Ghosts in asymptotically safe quantum gravity

Astrid Eichhorn

in collaboration with Holger Gies

based on: PRD80:104003 (2009), PRD81:104010 (2010)

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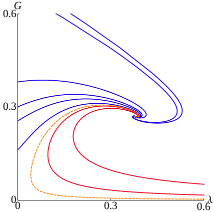
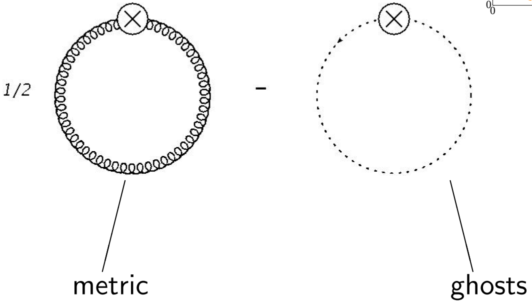
Research Training Group
Quantum and Gravitational Fields

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_t R_k$$

Wetterich, 1992
Reuter, 1996



$$\partial_t \Gamma_k =$$



Why are ghosts interesting?

Ghosts are not observable!

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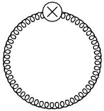
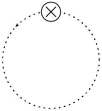
BUT:

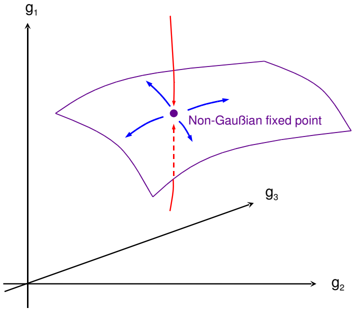
$\partial_t \Gamma_k \; = \; {}^{1/2}$  - 

Ghost fluctuations change running gravitational couplings

Ghosts may be crucial for fixed point properties

What do we want to understand about the ghost-sector?

- Relevant operators in the ghost sector?



→ Relevant directions: couplings have to be fixed by experiment!

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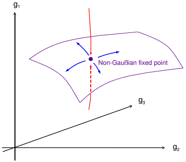
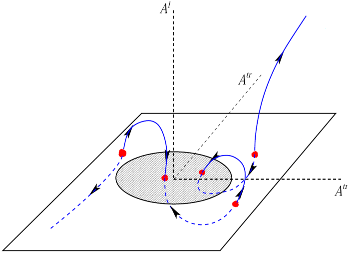


- Similarity to Yang-Mills theories in non-perturbative regime?
  - Gribov problem
  - Scaling solution for ghost and gluon propagator

Gribov problem and scaling solution in Landau gauge

$\mathcal{Z} = \int \mathcal{D}g\, \delta[F] \det(-\mathcal{M})\, e^{-S}$

Gribov, 1978



$\mathcal{M} \to 0$ at first Gribov horizon

strongly-interacting regime:

dominant field configurations
near Gribov horizon!

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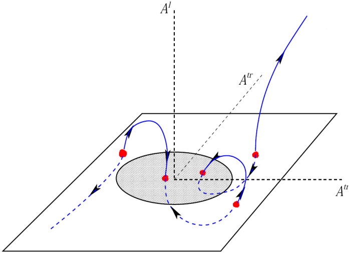

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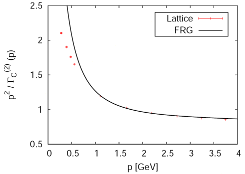
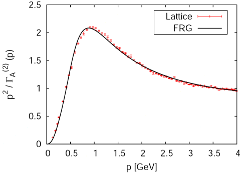


$$\left(\Gamma^{(2)}_{\rm gh}\right)^{-1} \sim \frac{1}{(p^2)^{-\kappa_C}\, p^2}$$



$$\left(\Gamma^{(2)}_{\rm gluon}\right)^{-1} \sim \frac{1}{(p^2)^{-\kappa_A}\, p^2}$$

scaling relation: $\kappa_A = -2\kappa_C$

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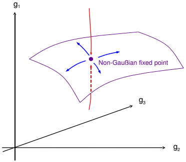

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Ghost wave function renormalisation

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$$\text{truncation}: \quad \Gamma_k \quad = \quad \Gamma_{\mathrm{EH}} + \Gamma_{\mathrm{gf}} + \Gamma_{\mathrm{gh}}$$

$$\Gamma_{\mathrm{EH}} \quad = \quad \frac{1}{16\pi G_N} Z_{\mathrm{N}}(k) \int d^4x \sqrt{g}(-R + 2\lambda(k))$$

$$\Gamma_{\mathrm{g}h} = -\sqrt{2} Z_c \int d^4x \sqrt{\bar{g}} \bar{c}_\mu \left(\bar{D}^\rho \bar{g}^{\mu\kappa} g_{\kappa\nu} D_\rho + \bar{D}^\rho \bar{g}^{\mu\kappa} g_{\rho\nu} D_\kappa - \tfrac{1+\rho}{2} \bar{D}^\mu \bar{g}^{\rho\sigma} g_{\rho\nu} D_\sigma \right) c^\nu$$

