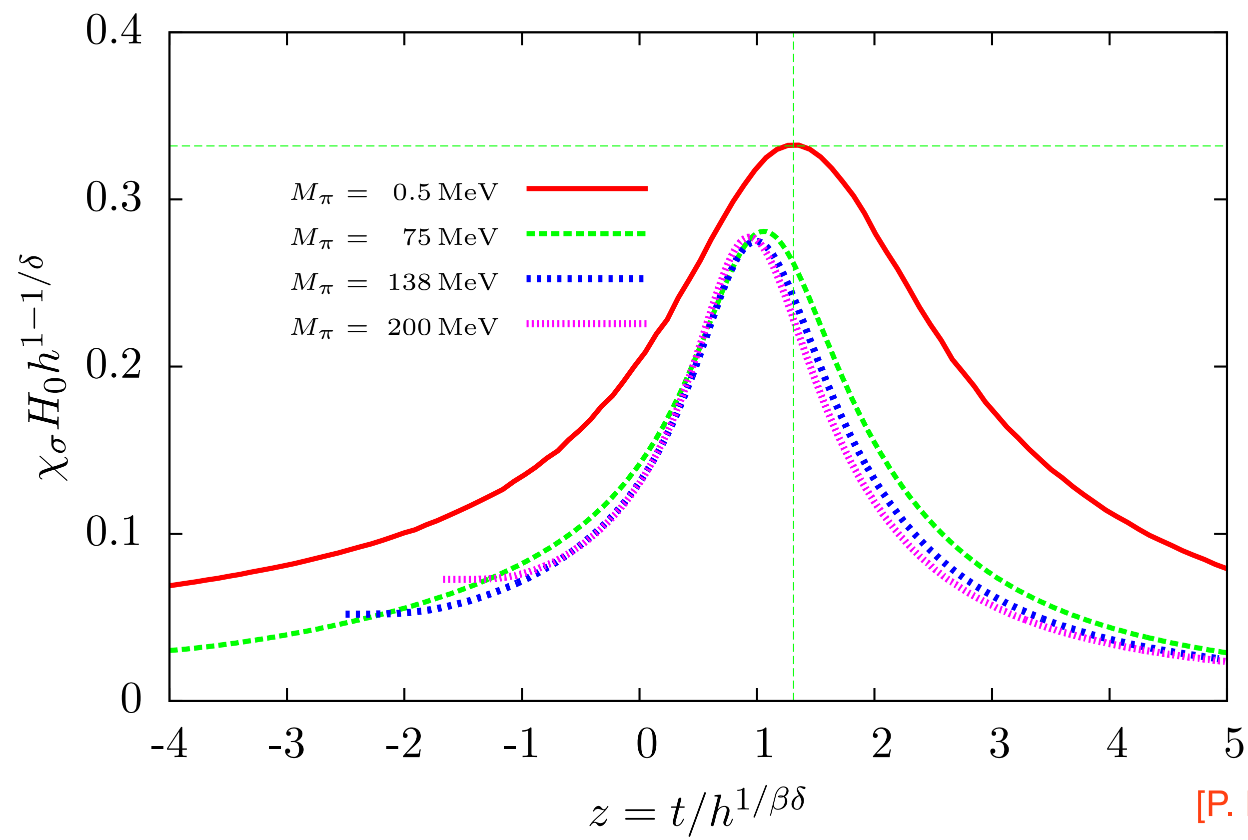

- apparent scaling for large quark masses
- differs from actual scaling function!
- sensitivity to quark mass larger than in QCD

