Fluctuations and the QCD phase diagram

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ERG 2010: 5th International Conference

on

the Exact Renormalization Group

Corfu, Greece

QCD Phase Transitions

QCD $\longrightarrow$ two phase transitions:

1. restoration of chiral symmetry

   $SU_{L+R}(N_f) \longrightarrow SU_L(N_f) \times SU_R(N_f)$

   order parameter:

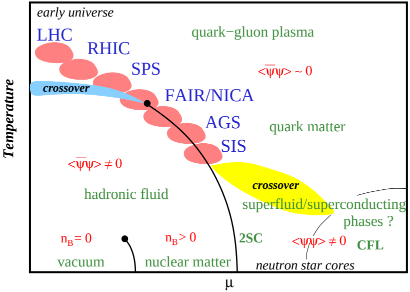
   $$\langle \bar{q}q \rangle \begin{cases} > 0 \Leftrightarrow \text{symmetry broken}, \ T < T_c \\ = 0 \Leftrightarrow \text{symmetric phase}, \ T > T_c \end{cases}$$

2. de/confinement

   order parameter: Polyakov loop variable

   $$\Phi \begin{cases} = 0 \Leftrightarrow \text{confined phase}, \quad T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase}, \ T > T_c \end{cases}$$

   $$\Phi = \left\langle \mathrm{tr}_c \mathcal{P} \exp \left(i \int_0^\beta d\tau A_0(\tau, \vec{x}) \right) \right\rangle / N_c$$



At densities/temperatures of interest only model calculations available

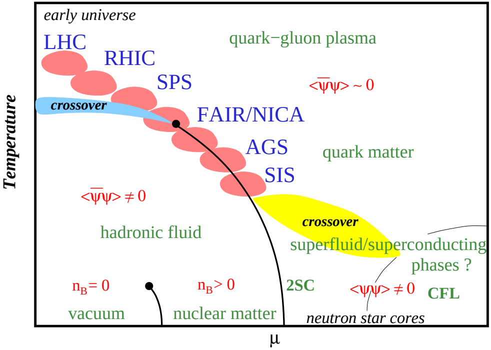
alternative: ➜ dressed Polyakov loop (or dual condensate)

it relates chiral and deconfinement transition to spectral properties of Dirac operator

effective models:

1. Quark-meson model — or other models e.g. NJL
2. Polyakov–quark-meson model — or PNJL models

The conjectured QCD Phase Diagram



At densities/temperatures of interest
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Open issues:

related to chiral & deconfinement transition

- ▷ existence of CEP?
- ▷ its location?
- ▷ additional CEPs? How many?
- ▷ coincidence of both transitions at $\mu = 0$?
- ▷ quarkyonic phase at $\mu > 0$?
- ▷ chiral CEP/ deconfinement CEP?
- ▷ so far only MFA results **effect of fluctuations** (e.g. size of crit. reg.)?
- ▷ ...

cf talk by Tina K Herbst (16.9.)

Outline

- ◦ **Three-Flavor Chiral Quark-Meson Model**
- ◦ **...with Polyakov loop dynamics**
- ◦ **The important role of fluctuations**
- ◦ **Functional Renormalization Group**

$\mathbf{N_f = 3}$ Quark-Meson (QM) model

- Model Lagrangian: $\mathcal{L}_{\text{qm}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

Quark part with Yukawa coupling $\mathbf{h}$:

$$\mathcal{L}_{\text{quark}} = \bar{q}(i\partial\!\!\!/ - h\frac{\lambda_a}{2}(\sigma_a + i\gamma_5\pi_a))q$$

Meson part: scalar $\sigma_{\mathbf{a}}$ and pseudoscalar $\pi_{\mathbf{a}}$ nonet

fields: $M = \sum_{a=0}^{8} \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$

$$\mathcal{L}_{\text{meson}} = \text{tr}[\partial_\mu M^\dagger \partial^\mu M] - m^2 \text{tr}[M^\dagger M] - \lambda_1(\text{tr}[M^\dagger M])^2 - \lambda_2 \text{tr}[(M^\dagger M)^2] + c[\det(M) + \det(M^\dagger)]$$
$$+ \text{tr}[H(M + M^\dagger)]$$

- explicit symmetry breaking matrix: $H = \sum_a \frac{\lambda_a}{2} h_a$
- $U(1)_A$ symmetry breaking implemented by 't Hooft interaction

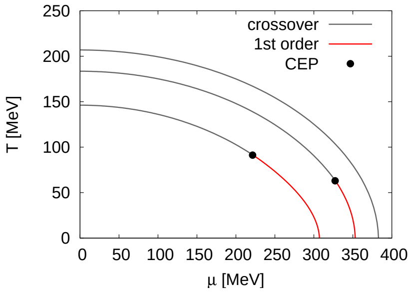
Phase diagram for $N_f = 2 + 1 \quad (\mu \equiv \mu_q = \mu_s)$

- Model parameter fitted to (pseudo)scalar meson spectrum:
- PDG: $f_0(600)$ mass=$(400 \ldots 1200)$ MeV $\rightarrow$ broad resonance

➜ existence of CEP depends on m_σ!

