Single impurity in a Luttinger liquid away from half filling

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Motivation

fRG – flexible method for large
one-dimensional systems

How precise is our implementation of fRG ?

Outline

- Model
- fRG
- Energy without impurity
- Energy with impurity
- Summary

Model

- interacting spinless fermions in a ring with impurity threaded by a magnetic field

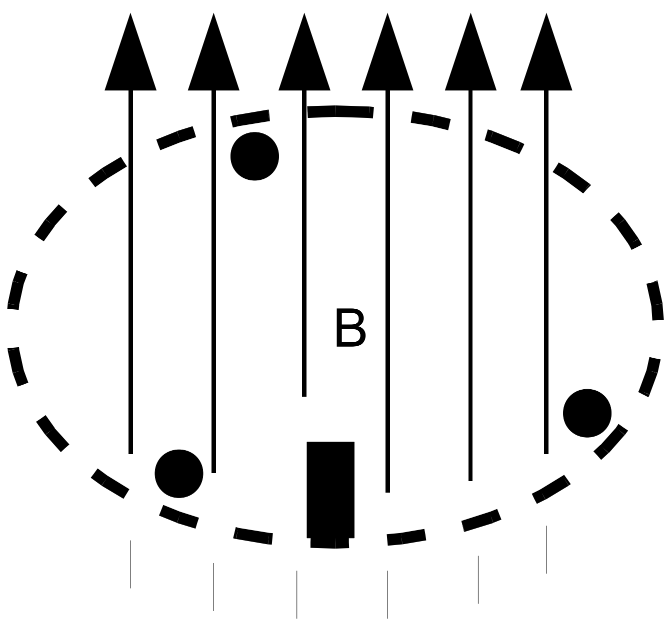
Hopping amplitude

Interaction amplitude

$$H = \sum_i \left[-t \left(e^{i\phi/L} a_i^+ a_{i+1} + e^{-i\phi/L} a_{i+1}^+ a_i \right) + U \, n_i n_{i+1} \right] + V \, n_1$$

Peierls term (magnetic field)

Impurity potential



- model without impurity exactly solvable by Bethe ansatz,Bosonization
- persistent current even with impurity

fRG

$$\partial_k \Gamma_k = -tr\left[\left(\frac{\delta^2 \Gamma_k}{\delta\theta\,\delta\bar{\theta}} + R_k\right)^{-1} \partial_k R_k\right]$$

Regulator $R_k \to 0, k \to 0$ Initial conditions $\Gamma_\infty = S$

ansatz for effective average action

$$\Gamma_k = \int d\tau\, a_0 + \sum_{i=1}^{L} \theta_i \partial_\tau \bar{\theta}_i +$$

$$a_{ii}\theta_i\bar{\theta}_i + a_{ii+1}\theta_i\bar{\theta}_{i+1} + a_{i+1\mathrm{i}}\theta_{i+1}\bar{\theta}_i + a_{ii+1\mathrm{ii}+1}\theta_i\theta_{i+1}\bar{\theta}_i\bar{\theta}_{i+1} + \ldots$$

