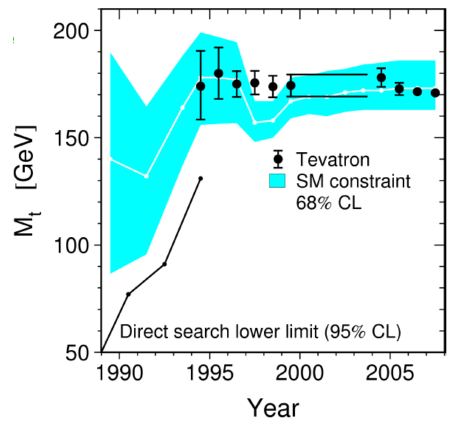
Heavy Quark Masses

André H. Hoang

Max-Planck-Institute for Physics
Werner-Heisenberg-Institute
Munich

Outline

- Masses and heavy quark mass schemes
  - Pole and short-distance masses
  - Renormalons



- Top mass measurement
  - Confinement & finite top lifetime
  - Why top reconstruction is conceptually nontrivial.

- What mass is implemented in Monte Carlos ?
  - A factorization theorem for e^+e^- as a guideline
  - A (partially complete) answer

- Conclusions

Present top mass: $m_t = 173.1 \pm 1.3 \text{ GeV}$

This talk shall make you aware of the conceptual subtleties that arise with ever decreasing errors.

Masses and Mass Schemes

Concepts of Mass

Classic Physics: Mass has absolute meaning.

$$F = m_i \left(\frac{d}{dt} v\right)$$ inertial mass

$$F = G \frac{m_{g,1}\, m_{g,2}}{|\vec{r_1} - \vec{r_2}|^2} = g\, m_{g,1}$$ gravitational mass

experimental fact

$$1 - \frac{m_i}{m_g} \sim 10^{-12}$$

Weak (Galilean) equivalence principle: $\frac{m_i}{m_g} = \kappa$ for any object

Special relativity: $p^2 = m^2$ rest mass, mass-shell

Concepts of Mass

Quantum Field Theory:

Particles: Field-valued operators made from creation and annihilation operators

Lagrangian operators constructed using correspondence principle

Classic action: m is the rest mass

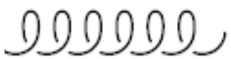
No other mass concept exists at the classic level.

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_{\rm classic} + \mathcal{L}_{\rm gauge-fix} + \mathcal{L}_{\rm ghost} \qquad (p^2 - m^2)\, q(x) = 0$$

$$\mathcal{L}_{\rm classic} = -\frac{1}{4} F^A_{\alpha\beta} F^{\alpha\beta}_A + \sum_{\text{flavors } q} \bar{q}_\alpha (i\not{D} - m_q)_{\alpha\beta}\, q_b \qquad D^\mu = \partial^\mu + i g T^C A^{\mu C}$$

$$i\frac{\not{p} + m}{p^2 - m^2 + i\epsilon}$$

classic particle poles

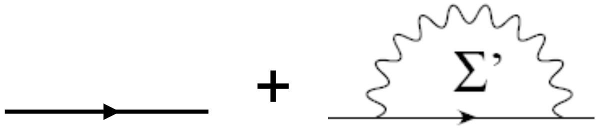


$$-i\frac{(g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}(\xi - 1))}{p^2 + i\epsilon}$$

Concepts of Mass

Renormalization: UV-divergences in quantum corrections

Fields, couplings, masses in classic action are bare quantities that need to be renormalized to have (any) physical relevance



$$= \not{p} - m^0 + \Sigma(p, m^0)$$

$$m^0 \, \frac{\alpha_s}{\pi} \left[-\frac{1}{\epsilon} + \text{finite stuff} \right]$$

sum over many quantum fluctuations

Mass Renormalization Schemes you certainly know:

Pole mass: mass = classic rest mass

$$m^0 = m^{\text{pole}} + \delta m^{\text{pole}} \qquad \delta m^{\text{pole}} = \Sigma(m, m)$$

$\overline{\text{MS}}$ mass: $m^0 = \overline{m}(\mu) - \frac{\alpha_s}{\pi} \frac{1}{\epsilon}$

Concepts of Mass

All mass schemes are related through a perturbative series.

$$m^{\text{schemeA}} - m^{\text{schemeB}} = \#\,\alpha_s + \#\,\alpha_s^2 + \#\,\alpha_s^3 + \dots$$

Lesson 1: Renormalization schemes are defined by what quantum fluctuations are kept in the dynamical matrix elements and by what quantum fluctuations are absorbed into the couplings and parameters.

Why do we have to care?

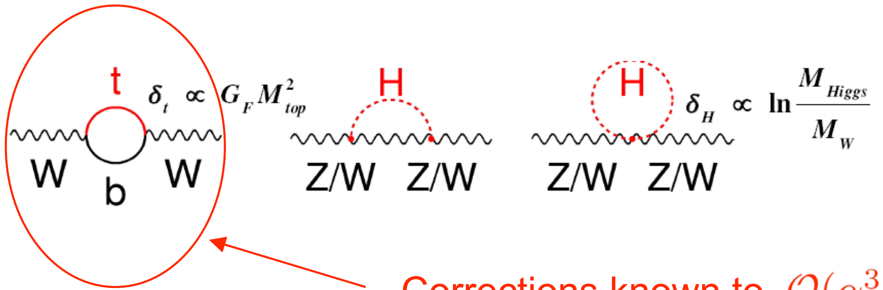
Different mass schemes are useful and appropriate for different applications.

Which is the best mass for a specific application?

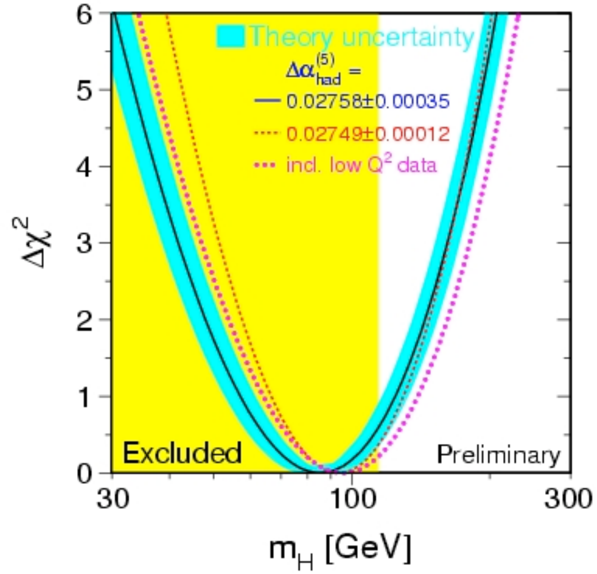
Lessson 2: A good scheme choice is one that gives systematically (not accidentally) good convergence. But there are almost always several alternatives one can use.

Precise Top Mass Needed!