Example: $m_\sigma = 600$ MeV (lower lines), 800 and 900 MeV (here mean-field approximation)

with $U(1)_A$ **[BJS, M. Wagner '09]**



Outline

- Three-Flavor Chiral Quark-Meson Model
- **...with Polyakov loop dynamics**
- The important role of fluctuations
- Functional Renormalization Group

Polyakov-quark-meson (PQM) model

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

- **polynomial Polyakov loop potential:** **Polyakov 1978, Meisinger 1996, Pisarski 2000**

$$\frac{\mathcal{U}(\phi, \bar{\phi})}{T^4} = -\frac{b_2(T, T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

$$b_2(T, T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

- **logarithmic potential:** **Rößner et al. 2007**

$$\frac{\mathcal{U}_{\text{log}}}{T^4} = -\frac{1}{2}a(T)\bar{\phi}\phi + b(T)\ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]$$

$a(T) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2$ and $b(T) = b_3(T_0/T)^3$

- **Fukushima** **Fukushima 2008**

$$\mathcal{U}_{\text{Fuku}} = -bT\left\{54e^{-a/T}\phi\bar{\phi} + \ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]\right\}$$

a controls deconfinement b strength of mixing chiral & deconfinement

Polyakov-quark-meson (PQM) model

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back reaction of the matter sector to the YM sector: N_f and μ-modifications

in presence of dynamical quarks: $T_0 = T_0(N_f, \mu)$ **BJS, Pawlowski, Wambach, 2007**

N_f	0	1	2	$2+1$	3
T_0 [MeV]	270	240	208	187	178

$\mu \neq 0 :$ $\bar{\phi} > \phi$

since $\bar{\phi}$ is related to free energy gain of antiquarks

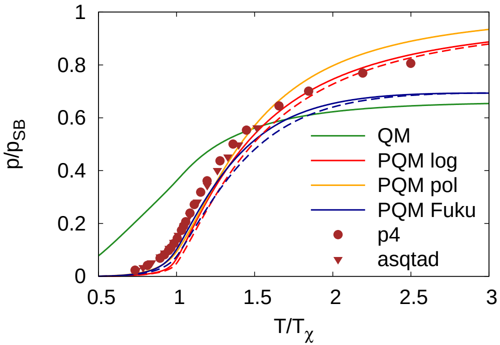
in medium with more quarks → antiquarks are more easily screened.

QCD Thermodynamics $N_f = 2+1$

[BJS, M. Wagner, J. Wambach '10]

SB limit: $\frac{p_{SB}}{T^4} = 2(N_c^2 - 1)\frac{\pi^2}{90} + N_f N_c \frac{7\pi^2}{180}$

(P)QM models (three different Polyakov loop potentials) versus QCD lattice simulations



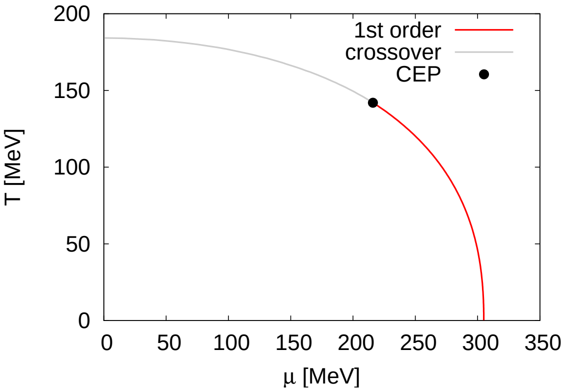
- ▷ solid lines:
  PQM with lattice masses (HotQCD)
  $m_\pi \sim 220, m_K \sim 503$ MeV
- ▷ dashed lines:
  (P)QM with realistic masses

lattice data: [Bazavov et al. '09]

$N_f = 2$ (P)QM phase diagrams

Summary of QM and PQM models in mean field approximation

chiral transition and 'deconfinement' coincide ■ for PQM model



[BJS, Pawlowski, Wambach '07]