Large set of coupled ODEs

Ground state energy : $E_0 = a_0 + \mu N$

Filling : $n_f = -\frac{1}{L}\frac{\partial a_0(\mu)}{\partial \mu}$

Half filling $\mu = U$

Ground state energy without impurity

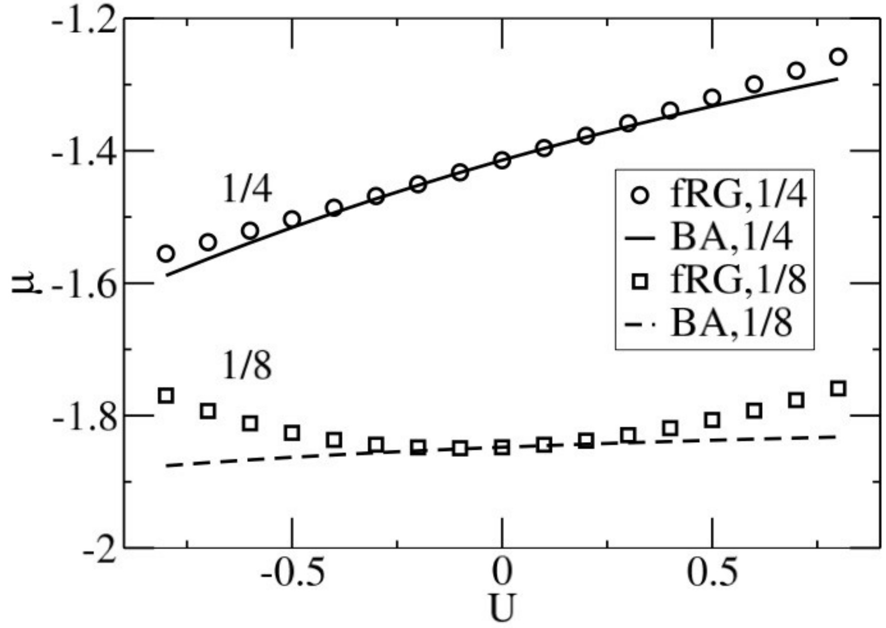
- low energy properties described by Luttinger liquid theory
- ground state energy can be written as

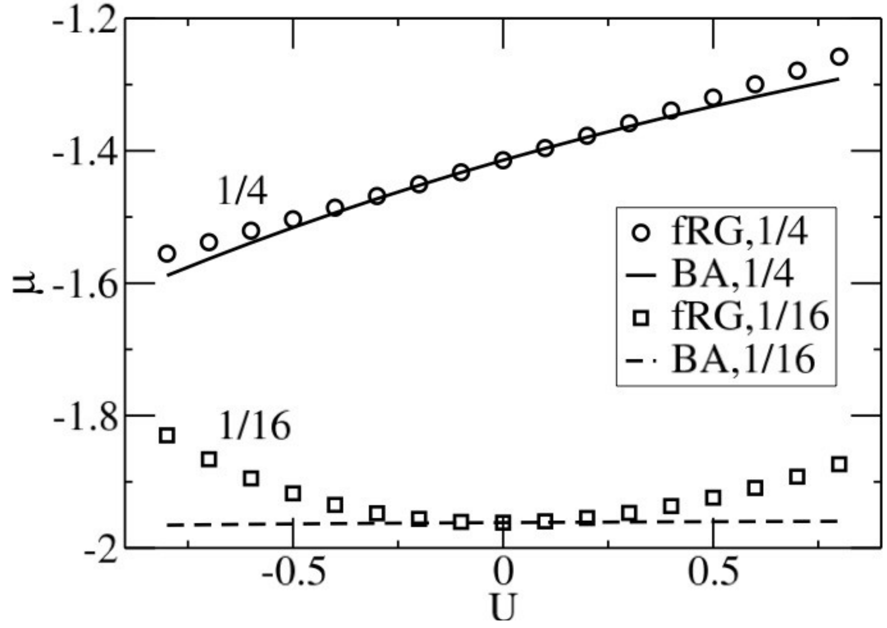
$$E_0 = \epsilon_0 L + \frac{\pi u}{2\mathrm{L}} \left[-\frac{1}{3} + \frac{1}{K} N^2 + K \left(J - \frac{\phi}{\pi} \right)^2 \right] + \dots$$