Fit to electroweak precision observables



Corrections known to $\mathcal{O}(\alpha_s^3)$



$$\sin\theta_W \times \Big(1 + \delta(m_t, m_H, \ldots)\Big)$$

$$= 1 - \frac{M_W^2}{M_Z^2}$$

$$m_t = 173.1 \pm 1.3 \text{ GeV}$$

$$m_H = 90 \pm 24 \text{ GeV}$$

2 GeV change: 15% change in m_H

$$m_H < 163 \text{ GeV} \ \text{(95\%CL)}$$

Best convergence using the $\overline{\text{MS}}$ top scheme:

$$\overline{m}_t(\overline{m}_t)$$

Precise Top Mass Needed!

Blick in die Zukunft:

Minimales Supersymmetrisches Standard Model

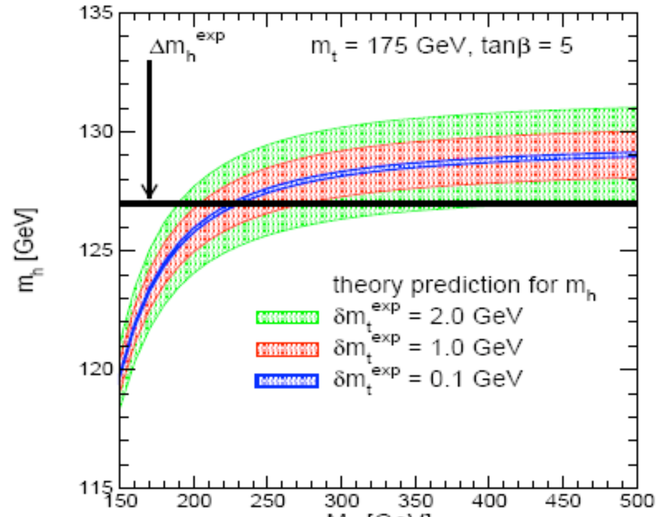
5 Higgs Bosonen:

m_h (skalar, neutral)

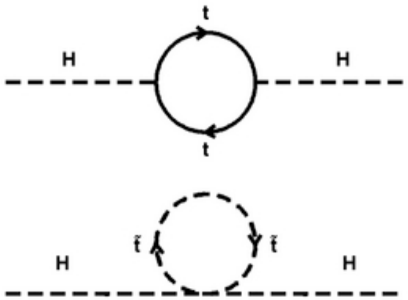
m_H (skalar, neutral)

m_A (speudoskalar, neutral)

$m_H^{\pm}$ (geladen)



Heinemeyer, Kraml, Porod, Weiglein



$$m_h^2 \simeq M_Z^2 + \frac{G_F\, m_t^4}{\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde t_1} m_{\tilde t_2}}{m_t^2}\right)$$

Corrections known to $\mathcal{O}(\alpha_s^3)$

Best convergence using the $\overline{\text{MS}}$ top scheme:

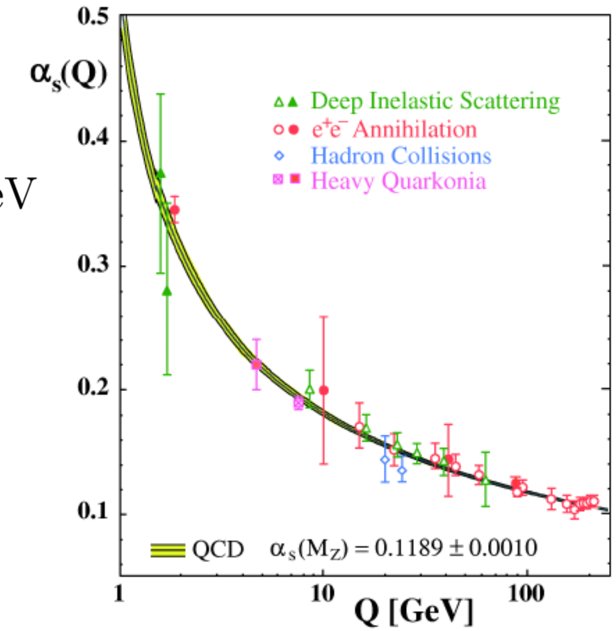
$$\overline{m}_t(\sqrt{M_{\rm SUSY}\overline{m}_t})$$

Haber, Hempfling, Hoang

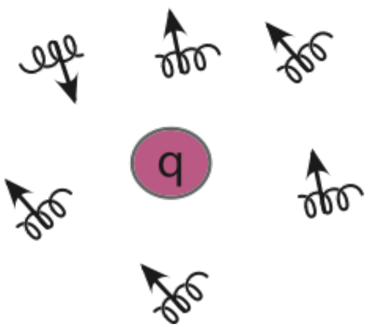
Confinement and Pole Mass

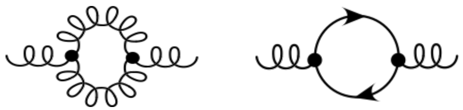
Low energies: QCD-effects cause nonperturbative **"Confinement"**

$\Lambda_{\mathrm{QCD}} \approx 0.3\ \mathrm{GeV}$



Concept of free quarks invalid.



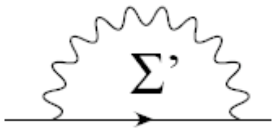


"Anti-Screening"

Lesson 3: Concept of a quark pole (rest) mass invalid a priori. Can only be used in the context of perturbation theory.

Renormalons

So … do we have to care?



$$\Sigma(m,m) \;=\; -\frac{4}{3}\int \frac{d^4q}{(2\pi)^4}\,\alpha_s\,\gamma^\mu\,\frac{\not q+\not k+m}{(q+k)^2-m^2}\gamma_\mu\frac{1}{q^2}$$

$$\overset{q\ll m}{=}\;\frac{2}{3}\int\frac{d^3q}{(2\pi)^3}\frac{\alpha_s(q)}{\vec q^{\,2}} \quad\Rightarrow\quad -\frac{1}{2}\int\frac{d^3q}{(2\pi)^3}\,V(\vec q^{\,2})$$

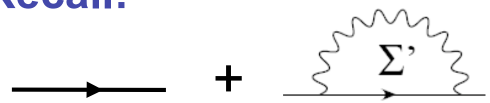
Linear sensitivity to infrared momenta leads to factorialy growing coefficients in perturbation theory (= renormalon ambiguity).

Bigi, Shifman, Uraltsev, Vainshtein

Beneke, Braun

$$\Sigma(m,m)\;\sim\;\mu\sum_n \alpha_s^{n+1}(2\beta_0)^n n! \qquad\qquad \int_0^\mu dq\,(-1)^n\,\ln^n\left(\frac{q}{\mu}\right)\;=\;n!$$

Recall:



$$= \not p - m^0 + \Sigma(\not p, m^0)$$
$$\sim \not p - m^{\text{pole}}$$

Isn't this just an argument in favor of the pole mass ?

