Cosmological vacuum selection and metastable susy breaking

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gauge mediation

flux stabilisation

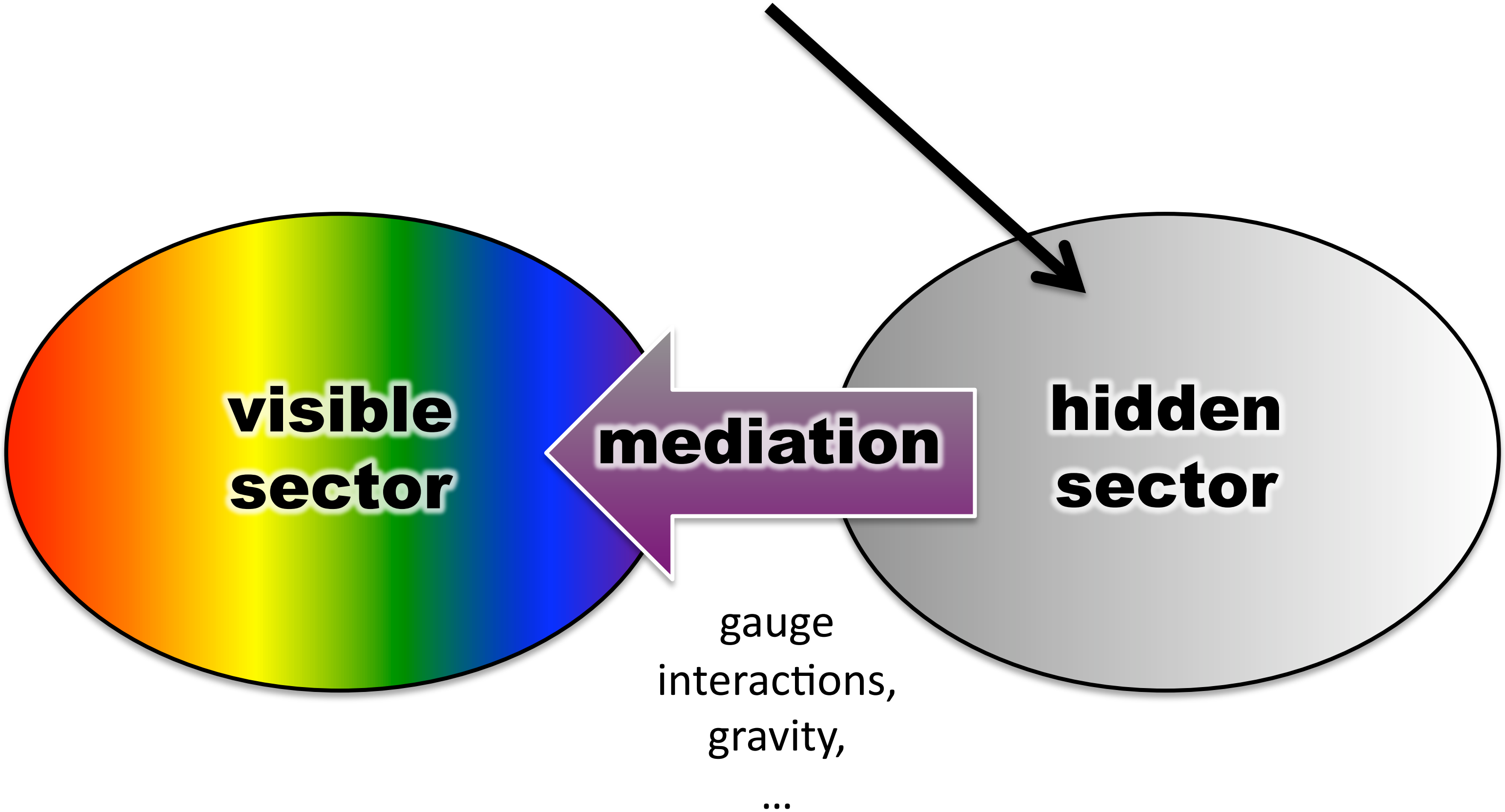
F-term uplifting

ISS sourcing gauge mediation

charged moduli

high T

Supersymmetry breaking



- gauge mediation: natural suppression of flavour changing processes, works in the flat limit
- but: light gravitino, problematic c.c. cancellation, problems with correct electroweak breaking
- gauge and gravity mediation - can one mix them arbitrarily?
- or: to what extent the hidden sector needs to be separated from the observable one?
- degree of cosmological tuning?

Mediation of supersymmetry breaking

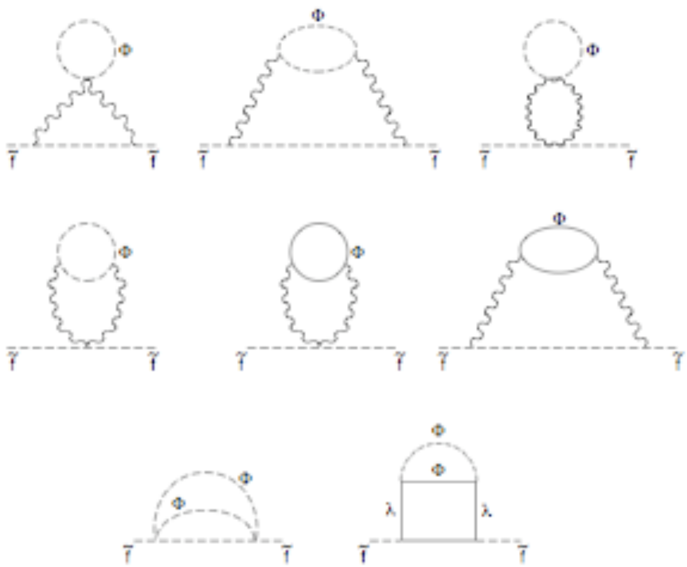
$$w = -\lambda X \tilde{\phi}\phi$$

singlet, breaks susy

$$\langle F_X \rangle \neq 0$$

nonsinglet, e.g. $\bar{\mathbf{5}}, \mathbf{5}$ of $SU(5)$

mass splitting between bosonic and fermionic components



$$M_i = \frac{\alpha_i}{4\pi} N \frac{\langle F_X \rangle}{\langle X \rangle}$$

10^5 GeV

$$m_s^2 = 2 \sum_i \left(\frac{\alpha_i}{4\pi}\right)^2 C_s^{(i)} N \left(\frac{\langle F_X \rangle}{\langle X \rangle}\right)^2$$