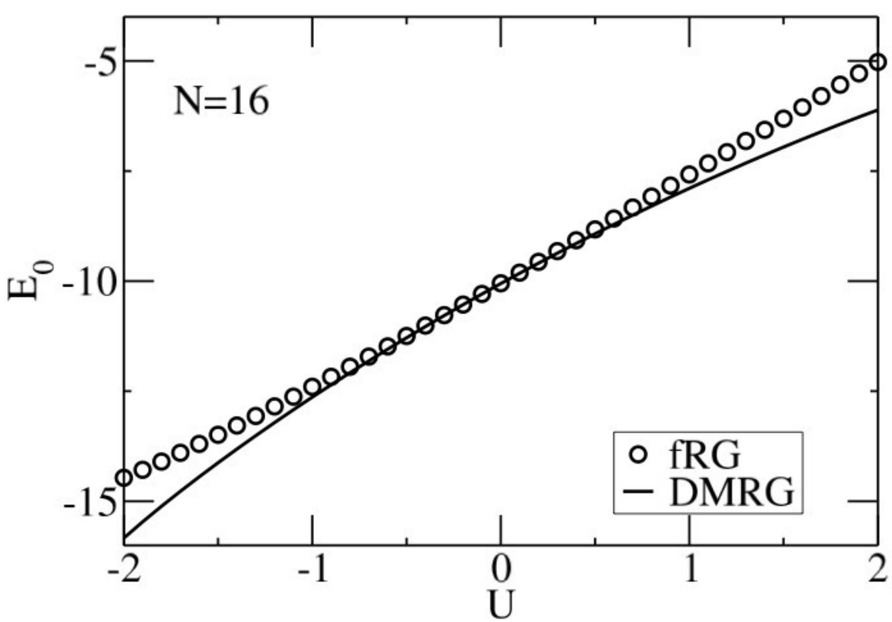
Parameters of the ground state energy

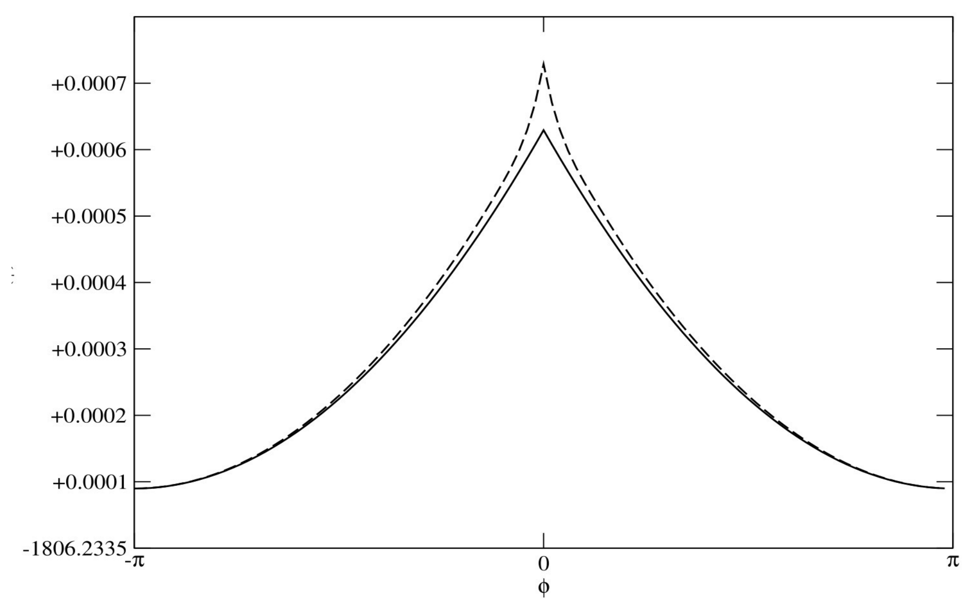
$$u = L \sqrt{\frac{\partial^2 E_0}{\partial N^2} \frac{\partial^2 E_0}{\partial \phi^2}} \qquad K = \pi \sqrt{\frac{\partial^2 E_0}{\partial \phi^2} / \frac{\partial^2 E_0}{\partial N^2}} \qquad \epsilon_0 = \lim_{L \to \infty} \frac{E_0}{L}$$