Ghost wave function renormalisation

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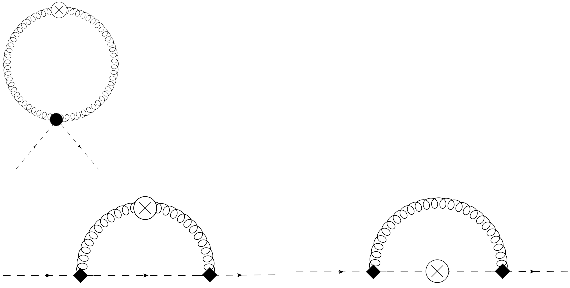
scale dependence of wave-function renormalisation:

anomalous dimension $\eta_c = -k\partial_k \ln Z_c$

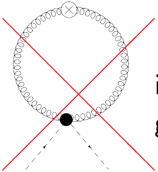
$$\Gamma^{(2)}_{gh} \sim (p^2)^{1-\eta_c/2}$$

K. Groh, F. Saueressig J.Phys.A 43:365403,2010; A.E., H. Gies PRD 81:104010,2010 diagrammatically:

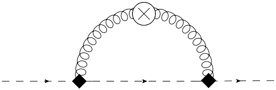


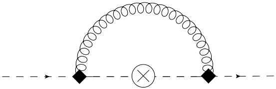
Ghost anomalous dimension

diagrammatically:



in our truncation:
ghost-antighost-two-graviton vertex vanishes!





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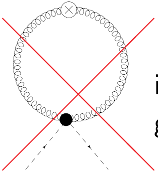

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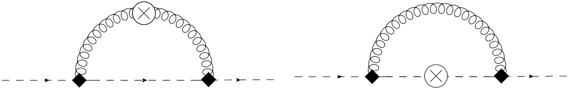


York decomposition of the metric:

$$h_{\mu\nu} = h^T_{\mu\nu} + D_\mu \xi_\nu + D_\nu \xi_\mu + D_\mu D_\nu \sigma - \frac{1}{d} D^2 \bar{g}_{\mu\nu} \sigma + \frac{1}{d} \bar{g}_{\mu\nu} h$$

$\Rightarrow$ 8 diagrams in total

Two good reasons for Landau deWitt gauge: ($\rho \to 0$, $\alpha \to 0$)

- fixed point of RG flow $\to$ "most physical" gauge
- technically favorable: not all metric modes can contribute:

$$h_{\mu\nu} = h^T_{\mu\nu} + D_\mu \xi_\nu + D_\nu \xi_\mu + D_\mu D_\nu \sigma - \tfrac{1}{d} D^2 \bar{g}_{\mu\nu}\sigma + \tfrac{1}{d}\bar{g}_{\mu\nu} h$$

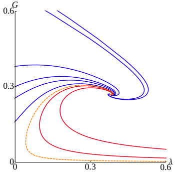
$$\to h^T_{\mu\nu} + \tfrac{1}{d}\bar{g}_{\mu\nu} h$$

$\to$ 4 diagrams

Results

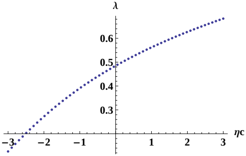
truncation	G_*	λ_*	$G_*\lambda_*$	$\theta_{1,2}$	η_c
$\eta_c = 0$	0.27	0.38	0.10	$2.1 \pm i1.69$	0
$\eta_c \neq 0$	0.29	0.32	0.09	$2.03 \pm i\ 1.50$	-0.78

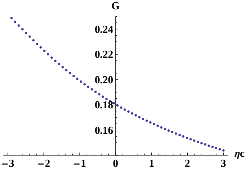
- NGFP exists also for $\eta_c \neq 0$
- positive $G_* \Rightarrow$ stability!
- critical exponents: two relevant directions



include effects beyond the current truncation (e.g. four-ghost operators) by varying η_c:

Fixed-point values:





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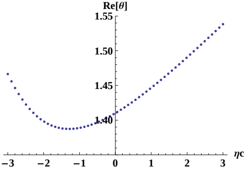

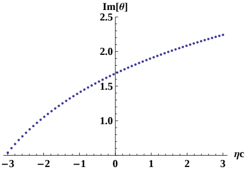

What does the anomalous dimension tell us?

full propagator: $(\Gamma^{(2)})^{-1} = \frac{1}{Z_c(k^2{=}p^2)p^2} = \frac{1}{(p^2)^{1-\frac{\eta}{2}}}$ $\qquad \eta_c \approx -0.8$

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Why is $\eta_c \neq \eta_N$?

metric: $-k\partial_k \ln Z_N = \eta_N = -2$

cancellation of non-physical metric modes? $\rightarrow$ anomalous scaling of vertices