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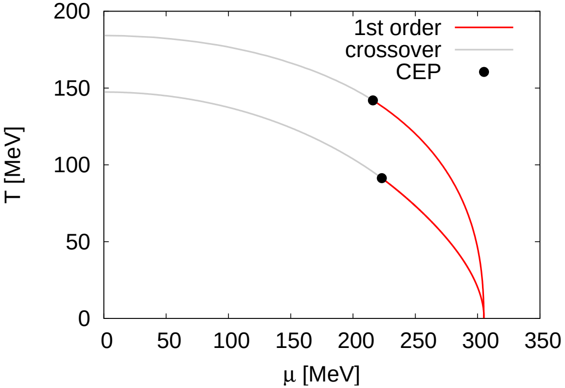

- for PQM model
- for QM model
  (lower lines)

[BJS, Pawlowski, Wambach '07]

$\mathbf{N_f = 2}$ (P)QM phase diagrams

Summary of QM and PQM models in mean field approximation

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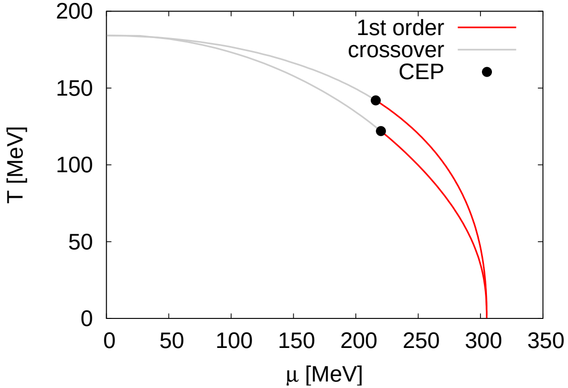


- for PQM model
- for PQM model **with** μ-modification in Polyakov loop potential (lower lines)

[BJS, Pawlowski, Wambach '07]

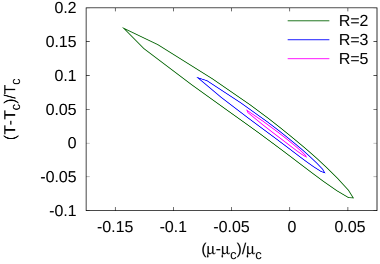
Critical region

contour plot of **size of the critical region** around CEP

defined via fixed ratio of susceptibilities: $R = \chi_q / \chi_q^{\text{free}}$

➜ compressed with Polyakov loop

QM model (2+1)



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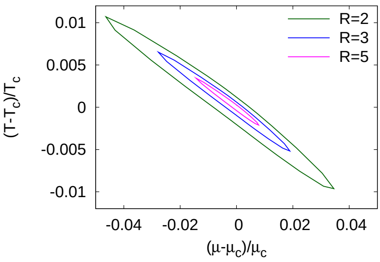

[BJS, M. Wagner; in preparation]

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- ○ Three-Flavor Chiral Quark-Meson Model
- ○ ...with Polyakov loop dynamics
- ○ **The important role of fluctuations**
- ○ Functional Renormalization Group

Importance of Dirac term

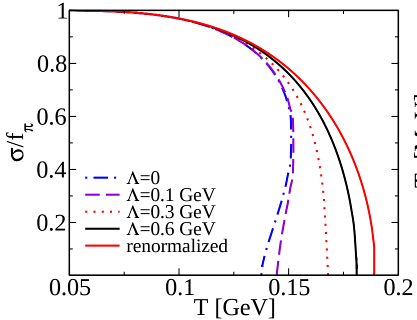
[V. Skokov, B. Friman, K.Redlich, BJS; arXiv:1005.3166]

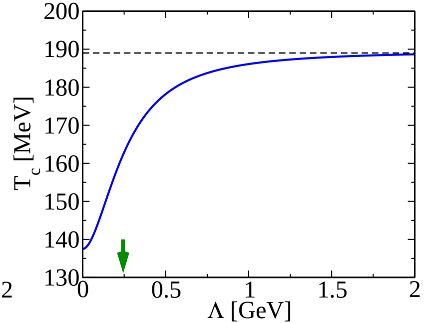
Thermodynamic potential (numerical results for $\mu = 0$)

$$\Omega = U_{\text{Pol}} + U_{\text{meson}} + \Omega_{q\bar{q}} \qquad \text{with}$$

$$\Omega_{q\bar{q}} = -2N_f \int \frac{d^3p}{(2\pi)^3} \left\{ N_c E_q \theta(p^2 - \Lambda^2) + T \ln N_q + T \ln N_{\bar{q}} \right\}$$

$$N_q = 1 + 3\Phi e^{-\beta(E_q-\mu)} + 3\bar{\Phi} e^{-2\beta(E_q-\mu)} + e^{-3\beta(E_q-\mu)}$$