Renormalons

Static energy of a static heavy quark-antiquark pair

$$E_{\rm static} = 2m^0 - 2\Sigma(m,m) + V(r)$$

$$\sim 2m^0 - \int \frac{d^3q}{(2\pi)^3} V(\vec{q}^{\,2}) + \int \frac{d^3q}{(2\pi)^3} V(\vec{q}^{\,2}) e^{i\vec{q}\vec{r}}$$

Renormalon behavior cancels in the sum of self and interaction energy but UV-divergent.

$$= 2m^{\rm pole} + V(r)$$

UV-renormalized, but renormalon behavior appears.

$$m^{\rm pole} = m^{\rm sd}(R) - \Sigma_R^{\rm lowE}(m,m)$$

Employ a short-distance mass scheme.

$$= 2m^{\rm sd}(R) + V(r) - 2\Sigma_R^{\rm lowE}(m,m)$$

Low Energy renormalon contributions are removed.

Short-Distance Mass Schemes

Short-distance mass schemes:

$$m^{\rm sd}(R) = m^{\rm pole} - R\left(a_1\frac{\alpha_s}{4\pi} + a_2\left(\frac{\alpha_s}{4\pi}\right)^2 + \ldots\right)$$

Generic form of a short-distance mass scheme.

$\overline{\text{MS}}$ mass: $R = \overline{m}(\mu)\,, \quad a_1 = \frac{16}{3} + 8\ln\frac{\mu}{m}$

Processes where heavy quarks are off-shell and energetic.

Threshold masses (1S, PS, kinetic masses)

$$R \sim m\alpha_s$$

Quarkonium & B physics: heavy quarks are close to their mass-shell.

Threshold masses (jet mass)

$$R \sim \Gamma_Q$$

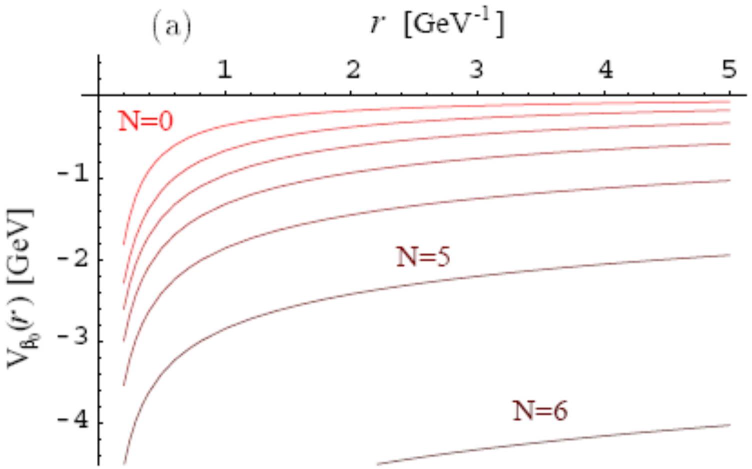
Single top resonance: heavy quark is very close to its mass-shell.

The a_i's are chosen such that the renormalon is removed.

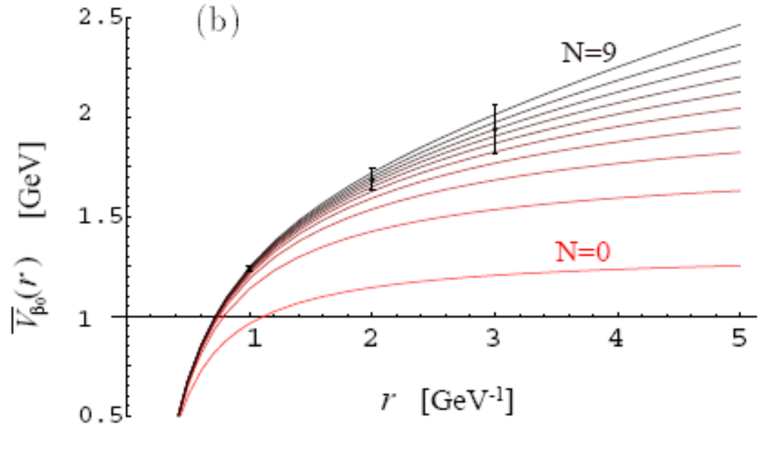
The scale R is of order the momentum scale relevant for the problem.

Renormalons

pole mass scheme



1S mass scheme



Lessson 4: When heavy quark masses need to be known with uncertainties below $\mathcal{O}(1)$ GeV, **short-distance** masses must be used. (B-physics, top quark physics, electroweak precision physics, …).

Renormalons

Top decay width

$$\Gamma_0 = \frac{G_F m_t^3 |V_{tb}|^2}{8\sqrt{2}\pi}$$

$$\Gamma_t(t \to bW) = \Gamma_0^{pole}[1 - 0.10\epsilon - 0.02\epsilon^2] \qquad m_t^{\rm pole}$$

$$\Gamma(t \to bW) = \bar{\Gamma}_0[1 - 0.04\epsilon - 0.003\epsilon^2] \qquad \overline{m}_t(\overline{m}_t)$$

Rho parameter

$$x_t \equiv 3\frac{G_F m_t^2}{8\sqrt{2}\pi^2}$$

$$\Delta\rho = x_t^{pole}[1 - 0.098\epsilon - 0.017\epsilon^2]$$

$$\Delta\rho = \bar{x}_t[1 - 0.007\epsilon - 0.007\epsilon^2]$$

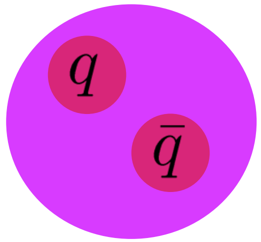
Why Top Mass Reconstruction is nontrivial conceptually.

Confinement

Confinement:

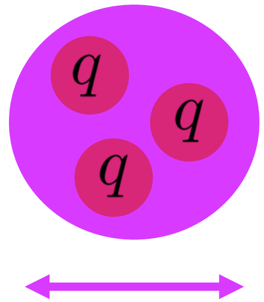
Mesons

$\pi, K, \rho, B, \ldots$



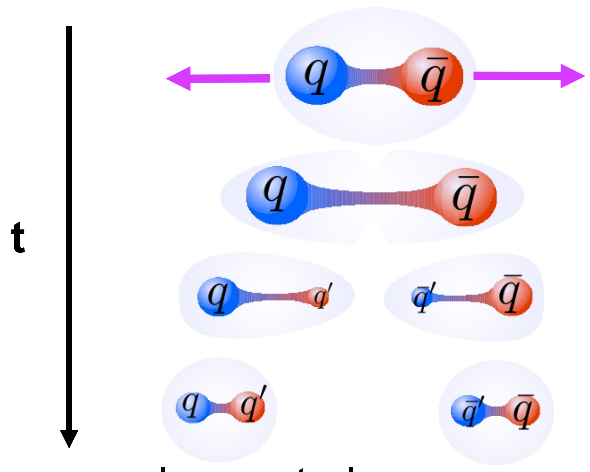
Baryons

$p, n, \Sigma, \Delta, \ldots$



$$r = \Lambda_{\rm QCD}^{-1} \simeq 1 \ {\rm fm}$$

Stable quarks only appear in the form of color-neutral hadrons.