$$m_{3/2} = \frac{F^X}{\sqrt{3}M_P}$$

$$m_{gaugino} = \frac{\alpha(m)}{4\pi}\frac{F^X}{<X>}$$

$$m_{scalar} = \frac{\alpha(m)}{4\pi}\boxed{\frac{\sqrt{3}M_P}{<X>}}m_{3/2}$$

$$\Rightarrow \quad m_{3/2,moduli} \ll m_{g,s}$$

$$K = \bar{X}X + \sum_{i=1}^{n}\left(\bar{\tilde{\phi}}_i\tilde{\phi}_i + \bar{\phi}_i\phi_i\right)$$

$$-\frac{1}{16\pi^2}f_4\frac{(\bar{X}X)^2}{\tilde{\Lambda}^2} - \frac{1}{16\pi^2}f_6\frac{(\bar{X}X)^3}{\tilde{\Lambda}^4} + \dots$$

$$W = \mu^2 X + \sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{\phi}_i(m_{ij} + \lambda_{ij}X)\phi_j + c$$

$$R(\tilde{\phi}_i) + R(\phi_j) = 0$$

$$R(\tilde{\phi}_i) + R(\phi_j) = 2$$

$$\delta K = -\frac{1}{16\pi^2}\mathrm{Tr}\left[\mathcal{M}^\dagger\mathcal{M}\ln\left(\frac{\mathcal{M}^\dagger\mathcal{M}}{Q^2}\right)\right]$$

$$m_g \sim f\partial_X det\mathcal{M}$$

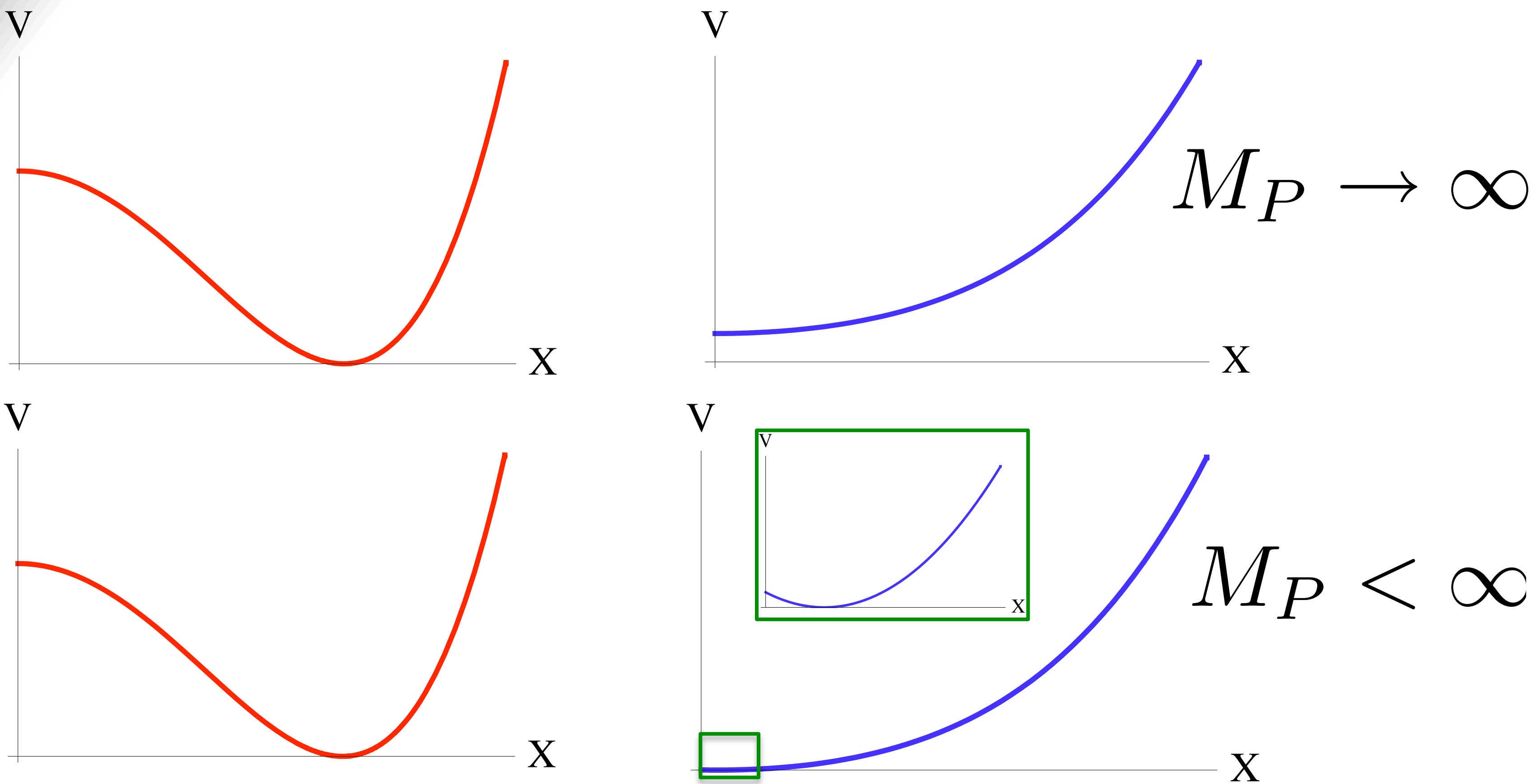
condensation + retrofitting

in string models:

- string instantons
- gauge instantons

$$W_{LE} = W(T) + \mu^2(\Lambda_{dyn})X + m(\Lambda_{dyn})\Phi^2 + \lambda XQq + \lambda' X\Phi^2 ...$$

supersymetry breakdown



$f_4 < 0$

supersymmetry breaking related to spontaneous R symmetry breaking

$f_4 > 0$

supersymmetry breaking related to soft R symmetry breaking transmitted through gravity