Chemical potential and ground state energy





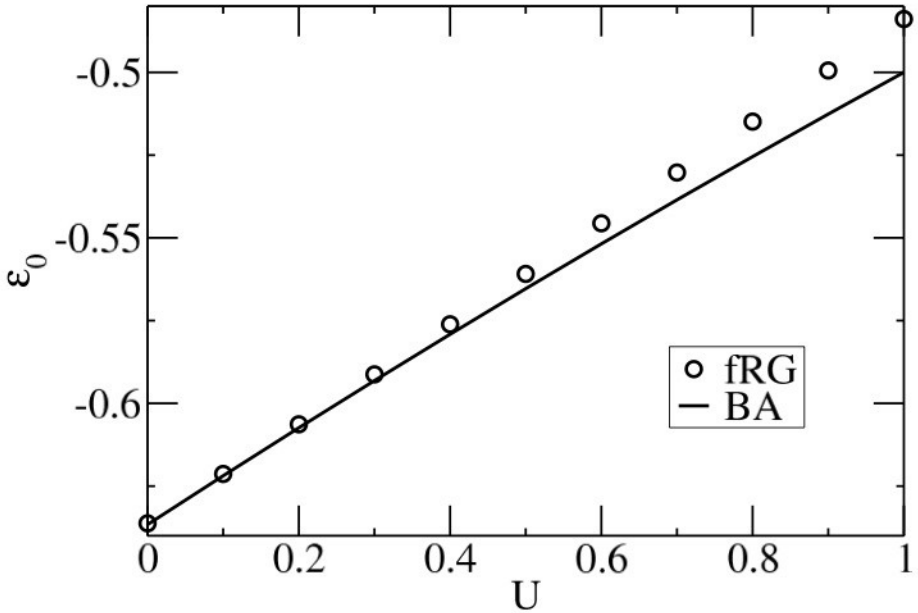




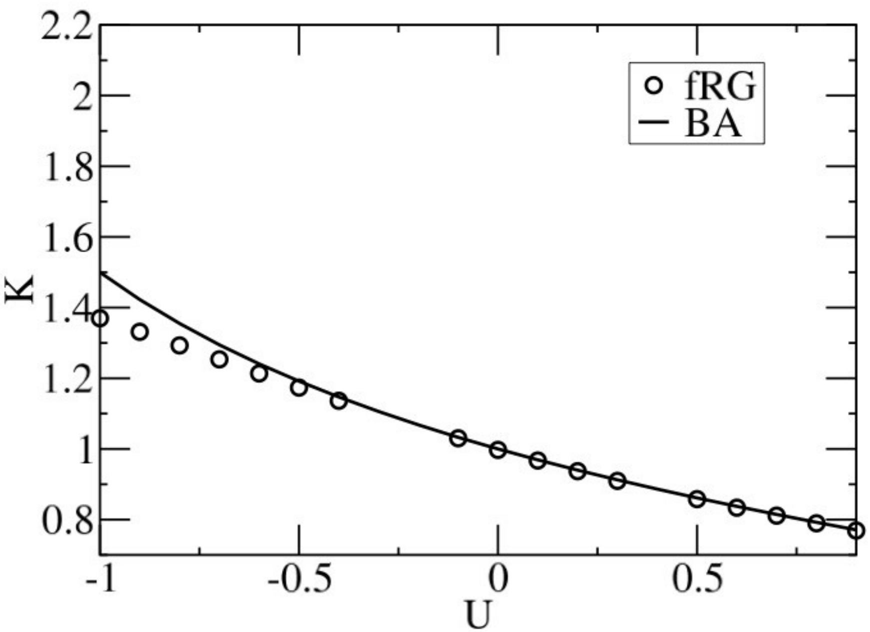
Parameters of ground state energy

Half filling

ϵ_0

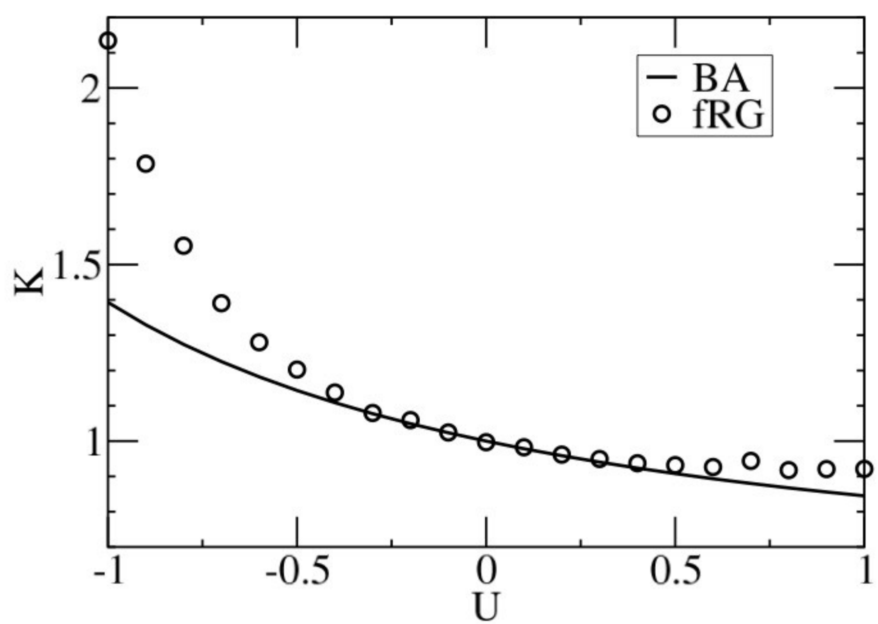


K



Quarter filling

U	E(BA)	E(fRG)	err
-1	-0.438618	-0.466357	6.00%
-0.5	-0.451108	-0.457022	1.00%
0	-0.449522	-0.450158	0.14%
0.5	-0.440358	-0.444977	1.00%