color-neutral
Meson

Hadronisation time:

$$\tau_{\rm had} = 10^{-23} \ {\rm s}$$

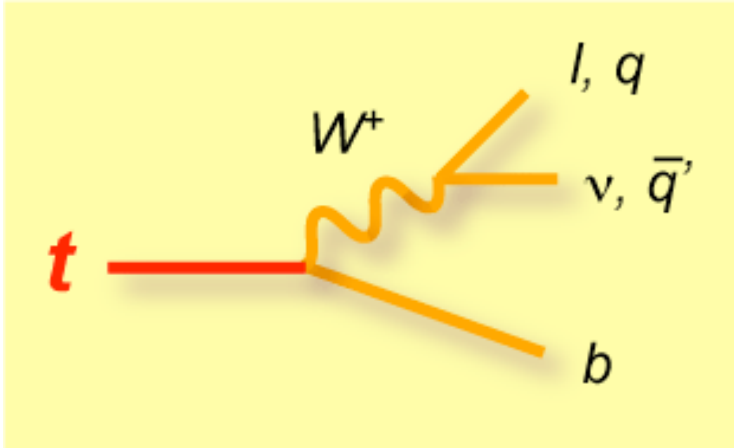
Top quark is not a physical observable object.

t

Top Decay

Decay of the top quark:

$$\Gamma(t \to bW) \approx 1.5 \text{ GeV}$$



Top quarks cannot ever form hadrons as they decay before that happens.

Color neutralization still relevant for the top quark via its decay products.

Top lifetime:

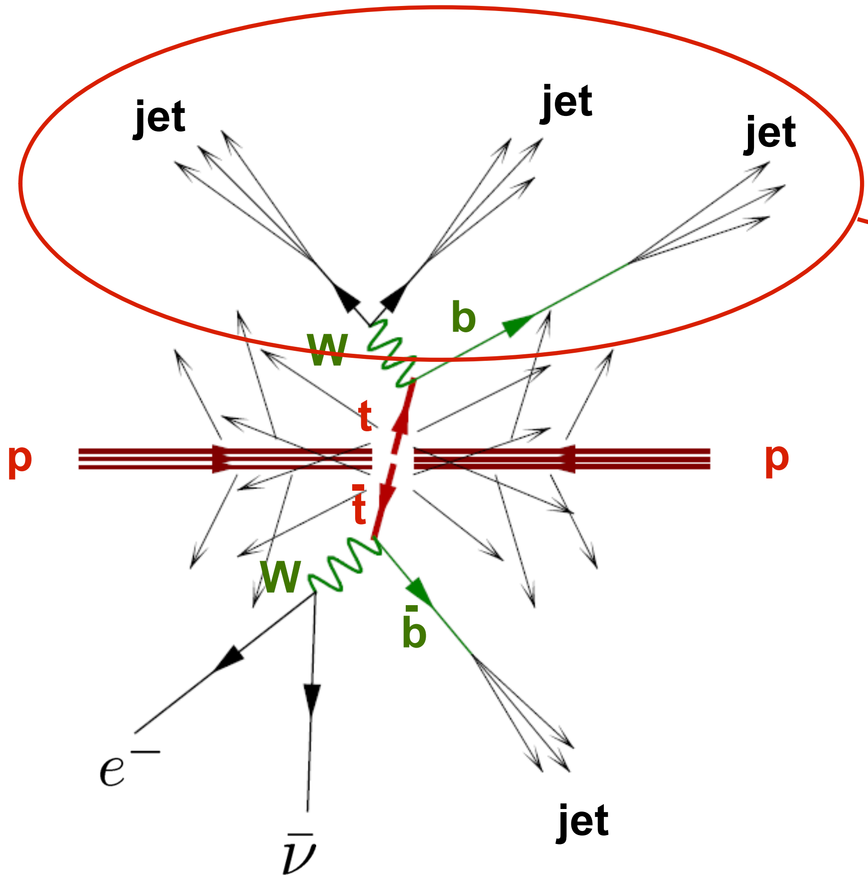
$$\tau_{\text{had}} = 10^{-24} \text{ s}$$

Hadronisation time:

$$\tau_{\text{had}} = 10^{-23} \text{ s}$$

Top Reconstruction

LHC:

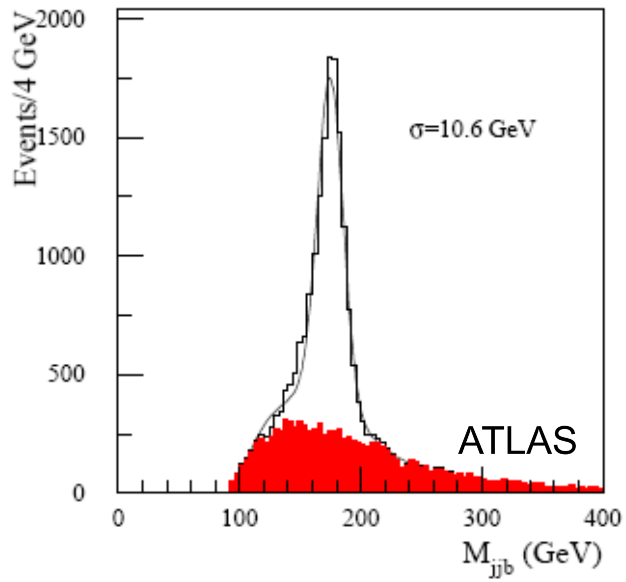


Idea:

Identify top decay products

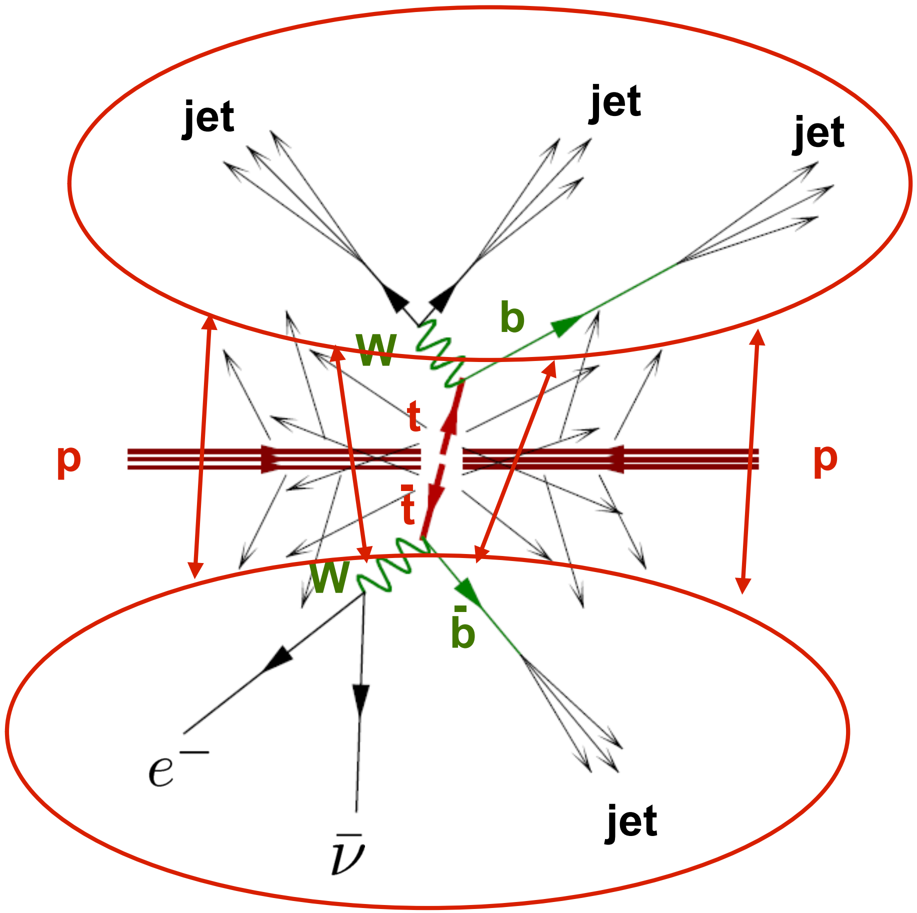
$$\text{“ } m_{\text{top}}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2 \text{ ”}$$