Solutions

$$X = \frac{1}{2\sqrt{3}} \frac{\tilde{\Lambda}^2}{f_4 M_P}$$ if $f_4 > 0$ and dominant

$$X^2 = \frac{8|f_4|}{9 f_6} \tilde{\Lambda}^2$$ if $f_4 < 0$ and $f_6 > 0$

$$X^3 = \frac{16\pi^2}{9\sqrt{3} f_6} \frac{\tilde{\Lambda}^4}{M_P}$$ if $f_4 \approx 0$

in all cases solutions exist if $\langle X \rangle \lesssim 10^{-3}\, M_P$

above this value of X the gravitational term gives to large negative slope

Gauging the hidden sector - introducing moduli

$$K = |X|^2 - \frac{|X|^4}{\Lambda^2} + \frac{m_V^2}{2}(T + \bar{T} - V)^2, \quad W = W_0 + f X e^{-T}$$

$$D = (|X|^2 - 2\frac{|X|^4}{\Lambda^2}) - m_V^2 (T + \bar{T})$$

$$X = \frac{\Lambda^2}{2\sqrt{3}(1 + \frac{3\Lambda^2}{2m_V^2})}, \quad T = \frac{\Lambda^4}{24 m_V^2 (1 + \frac{3\Lambda^2}{2m_V^2})^2}$$

$$\text{masses of } (Re(X), Im(X), Re(T)):$$

$$(\frac{2\sqrt{2}f}{\Lambda}\sqrt{1 + \frac{3\Lambda^2}{2m_V^2}}), \ \frac{2\sqrt{2}f}{\Lambda}\sqrt{1 + \frac{3\Lambda^2}{2m_V^2}}), \ 2\, m_V)$$

$$\langle D \rangle \sim f^2 \qquad\qquad m_{3/2} \sim f$$

Adding untwisted moduli

$$W = \mu^2 X - \lambda X Q q + A e^{-aT} + B$$

$$x = aT$$

$$m_{soft} = \frac{g^2}{16\pi^2}\frac{\mu^2}{\tilde{\Lambda}^2}\frac{a^{3/2}}{x^{3/2}} = \frac{g^2}{16\pi^2}\frac{\mu^2}{\tilde{\Lambda}^2}\frac{1}{(Nx)^{3/2}}$$

$$N = 10,\ g^2 = 0.01$$

request that $m_{soft} = \gamma\,\text{TeV} = \gamma\,10^{-15}$

cc=0: $$\frac{2}{\sqrt{3}}|A| x e^{-x} = \mu^2$$

$$\langle X \rangle = \tilde{\Lambda}^2 = 2 \times 10^9 \gamma^{-1} x^{-1/2} e^{-x}$$

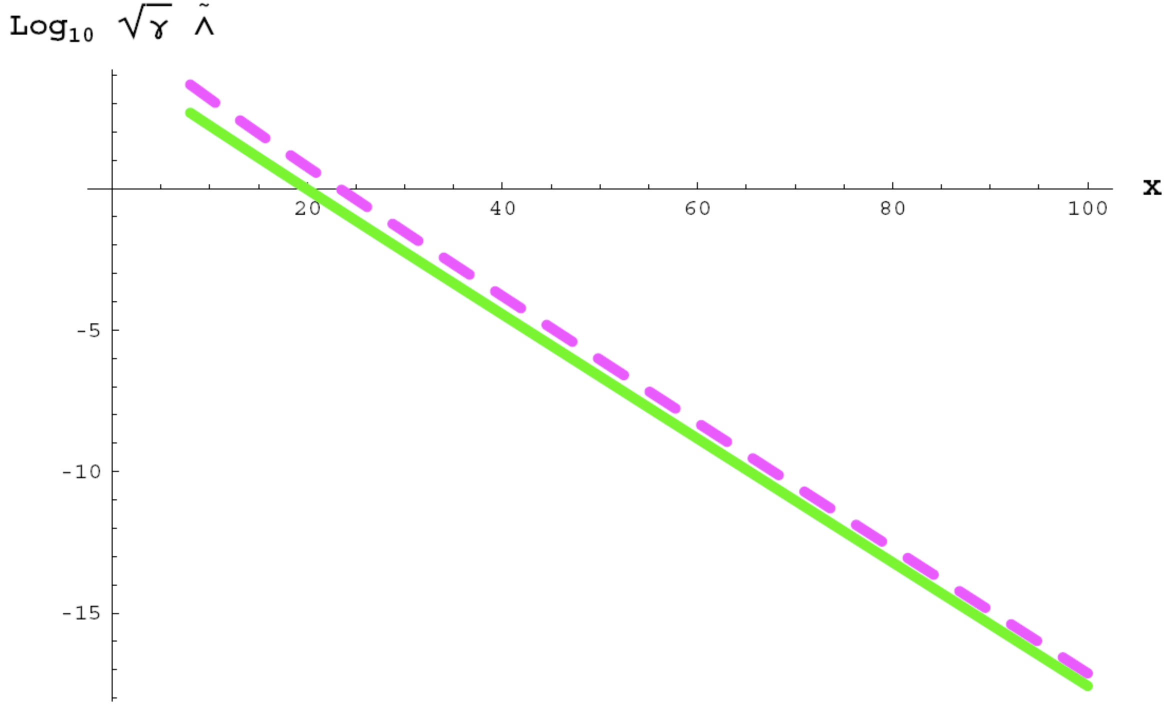


Figure 1: *Plot of* $\log_{10}(\sqrt{\gamma}\tilde{\Lambda})$ *as the function of* $x = T/N$. *The lower curve corresponds to a fixed value of* g^2, *and the upper (dashed) to* $g^2 = 8\pi^2 a/x$

if gravitational contribution to scalar masses amounts to 10% and 30% of the gauge mediated contribution, one obtains
$\tilde{\Lambda} = 6\,10^{-3}\,M_P$ and $\tilde{\Lambda} = 10^{-2}\,M_P$ respectively

$$|F_X|/|F_T| \sim aT^2$$

$$m^2_{\mathrm{soft}}/m^2_T \sim 10^{-4}/(\tilde{\Lambda}^4 (aT)^2)$$

A way out: (Problematic, with S. de Alwis)