1	-0.426673	-0.450158	5.20%



Ground state energy with impurity

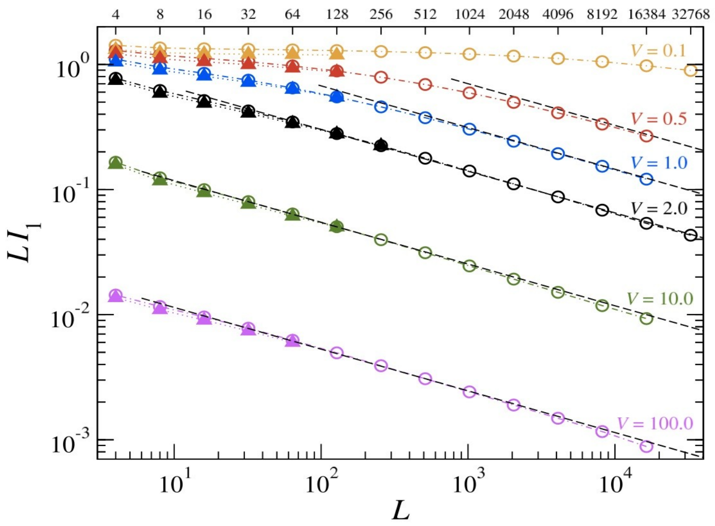
-inserting of impurity changes form of the energy

$$E_0 = \epsilon_0 L + \frac{\pi u}{2L}\left[-\frac{1}{3} + \frac{1}{K} N^2 \pm \frac{A}{L^{1/K-1}} \cos(\phi)\right] + \ldots$$

Persistent current

$$I = -\frac{\partial E_0}{\partial \phi} = \mp \frac{\pi u A}{2L^{1/K}} \sin(\phi)$$