Invariant mass distribution



Top Reconstruction

LHC:



Idea:

Identify top decay products

$$\text{“}\; m_{\rm top}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2 \;\text{”}$$

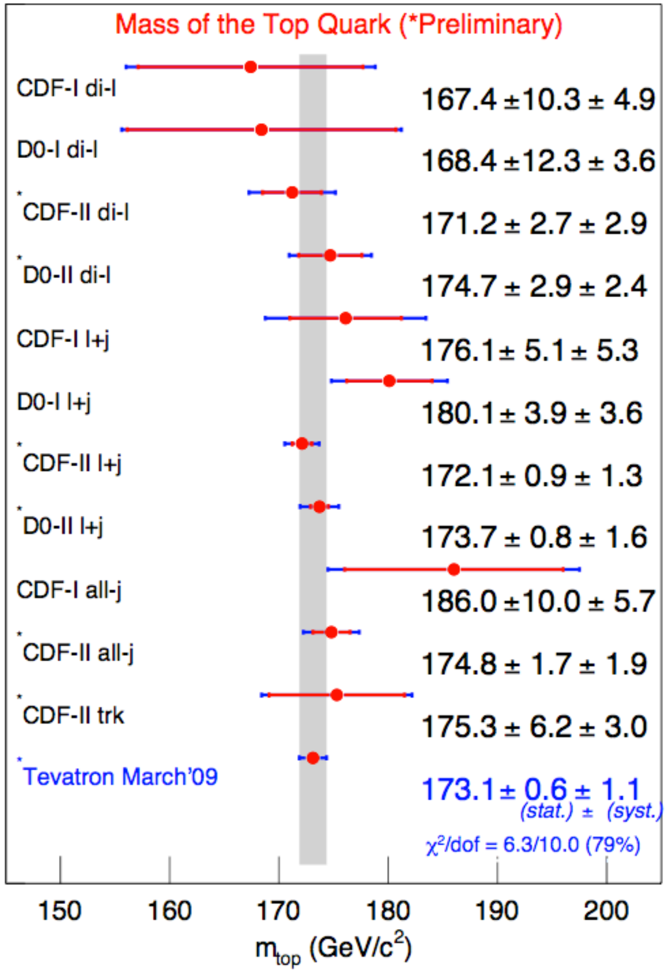
Conceptually this is quite subtle!

The measured quantity does not exist a priori. It is defined only through the experimental prescription.

The idea of an a priori physical object with a well defined mass is not correct.

Details of the color neutralization and hadronization models in MC's affect the simulation of reconstruction and thus the top mass measurement at leading order.

Top Mass at Tevatron



Experimental distributions are compared to Monte Carlo predictions.

Top mass parameter in the Monte Carlo is fitted to the distributions.

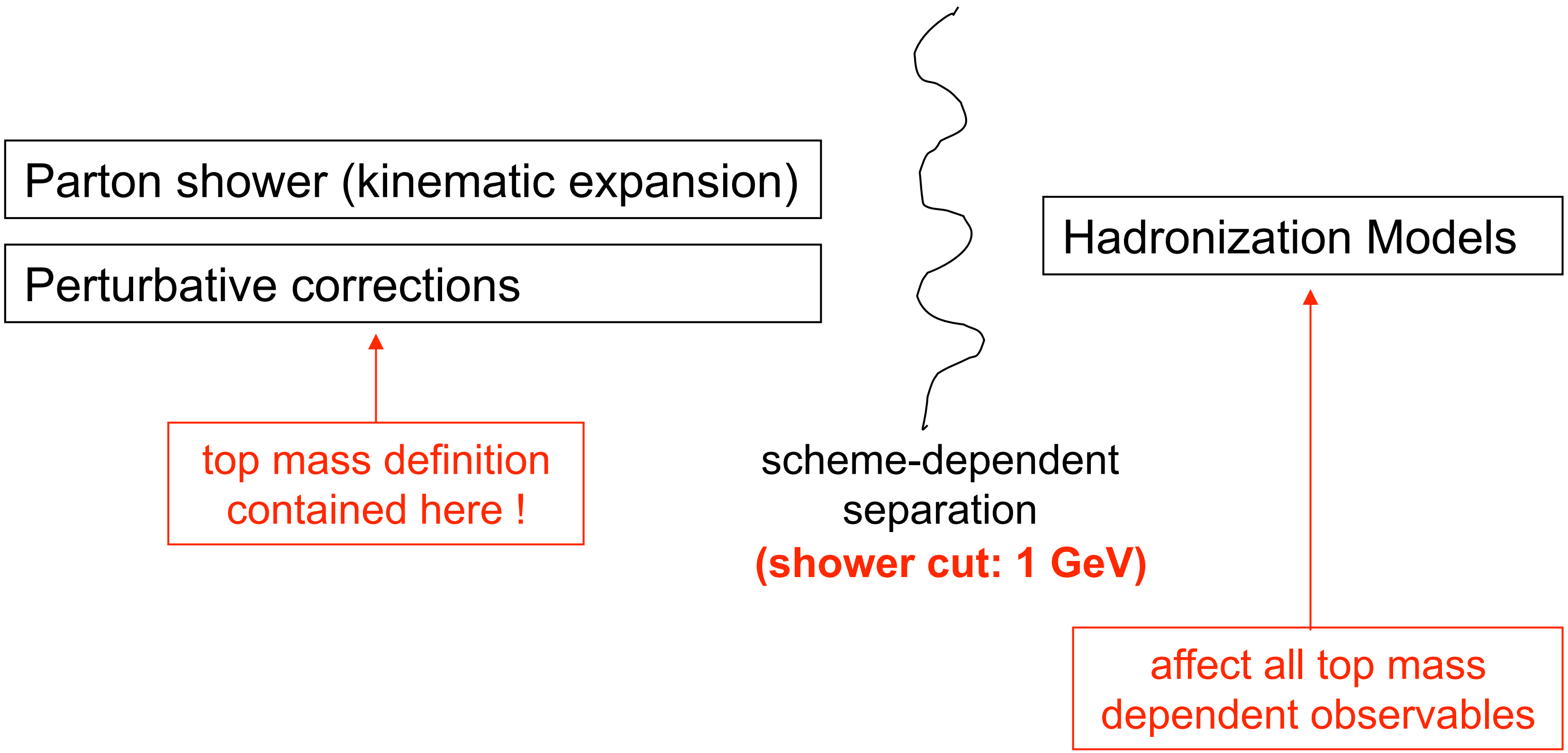
Aspects of gluon radiation and color neutralization must be described correctly in the Monte Carlo.

$$m_t^{\rm TeV} = 173.1 \pm 1.3 \text{ GeV} \quad = \quad m_t^{\rm Pythia}$$

What top mass is contained in Monte Carlos ?