$$W_{npert} = ANe^{-T/N} - B(N+1)e^{-T/(N+1)} + W_0$$

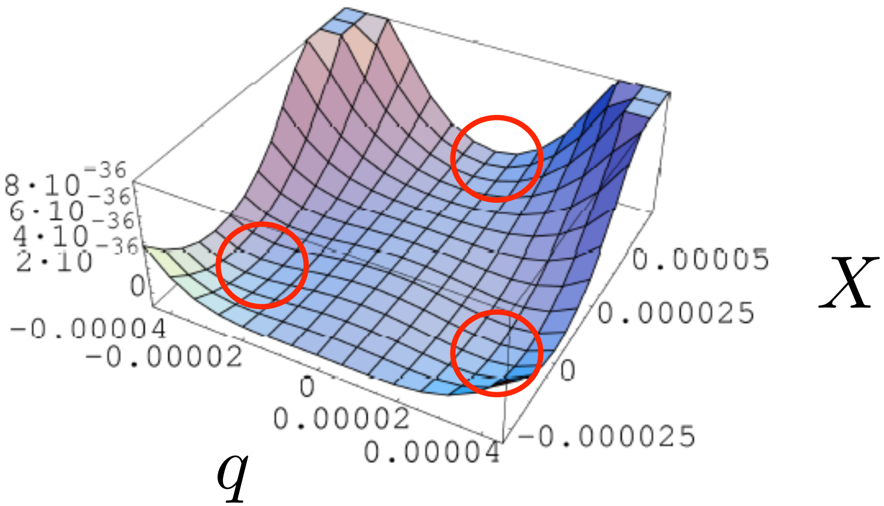
$$W_{eff}(T) = \frac{\Delta}{2}(T - T_0)^2 + C$$

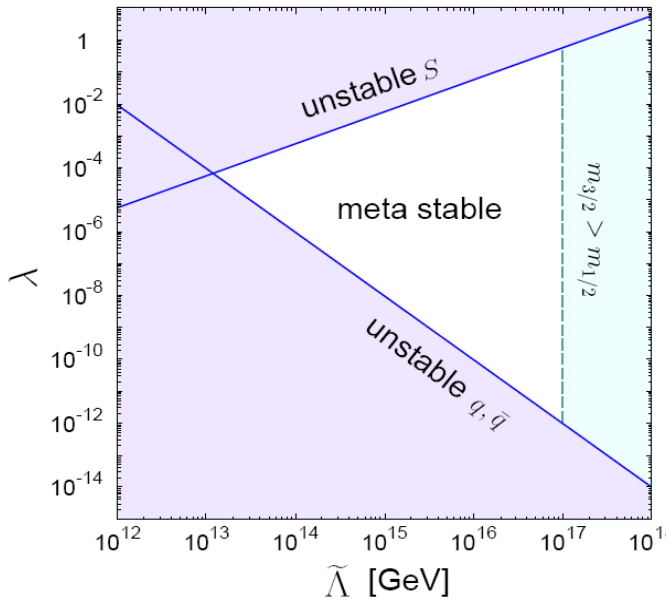
$$t_0 = N(N+1)\log\left(\frac{A}{B}\right),\ \Delta = \frac{A}{N}e^{-t_0/N}(1 - e^{-t_0/N^2})$$

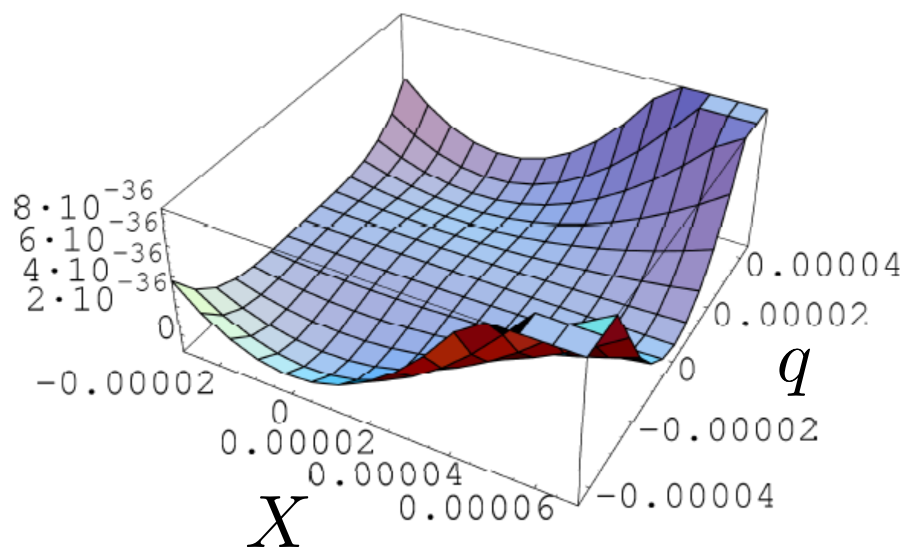
no relation between t_0 and $\tilde{\Lambda}$

but $t_0 \gg 1$ and cc cancellation $\rightarrow$ $F^T/F^X \ll 1$

Cosmological vacuum selection

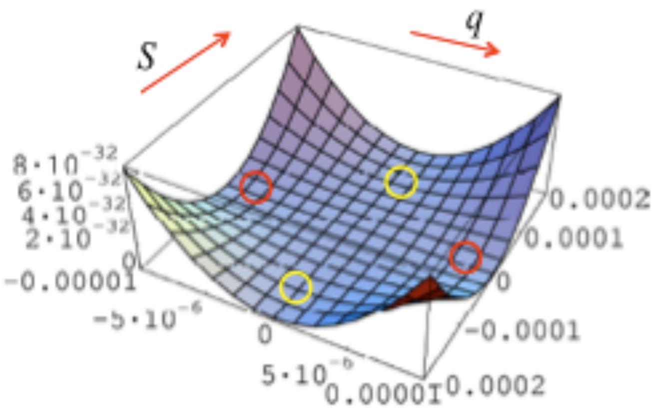






$$g_4 = -1/\Lambda_1^2 \;\; \text{and} \;\; g_6 = 1/\Lambda_2^4$$

When g_4 is negative and $\Lambda_1^3 < \Lambda_2^2$ we have a supersymmetry breaking minimum which survives in the global susy limit:



$$S^2 = \frac{2}{9}\frac{|g_4|}{g_6} \sim \frac{\Lambda_2^4}{\Lambda_1^2}$$

$$q^2 = \frac{\mu^2}{\lambda}$$

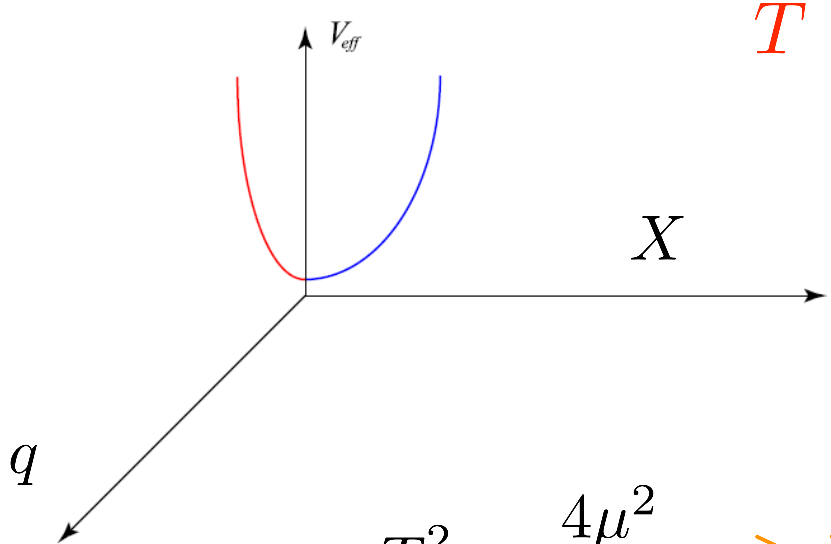
For vacuum stability: $\frac{\Lambda_2^4}{\Lambda_1^2} > \frac{\mu^2}{\lambda}$

When g_4 is negative and $\Lambda_1^3 > \Lambda_2^2$ (or g_4 negligible) then we have a supersymmetric breaking minimum:

$$S^3 = \frac{1}{9}\frac{c}{\mu^2}\frac{1}{g_6} \sim \Lambda_2^4$$

and the stability of this vacuum in the q-direction implies: $\Lambda_2^{8/3} > \frac{\mu^2}{\lambda}$

$$T >> \tilde{\Lambda}$$

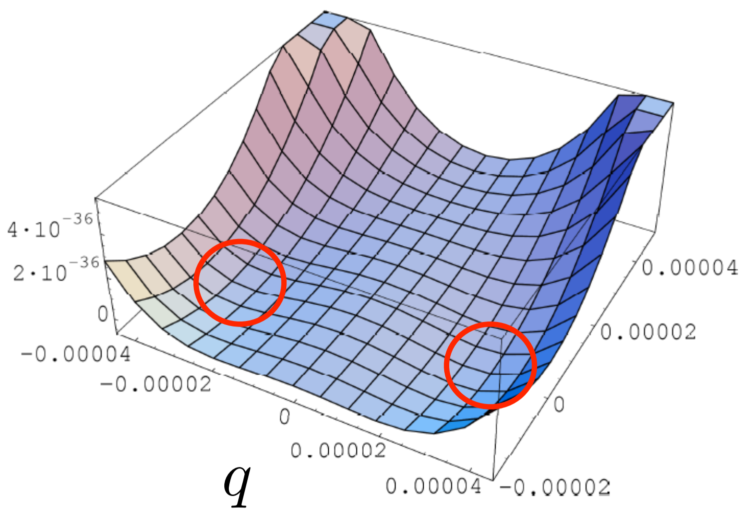


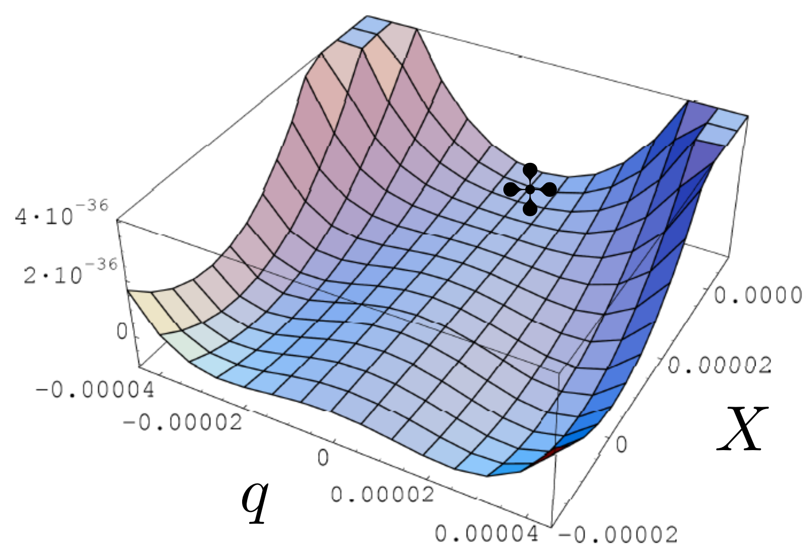
$$X = \frac{4\mu^2 c}{3\lambda^2\tilde{\Lambda}^2} \sim \frac{\mu^4}{\lambda^2\tilde{\Lambda}^2}$$

$$q_c^{min} = \bar{q}_c^{min} = 0, \qquad T > \tilde{\Lambda}$$

$$\delta W = \lambda X q \bar{q}$$

$$T_{cr}^2 = \frac{4\mu^2}{\lambda} \quad >> \quad T_S^2 = \frac{\mu^3}{\sqrt{\lambda^3}}$$





The *second critical temperature* at which the susy breaking minima are formed:

$$1)\ g_4 > 0 \qquad T_S \sim \left(\frac{\mu^3}{\sqrt{\lambda^3}}\right)^{1/2} = T_{cr}^{3/2}$$

$$2)\ g_4 < 0 \text{ and } g_6 > 0 \qquad T_S \sim \frac{\mu^2}{\lambda}\frac{1}{\Lambda_1}$$

$$3)\ g_4 \sim 0 \qquad T_S \sim \frac{\mu^2}{\lambda}\frac{1}{\Lambda_2^{2/3}}$$

For all the cases we find that this second critical temperatur is lower than the first critical temperature which drives the system to the susy minima.

non-supersymmetric vacua disfavoured!