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negative η_c

- potentially more relevant ghost operators:
  $g\, c^\mu \mathcal{O}_{\mu\nu} c^\nu \Rightarrow \partial_t g \sim \eta_c g + \ldots$

truncation:

$$\Gamma_k = \Gamma_{\rm EH} + \Gamma_{\rm gf} + S_{\rm gh} + \zeta_k \int d^4x \sqrt{g}\, \bar{c}^\mu\, R\, c_\mu$$

A.E., H. Gies, M.M. Scherer, 2009

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A.E., H. Gies, M.M. Scherer, 2009

transverse traceless approximation

expectation: transverse traceless graviton mode carries essential physical information

York decompose metric:
$h_{\mu\nu} = h^T_{\mu\nu} + D_\mu \xi_\nu + D_\nu \xi_\mu + D_\mu D_\nu \sigma - \frac{1}{d} D^2 \bar{g}_{\mu\nu} \sigma + \frac{1}{d} \bar{g}_{\mu\nu} h$
$\rightarrow$ let only $h^T_{\mu\nu}$ fluctuate

existence of NGFP and number of relevant directions conserved in Einstein-Hilbert-truncation

Ghost-curvature coupling: Result

- at NGFP in Einstein-Hilbert sector: $\zeta_* = 0$
- NGFP seems compatible with "simple" Faddeev-Popov gauge fixing
- critical exponent: $-\left(\partial_\zeta(k\partial_k\zeta)\right)\big|_{\zeta_*} \approx 1.4$

  $\longrightarrow$ possibly relevant coupling

  to which IR-measurable coupling is ζ related?

two-derivative operators in the ghost sector

- $Z_c \bar{c}^\mu D^2 g_{\mu\nu} c^\nu$
- $\bar{c}^\mu [D_\mu, D_\nu] c^\nu = \bar{c}^\mu R_{\alpha\mu} c^\alpha$
- $\bar{c}^\mu \{D_\mu, D_\nu\} c^\nu$
- $\bar{c}^\mu R c_\mu$

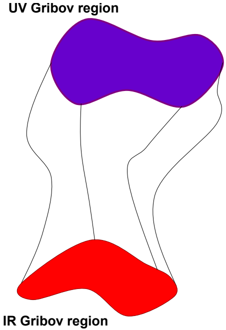
truncation:

$$-\sqrt{2} Z_c \int d^4x \sqrt{\bar{g}} \bar{c}^\mu \left(\bar{D}^\rho \bar{g}^{\mu\kappa} \gamma_{\kappa\nu} D_\rho + \delta Z_c \left(\bar{D}^\rho \bar{g}^{\mu\kappa} \gamma_{\rho\nu} D_\kappa - \frac{1+\rho}{2} \bar{D}^\mu \bar{g}^{\rho\sigma} \gamma_{\rho\nu} D_\sigma \right) \right) c^\nu$$

$$\overset{g_{\mu\nu} \to \bar{g}_{\mu\nu}}{\longrightarrow} \; -2\sqrt{2} Z_c \int d^4x \sqrt{\bar{g}} \bar{c}^\mu \left(\bar{D}^2 \bar{g}_{\mu\nu} + \delta Z_c \left(\frac{3+\rho}{4} [\bar{D}^\nu, \bar{D}^\mu] + \frac{1+\rho}{4} \{\bar{D}^\mu, \bar{D}^\nu\} \right) \right) c^\nu$$

Preliminary results: Two Non-Gaussian fixed points

- $\delta Z = 1$
  - $\{G_*, \lambda_*\} = \{0.287, 0.317\}$
  - new relevant direction: $\theta_3 = 0.646$
  - flow towards the IR: δZ changes $\rightarrow$ FP det changes $\rightarrow$ shape of Gribov region changes



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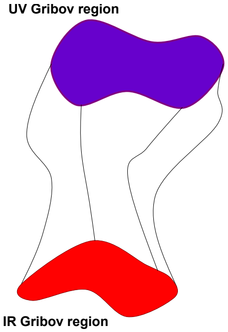

- $\delta Z = 1.56154$
  - $\{G_*, \lambda_*\} = \{0.262, 0.372\}$
  - new irrelevant direction: $\theta_3 = -0.83$
  - $\eta_c = 0.24$
  - no simple Faddeev-Popov gauge fixing?

Summary

- Ghosts sector can have interesting consequences for FP
- Relevant operators in ghost sector have to be connected to measurable couplings
- Ghost wave-function renormalisation:
  - negative anomalous dimension
  - Ghost operators potentially relevant
- Ghost-curvature coupling: Possibly relevant coupling
- Preliminary results on ghost-derivative couplings: Two NGFPs

Outlook

still many open issues in the ghost sector:

- How does the Gribov region look like in gravity?
- How many relevant operators?
- To which IR-measurable couplings are these related?
- NGFP compatible with simple Faddeev-Popov gauge fixing?

Thank you for your attention!