Black holes and the renormalisation group [1]

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[1] based on KF, D. F. Litim and A. Raghuraman, arXiv:1002.0260 [hep-th] also KF, D. F. Litim; KF, G. Hiller, D. F. Litim, to appear

Classical Black holes

• Event Horizon: Surface separating null geodesics that cannot reach arbitrarily large distances.

• No hair Q, J, M. Static solution

$$ds^2 = -\left(1 - \frac{2G_N M}{r}\right) dt^2 + \frac{1}{1 - \frac{2G_N M}{r}} dr^2 + r^2 d\Omega^2 \,. \tag{1}$$

• Singularities $\rightarrow$ break down of the evolution of the equations of motion.

• Hoop conjecture.

• Planck energy scattering $\rightarrow$ Black holes.

• Semi-classical theory $\rightarrow$ thermodynamics $\rightarrow$ information paradox.

$$S_{BH} = \frac{A}{4G_N} \tag{2}$$

Asymptotic Safety[2]

• Conservative approach to quantum gravity.
• Functional RG approach [3]

$$\beta_g = (d - 2 + \eta) g(k) \tag{3}$$

• Absence of unphysical singularities (interacting fixed point)

$$\eta_* = 2 - d \tag{4}$$

• Predictive (finite dimensional critical surface).
• Q: How are the singularities and the information paradox resolved in AS?

[2] S. Weinberg, in *General Relativity: An Einstein centenary survey*, Eds. S.W. Hawking and W. Israel, Cambridge University Press (1979), p. 790.

[3] M. Reuter '96

Experimental Observations?

• The Planck scale is inaccessible to current particle accelerators $M_{Pl} \gg M_{EW}$.

• Large extra dimensions [4].

$$M_{\rm Pl}^2 \approx M_D^2 (M_D\, L)^n\,. \tag{5}$$

• $M_D \sim 1\, TeV$

• Black hole production at colliders.

$$\hat{\sigma}_{\rm cl}(s) \approx \pi\, r_{\rm cl}^2 (M = \sqrt{s})\, \theta(\sqrt{s} - M_{\rm min})\,. \tag{6}$$

• Fixed points in higher dimensions [5].

• Higher dimensional black hole solutions, Myers-Perry (talk by Nikolakopoulos), Black rings.

[4] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali. [hep-ph/9803315]

[5] P. Fischer and D. F. Litim

Improved metric

• Quantum corrections to classical metric via RG improvement [6]

For $d \geq 4$

$$ds^2 = -f(r)\, dt^2 + f^{-1}(r)\, dr^2 + r^2\, d\Omega_{d-2}^2\,. \qquad (7)$$

where the lapse function $f(r)$ is given by

$$f(r) = 1 - c_d \frac{G(r)M}{r^{d-3}} \qquad (8)$$

where $c_d = \frac{16\pi}{(d-2)\Omega_{d-2}}$ is a constant.

The function $G(r)$ is obtained by a scale identification from $G(k)$

$$k = \frac{\xi}{r} \qquad (9)$$

[6] A. Bonanno and M. Reuter, Renormalization group improved black hole spacetimes. Phys. Rev. D **62** (2000) 043008, KF, D. F. Litim and A. Raghuraman, arXiv:1002.0260 [hep-th]

Results

The horizons are found by solutions to

$$r_s^{d-3}(M) = c_d\, G(r_s(M))M \tag{10}$$

- Inner and outer horizons.
- Minimum mass $M_c \sim M_D$ below which no solutions exist.
- Same qualitative behaviour independent of $d \geq 4$.

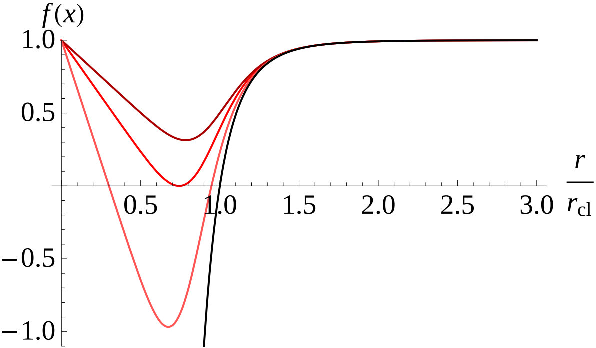

Figure: . Lapse function in $d = 6$ for various M. Black line corresponds to the classical metric.

- Reproduce the classical Schwarzschild radius $r_s \to r_{\rm cl}$ in the limit $\frac{M}{M_D} \to \infty$, where $G(r \to \infty) = G_N$.

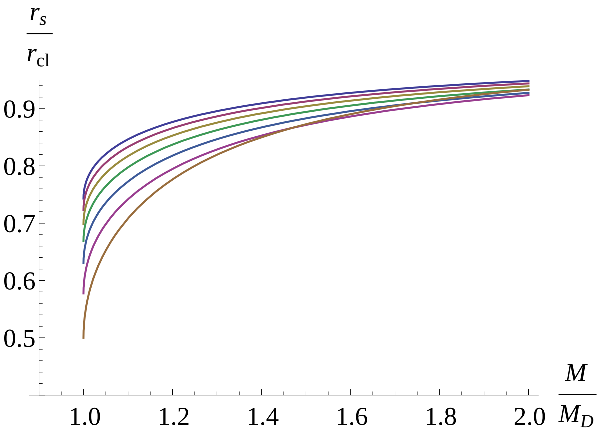

Figure: . Comparison of the improved Schwarzschild radius to the classical one in $d = 4, 5...10$. Here we set $M_c = M_D$.