Conditions for thermal equilibrium

Messengers:

- $\langle \sigma v n \rangle \sim \alpha_{SM} T$
- $T_{up} \sim 10^{14} - 10^{15}$ GeV
- $T_q \sim m_q/20$

$$\delta W = \lambda S q \bar{q}$$

$$\lambda^2 |S|^2 \langle |q|^2 \rangle_T \sim \lambda^2 |S|^2 T^2$$

Spurion:

- For $T > m_q$ the thermally averaged cross section for 2-2 processes with two spurions is of the order of

$$\Gamma_{int} =< \sigma v n >_T \sim \frac{\lambda^4 \alpha^2}{16\pi^2} T$$

- equilibrium possible below

$$T_{eq} \sim \frac{\lambda^4 \alpha^2}{16\pi^2 \sqrt{g_*}} = \mathcal{O}(10^{-3}) \lambda^4 \alpha^2$$

- for $T < m_q$, the thermalization could also be achieved and the relevant averaged cross section becomes $\langle \sigma v n \rangle_T \sim \alpha^2 \lambda^4 T^5 / m_q^4$
  This gives a lower bound for the equilibrium window

- For the case of mixed gauge/gravity mediation spurion doesn't achieve equilibrium

Conditions for the selection of the susy breaking vacuum

- Biased initial conditions:

$$\Lambda^2 < S_{init} < \Lambda$$

1) $W = W_I\,(1 - \xi Q_I S + ...) \rightarrow$
$V \simeq 3H^2 \left(|S|^2 - \mathcal{O}\left(\frac{|S|^4}{\tilde{\Lambda}^2}\right)\right) + 3H^2\,|1 - \xi Q_I S|^2$
(for instance $Q_I = I_e - I_0$) $\quad \rightarrow S \sim Q_I$
2) $\delta K = -q^\dagger q I^\dagger I$

- and

$$T_{reheating} < m_q \sim \lambda \langle S \rangle$$

- or, for sufficiently small λ,

$$m_q < T_{reheating} < T_S$$

(Λ, λ)	$m_{q,\,\max}$	$m_{q,\,\min}$	T_S	$T_{\rm rh}$
$(10^{-2}, 10^{-7})$	10^{-9}	10^{-11}	$10^{-8.25}$	$10^{-(9-11)} < T_{\rm rh} < 10^{-8.25}$
$(10^{-2}, 10^{-8})$	10^{-10}	10^{-12}	$10^{-7.5}$	$10^{-(10-12)} < T_{\rm rh}$
$(10^{-2}, 10^{-9})$	10^{-11}	10^{-13}	$10^{-6.75}$	$10^{-(11-13)} < T_{\rm rh}$
$(10^{-3}, 10^{-7})$	10^{-10}	10^{-13}	$10^{-9.75}$	$10^{-(10-13)} < T_{\rm rh} < 10^{-9.75}$

Table 3: The range of values ($M_P = 1$) of the reheating temperature that favour the selection of the susy breaking vacuum by the system of fields for the first case i.e. the Kähler potential for the spurion is $K = |S|^2 - |S|^4/\Lambda^2$. This can be achieved for sufficiently small coupling λ. Such small values of the coupling don't destabilize the metastable susy breaking vacuum only if the cut-off scale Λ takes values larger than $\sim 10^{-3}$. However, the Λ cannot exceed $\sim 10^{-2}$ which is the maximum value allowed by the requirement of gauge mediation domination. The reheating temperature is *not* bounded from above for couplings $\lambda < T_S$. The lower bound on the reheating temperature is the mass of messengers $m_q = \lambda S_{\rm rh}$. The conservative bound applies when the spurion has the maximal initial displacement $S_{\rm rh} \sim \Lambda$. In the optimistic case, $S_{\rm rh} \sim \Lambda^2$, the lower bound on the reheating temperature is Λ^{-1} times smaller. It is interesting to remark that e.g. for $\lambda = 10^{-8}$ and $\Lambda = 10^{-2}$ the gravitino mass is at the GeV range and the reheating temperatures can take rather plausible values consistent with cosmological models.