MC and the Top Mass

- MC is a tool designed to describe many **physical** final states, cross sections, etc..
- The concept of mass in the MC depends on the **structure of the perturbative part** and the **interplay of perturbative and nonperturbative part** in the MC.



MC Top Mass

Isn't it the pole mass?

NO, it is not the pole mass.

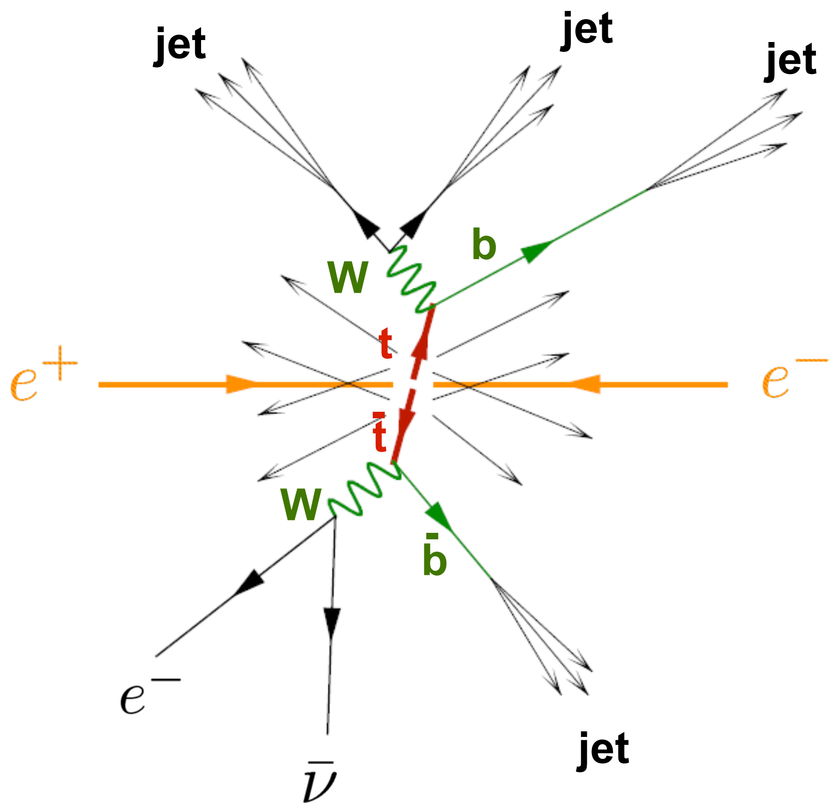
In the MC the low energy perturbative contributions in the parton shower are switched off by the shower cutoff.

The MC mass is in principle a short-distance mass, but it is difficult to identify with only have leading order showers/matrix elements implemented.

Toy Model

Top Invariant Mass Distribution at the ILC:

Fleming, Mantry, Stewart, AH

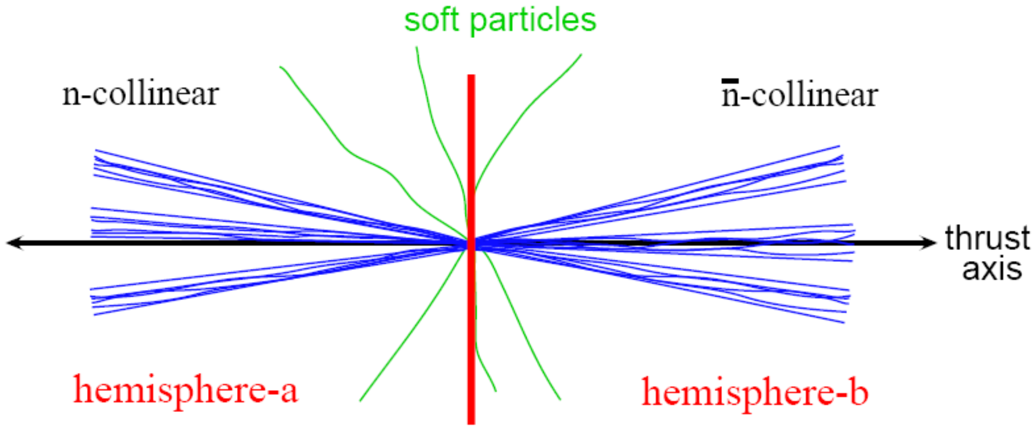


$$Q \gg m_t \quad (p_T \gg m_t)$$

QCD-Factorization

Top invariant mass distribution:

Definition of the observable

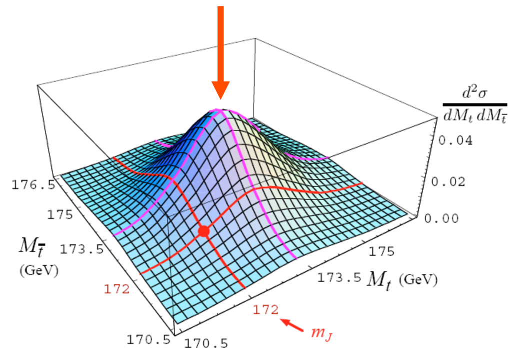


$$M_t^2 = \left(\sum_{i \epsilon a} p_i^\mu\right)^2 \qquad M_{\bar{t}}^2 = \left(\sum_{i \epsilon b} p_i^\mu\right)^2$$

$$\frac{d^2\sigma}{dM_t dM_{\bar{t}}}$$

Resonance region:

$$M_{t,\bar{t}} - m_t \sim \Gamma$$



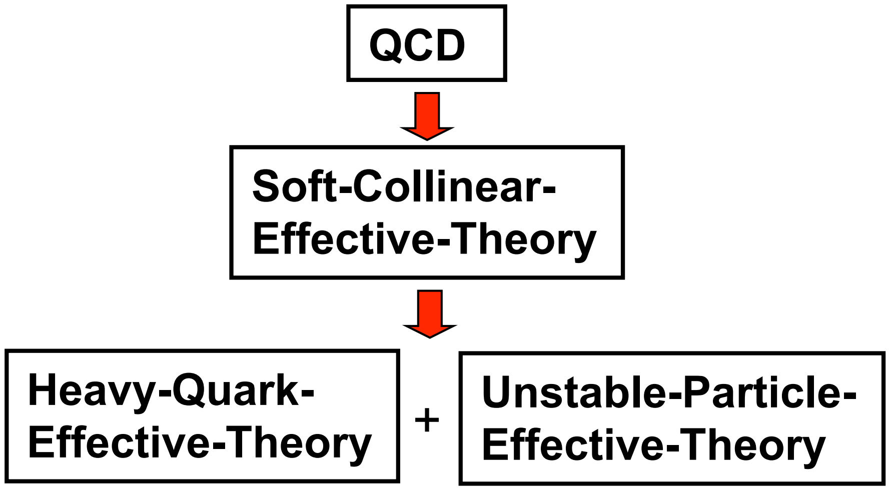
QCD-Faktorisierung

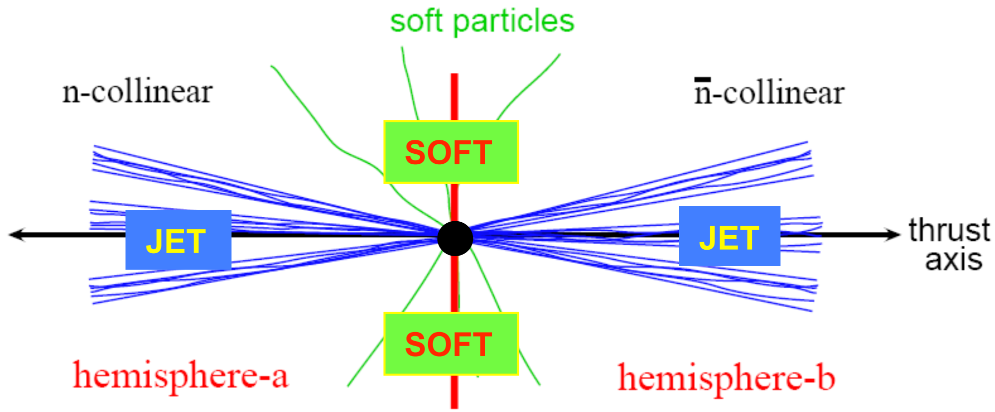
Fleming, Mantry, Stewart, AHH

Phys.Rev.D77:074010,2008

Phys.Rev.D77:114003,2008

$$Q \gg m_t \gg \Gamma_t > \Lambda_{\rm QCD}$$





Faktorization Formula

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \qquad \hat{s} = \frac{M_t^2 - m_J^2}{m_J}$$

$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

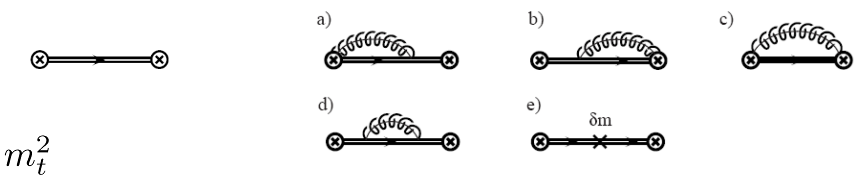
JET JET SOFT

Factorization Theorem

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Jet functions: $$B_+(\hat s, \Gamma_t, \mu) = {\rm Im}\left[\frac{-i}{12\pi m_J} \int d^4x\, e^{ir.x} \langle 0|\, T\{\bar h_{v_+}(0) W_n(0)\, W_n^\dagger(x) h_{v_+}(x)\}\, |0\rangle\right]$$

- perturbative, mass definition contained here
- depends on m_t, Γ_t
- Breit-Wigner at tree level



$$B_\pm(\hat s, \Gamma_t) = \frac{1}{\pi m_t} \frac{\Gamma_t}{\hat s^2 + \Gamma_t^2} \qquad \hat s = \frac{M^2 - m_t^2}{m_t}$$

Soft function: $$S_{\rm hemi}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_s} \delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b}) \langle 0|\overline{Y}_{\bar n}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger(0)|0\rangle$$

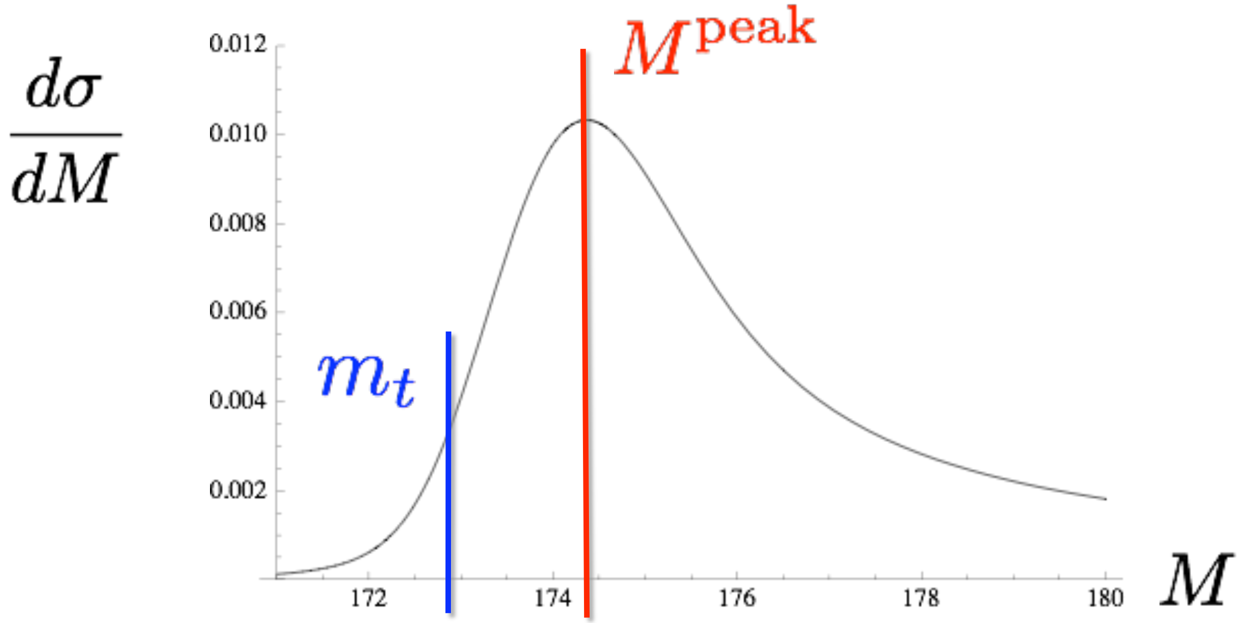
- non-perturbative
- mass+flavor independent
- also governs **massless dijet thrust and jet mass event distributions**

Korshemsky, Sterman, etal.
Bauer, Manohar, Wise, Lee

➡ **Short distance top mass can (in principle) be determined to better than $\Lambda_{\rm QCD}$.**

