Some exact results in out of equilibrium systems

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ERG 2010, Corfu, Greece

Outline

1. Branching and annihilating random walks
2. Pure annihilation
3. Expansion in the branching ratio σ

The problem

- Branching and annihilating random walks
- Diffussing particles suffering chemical reactions

$$A \xrightarrow{\sigma} 2A$$

$$2A \xrightarrow{\lambda} \emptyset$$

- No temperature, no Boltzmann probability distribution
- Existence of stationary probability distribution

Phase transitions in BARW

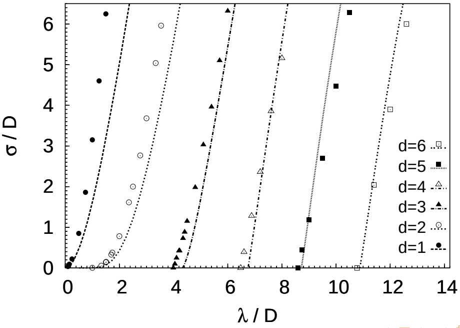
- Depending on reaction rates there exists an active-to-absorbing second order phase transition
- Mean field result $\rightarrow$ mass action law:

$$\frac{\partial \rho}{\partial t} = \sigma \rho - \lambda \rho^2$$

- With stationary solutions $\rho = 0$ (unstable) and $\rho = \sigma/\lambda$
- No phase transition! $\rightarrow$ fluctuation induced phase transition
- Perturbative RG: existence of phase transitions for $d = 1$ and $d = 2$ but not above

Phase transitions in BARW

- MonteCarlo shows a phase transition in all dimensions, for small branching ratio σ but for $\lambda > \lambda_c$ threshold value
- Belongs to the Directed Percolation universality class ($d_c = 4$)
- But here we are interested in non-universal properties



Expansion in the branching rate σ

- For studying the existence of a phase transition one can perform a perturbative expansion in the branching rate σ
- We start with the case $\sigma = 0$
- We first solve Pure Annihilation $\rightarrow$ exactly solvable

NPRG flow equations for $\Gamma_k^{(n,m)}$

- $\varphi(x) = \langle \phi(x) \rangle$, (average density) $\hat{\varphi}(x) = \langle \phi^*(x) \rangle$ (average response field) real fields $\rightarrow$ usual Legendre transform
- We work with NPRG equations for the (n, m)- correlation functions $\Gamma_k^{(n,m)}$

$$\Gamma_k^{(n,m)} = \frac{\delta^{n+m}\Gamma_k[\varphi, \hat{\varphi}]}{\delta^n \varphi \delta^m \hat{\varphi}}\bigg|_{\varphi,\hat{\varphi}=const}$$

- In principle we have to solve an infinite hierarchy of equations
- Must enforce causality (Itô prescription)

Pure annihilation

- Simple system $2A \to \emptyset$, no active phase
- Microscopic action:

$$S^{PA}[\bar{\phi}, \phi] = \int d^d x dt \left\{ \bar{\phi}(\partial_t - D\nabla^2)\phi + \lambda\bar{\phi}(\bar{\phi} + 2)\phi^2 \right\}.$$

- Hierarchy of NPRG equations can be closed
- Simplifying properties: $\Gamma_k^{(n,m)} = 0$ for $n < m$ and $\Gamma_k^{(n,0)} = 0$ for all n
- No field renormalization!

Pure annihilation

- Solving the flow of $\Gamma^{(2,2)}$

$$\partial_s \Gamma_k^{(2,2)}(p_1, p_2, p_3, p_4) = \int_q \partial_s R_k(q) G_k^2(q) G_k(p_1 + p_2 - q)$$
$$\times \Gamma_k^{(2,2)}(p_1, p_2, -q, -p_1 - p_2 + q) \Gamma_k^{(2,2)}(q, p_1 + p_2 - q, -p_3, -p_4)$$

- So that one can define $\lambda_k(p_1 + p_2) = \frac{1}{4}\Gamma_k^{(2,2)}(p_1, p_2, p_3, p_4)$
- With final solution

$$\lambda_k(p) = \frac{\lambda}{1 + 2\lambda \int_q \left\{ G_k(q) G_k(p - q) - G_\Lambda(q) G_\Lambda(p - q) \right\}}$$

Expansion in the branching ratio σ

- Microscopic action

$$S^{odd} = \int d^d x dt \left\{ \bar{\phi}(\partial_t - D\nabla^2)\phi + \lambda\bar{\phi}(\bar{\phi}+2)\phi^2 - \sigma\bar{\phi}(\bar{\phi}+1)\phi \right\}$$

- Bare level power counting is preserved along the flow
- Flow can again be closed $\rightarrow$ finite hierarchy

Threshold λ_c for $d > 2$

- For finding conditions for criticality one imposes

$$\Gamma_{k=0}^{(1,1)}(p=0)=0$$

- at order σ one finds

$$\Gamma_k^{(1,1)}(p)=-\sigma-2\frac{\sigma}{\lambda}(\lambda_k(p)-\lambda)$$

- With $\lambda_k(p)$ the same as in pure annihilation
- Regulator dependence is natural: λ_c is non-universal and depends on the UV details of the theory
- In this work → hypercubic lattice & continuum theory with cut-off Λ

Threshold λ_c in lattice & continuum

Hypercubic lattice dimension	3	4	5	6
λ_c/Da^{d-2} (this work)	3.96	6.45	8.65	10.7
λ_c/Da^{d-2} (MonteCarlo) [Canet04]	3.99	6.48	8.6	10.8

- With large d limit

$$\lambda_c/Da^{2-d} \overset{d\to\infty}{\sim} 2d.$$

- Whereas for the continuum one finds

$$\tilde{\lambda}_c = \frac{D\Lambda^{2-d}(4\pi)^{d/2}\Gamma(d/2)\, d(d-2)}{4}$$

- In agreement with previous NPRG numerical results

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Generalizations (work in progress)

- Parity Conserving universality class (in particular d_c)
- Higher order σ-expansion
- Other out of equilibrium systems