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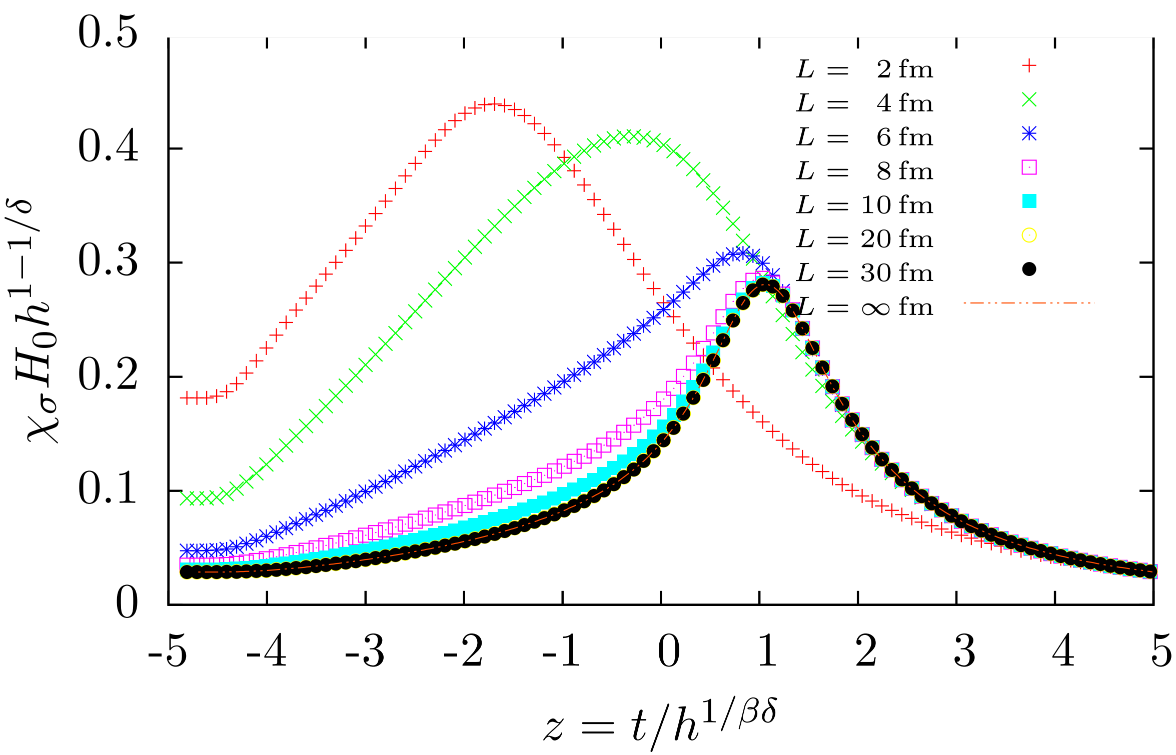
Infinite volume scaling in finite volume?

- rescaled susceptibility $\chi_\sigma H_0 h^{1-1/\delta}$ in finite volume



[P. Piasecki, J. Braun, and B. Klein (2010), arXiv:1008.2155]

- $m_\pi = 75$ MeV
- deviations from infinite-volume scaling for $L < 6$ fm
- effects probably weaker in lattice QCD

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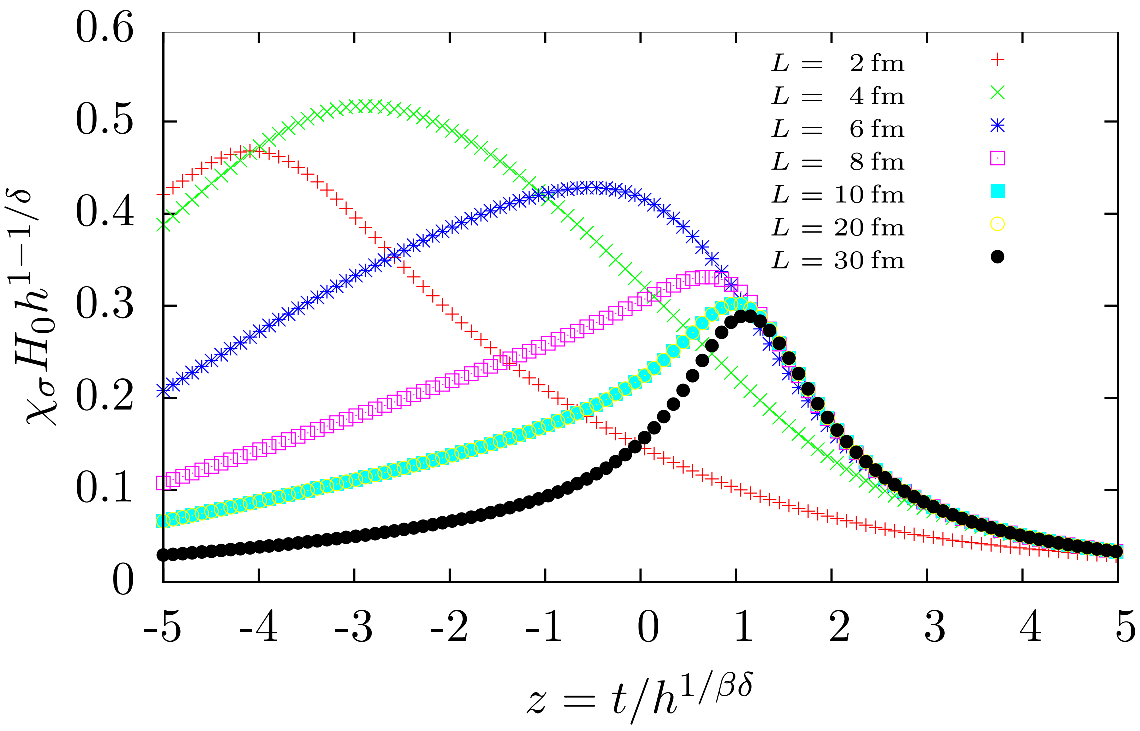