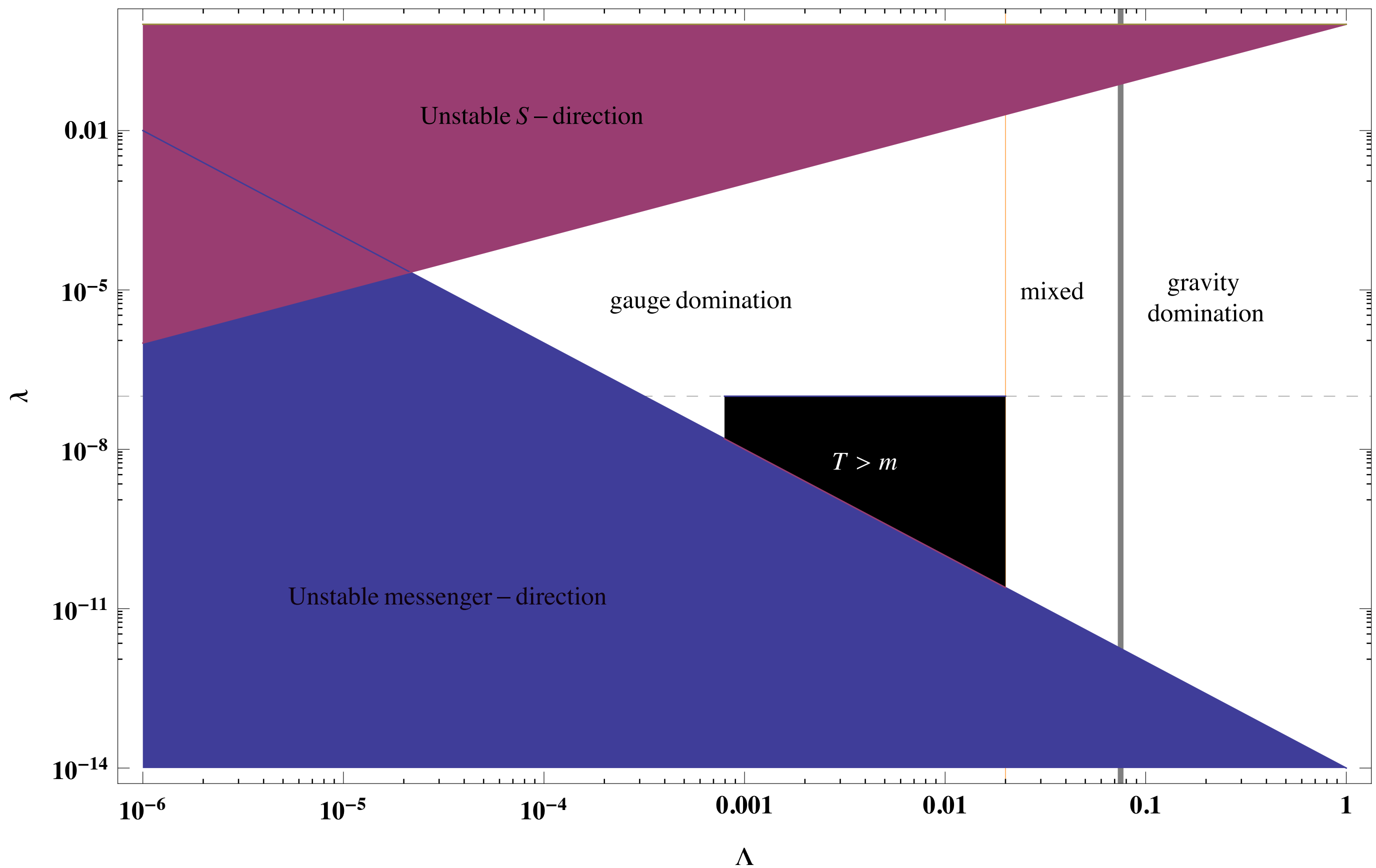


Figure 2: The parameter space spanned by the energy scale $\Lambda \approx \sqrt{\langle S \rangle}$ (horizontal axis with $M_P = 1$) and the coupling λ (vertical axis) for the case of the Kähler potential $K = |S|^2 - |S|^4/\Lambda^2$. The white triangle region (including the black trapezium) gives a meta-stable supersymmetry breaking minimum. It is divided into three parts. The part on the right of the parameter space corresponds to $m_{3/2} > m_{\rm gaugino}$ i.e. to gravity domination in the mediation of susy breaking. The part on the left corresponds to $m_{3/2} < 0.1 m_{\rm gaugino}$ according to (75). The part in between gives susy breaking minima with mixed gauge-gravity mediation. The *black region* is the parameter space that allows the realization of the metastable minimum for thermalized messengers, $T_{\rm rh} > m_q$, see Table 3. It could be extended to the right of the parameter space, i.e. towards larger Λ, if the requirement of gauge mediation domination is waved.

Moving vacua around

$$W = \mu^2 S - \lambda S q\bar{q} \pm M q\bar{q} + c$$

In the global susy the theory has a susy minimum at

$$S = \mp \frac{M}{\lambda}, \quad q\bar{q} = \frac{\mu^2}{\lambda},$$

and a susy breaking minimum at

$$S = q = \bar{q} = 0$$

In sugra

$$V_0 \approx -2c\mu^2(S + S^\dagger) + 4\mu^4 \frac{|S|^2}{\Lambda^2}$$

hence

$$S \approx c\Lambda^2/(2\mu^2)$$

Finite temperature potential, assuming a temperature higher than the messenger scale M is

$$V^T = \frac{T^2}{12}\left[4\frac{\mu^4}{\Lambda^2} + 4\mu^2 c\frac{S+S^\dagger}{\Lambda^2} + 3|\lambda S \pm M|^2 + 3\lambda^2(|q|^2+|\bar{q}|^2)\right]$$

$$m_q^2 \approx -2\lambda\mu^2 + \frac{1}{2}T^2\lambda^2$$

$$T_{cr} \approx 2\mu/\sqrt{\lambda}$$

Note:

$\delta W = D^2(\lambda S - \Gamma)$ leads to $\lambda S \approx \Gamma$ if $\Gamma \gg M$

since $\delta V_T \sim T^2(\lambda S - \Gamma)^2$

Ellis, Llewellyn Smith and Ross '82

Moduli at finite temperature

scaling by moduli dependent factors:

- $\lambda \longrightarrow \lambda e^{K(T,\bar{T})/2}$
- $\mu^2 \longrightarrow \mu^2 e^{K(T,\bar{T})/2}$
- $c \longrightarrow c\, e^{K(T,\bar{T})/2}$
- $\Lambda \longrightarrow \Lambda\, e^{K(T,\bar{T})/2}$

where e.g. $e^{K(T,\bar{T})/2} = \frac{1}{(T+\bar{T})^{3/2}}$

all critical temperatures are moduli independent

e.g. $T_c = \frac{\mu}{\sqrt{\lambda}} \longrightarrow \frac{\mu e^{K/4}}{\sqrt{\lambda\, e^{K/2}}}$

Backreaction of the temperature effects on moduli

$$V_{tot}(t) = V_0 + V_{Temp} + F(T, g^2 = \frac{1}{2t})$$

$$V_{tot} = \Delta^2 (t - t_0)^2 + \frac{1}{(2t)^3} \frac{(S^2 + 2q^2)\lambda^2 T^2}{4} + \frac{\pi^2 T^4}{24} \frac{a_2}{2t}$$

where $\Delta = \frac{1}{N} e^{-t/N}$, $a_2 \approx \frac{3N^3}{8\pi^2}$

conditions: $V'_{tot} = 0$ and $|t - t_0| \sim t_0$
give new critical temperature T_*:

$$\Delta^2 t_0 = \frac{3}{16 t_0^4} (\langle S^2 \rangle + 2\langle q^2 \rangle) \lambda^2 T_*^2 + \frac{\pi^2 a_2}{96 t_0^2} T_*^4$$

additional condition: $T_R < T_*$

Summary

- Thermalization usually makes metastable supersymmetry breaking cosmologically disfavoured
- Biased initial conditions and low reheating temperature help to drive the system towards the susy breaking vacuum
- Cosmological history of susy breakdown is rather sensitive to the nature of the hidden/transmission sectors