• Black hole solutions exist for $M \geq M_c$. The critical mass M_c is defined implicitly via the simultaneous vanishing of $f(r_s(M_c), M_c) = 0$ and $f'(r_s(M_c), M_c) = 0$

$$(d-3)G(r_c) = r_c\, G'(r_c)\,, \tag{11}$$

$$r_c = r_s(M_c)\,, \tag{12}$$

which serves as a definition for M_c. Corresponds to $\eta = 3 - d$.

• Tempting to interpret M_c as a renormalised Planck scale

• Even in the absence of a UV fixed point sufficient weakening of gravity at short distances allows for a smallest black hole mass.

Thermodynamics

- Hawking temperature associated to the surface gravity at outer horizon

$$T = \frac{d-3}{4\pi r_s}\left(1 + \frac{\eta(r_s)}{d-3}\right) \tag{13}$$

$$C_V = \frac{\partial M}{\partial T} \tag{14}$$

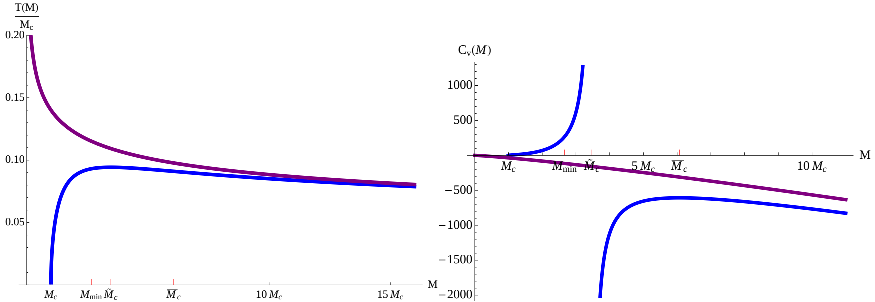

Figure: . Temperature and Specific heat in $d = 8$

Entropy

• The associated entropy is defined by $dM = TdS$. Via a change of variables $A = \Omega_{d-2} r_s^{d-2}$ we find

$$dS = \frac{1}{4G(A)} dA \tag{15}$$

For an explicit approximation of $G(k)$ with a UV fixed point we obtain upon integration

$$S = \frac{A}{4G_N} + \frac{\xi^{d-2}\Omega_{d-2}}{4g^*} \log \frac{A}{G_N} + c \tag{16}$$

• remnant? High thermal fluctuations as $C_V \to 0$

Black hole production

- Apply our results to phenomenological black hole production models
- Cross section is reduced due to the decreased radius r_s
- $M_{min} \approx M_c$

$$\hat{\sigma}_{\rm cl}(s) = \hat{\sigma}_{\rm cl} F(\sqrt{s}) \tag{17}$$

$F(\sqrt{s}) = (r_s/r_{\rm cl})^2|_{M=\sqrt{s}}$

- Qualitatively different results obtained by Koch [7] using scale identification $k \propto \sqrt{s}$

$\rightarrow$ however no semi-classical limit achieved in the trans-Planckian limit

- Our prescription

$$G_N \rightarrow G(k) \quad \text{with} \quad k \propto M_D \left(\frac{M_D}{\sqrt{s}}\right)^{1/(d-3)} \tag{18}$$

[7] B. Koch. Renormalization group and black hole production in large extra dimensions. Phys. Lett. B **663** (2008) 334 [0707.4644 [hep-ph]].

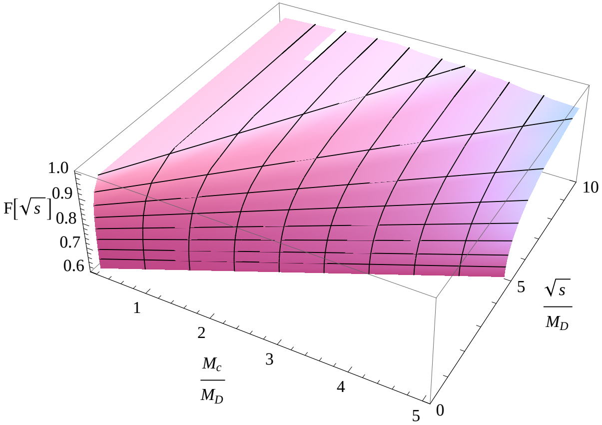

Figure: . Form factor with $n = 4$ extra dimensions

Summary

• Applied renormalisation group improvement to black hole solutions in $d \geq 4$

• A sufficient weakening of the gravitational coupling at short distances $\rightarrow$ smallest black hole

• Thermodynamical corrections lead to a maximum temperature and a logarithmic correction to the Bekenstein-Hawking entropy

• Results are applied to models of black hole production at colliders

• Thank you.