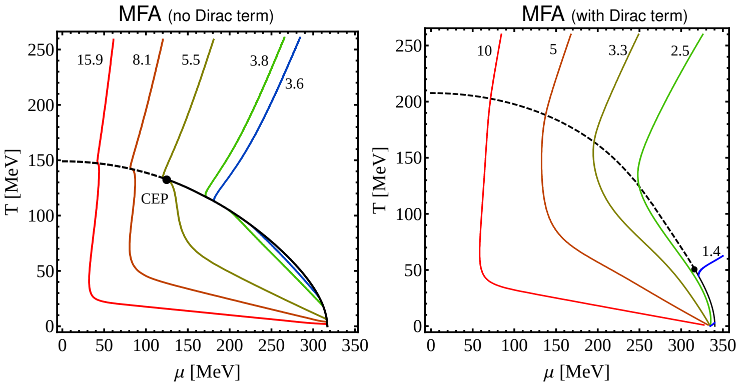
Isentropes $s/n = const$ and Focussing

[E. Nakano, BJS, B.Stokic, B.Friman, K.Redlich '10]

here: $N_f = 2$ QM model: kink in MFA are washed out in FRG

→ no focussing if fluctuations taken into account

a) influence of Dirac term   b) smallnest of critical region



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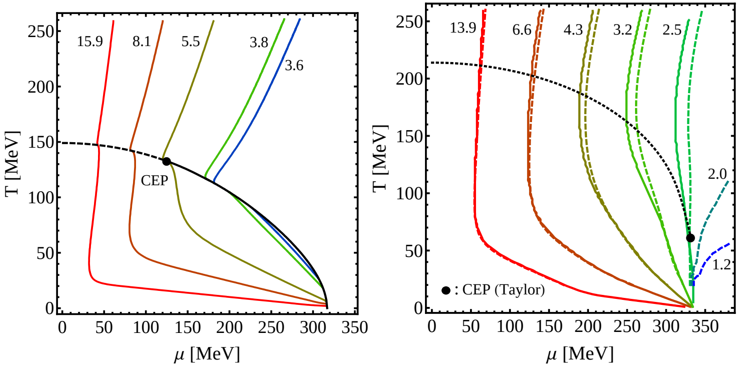
MFA (no Dirac term) FRG (full grid and Taylor expansion)



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a) influence of Dirac term   b) smallnest of crit region

kink structure at boundary in mean field approximation

$\Rightarrow$ remnant of first-order transition in chiral limit

if Dirac term neglected

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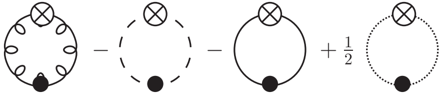
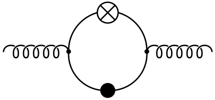
$\mathbf{T}_0(N_f, \mu)$ modification

full QCD FRG flow: gluon , ghosts, quark and meson (via hadronization) fluctuations
[J. Braun, H. Gies, L.M. Haas, F. Marhauser, J.M. Pawlowski et al.]

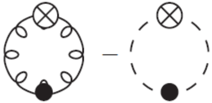
➜talks by T.K. Herbst (16.9) and J.M. Pawlowski (18.9)

$\partial_t \Gamma_k[\phi] = \frac{1}{2}$ 

in presence of dynamical quarks
gluonic contribution modified:



pure YM flow
($\rightarrow$ Polyakov loop potential):

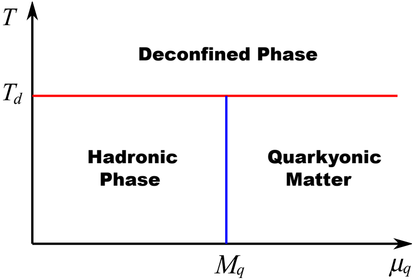
$\partial_t \Gamma_k[\phi] = \frac{1}{2}$ 

$$T_0 \leftrightarrow \Lambda_{QCD} \qquad : \qquad T_0 \rightarrow T_0(N_f, \mu)$$

[BJS, Pawlowski, Wambach, 2007]
[Herbst, Pawlowski,BJS; arXiv:1008.0081]

Quarkyonic Phase

large N_c world



[McLerran, Pisarski '07]

- if $\mu < M_q \sim M_B/N_c \sim O(1)$

  ➜

  hadronic phase with zero baryon density
- if $T > T_d \sim \Lambda_{\text{QCD}}$

  ➜

  # d.o.f. jump from $O(1)$ to $O(N_c^2)$ (gluons)

  deconfined phase
- since quark loops are suppressed by $1/N_c$

  T_d is **μ-independent**
- if $\mu > M_q$ ➜

  non-zero baryon density

quarkyonic phase is confining but chirally restored (➜parity-doubled hadrons)

What happens at $N_c = 3$?