Backup

Vacua of microscopic O'R models

$$W_1 = \frac{\lambda}{\sqrt{2}} X\phi_1\phi_2 + \frac{m}{\sqrt{2}}\phi_1\phi_3 + \frac{1}{2\sqrt{2}} m_2\phi_2^2 + fX + c$$

$$K_{\text{eff}} = \frac{m^2}{128\pi^2}\left[f_0(R, m^2/\Lambda^2) + f_2(R, m^2/\Lambda^2)a^2 + f_4(R)a^4 + f_6(R)a^6 + \mathcal{O}(a^8)\right]$$

$$a \equiv \lambda X/m \quad \text{and} \quad R \equiv m_2/m$$

$$\begin{aligned}
f_0(R, m^2/\Lambda^2) &= (2+R^2)\ln\left(\frac{m^2}{4\Lambda^2}\right) + R^2 \ln R^2 \\
f_2(R, m^2/\Lambda^2) &= -2 - 2\ln\left(\frac{m^2}{4\Lambda^2}\right) + \frac{2R^2}{R^2-1}\ln R^2 \\
f_4(R) &= \frac{1 + 2R^2 - 3R^4 + R^2(R^2+3)\ln R^2}{(R^2-1)^3} \\
f_6(R) &= -\frac{1 + 27R^2 - 9R^4 - 19R^6 + 6R^2(R^4+5R^2+2)\ln R^2}{3(R^2-1)^5}
\end{aligned}$$

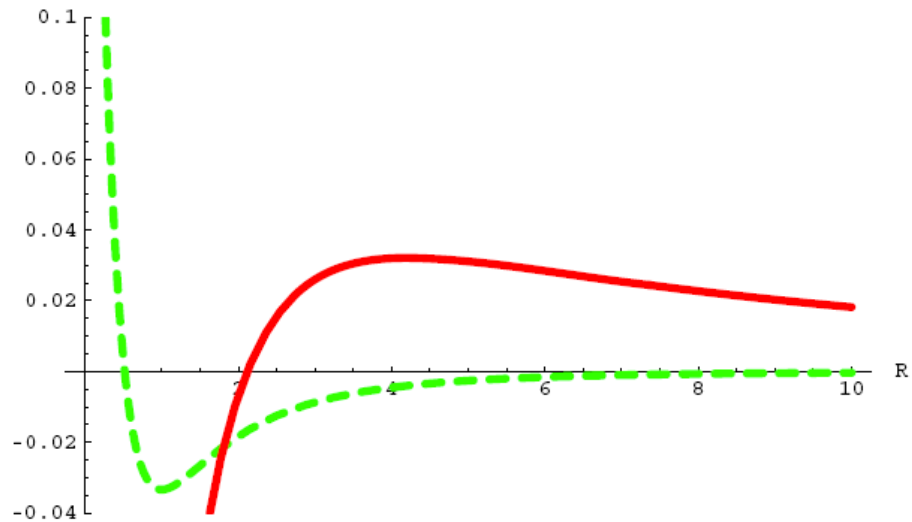


Figure 3: *Values of the coefficients f_4 and f_6 in the Kähler potential (67), solid and dashed lines, respectively.*

$$\tilde{\Lambda}^{-2} \equiv \lambda^4/(128\pi^2 m^2)\,|f_4(R)|$$

$R < 2.11$

$$X \approx \tilde{\Lambda}^2/\sqrt{12}$$

$R > 2.11$

$$X^2 = -\frac{2f_4(R)}{9f_6(R)}\frac{m^2}{\lambda^2}$$

this assumes sugra corrections null

if: $\tilde{\Lambda}^2 > (\lambda^2/48\sqrt{3}\pi^2)(f_4^2/|f_6|)$

the term linear in X, sourced by sugra, becomes important, and :

$$X^3 = -\frac{128\pi^2}{9\sqrt{3}f_6(R)}\frac{m^4}{\lambda^6}$$

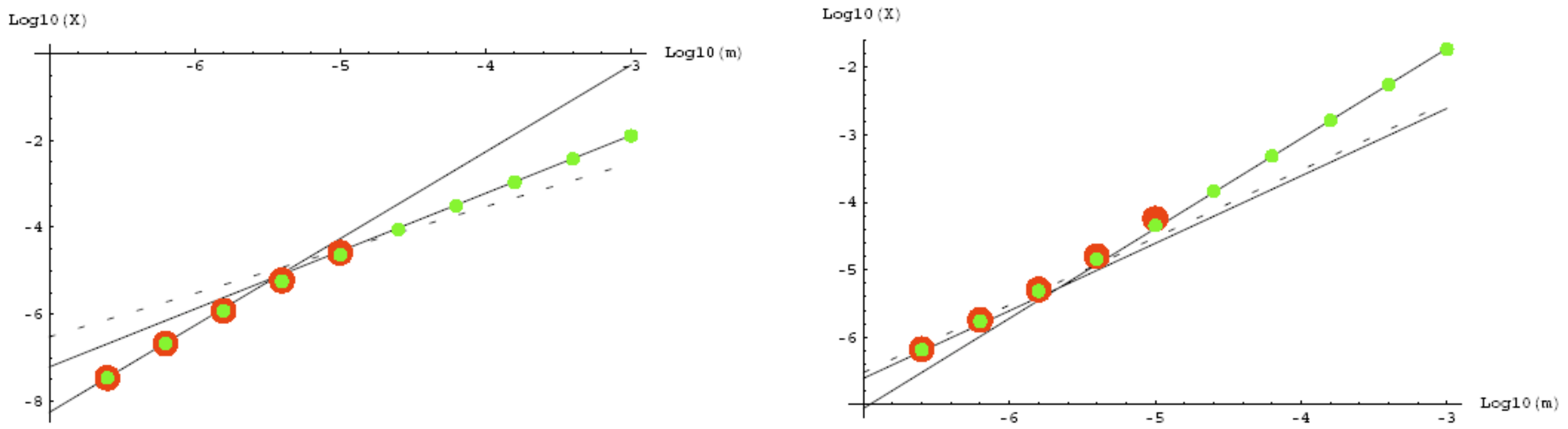


Figure 4: *Position of the minimum of the full supergravity potential for $R = 1.5$ (left panel) and $R = 3$ (right panel) shown as large red dots. Small green dots represent the results obtained with the use of the expansion (67). In the left panel, the approximate result (82) is shown as the steeper solid line and the the approximate result (85) as the less steep one. In the right panel, the approximate solutions (84) and (85) are shown as less and more steep solid lines, respectively. In both cases, the minimum with $X \neq 0$ does not exist if m is too large.*

$$< X > \lesssim 10^{-3}\, M_P$$