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- $m_\pi = 48$ MeV
- deviations from infinite-volume scaling for $L < 10$ fm

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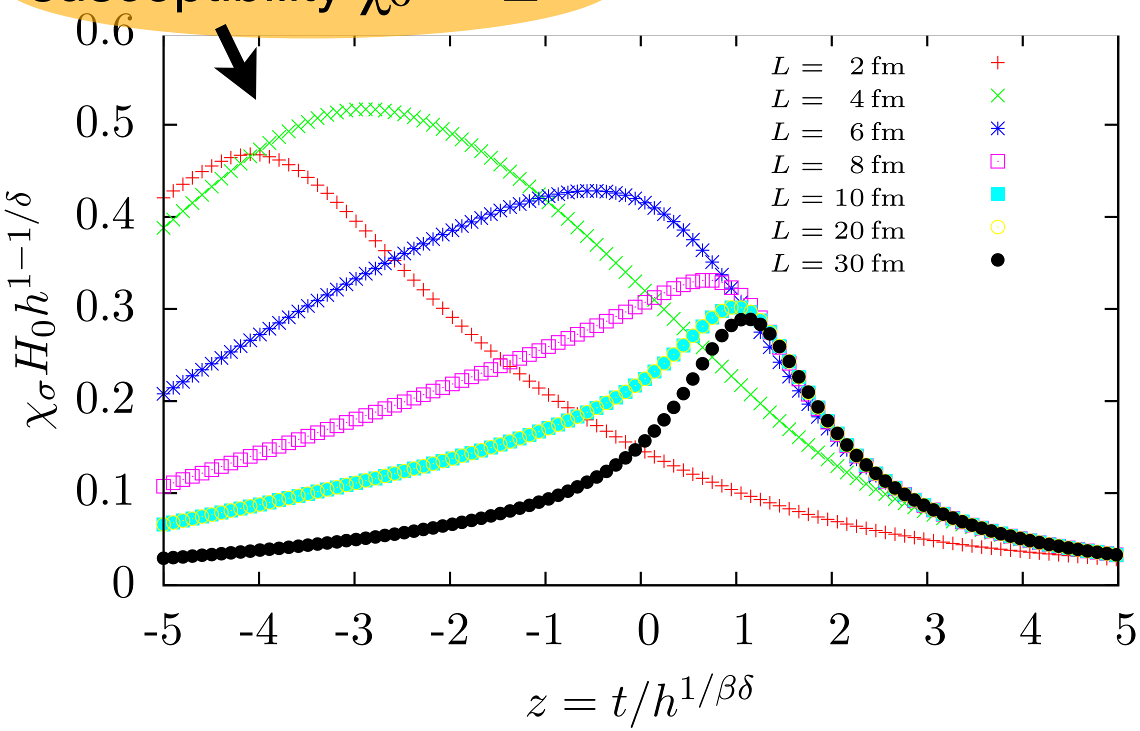