Phase diagram $N_f = 2 + 1$

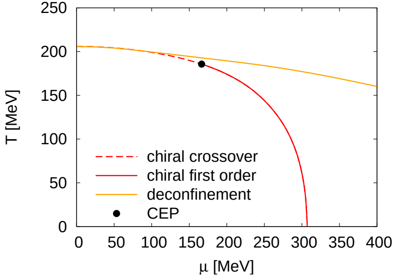
[BJS, M. Wagner; in preparation]

influence of Polyakov loop

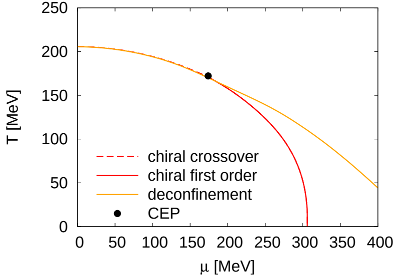
Logarithmic Polyakov loop potential

Mean-field approximation

$T_0 = 270$ MeV (constant)



$T_0(\mu)$ (i.e. with μ corrections)



shrinking of possible quarkyonic phase

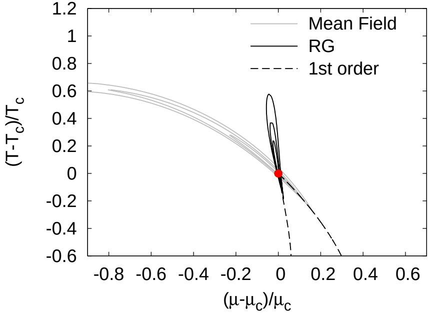
Critical region

similar conclusion if **fluctuations** are included

fluctuations via Functional Renormalization Group

comparison: $N_f = 2$ **QM model**

Mean Field $\leftrightarrow$ RG analysis

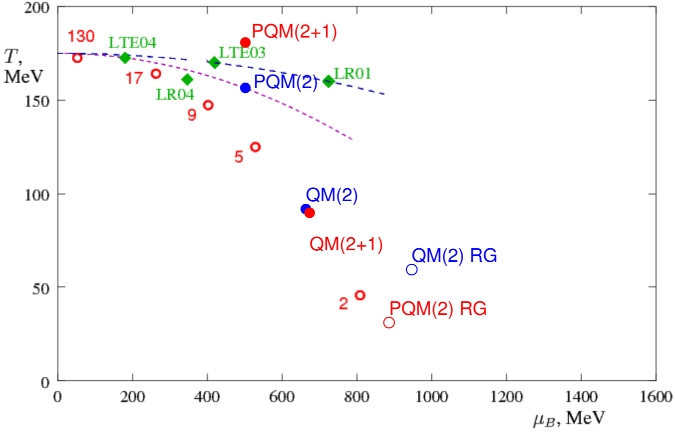


[BJS, J. Wambach '06]

Critical Endpoints

model studies vs. lattice simulations

Blue points: models   Lines & green points: lattice   Red circles: Freezeout points for HIC



lattice methods:

- reweighting
- imaginary μ_B
- Taylor expansion around $\mu_B = 0$

Summary

- $N_f = 2$ and $N_f = 2 + 1$ chiral (Polyakov)-quark-meson model study

➜ Mean-field approximation and FRG

➜ **fluctuations are important**

functional approaches (such as the presented FRG) are suitable and controllable tools to investigate the QCD phase diagram and its phase boundaries

Findings:

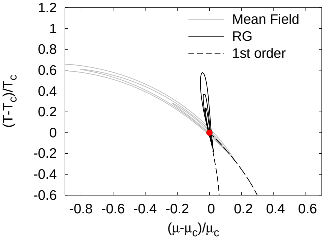
▷ matter **back-reaction to YM sector**: $T_0 \Rightarrow T_0(N_f, \mu)$

▷ **FRG with PQM truncation**: Chiral & deconfinement transition **coincide** for $N_f = 2$ with $T_0(\mu)$-corrections

▷ same conclusion for $N_f = 2 + 1$?

▷ **role of quantum fluctuations**

effects of Dirac term in a mean-field approximation



Outlook:

▷ include **glue dynamics** with FRG

$\rightarrow$ towards full QCD

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Schladming, Styria, Austria, February 26 - March 5, 2011

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