Numerical Analysis

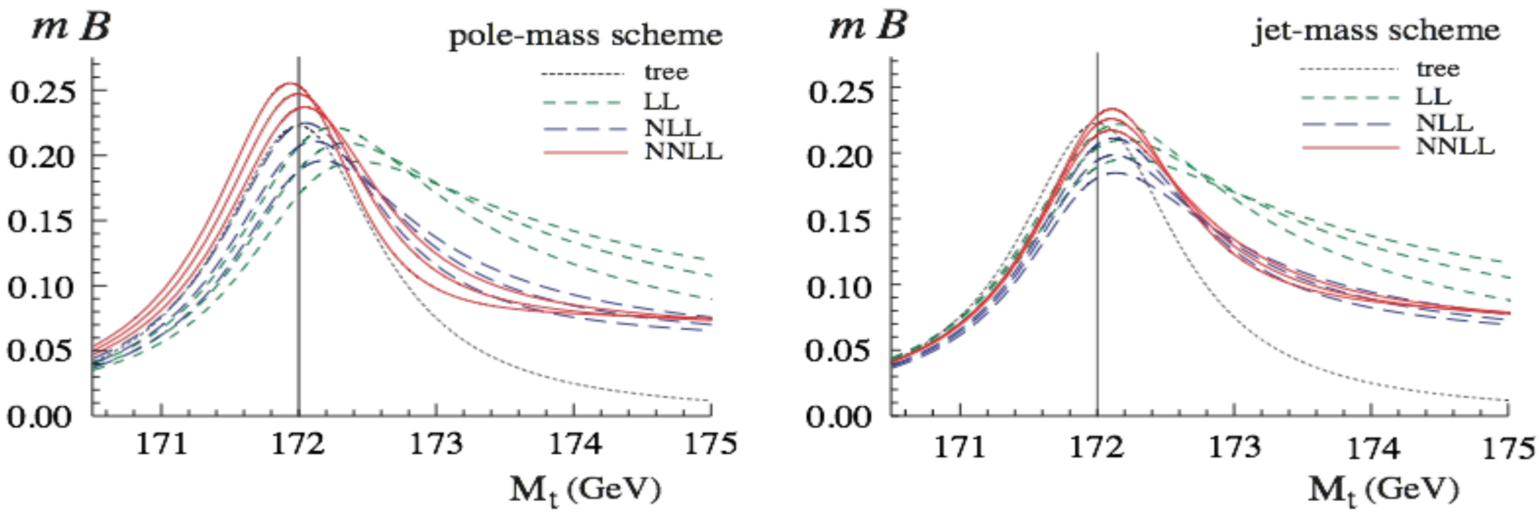
Peak Position und Topquark-Masse:



$$M^{\rm peak} = m_t + \Gamma_t(\alpha_s + \alpha_s^2 + \ldots) + \frac{Q\,\Omega_1}{m_t} + \mathcal{O}\Big(\frac{m_t \Lambda_{\rm QCD}}{Q}\Big)$$

Numerical Analysis

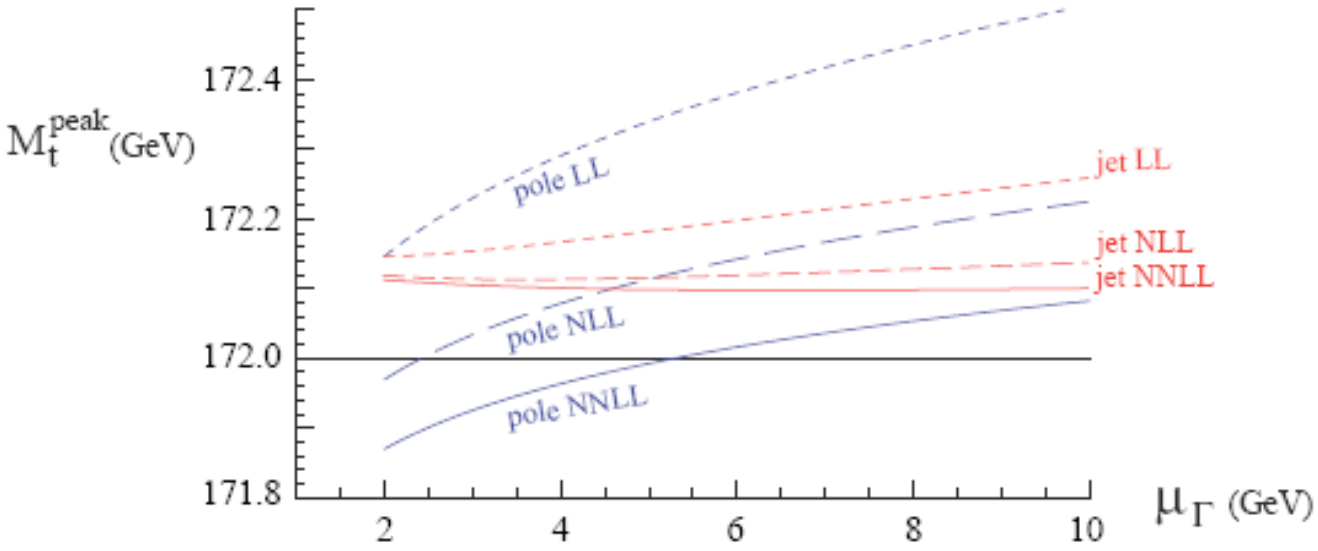
Higher orders and top mass schemes:



$$m_J(R,\mu) \;=\; m_t^{\rm pole} - R\,\frac{\alpha_s}{\pi}\,C_F e^{\gamma_E}\left[\frac{1}{2}+\ln\frac{\mu}{m}\right] + \ldots \qquad\qquad R = \Gamma_t$$

NLL Numerical Analysis

Scale-dependence of peak position



- Jet mass scheme: significanly better perturbative behavior.
- Renormalon problem of pole scheme already evident at NLL.

MC Top Mass

→ Use analogies between MC set up and factorization theorem

Monte Carlo

- Parton shower evolution
- Shower cutoff $R_{sc} \sim 1\ \mathrm{GeV}$
- Hadronization models fixed from reference processes

Factorization Theorem

- Renormalization group evolution from Q to sum large logarithms.
- Subtraction scale R in jet function that defines the mass scheme
- Soft function extracted from event shape distributions with the same soft function

Additional Complications:

Initial state shower, underlying events, combinatorial background, etc

We ignore these issues for now, as they are not included in the factorization theorem yet.

MC Top Mass

Conclusion:

unknown constant of order unity

$$m_t^{\rm MC}(R_{sc}) = m_t^{\rm pole} - R_{sc}\, c \left[\frac{\alpha_s}{\pi}\right] + \mathcal{O}(\alpha_s)$$

$$R_{sc} = 1 \text{ GeV}$$

correction from 1-loop matrix elements

$$\overline{m}_t(\overline{m}_t) = 162.5 \pm 1.3 \pm \mathcal{O}(1) \text{ GeV}$$

Further work to do to complete the relation. Current uncertainty: $\mathcal{O}(1\,\text{GeV})$

There is also work to do to improve the Monte Carlos to make the relation more rigorous!

Conclusions

Be careful thinking about (top) mass definitions. Short-distance masses definition are the better choice if you ask for precision.

The top quark mass is a scheme-dependent parameter. There is no a priori physical quantity associated to the top quark mass.

The exact scheme of the current Tevatron top quark mass is unknown.

It seems possible to determine what top mass scheme is in Monte Carlos for the LHC, but at present we do not even know the 1-loop terms.