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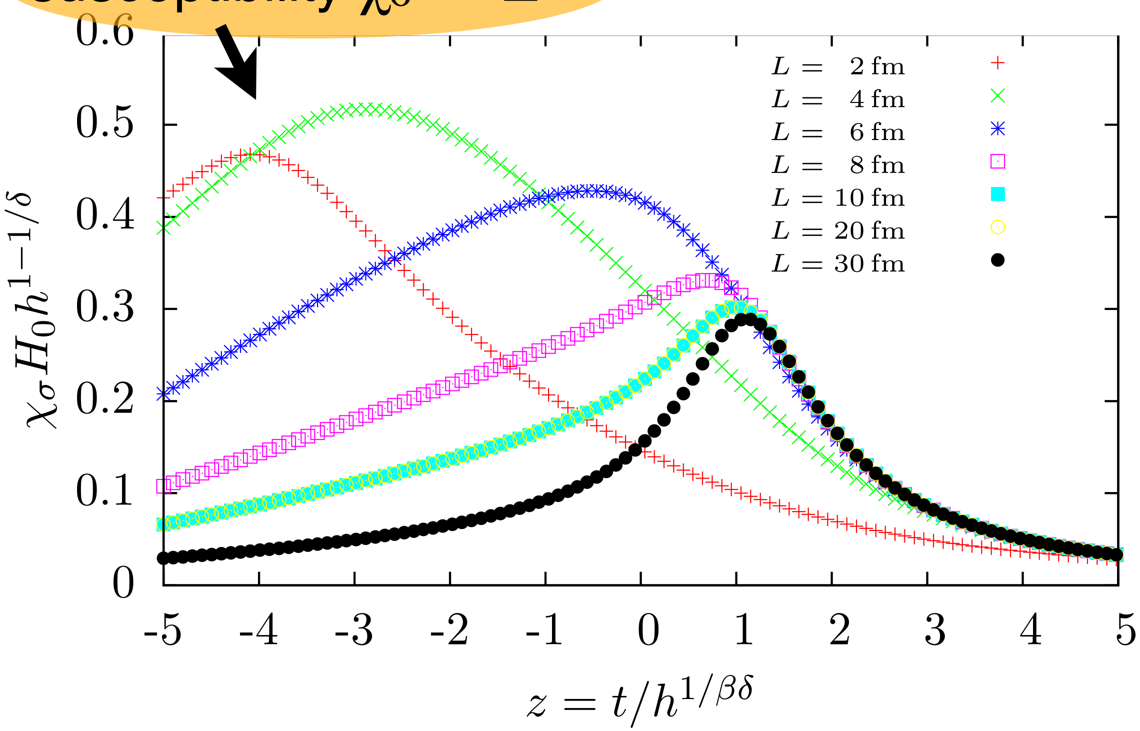

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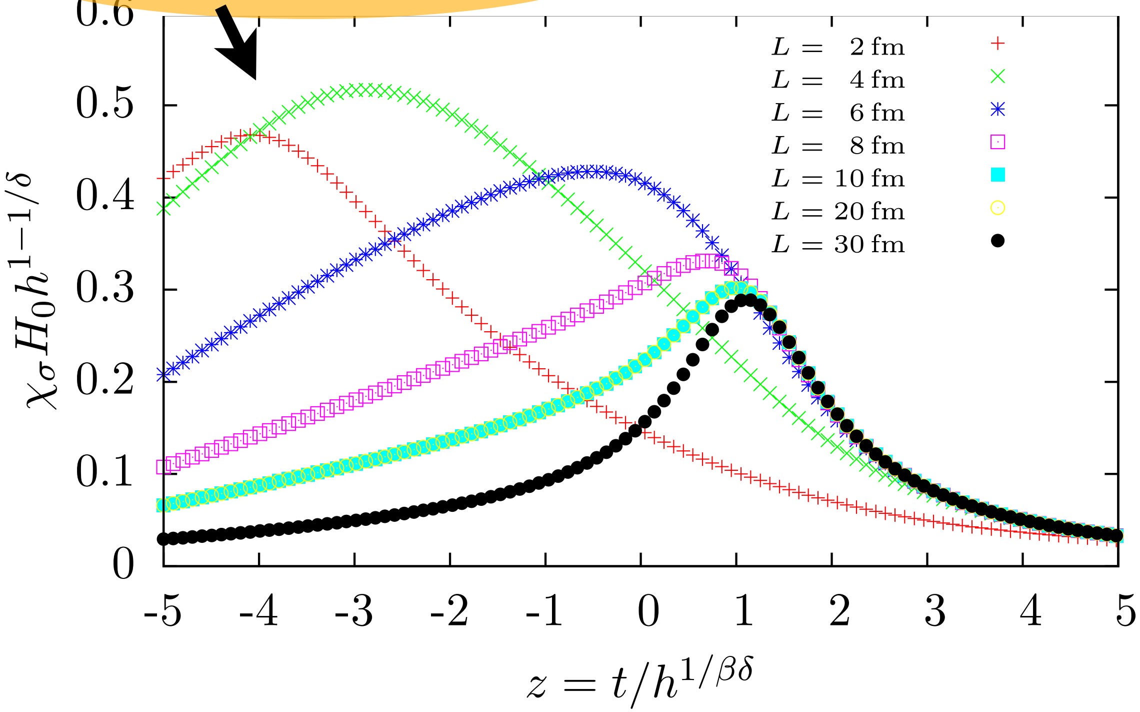

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→ expect finite-volume effects in lattice QCD scaling analysis

Conclusions

- Scaling functions from the functional renormalization group for the analysis of the QCD chiral phase transition
- Results from a model for the chiral phase transition
- Current quark masses used in lattice simulations lead to significant deviations from expected scaling behavior
- Current volumes used in lattice simulations lead to significant deviations