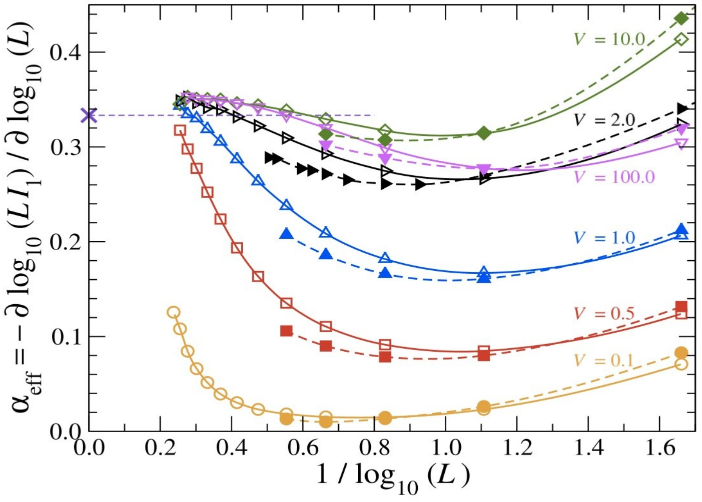
Persistent currents for half filling



Persistent Current for half filling

Effective α for half filling

$$\alpha = 1/K - 1$$

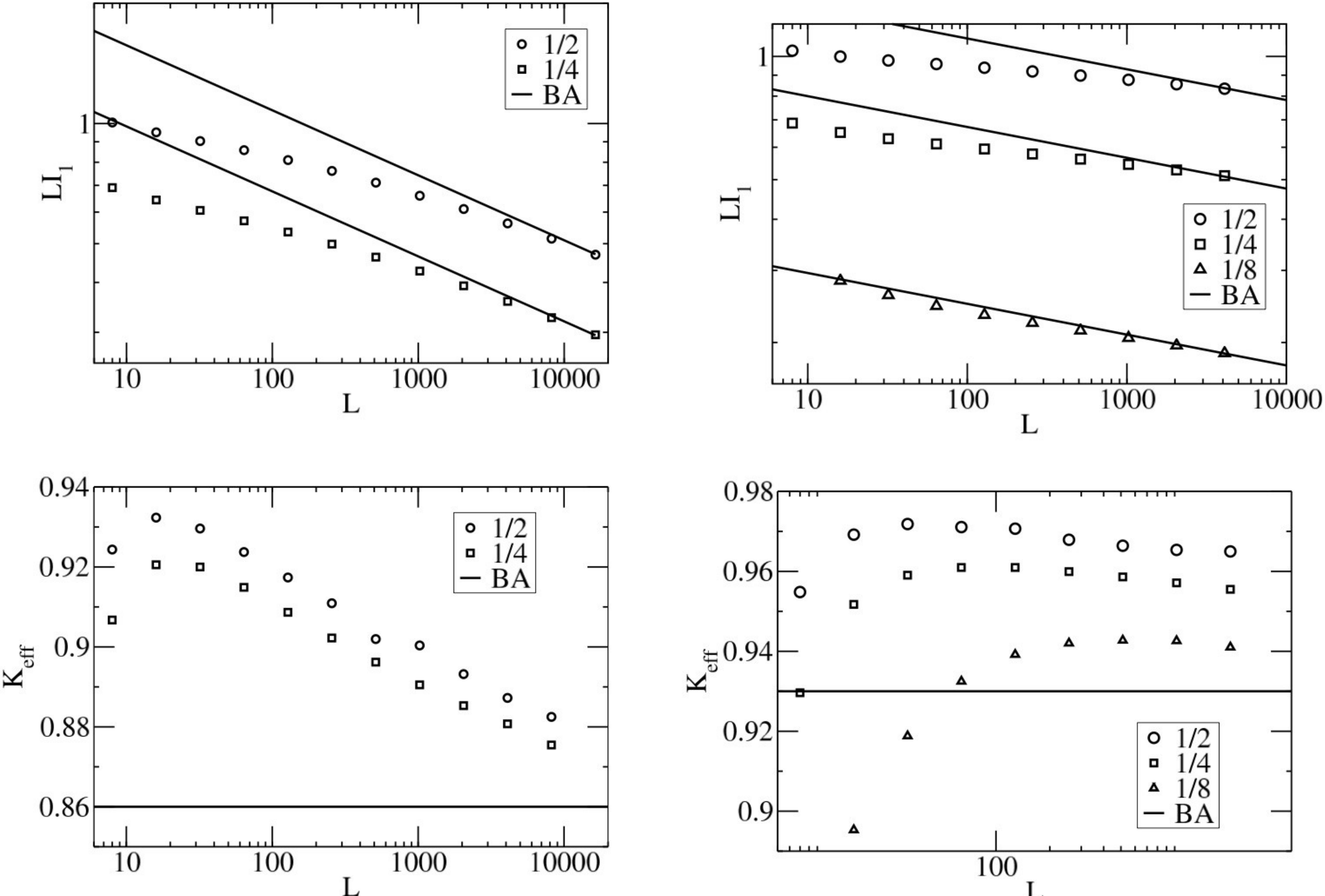


Persistent currents off half filling

K=0.86 K=0.93

I

K



Summary

We calculated ground state energy and persistent current of 1D rings of spinless fermions using fRG,Bethe ansatz and DMRG .

We obtained coefficients ϵ_0, u and K which characterize ground state energy.

fRG can be used to describe large systems (L>256) with reasonable accuracy (5%<) .